CHAPTER 4
Structure of the Atom

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4.1 The Atomic Models of Thomson and Rutherford

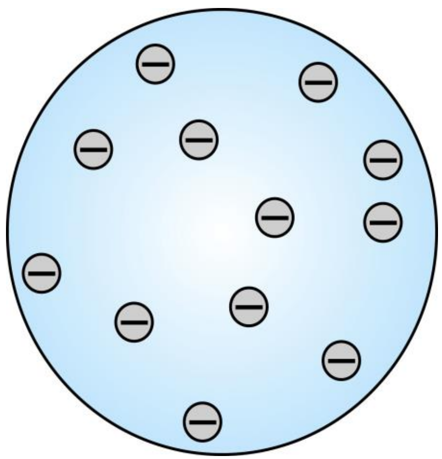
Pieces of evidence that scientists had in 1900 to indicate that the atom was not a fundamental unit:

1) There seemed to be too many kinds of atoms, each belonging to a distinct chemical element.
2) Atoms and electromagnetic phenomena were intimately related.
3) The problem of **valence**. Certain elements combine with some elements but not with others, a characteristic that hinted at an internal atomic structure.
4) The discoveries of radioactivity, of x rays, and of the electron

Thomson’s Atomic Model

(turned out to be not correct)

- Thomson’s “plum-pudding” model of the atom had the positive charges spread uniformly throughout a sphere the size of the atom with, the newly discovered “negative” electrons embedded in the uniform background.



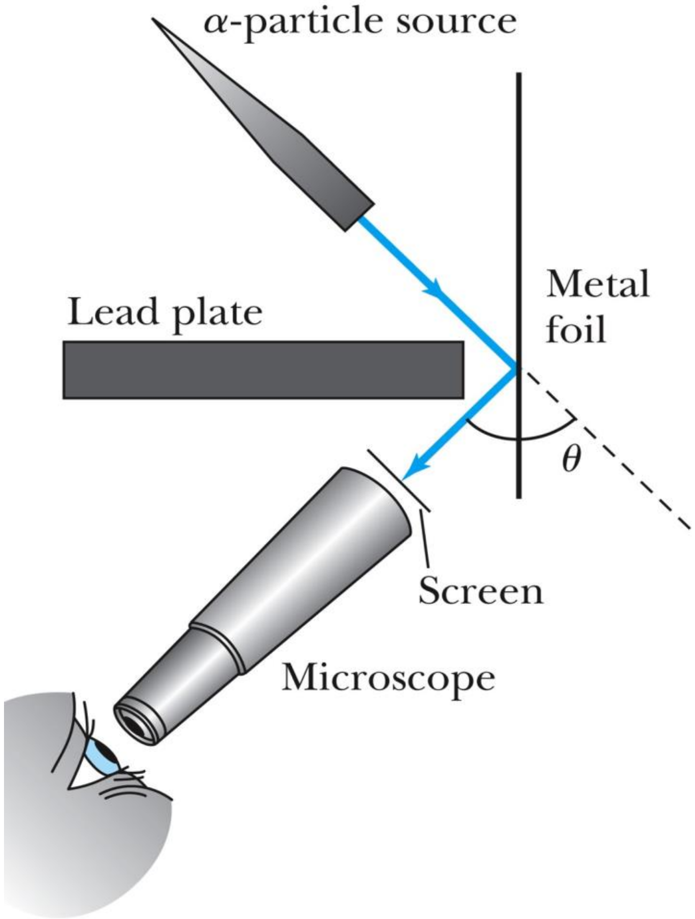
- In Thomson’s view, when the atom was heated, the electrons could vibrate about their equilibrium positions, thus producing electromagnetic radiation.

Alpha particles cannot be scattered through large angles in this model

Experiments of Geiger and Marsden

- Rutherford, Geiger, and Marsden conceived a new technique for investigating the structure of matter by scattering α particles from atoms.

- Geiger showed that many α particles were scattered from thin gold-leaf targets at backward angles greater than 90°.

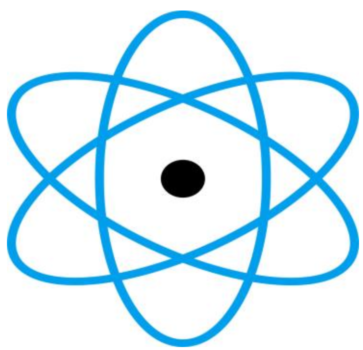


Rutherford's Atomic Model (correct)

- $\langle \theta \rangle_{\text{total}} = 6.8°$ even if the α particle scattered from all 79 electrons in each atom of gold

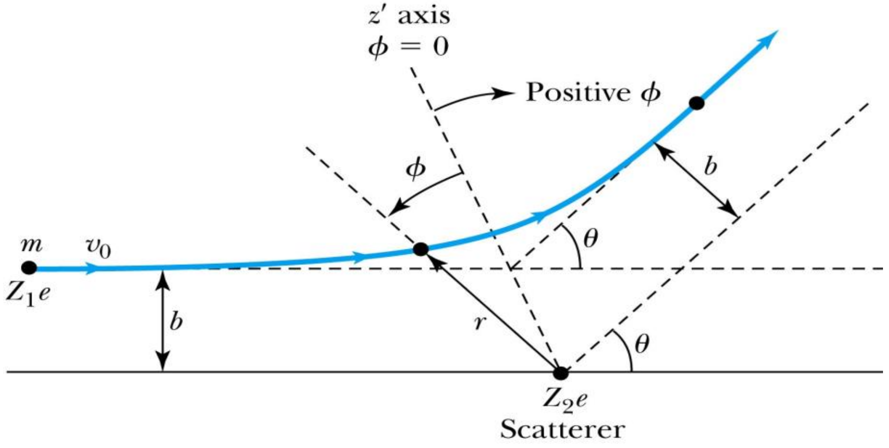
  The experimental results were not consistent with Thomson's atomic model.

- Rutherford proposed that an atom has a positively charged core (nucleus) surrounded by the negative electrons.



Rutherford Scattering

- Scattering experiments help us study matter too small to be observed directly.
- There is a relationship between the impact parameter *b* and the scattering angle *θ*.



When *b* is small,

➞ *r* gets small.

➞ Coulomb force gets large.

➞ *θ* can be large and the particle can be repelled backward.

$$b = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 K}\cot\frac{\theta}{2} \quad \text{where } K = m{v_0}^2/2$$

The Relationship Between the Impact Parameter b and the Scattering Angle α

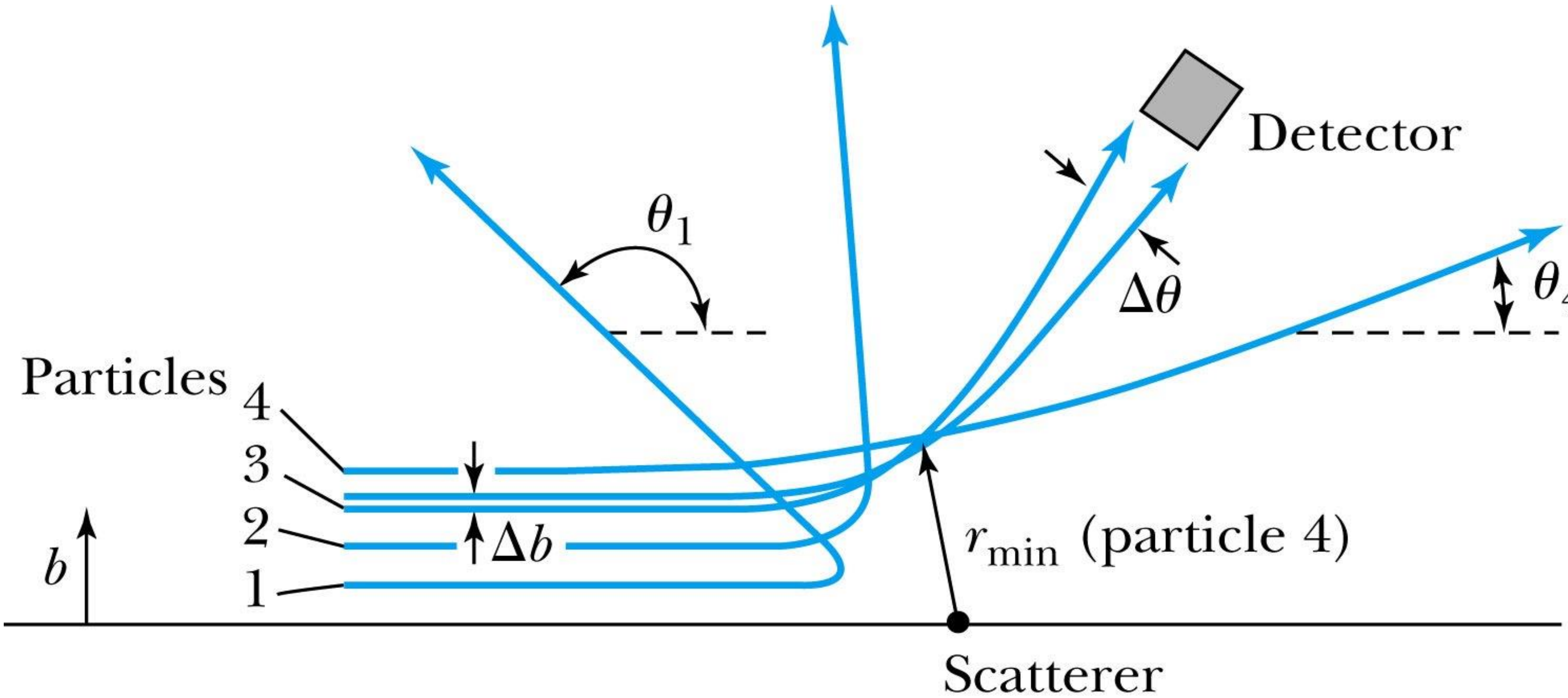
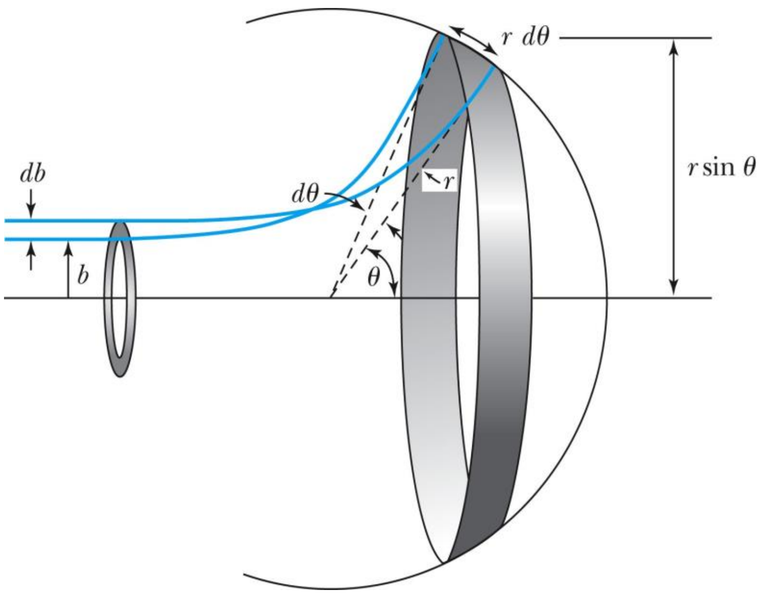
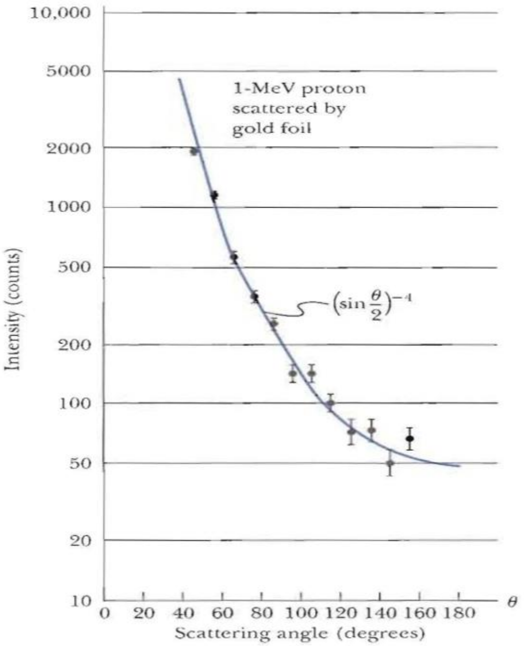


Figure 4.7 The relationship between the impact parameter b and scattering angle u. Particles with small impact parameters approach the nucleus most closely (rmin) and scatter to the largest angles. Particles within the range of impact parameters b will be scattered within u.

Rutherford Scattering Equation

- In actual experiment a detector is positioned from θ to $\theta + d\theta$ that corresponds to incident particles between b and $b + db$.





- The number of particles scattered per unit area is

$$N(\theta) = \frac{N_i n t}{16} \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 \frac{Z_1^{\,2} Z_2^{\,2}}{r^2 K^2 \sin^4(\theta/2)}$$

The Important Points

1. The scattering is proportional to the square of the atomic number of *both* the incident particle (Z_1) and the target scatterer (Z_2).
2. The number of scattered particles is inversely proportional to the square of the kinetic energy of the incident particle.
3. For the scattering angle θ, the scattering is proportional to 4^{th} power of sin(θ/2).
4. The Scattering is proportional to the target thickness for thin targets.

The distance of closest approach is in a head-on collision

Consider an ∝-particle colliding head on with an *Al*– nucleus ($Z_2 = 13$) ∝-particle ($Z_1 = 2$) and kinetic energy K = 7.7MeV

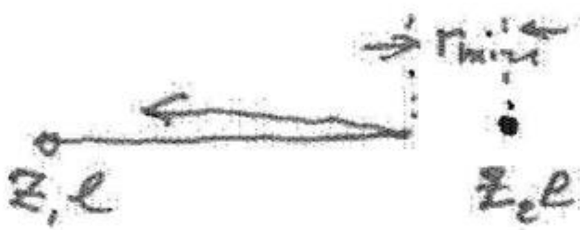
$$k = \frac{1}{2}mv^2 = \frac{Z_1 e Z_2 e}{4\pi\varepsilon_o r_{min}}$$

$$r_{min} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_o k}$$

$$r_{min} > r_{\propto} + r_{nucleus}$$

$$= \frac{2*13*(1.6*10^{-19}C)^2*9*10^9 Nm^2C^{-2}}{7.7MeV*1.6*10^{-13}J/MeV}$$

$$= 5*10^{-15}m$$



Note:

At very short distances a new force (additional to the Coulomb) comes into play: The attractive nuclear force. When this is taken into account it follows;

$$r_{nucleus} = R_o A^{\frac{1}{3}}$$

$$R_o = 1.8x10^{-15}m$$

$$A = mass\ number$$

note: 10^{-15}m = 1 Fermi

Clicker - Questions

7) Indicate the true statement concerning Rutherford's model of the atom

a) The electrons are in the center forming a negative nucleus with positive charges filling a larger volume.

b) All the positive charges are in the center forming a positive nucleus, which can deflect α-particles through a large angle.

c) α-particles scattering produces small angle scattering from the nucleus.

d) α-particles are attracted by the positive nucleus.

4.3: The Classical Atomic Model

As suggested by the Rutherford Model the atom consisted of a small, massive, positively charged nucleus surrounded by moving electrons. This then suggested consideration of a planetary model of the atom.

Let's consider atoms as a planetary model.

- The force of attraction on the electron by the nucleus and Newton's 2nd law give

$$\vec{F}_e = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\hat{e}_r = \frac{mv^2}{r}$$

Radial acceleration
a=v^2 / r

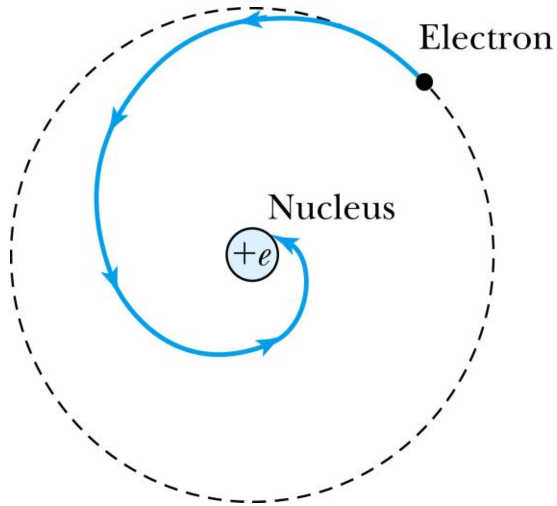
where *v* is the tangential velocity of the electron.

- The total energy is

$$E = K + V = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = \frac{-e^2}{8\pi\varepsilon_0 r}$$

The Planetary Model is Doomed

- From classical E&M theory, an accelerated electric charge radiates energy (electromagnetic radiation) which means total energy must decrease. ⟶ *Radius r must decrease!!*



Electron crashes into the nucleus!?

- Physics had reached a turning point in 1900 with Planck's hypothesis of the quantum behavior of radiation.

4.4: The Bohr Model of the Hydrogen Atom

Bohr's dramatic general assumptions:

A. "Stationary" states or orbits must exist in atoms, i.e., orbiting electrons *do not radiate* energy in these orbits. These orbits or stationary states are of a fixed definite energy E.

B. The emission or absorption of electromagnetic radiation can occur only in conjunction with a transition between two stationary states. The frequency, f, of this radiation is proportional to the *difference* in energy of the two stationary states:

C. $$E = E_1 - E_2 = hf$$

where h is Planck's Constant

D. Classical laws of physics do not apply to transitions between stationary states.

A. The mean kinetic energy of the electron-nucleus system is $K = nhf_{orb}/2$, where f_{orb} is the frequency of rotation. This is equivalent to the angular momentum of a stationary state to be an integral multiple of $h/2\pi$

$$L = mvr = nh/2\pi$$

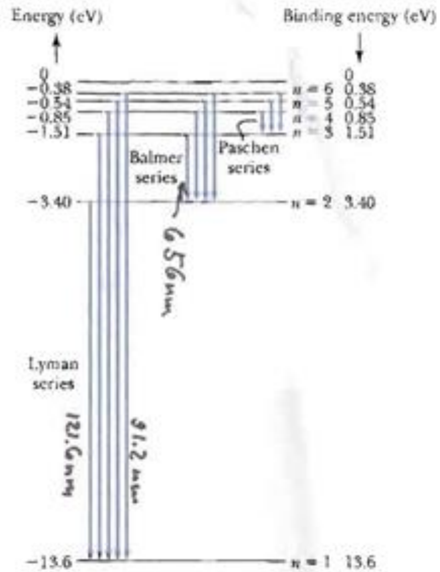
Summary: Bohr's model of the hydrogen atom

$\lambda f = c \qquad \frac{1}{\lambda} = \frac{f}{c} = \frac{E_a - E_e}{h c}$

Bohr's model of the hydrogen atom

assumptions:

1. There exist 'stationary states' of definite total energy
2. Emission and absorption occurs in transitions between the 'stationary states' and $E = E_a - E_e = hf$
3. Classical dynamics governs the 'stationary states' but not the transitions
4. Only electron orbits occur for which $\vec{L} = n\hbar$, $\hbar = h/2\pi$



The force between the atom and the electron is

$$F = \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{Ze\,e}{r^2}}_{\text{electrostatic}} = \underbrace{m_e a_r = \frac{m_e v^2}{r}}_{\text{centripedal}} \qquad v^2 = \frac{Ze^2}{4\pi\varepsilon_0 m_e \cdot r}$$

$$\left.\begin{aligned}\vec{L} = \vec{r} \times \vec{p} = m_e v \cdot r = n\hbar \\ v = \frac{n\hbar}{m_e r}\end{aligned}\right\} \qquad v^2 = \frac{n^2\hbar^2}{m_e^2 r^2}$$

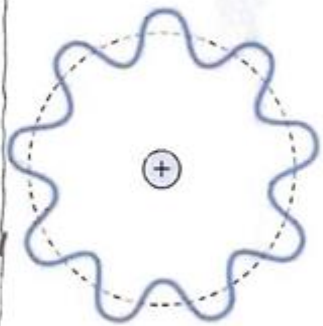
Z = 1 for hydrogen: atom radius $= r = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{m_e e^2} = n^2 a_0$ (quantized)

$$0.5 \times 10^{-10}\,\mathrm{m} = a_0 = \frac{(1.055 \times 10^{-34}\,\mathrm{J \cdot s})^2}{9 \times 10^9 \frac{\mathrm{Nm^2}}{\mathrm{C}} \times 9.1 \times 10^{-31}\,\mathrm{kg} \times (1.6 \times 10^{-16}\,\mathrm{C})^2}$$

Bohr radius for n = 1

$$E_n = K + V = \frac{1}{2} m v^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$= \frac{1}{8\pi\varepsilon_0}\frac{e^2}{r} - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -\frac{e^2}{8\pi\varepsilon_0 a_0 n^2} = -\frac{E_0}{n^2} \qquad E_0 = 13.6\,\mathrm{eV}$$



represent the electron as a standing wave

$n\lambda = 2\pi r$

De Broglie $\lambda = \frac{h}{p}$

$2\pi r = n\frac{h}{p}$

$\vec{L} = rp = \frac{nh}{2\pi} = n\hbar$

Bohr Radius

- The diameter of the hydrogen atom for stationary states is

$$r_n = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{me^2} \equiv n^2 a_0$$

Where the **Bohr radius** is given by

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{me^2} = \frac{(1.055\times 10^{-34}\ \mathrm{J\cdot s})^2}{\left(8.99\times 10^9 \dfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)(9.11\times 10^{-31}\ \mathrm{kg})(1.6\times 10^{-16}\ \mathrm{C})^2} = 0.53\times 10^{-10}\ \mathrm{m}$$

- The smallest diameter of the hydrogen atom is

$$2r_1 = 2a_0 \approx 10^{-10}\mathrm{m}$$

- n = 1 gives its lowest energy state (called the "ground" state)

$$= \frac{(1.055\times 10^{-34}\ \mathrm{J\cdot s})^2}{\left(8.99\times 10^9 \dfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)(9.11\times 10^{-31}\ \mathrm{kg})(1.6\times 10^{-16}\ \mathrm{C})^2}$$

Chapter 4 quiz question

- In his calculations, Bohr came across a number now called
- the "Bohr radius". This number is 5.29 x 10-11 m and is

- a.The maximum radius of the hydrogen atom.
- b.The minimum radius of the hydrogen atom.
- c.The average radius of the hydrogen atom.
- d. The minimum radius of atoms of all elements.
- e.The maximum radius of atoms of all elements.

Rydberg equation

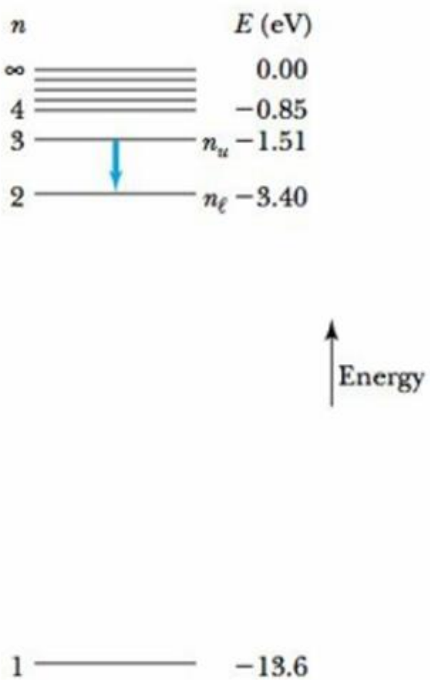


Figure 4.15 The energy-level diagram of the hydrogen atom. The principal quantum numbers n are shown on the left, with the energy of each level indicated on the right. The ground-state energy is −13.6 eV; negative total energy indicates a bound, attractive system. When an atom is in an excited state (for example, $n_u = 3$) and decays to a lower stationary state (for exam-

state ($n = n_\ell$). A transition between two energy levels is schematically illustrated in Figure 4.15. According to Assumption B we have

$$hf = E_u - E_\ell \tag{4.27}$$

where f is the frequency of the emitted light quantum (photon). Because $\lambda f = c$, we have

$$\begin{aligned} \frac{1}{\lambda} &= \frac{f}{c} = \frac{E_u - E_\ell}{hc} \\ &= \frac{-E_0}{hc}\left(\frac{1}{n_u^{\,2}} - \frac{1}{n_\ell^{\,2}}\right) = \frac{E_0}{hc}\left(\frac{1}{n_\ell^{\,2}} - \frac{1}{n_u^{\,2}}\right) \end{aligned} \tag{4.28}$$

where

$$\frac{E_0}{hc} = \frac{me^4}{4\pi c\hbar^3(4\pi\epsilon_0)^2} \equiv R_\infty \tag{4.29}$$

This constant R_∞ is called the **Rydberg constant** (for an infinite nuclear mass). Equation (4.28) becomes

$$\frac{1}{\lambda} = R_\infty\left(\frac{1}{n_\ell^2} - \frac{1}{n_u^{\,2}}\right) \tag{4.30}$$

which is similar to the Rydberg equation (3.13). The value of $R_\infty = 1.097373 \times 10^7\ \text{m}^{-1}$ calculated from Equation (4.29) agrees well with the experimental values given in Chapter 3, and we will obtain an even more accurate result in the next section.

Calculate the shortest wavelength of the Li ++ ion

$$\frac{1}{\lambda} = (3)^2 R\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = 9R$$

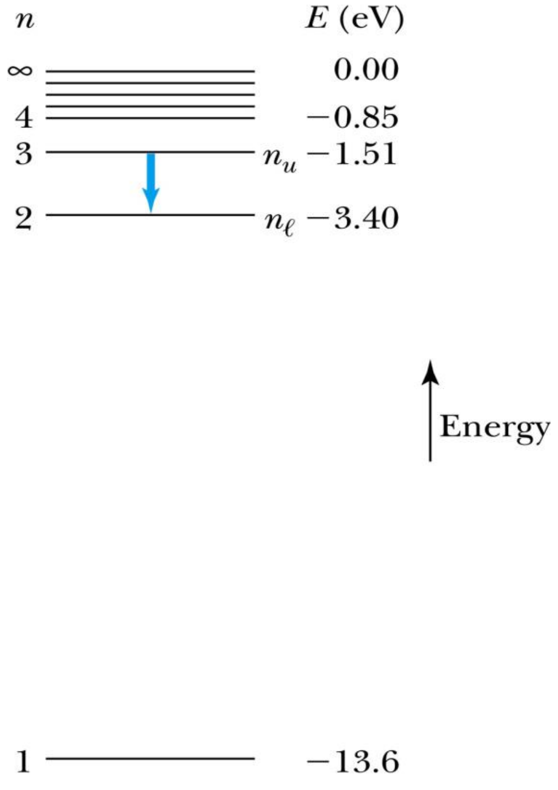
$$\lambda = \frac{1}{9R} = 10.1\ \text{nm}$$

The Hydrogen Atom

- The energies of the stationary states

$$E_n = -\frac{e^2}{8\pi\varepsilon_0 r_n} = -\frac{e^2}{8\pi\varepsilon_0 a_0 n^2} \equiv -\frac{E_0}{n^2}$$

where E_0 = 13.6 eV



- Emission of light occurs when the atom is in an excited state and decays to a lower energy state ($n_u \rightarrow n_\ell$).

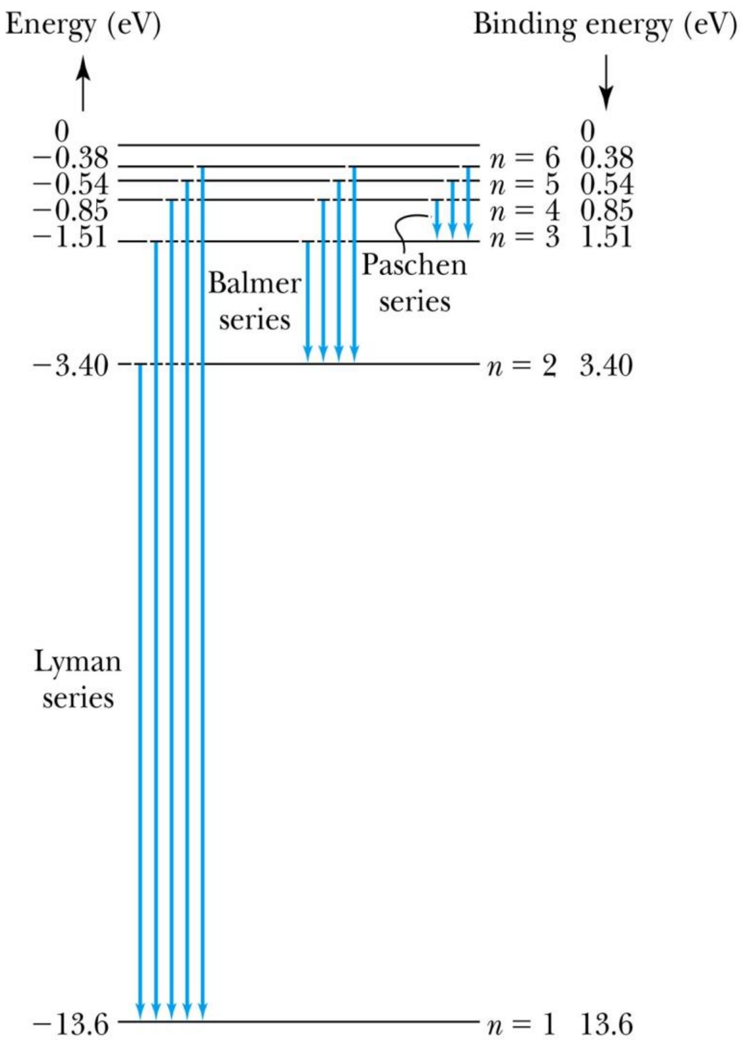
$$hf = E_u - E_\ell$$

where f is the frequency of a photon.

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{E_u - E_\ell}{hc} = R_\infty \left(\frac{1}{n_\ell^2} - \frac{1}{n_u^2} \right)$$

R_∞ is the **Rydberg constant**.

Transitions in the Hydrogen Atom



Lyman series

The atom will remain in the excited state for a short time before emitting a photon and returning to a lower stationary state. All hydrogen atoms exist in n = 1 (invisible).

Balmer series

When sunlight passes through the atmosphere, hydrogen atoms in water vapor absorb the wavelengths (visible).

Fine Structure Constant

- The electron's velocity in the Bohr model:

$$v_n = \frac{n\hbar}{mr_n} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0\hbar}$$

- On the ground state,

  v_1 = 2.2 × 10^6 m/s ~ less than 1% of the speed of light

- The ratio of v_1 to c is the **fine structure constant**.

$$\alpha \equiv \frac{v_1}{c} = \frac{\hbar}{ma_0c} = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$$

Calculate the speed and radial acceleration for a ground-state electron in the hydrogen atom. Do the same for the He^+ ion and the Li^{++} ion.

$$F = \frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \rightarrow v^2 = \frac{Ze^2}{4\pi\varepsilon_0 m}$$

$$v = \frac{ec\sqrt{Z}}{\sqrt{4\pi\varepsilon_0 mc^2 r}}$$

$$= \frac{\sqrt{1.44\,eV\cdot nm}\,\sqrt{Z}\cdot c}{\sqrt{(511\times10^3 eV)\,0.053\,nm}}$$

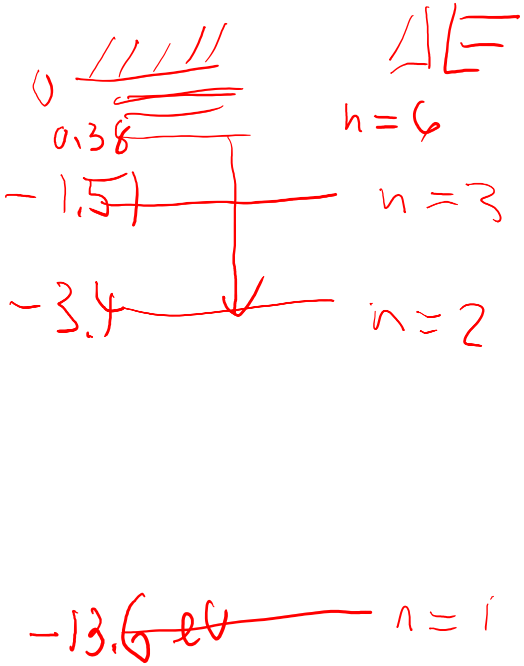
$$v_H = 7.3\times10^{-3}\,c = 2.2\times10^6\,\frac{m}{s}$$

$$v_{He^+} = \qquad = 3.1\times10^6\,\frac{m}{s}$$

$$v_{Li^{++}} = 6.6\times10^6\,m/s = 3v_H$$

$$a_H = \frac{v^2}{r} = \frac{(2.2\times10^6)^2}{0.529\times10^{-10}} = 9.07\times10^{22}\,m/s^2$$

23. A hydrogen atom in an excited state absorbs a photon of wavelength 410 nm. What were the initial and final states of the hydrogen atom?



$$\Delta E = \frac{hc}{\lambda} = \frac{124\ \text{eV}\cdot\text{nm}}{410\ \text{nm}} = 3.02\ \text{eV}$$

ΔE between $n=2$ and $n=6$

$= 3.4 - 0.38 = 3.02\ \text{eV}$

Clicker - Questions

1) Indicate in which atom the electron is most strongly bound?

a) H

b) He^{+}

c) Li^{++}

Clicker - Questions

4) The diameter of the stationary states of the H-atom is increasing with n

a) Linearly

b) Quadratically

c) With the varying Bohr's radius

d) Inversely

Clicker - Questions

5) The Bohr radius is 0.5×10^{-10} m.What is the radius of the stationary state with n=2 ?

a) 2×10^{-10} m

b) 1×10^{-10} m

c) 4.5×10^{-10} m

d) 0.25×10^{-10} m

The Correspondence Principle

Classical electrodynamics + Bohr’s atomic model

Determine the properties of radiation

Need a principle to relate the new modern results with classical ones.

Bohr’s correspondence principle → In the limits where classical and quantum theories should agree, the quantum theory must reduce the classical result.

Clicker - Questions

7) Indicate the true statement concerning Rutherford's model of the atom

a) The electrons are in the center forming a negative nucleus with positive charges filling a larger volume.

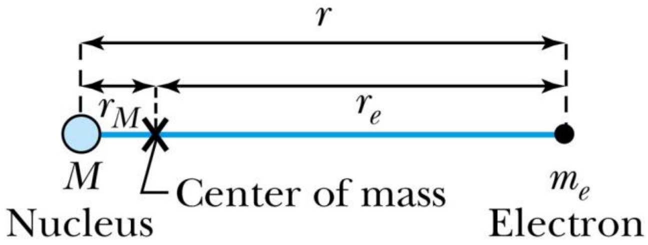
b) All the positive charges are in the center forming a positive nucleus, which candeflect α-particles through a large angle.

c) α-particles scattering produces small angle scattering from the nucleus.

d) α-particles are attracted by the positive nucleus.

4.5: Successes and Failures of the Bohr Model

- The electron and hydrogen nucleus actually revolved about their mutual center of mass.



- The electron mass is replaced by its **reduced mass**.

$$\mu_e = \frac{m_e M}{m_e + M} = \frac{m_e}{1 + \frac{m_e}{M}}$$

- The Rydberg constant for infinite nuclear mass is replaced by *R*.

$$R = \frac{\mu_e}{m_e} R_\infty = \frac{1}{1 + \frac{m_e}{M}} R_\infty = \frac{\mu_e e^4}{4\pi c \hbar^3 (4\pi\varepsilon_0)^2}$$

EXAMPLE 4.8

Calculate the wavelength for the $n_u = 3 \rightarrow n_\ell = 2$ transition (called the H_α *line*) for the atoms of hydrogen, deuterium, and tritium.

Strategy We use Equation (4.30) but with R_∞ replaced by the Rydberg constant expressed in Equation (4.37). In order to use Equation (4.37) we will need the masses for hydrogen, deuterium, and tritium.

Solution The following masses are obtained by subtracting the electron mass from the atomic masses given in Appendix 8.

$$\text{Proton} = 1.007276 \text{ u}$$

$$\text{Deuteron} = 2.013553 \text{ u}$$

$$\text{Triton (tritium nucleus)} = 3.015500 \text{ u}$$

The electron mass is $m_e = 0.0005485799$ u. The Rydberg constants are

$$R_H = \frac{1}{1 + \dfrac{0.0005486}{1.00728}} R_\infty = 0.99946 R_\infty \quad \text{Hydrogen}$$

$$R_D = \frac{1}{1 + \dfrac{0.0005486}{2.01355}} R_\infty = 0.99973 R_\infty \quad \text{Deuterium}$$

$$R_T = \frac{1}{1 + \dfrac{0.0005486}{3.01550}} R_\infty = 0.99982 R_\infty \quad \text{Tritium}$$

The calculated wavelength for the H_α line is

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 0.13889R$$

$$\lambda(H_\alpha, \text{hydrogen}) = 656.47 \text{ nm}$$

$$\lambda(H_\alpha, \text{deuterium}) = 656.29 \text{ nm}$$

$$\lambda(H_\alpha, \text{tritium}) = 656.23 \text{ nm}$$

Deuterium was discovered when two closely spaced spectral lines of hydrogen near 656.4 nm were observed in 1932. These proved to be the H_α lines of atomic hydrogen and deuterium.

Problem 4.36

Calculate the Rydberg constant for the single-electron (hydrogen-like) ions of helium, potassium, and uranium. Compare each of them with R and determine the percentage difference.

-

$$R = \frac{1}{1+\frac{m}{M}} R_\infty \quad \text{where } R_\infty = 1.0973731534\times10^{7}\,\text{m}^{-1} \text{ and } m = 0.0005485799\,\text{u}.$$

^{4}He (M = 4.0026 u), $R = 0.999863\ R_\infty = 1.097223\times10^{7}\,\text{m}^{-1}$ (off by 0.14%).

^{39}K (M = 38.963708 u), $R = 0.9999859\ R_\infty = 1.097358\times10^{7}\,\text{m}^{-1}$ (off by 0.0014%)

^{238}U (M = 238.05078 u), $R = 0.9999977\ R_\infty = 1.097371\times10^{7}\,\text{m}^{-1}$ (off by 0.00023%)

32. Positronium is an atom composed of an electron and a positron (mass $m = m_e$, charge $q = +e$). Calculate the distance between the particles and the energy of the lowest energy state of positronium. (*Hint:* what is the reduced mass of the two particles? See Problem 53.)

$m_e = m_p = m$

$$\mu = \frac{m \cdot m}{m + m} = m/2$$

$$r = \frac{4\pi \varepsilon_0 \hbar^2}{\mu e^2} = 2a_0$$

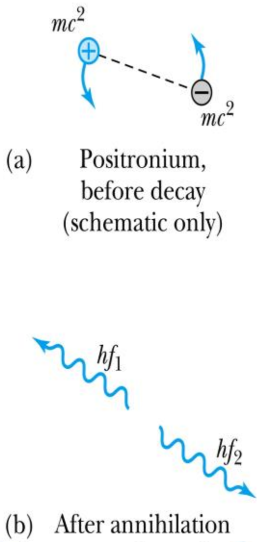
$$E = -\frac{e}{8\pi\varepsilon_0 r} = -\frac{e}{8\pi\varepsilon_0 2a_0} = -\frac{E_0}{2} = -6.8\,eV$$

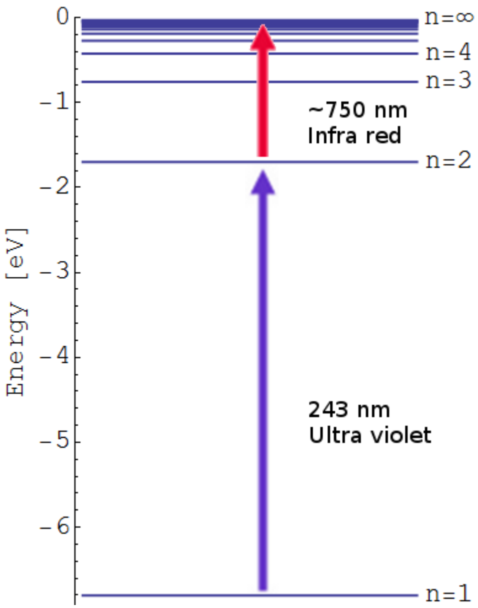
Find the wavelength for $n=2 \rightarrow n=1$

$$\Delta E = -\frac{E_0}{2^2} - \left(-\frac{E_0}{1^2}\right) = \frac{3E_0}{4} = 5.1\,eV \qquad \lambda = \frac{hc}{\Delta E} = \frac{1240\,eV\cdot nm}{5.1\,eV} = 243\,nm$$

Positronium energy levels

Use reduced mass





While precise calculation of positronium energy levels uses the Bethe–Salpeter equation or the Breit equation, the similarity between positronium and hydrogen allows a rough estimate. In this approximation, the energy levels are different because of a different effective mass, *m**, in the energy equation (see electron energy levels for a derivation):

$$E_n = -\frac{\mu q_e^4}{8h^2\varepsilon_0^2}\frac{1}{n^2},$$

where:

q_e is the charge magnitude of the electron (same as the positron),

h is Planck's constant,

ε_0 is the electric constant (otherwise known as the permittivity of free space),

μ is the reduced mass:

$$\mu = \frac{m_e m_p}{m_e + m_p} = \frac{m_e^2}{2m_e} = \frac{m_e}{2},$$

where m_e and m_p are, respectively, the mass of the electron and the positron (which are *the same* by definition as antiparticles).

Thus, for positronium, its reduced mass only differs from the electron by a factor of 2. This causes the energy levels to also roughly be half of what they are for the hydrogen atom.

So finally, the energy levels of positronium are given by

$$E_n = -\frac{1}{2}\frac{m_e q_e^4}{8h^2\varepsilon_0^2}\frac{1}{n^2} = \frac{-6.8\ \text{eV}}{n^2}.$$

muonic atom

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{\mu e^2} = \frac{(6.58\times10^{-16}\ eVs)^2 \frac{(3\times10^8\ \frac{m}{s})^2}{c^2}}{(1.44\times10^{-3} e)\ 95.2\ \frac{MeV}{c^2}}$$

$$\mu = \frac{mM}{m+M} = \frac{106\ \frac{MeV}{c^2}\ 938\ \frac{MeV}{c^2}}{106+938} = 95.2\ \frac{MeV}{c^2}$$

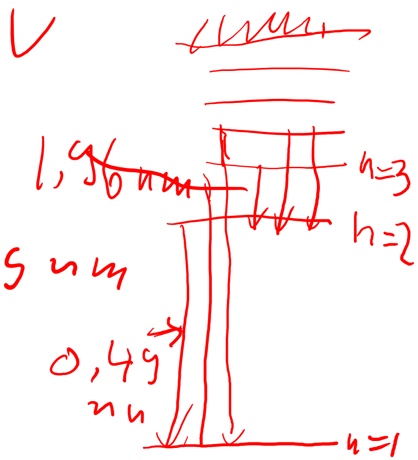
$$a_0 = 2.84\times10^{-13}\ m$$

$$E = \frac{e^2}{8\pi\epsilon_0 a_0} = \frac{1.44\times10^{-9}\ eVm}{2\times2.84\times10^{-13}\ m} = 2535\ eV$$

$n=1$ first series $\lambda = \frac{hc}{E} = \frac{1240\ eV\ nm}{2535\ e} = 0.49\ nm$

$n=2$ second series $\lambda = \frac{4hc}{E} = 1.96\ nm$

$n=3$ third series $\lambda = \frac{9hc}{E} = 4.40\ nm$



Limitations of the Bohr Model

The Bohr model was a great step of the new quantum theory, but it had its limitations.

1) Works only to single-electron atoms
2) Could not account for the intensities or the fine structure of the spectral lines
3) Could not explain the binding of atoms into molecules

X-rays revisited

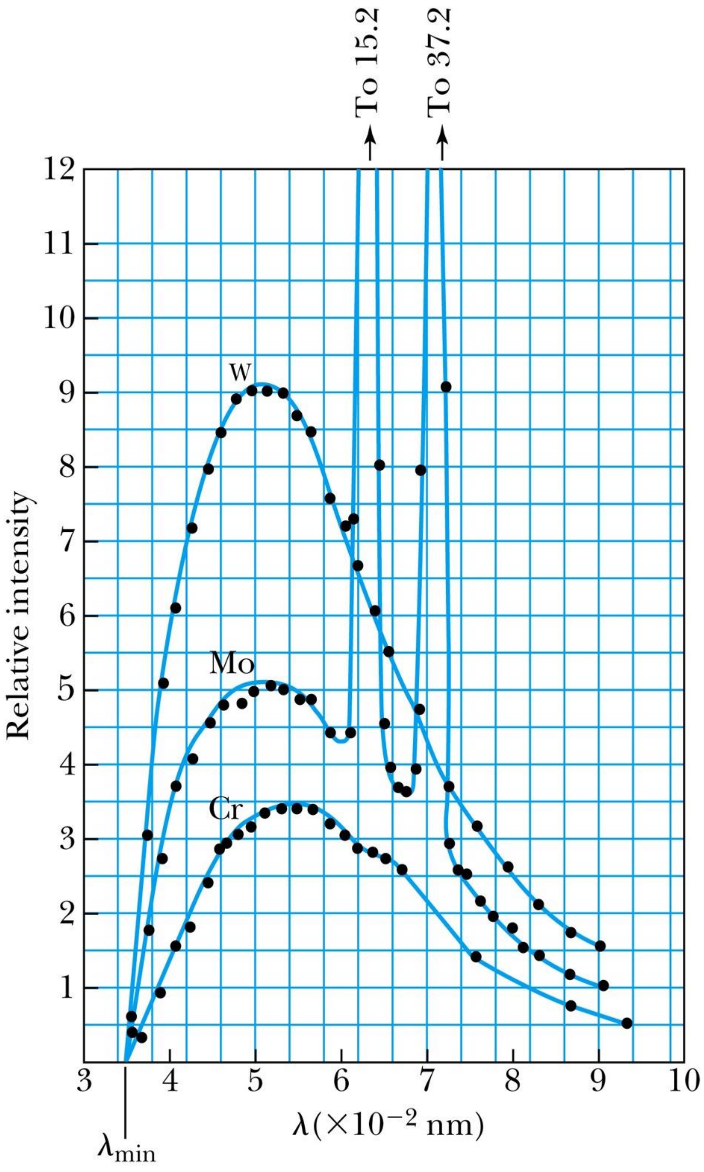
Bremsstrahlung=braking radiation has sharp lines in it

Inverse Photoelectric Effect (slide from Chapter3).

- Conservation of energy requires that the electron kinetic energy equal the maximum photon energy where we neglect the work function because it is normally so small compared to the potential energy of the electron. This yields the **Duane-Hunt limit** which was first found experimentally. The photon wavelength depends only on the accelerating voltage and is the same for all targets.

$$eV_0 = hf_{\max} = \frac{hc}{\lambda_{\min}}$$

$$\lambda_{\min} = \frac{hc}{e}\frac{1}{V_0} = \frac{1.240 \times 10^{-6}\ \mathrm{V \cdot m}}{V_0}$$



4.6: Characteristic X-Ray Spectra and Atomic Number

- Shells have letter names:

  K shell for $n = 1$

  L shell for $n = 2$

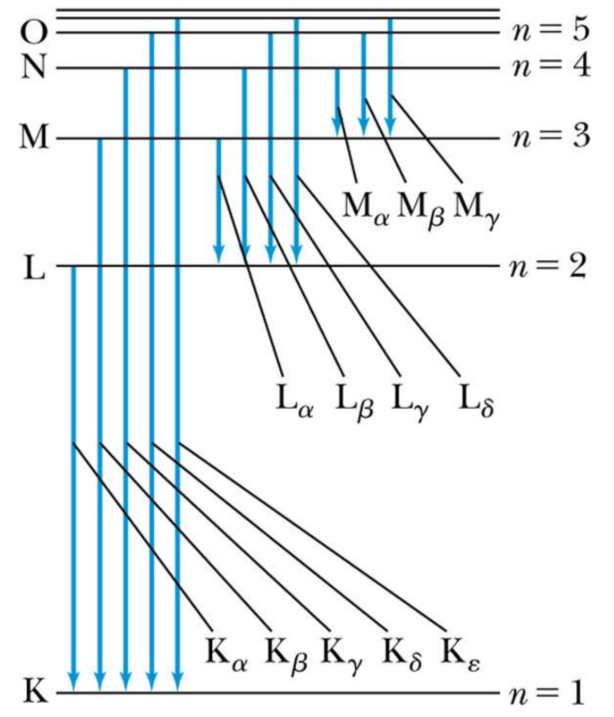
  ⋮

- The atom is most stable in its ground state.

→ An electron from higher shells will fill the inner-shell vacancy at lower energy.

- When it occurs in a heavy atom, the radiation emitted is an **x ray**.
- It has the energy E (x ray) $= E_u - E_\ell$.
- For this to happen an inner shell vacancy has to be produced(for instance by a high energy electron collision) in Roentgen's discovery of X-rays

Atomic Number



L shell to K shell → K_α x ray

M shell to K shell → K_β x ray

- *Atomic number Z = number of protons in the nucleus*
- Moseley found a relationship between the frequencies of the characteristic x ray and Z.

  This holds for the K_α x ray

$$f_{K_\alpha} = \frac{3cR}{4}(Z-1)^2$$

Chapter4 quiz question

Characteristic x-ray spectra come from

a. bremsstrahlung processes for electrons close to the nucleus.

b. electrons transitioning down from an outer shell replace electrons ejected from an inner shell.

c. random excitations of electrons when atoms are near room temperature.

d. photons scattering off of electrons, thereby losing energy and emitting radiation.

Clicker - Questions

9) The k_α-xray comes from transition of an electron;

a) From the L-shell to a vacancy in the k-shell

b) From the M-shell to a vacancy in the k-shell

c) From the M-shell to a vacancy in the L-shell

d) From the U-shell to a vacancy in the k-shell

Moseley's Empirical Results

- The K_α x ray is produced from n = 2 to n = 1 transition.
- In general, the K series of x ray wavelengths are

$$\frac{1}{\lambda_K} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right) = R(Z-1)^2\left(1 - \frac{1}{n^2}\right)$$

An electron in the L shell feels the effective charge Z-1 due +Ze of the nucleus and –e from the remaining electron in the K shell electron

Moseley's research clarified the importance of the electron shells for all the elements, not just for hydrogen.

(43) Calculate K_α and K_β wavelengths for He and Li

$$\frac{1}{\lambda(K_\alpha)} = R(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3}{4}R(Z-1)^2$$

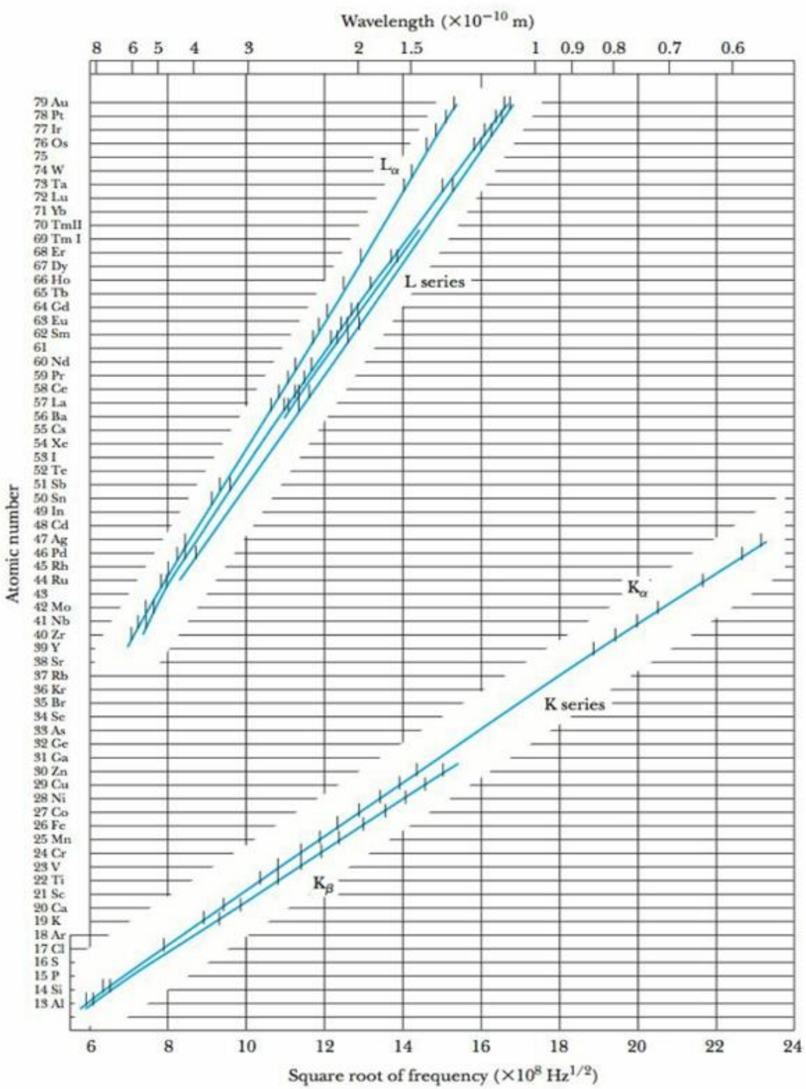
He:
$$\lambda(K_\alpha) = \frac{4}{3R(Z-1)^2} = 122\ nm \qquad \lambda(K_\beta) = \frac{9}{8R(Z-1)^2}$$

$$= 103\ nm$$

Li
$$\lambda(K_\alpha) = \frac{4}{3R(Z-1)^2} = 30.4\ nm$$

$$\lambda(K_\beta) = 25.6\ nm$$

Mosely plot the atomic number Z is responsible for ordering the periodic table



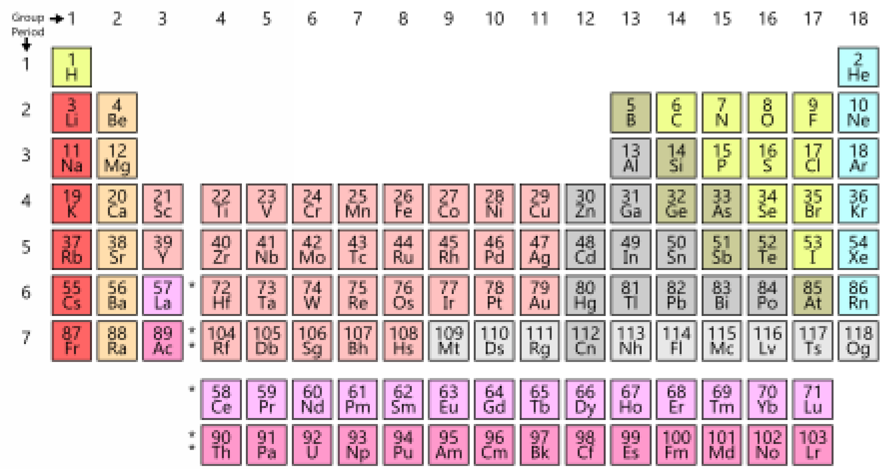
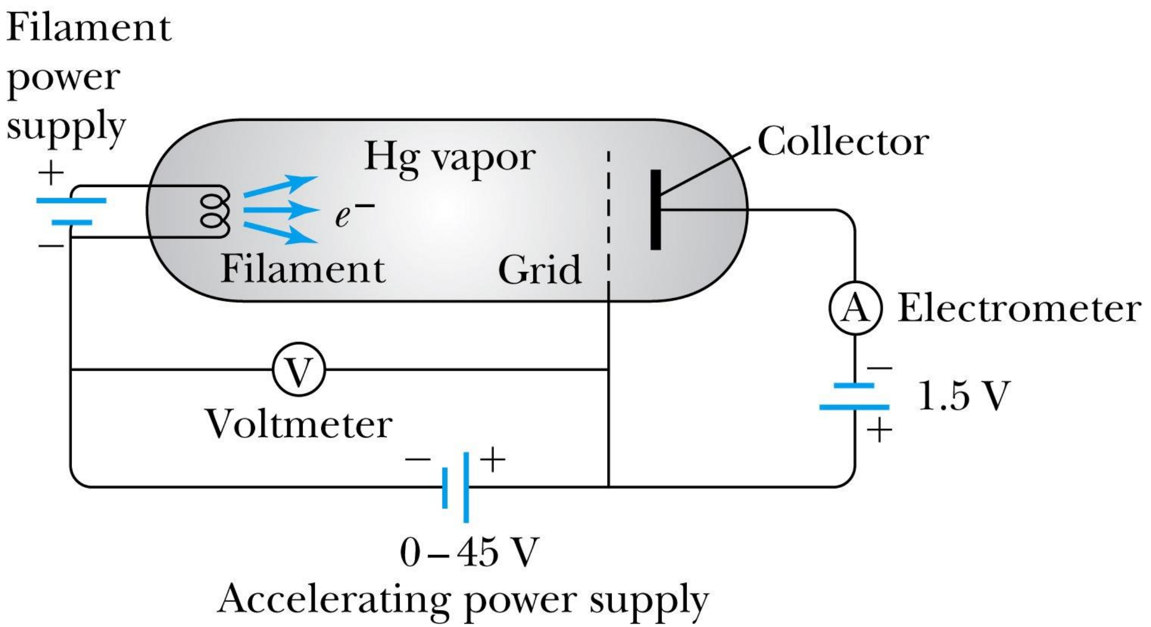


Figure 4.19 Moseley's original data indicating the relationship between the atomic number Z and the characteristic x-ray frequencies. Notice the missing entries for elements $Z = 43$, 61, and 75, which had not yet been identified. There are also a few errors in the atomic number designations for the elements. © *From H. G. J. Moseley*, Philosophical Magazine *(6)*, ***27****, 703 (1914).*

4.7: Atomic Excitation by Electrons

- Franck and Hertz studied the phenomenon of ionization.



Accelerating voltage is below 5 V

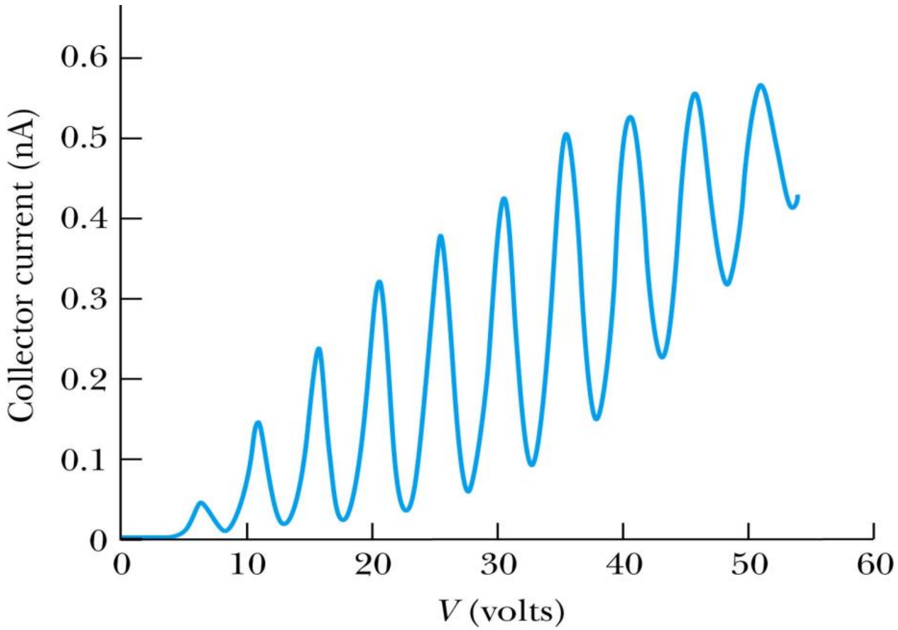
⟶ electrons did not lose energy

Accelerating voltage is above 5 V

⟶ sudden drop in the current

Atomic Excitation by Electrons

- Ground state has E_0 to be zero.
  First excited state has E_1.
  The energy difference $E_1 - 0 = E_1$ is the **excitation energy**.



- Hg has an excitation energy of 4.88 eV in the first excited state
- No energy can be transferred to Hg below 4.88 eV because not enough energy is available to excite an electron to the next energy level

- Above 4.88 eV, the current drops because scattered electrons no longer reach the collector until the accelerating voltage reaches 9.8 eV and so on.

Problem 4.46 Why is the small negative potential difference between grid and collector plate necessary??

- Without the negative potential an electron with any energy, no matter how small, could drift into the collector plate. As a result the electron could give up its kinetic energy to a Hg atom and still contribute to the plate current. The Franck-Hertz curve would not show the distinguishing periodic drops, but rather would rise monotonically.

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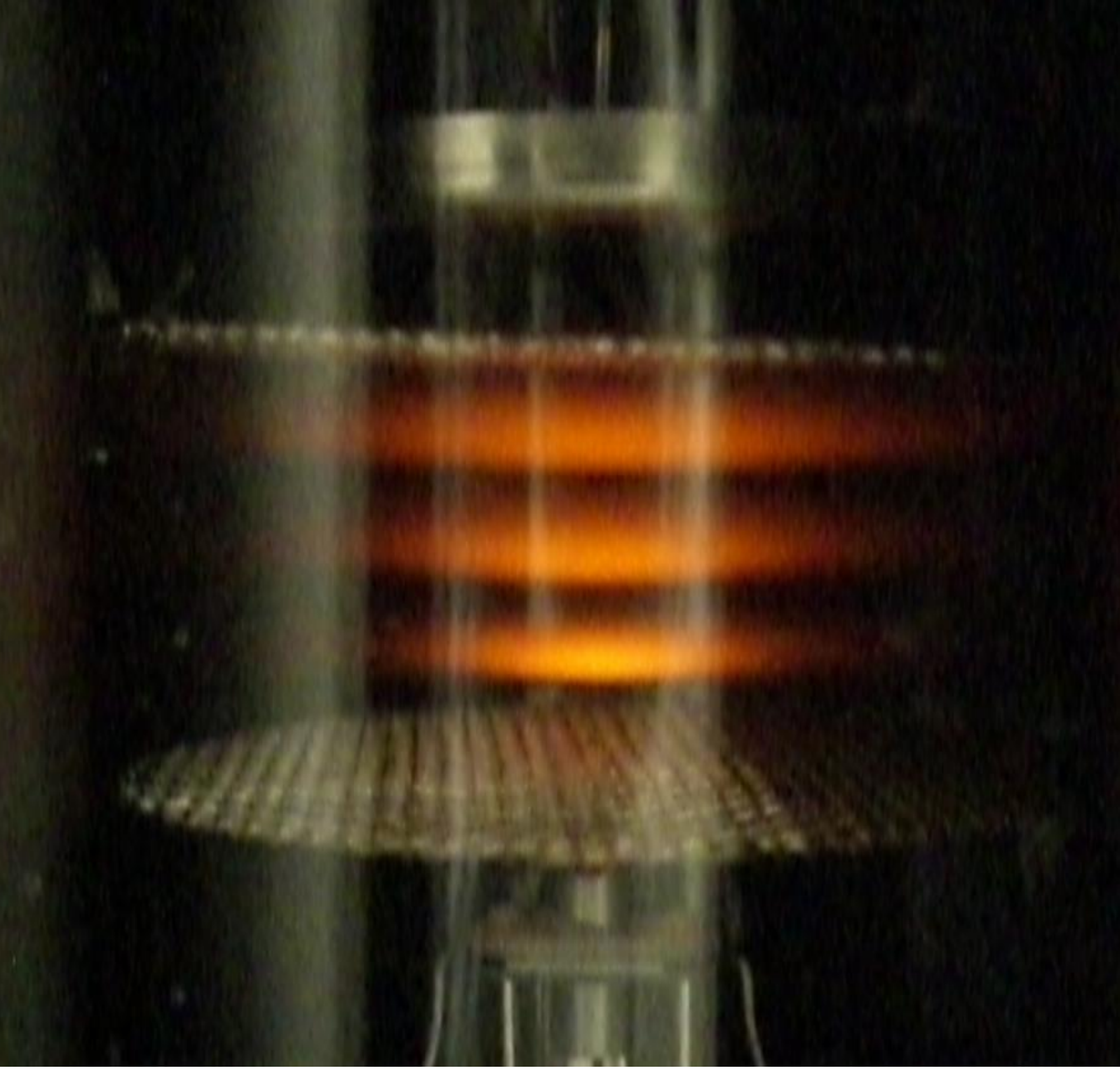

Clicker - Questions

2) In the Franck-Hertz experiment the electron loses energy by

a) Ionization of a Hg atom

b) Excitation of a Hg atom to the first excited state

c) Excitation to the second excited state of a Hg atom

d) By being captured by a Hg atom

Clicker - Questions

3) In the Franck-Hertz experiment the electron collides with a Hg atom in an

a) Elastic Collision

b) Inelastic Collision

c) Grazing Collision

d) Orbiting Collision

Problem 4.47

Determine Planck's constant from the Frank Hertz experiment in Hg vapor

$$\frac{1}{\lambda} = \frac{E_2 - E_1}{hc} \qquad \Delta E = \frac{hc}{\lambda} \qquad h = \frac{\Delta E . \lambda}{c}$$

Using $\Delta E = hc/\lambda$ we find $h = \dfrac{\lambda \Delta E}{c} = \dfrac{(254\text{ nm})(4.88\text{ eV})}{3.00\times 10^{8}\text{ m/s}} = 4.13\times 10^{-15}\text{eV}\cdot\text{s}$