









$$\varphi_{(g,i)}(\omega) = \varphi_{p,(g,i)}(\omega, a_{2,(g,i)}, a_{3,(g,i)}, a_{4,(g,i)}) + \varphi_{r,(g,i)}(\omega), \quad (4)$$

where the  $\varphi_{r,(g,i)}(\omega)$  was the interpolation function of  $\varphi_{p,(g,i)}(\omega_j)$ . Then the electric field was obtained by

$$E_{(g,i)}(t) = \int \left| \tilde{E}(r_{(g,i)} + \omega) \right| e^{i\varphi_{(g,i)}(\omega)} e^{i\omega t} d\omega. \quad (5)$$

The SHIAC was calculated according to

$$I_{ac\_cal(g,i)}(\tau) = \int_{-\infty}^{\infty} \left[ E_{(g,i)}(t - \tau) + E_{(g,i)}(t) \right]^2 dt. \quad (6)$$

Thus, the rms error can be obtained

$$\Delta_{(g,i)} = \sqrt{\sum_{i=1}^{N_{\tau}} [I_{ac\_cal(g,i)}(\tau_i) - I_{ac\_imuse}(\tau_i)]^2 / N_{\tau}}, \quad (7)$$

where  $N_{\tau}$  is the number of total sample points in the error calculation. Equation (7) is the same as Eq. (3), except that (g,i) is added as subscript to indicate the numbers of generation and individual.  $\Delta_{(g,i)}$  was used as a criterion for judging which individual could survive in GA.

In summary,  $\varphi_{r,(g,i)}(\omega_j)$ ,  $a_{2,(g,i)}$ ,  $a_{3,(g,i)}$ ,  $a_{4,(g,i)}$  and  $r_{(g,i)}$  are the genes in our GA. We call them  $x_{k,(g,i)}$ . Our goal of GA is to adjust  $x_{k,(g,i)}$  to minimize the fitness function  $\Delta_{(g,i)}$ . Figure 1 shows the scheme of our GA.

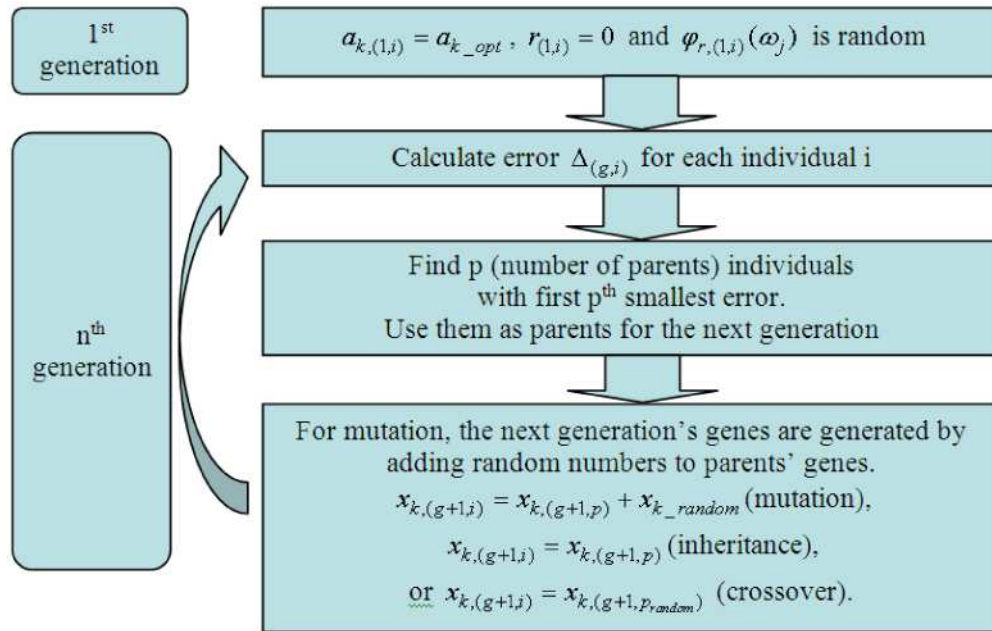


Fig. 1. The scheme of the GA. Each parent will reproduce itself as one of the individuals in the next generation without any change to genes, which is called inheritance. Each of them also produces a certain number of individuals of the next generation by mutation and crossover.

### 3. Results

#### 3.1. Results for experimentally available pulses

As the first example of GOA + GA, we retrieved the spectral phase of a sub-10 fs pulse generated by a mode-locked Ti:Sapphire oscillator (FemtoLasers, Rainbow). The SHIAC was measured by a commercial interferometric autocorrelator (FemtoLasers, 5 – 150 fs). The spectral intensity was measured by an Ocean Optics spectrometer (550 – 1050 nm). In the computation, each pulse is characterized by 2048 points; the time step we used was 0.25 fs, and the time window was 512 fs.

In the GOA,  $a_2$  was scanned from 0 to 500,  $a_3$  from –1000 to 1000 and  $a_4$  from –1000 to 1000. The numbers we chose above are more or less arbitrary. This process takes about 6.6 seconds on a computer with Intel® Core 2 Duo E8400 CPU. After the graduated optimization, the retrieved SHIAC and the measured SHIAC (after data preparation) are shown in Fig. 2 (a). There are still visible discrepancies between them.

For GA, we used the following parameters: the number of individuals 50, the number of parents 5, children produced from inheritance 1, mutation 7, and crossover 2, the number of generations 200, a mutation rate of 10% and a crossover rate of 25%. The mutation process is like this: with 10% mutation rate, 10% of a parent's genes will change their values and form an individual of the next generation. The process of crossover is like this: with crossover rate of 25%, a parent takes 75% of its genes and combined with 25% genes from other parents to form an individual of the next generation. The spectral phase was sampled by  $N_\omega = 80$  points. The choice of  $N_\omega$  is discussed in sSection 4. The GA was programmed with parallel computing function and took about 413 seconds on the same computer (described above). The result of the GOA + GA is shown in Fig. 2. Black dotted curve in (a) and (b) are measured SHIAC after data processing. The red solid curve in (a) is the calculated SHIAC with the spectral phase obtained from GOA coarse global search. The red solid curve in (b) is the calculated SHIAC from the spectral phase obtained by GA. Measured spectral intensity and retrieved spectral phase from GOA and the GA after GOA are shown in (c). The error of each individual was calculated by Eq. (7) and the minimum of errors of individuals in each generation was plotted in Fig. 2 (d).

As a second example, we sent the pulse, described above, through a thin solution of an infrared absorber contained between two cover slips. The infrared absorber is Exciton NP800 and the solvent is 1,1,2,2-tetrachloroethane. The infrared absorber has a very strong absorption peak around 800 nm. After the pulse transmitted through the absorber and cover slips, its spectral intensity around 800 nm is greatly reduced due to the absorption of the absorber, and the spectral phase is also modified due to the dispersion from the absorber and the cover slips. The pulse's spectral phase and intensity are more complicated than those in the first example which provides a challenging test of our combined algorithms. In the GOA,  $a_2$  was scanned from 0 to 500.  $a_3$  was scanned from –1000 to 1000. We scanned  $a_4$  from –5000 to 5000. These numbers are also more or less arbitrary. Parameters used in the GA were the same as those used in example 1. The computation time for GOA was about 6.9 seconds and the GA took about 407.8 seconds to finish. The result is shown in Fig. 3. It can be seen from Fig. 3 (b) that the retrieved SHIAC matched the experimental SHIAC quite well.

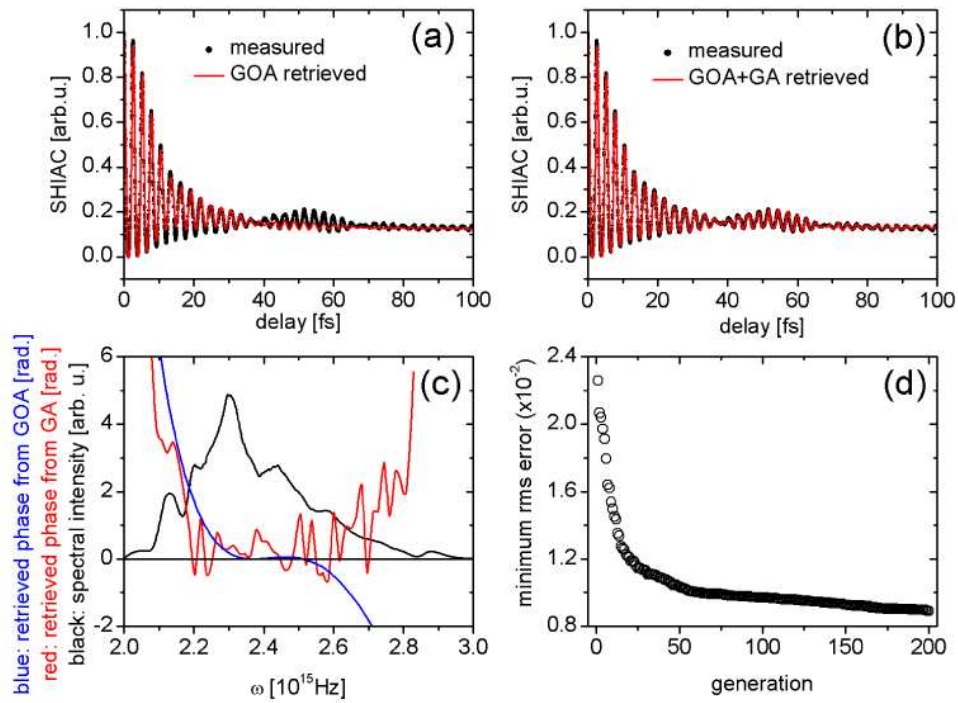


Fig. 2. First example of phase retrieval by the combined GOA and GA. (a) The red solid curve is the calculated SHIAC with optimized phase from GOA. (b) The red solid curve is the calculated SHIAC with retrieved phase from GA with GOA's result as starting point. In (a) and (b), the black dotted curve is the experimental SHIAC after adjustment. The retrieved autocorrelation is symmetric, therefore only half of the curve was plotted. (c) The measured spectral intensity (black), the retrieved phase from GOA (blue) and the retrieved spectral phase from GA (red). (d) The evolution of minimum error for each generation. The unit of the error is the same as the unit in (a) and (b).

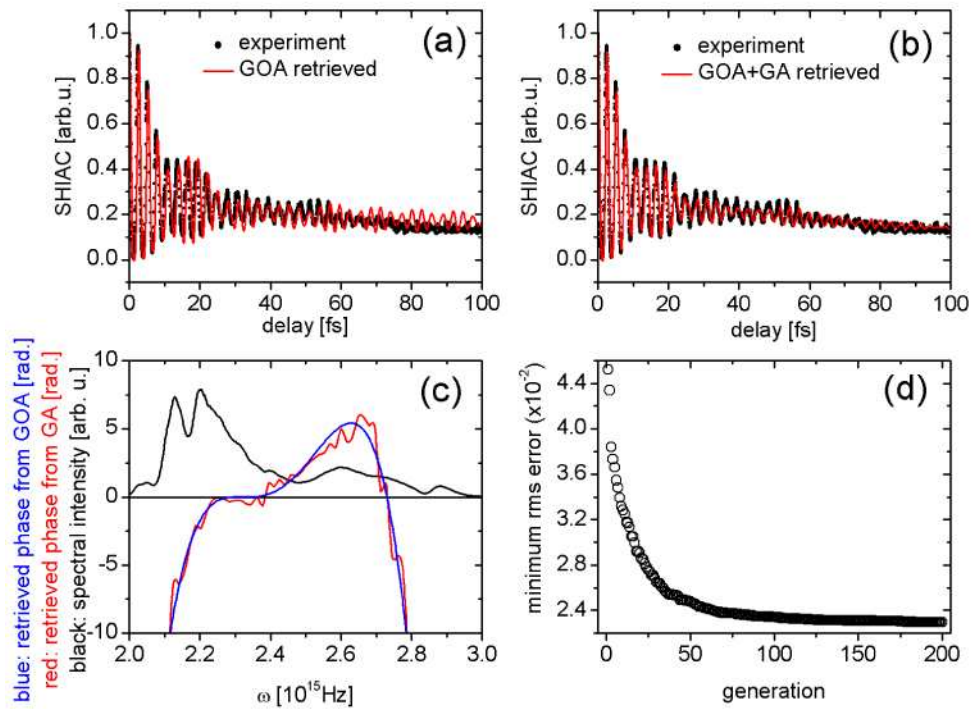


Fig. 3. Second example of phase retrieval by the combined GOA and GA. (a) The red solid curve is the calculated SHIAC with optimized phase from GOA. (b) The red solid curve is the calculated SHIAC with retrieved phase from GA with GOA's results as starting point. In both (a) and (b), the black dotted curve is the experimental SHIAC after adjustment. The retrieved autocorrelation is symmetric, therefore only half of the curve was plotted. (c) The measured spectral intensity (black), the retrieved phase from GOA (blue) and the retrieved spectral phase from GA (red). (d) The evolution of minimum error for each generation. The unit of the error is the same as the unit in (a) and (b).

Using the inverse Fourier transform, we can calculate the retrieved electric field (Fig. 4) from the measured spectral intensity and the retrieved spectral phase. As is expected, the center peak of the electric field  $E(t)$  is sharp and strong. However, the tails, which are expected to be weak, are stronger than expected. This will be explained in the discussion part in Section 4.

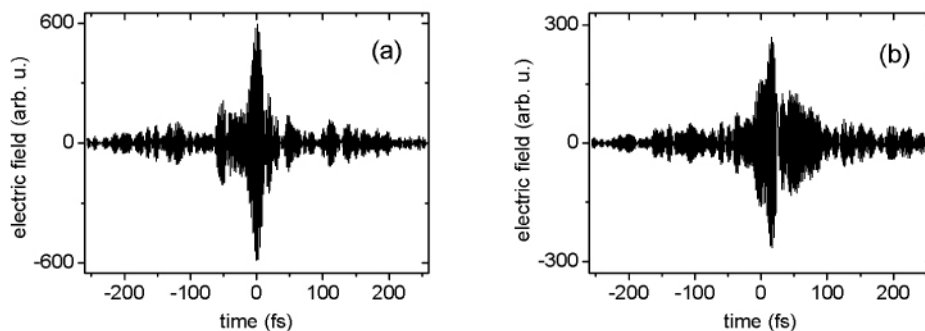


Fig. 4. (a) Retrieved electric field from combined GOA + GA in example 1. (b) Retrieved electric field from combined GOA - GA in example 2.



GOA + GA's results are listed in Table 1.  $\Delta_{GOA}$  is the minimum rms error achieved by GOA and  $a_{i\_opt}$  are chirp parameters for the optimized pulse shape of GOA.  $b$  is defined by Eq. (2).  $\Delta_{GOA-GA}$  is the minimum rms errors achieved by GOA + GA. However, GA's results  $a_i$  are not very meaningful, because the spectral phase is a combination of low order chirps and phases for individual frequencies. For example, the optimized spectral phase in Fig. 2 (c) looks noisy and cannot be well described by  $a_i$ . We list the  $a_i$  here just to show that our GA can fine-tune  $a_i$  and  $r$ .

**Table 1. Results of GOA and GOA + GA for example 1 and 2**

Results for example 1 in section 3.1									
GOA's results					GOA + GA's results				
$a_{2\_opt}$	$a_{3\_opt}$	$a_{4\_opt}$	$b^*$	$\Delta_{GOA}$	$a_2$	$a_3$	$a_4$	$r$	$\Delta_{GOA-GA}$
22	-182	194	1.04	$2.2 \times 10^{-2}$	22	-90.28	194	0	$9 \times 10^{-3}$
Results for example 2 in Section 3.1									
GOA's results					GOA + GA's results				
$a_{2\_opt}$	$a_{3\_opt}$	$a_{4\_opt}$	$b^*$	$\Delta_{GOA}$	$a_2$	$a_3$	$a_4$	$r$	$\Delta_{GOA-GA}$
33	653	-1877	1.04	$4.5 \times 10^{-2}$	33	653.48	-1878.4	-0.0055	$2.3 \times 10^{-2}$

\*  $b$  is not obtained from GOA.

The necessity of GOA can be shown by comparing the results of GA with and without GOA on both examples in this section. We applied the GA with and without the help of the GOA six times for each example, and the minimum errors for each generation in the six runs are shown in Fig. 5. For the GA without GOA, the starting individuals have random phases. For example 1 (Fig. 5(a)), the average error at 50<sup>th</sup> generation of GOA + GA is the same as the average error of 56<sup>th</sup> generation of GA without GOA. (Here, the average means the average of the minimum errors of the six runs.) The computation time for one generation is about 2.1 seconds and the GOA takes 6.6 seconds. So GOA + GA leads GA without GOA by 5.8 seconds. In Fig. 5(a), the chirp of the pulse is small, thus the GA without GOA catches up with the GA with GOA after a couple of generations, which also shows the strong searching ability of our GA. For example 2 (Fig. 5(b)), GA without GOA's average error at 200<sup>th</sup> generation is the same as the average error of GOA + GA at 106<sup>th</sup> generation, which means GA without GOA lags GOA + GA by 188 seconds. The error of GOA with GA at 100<sup>th</sup> generation is close to the error of GOA + GA at 70<sup>th</sup> generation, which means GA without GOA lags GOA + GA by 60 seconds. In this example, because the pulse has a bigger chirp, GOA + GA is much faster than GA without GOA.

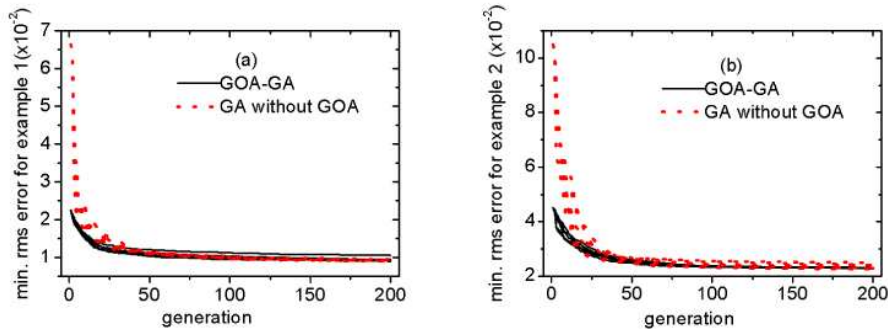


Fig. 5. (a) Comparison of GA with and without GOA with the data in example 1. (b) Comparison of GA with and without GOA with the data in example 2. The unit of the errors is the same as the unit of (a) and (b) in Fig. 2 and Fig. 3. The scales of (a) and (b) are made different due to the different ranges of the data.

### 3.2. Results for noise-free simulated pulses

The strength of GOA + GA can be better shown by using GOA + GA to retrieve the spectral phase of noise-free simulated pulses. An 800nm 40 fs transform limited pulse was used as the seed pulse. Arbitrary chirps ( $a_2$ ,  $a_3$  and  $a_4$ ) were added to the seed pulse and GOA + GA was then used to retrieve the spectral phases of them.

For a pulse with only linear chirp  $a_2 = 483$  the GOA gave exact value of the chirp with zero error, this is as expected because GOA starts from searching  $a_2$ , so if the pulse has only linear chirp, GOA can find the chirp with zero error. The results of another three simulated pulses are listed in Table 2. From the data in Table 2, the GOA + GA can make a retrieve with error between  $10^{-3}$  to  $10^{-4}$  on these noise-free simulated pulses. The  $a_i$  retrieved by GA is not listed here because the total phase is the sum of the polynomial part and the phase for individual frequencies.

**Table 2. GOA + GA's results for noise-free simulated pulses**

Pulse 1's parameters			GOA's results				GOA + GA's results
$a_2$	$a_3$	$a_4$	$a_{2\_opt}$	$a_{3\_opt}$	$a_{4\_opt}$	$\Delta_{GOA}$	$\Delta_{GOA-GA}$
23	120	1532	48	17	64	$9.3 \times 10^{-4}$	$3.1 \times 10^{-4}$
Pulse 2's parameters			GOA's results				GOA + GA's results
$a_2$	$a_3$	$a_4$	$a_{2\_opt}$	$a_{3\_opt}$	$a_{4\_opt}$	$\Delta_{GOA}$	$\Delta_{GOA-GA}$
135	-1563	-2831	132	-1420	-1908	$1.2 \times 10^{-3}$	$1.6 \times 10^{-4}$
Pulse 3's parameters			GOA's results				GOA + GA's results
$a_2$	$a_3$	$a_4$	$a_{2\_opt}$	$a_{3\_opt}$	$a_{4\_opt}$	$\Delta_{GOA}$	$\Delta_{GOA-GA}$
135	563	-11953	68	-292	-116	$2.5 \times 10^{-3}$	$1.04 \times 10^{-3}$

### 3.3. About the usefulness of crossover

Crossover is considered as one of the most important features of GA. It is worthy to study how helpful crossover process is in phase retrieval problems. We ran the GA without crossover six times and the minimum errors of each generation of the six runs are plotted in Fig. 6 together with those of the six GA runs with crossover. We found that there is no big difference whether we use crossover or not in GA in our phase retrieval problem.

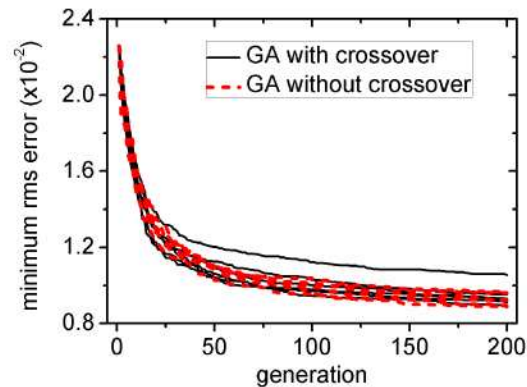


Fig. 6. GA with and without crossover. The red dashed lines are the minimum errors of each generation in the six runs of GA without using crossover and the black solid line are that of GA with crossover. There is no obvious difference between the results of GA with or without crossover. There is an unlucky case in the GA with crossover whose minimum errors are higher than those of any other runs.

#### 4. Conclusions and discussion

The GOA + GA combined algorithm can effectively avoid local extrema and retrieve the spectral phase from the SHIAC and spectral intensity successfully. Chirp parameters  $a_i$  were introduced into our GA, which strengthened the ability of our GA to search for global extrema. For the noise-free simulated strongly chirped pulses, the rms errors between retrieved SHIAC and the simulated SHIAC can reach  $10^{-3}$  to  $10^{-4}$ . For the experimentally available pulses, errors around  $10^{-2}$  are achieved. For pulses with small chirp parameters, the GA itself works as well as GOA + GA. For pulses with big chirps the GOA + GA converges faster than GA without GOA. The crossover is not found to play an important role in our GA.

There are several issues that need to be noticed. (1). Due to time reversal invariance, it is impossible to know the sign of the total phase; thus, we defined  $a_2$  to be a positive number and  $a_3$  and  $a_4$  were scanned from negative to positive numbers. (2). Due to the Nyquist–Shannon sampling theorem,  $N_\omega$  determines the pulse's temporal length that can be retrieved accurately. However, increasing  $N_\omega$  will also decrease the step size used in the discrete Fourier transform, which will increase the runtime of the algorithm. In our experiment, we are mainly interested in the central part of the pulse and the measured SHIAC is between  $-100$ fs to  $100$ fs, so 80 points is sufficient for our application. However, this will cause the retrieved electric fields (see Fig. 4.) to have some spurious tails. (3). GA is highly parallelizable. The parallelized program is about 25% faster than serial program on the dual core computer described in the second paragraph of Section 3.1. With multi-core computers, the parallelized program should be faster. However, the communication between CPUs also takes time; thus, the efficiency of this program on multi-core computer needs further investigation. (4). The correction of the SHIAC data is crucial to the successful retrieval of the spectral phase. The spectrum was not only a set of data that was used in the algorithm, but was also used as a criterion to prepare the SHIAC data. We corrected only the linear shift of the SHIAC. (5). The GOA can also be done more than once by using optimized  $a_i$  instead of zeros as starting values. (6). As is easily notice in Fig. 5 and Fig. 6, one set of err for GOA + GA converges slower than all others. It can be regarded as an unlucky case, which shows that luck can also affect the result of GA. (7). Given the spectral intensity, our GOA + GA combined algorithm provides a general method to retrieve spectral phase from measured data, and could be used to retrieve spectral phase from FROG or MOSAIC. In order to retrieve the phase from another kind of measurement, one only needs to change the error function to be the difference between the calculated result and measured results.

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