CHAPTER 3
The Experimental Basis of Quantum Physics

- 3.1 Discovery of the X Ray and the Electron
- 3.2 Determination of Electron Charge
- 3.3 Line Spectra
- 3.4 Quantization
- 3.5 Blackbody Radiation
- 3.6 Photoelectric Effect
- 3.7 X-Ray Production
- 3.9 Pair Production and Annihilation
Light = electromagnetic radiation
3.5: Blackbody Radiation

- When matter is heated, it emits radiation.
- A blackbody is a cavity in a material that only emits thermal radiation. Incoming radiation is absorbed in the cavity.

- Blackbody radiation is theoretically interesting because the radiation properties of the blackbody are independent of the particular material. Physicists can study the properties of intensity versus wavelength at fixed temperatures.
Wien’s Displacement Law

- The intensity $I(\lambda, T)$ is the total power radiated per unit area per unit wavelength at a given temperature.
- **Wien’s displacement law**: The maximum of the distribution shifts to smaller wavelengths as the temperature is increased.

\[ \lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K} \]

(where $\lambda_{\text{max}} = $ wavelength of the peak)
Problem 21. Calculate the maximum wavelength for blackbody radiation (a) liquid helium at 4.2 K (b) room temperature at 293 K, (c) a steel furnace at 2500 K, (d) a blue star at 9000 K

1. (a) \( \lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{4.2 \text{ K}} = 0.69 \text{ mm} \)

(b) \( \lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{293 \text{ K}} = 9.89 \text{ \mu m} \)

(c) \( \lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{2500 \text{ K}} = 1.16 \text{ \mu m} \)

(d) \( \lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{9000 \text{ K}} = 0.322 \text{ \mu m} \)
When do you have fever?

<table>
<thead>
<tr>
<th>Conversion</th>
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<td><strong>Celsius to Fahrenheit</strong></td>
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The total power radiated increases with the temperature:

\[ R(T) = \int_0^\infty \mathcal{L}(\lambda, T) \, d\lambda = \varepsilon \sigma T^4 \]

This is known as the **Stefan-Boltzmann law**, with the constant \( \sigma \) experimentally measured to be \( 5.6705 \times 10^{-8} \, \text{W} / (\text{m}^2 \cdot \text{K}^4) \).

The **emissivity** \( \varepsilon \) (\( \varepsilon = 1 \) for an idealized blackbody) is simply the ratio of the emissive power of an object to that of an ideal blackbody and is always less than 1.
19. (a) A blackbody’s temperature is increased from 900 K to 2300 K. By what factor does the total power radiated per unit area increase? (b) If the original temperature is again 900 K, what final temperature is required to double the power output?

(a) \[ \frac{P_1}{P_0} = \frac{\sigma T_1^4}{\sigma T_0^4} \]; so \( P_1 = P_0 \frac{T_1^4}{T_0^4} = \left( \frac{2300 \text{ K}}{900 \text{ K}} \right)^4 P_0 = 42.7 \ P_0 \); The power increases by a factor of 42.7.

(b) To double the power output, the ratio of temperatures to the fourth power must equal

2. \[ \left( \frac{T_1}{900 \text{ K}} \right)^4 = 2 \]. Solving we find \( T_1 = 900 \times 2^{1/4} = 1070 \text{ K} \).
Problem 20  

(a) At what wavelength will the human body radiate the maximum radiation? (b) Estimate the total power radiated by a person of medium build (assume an area given by a cylinder of 175-cm height and 13-cm radius). (c) Using your answer to (b), compare the energy radiated by a person in one day with the energy intake of a 2000-kcal diet

1.  

(a) \[ \lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{310\text{K}} = 9.35 \mu\text{m} \]

(b) At this temperature the power per unit area is

\[ R = \sigma T^4 = \left(5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^4\right)(310\text{K})^4 = 524\text{W/m}^2. \]

The total surface area of a cylinder is

\[ 2\pi r (r + h) = 2\pi (0.13 \text{ m})(1.75 \text{ m} + 0.13 \text{ m}) = 1.54 \text{ m}^2 \]

so the total power is

\[ P = (524 \text{ W/m}^2)(1.54 \text{ m}^2) = 807 \text{ W}. \]

(c) The total energy radiated in one day is the power multiplied by the time;

\[ E = P \cdot t = (807 \text{ W}) \cdot (86400 \text{ s}) = 6.97 \times 10^7 \text{ J}. \]

There is more energy radiated away than consumed by eating

2000 kcal = \(2 \times 10^6 \text{ cal}\) \(\cdot\) \(4.186\text{J/cal}\) = \(8.37 \times 10^6 \text{ J}\).

There are several assumptions. First, a cylinder may overestimate the total surface area; second, radiation is minimized by hair covering and clothing.
Planck assumed that the radiation in the cavity was emitted (and absorbed) by some sort of "oscillators" that were contained in the walls. He used Boltzman’s statistical methods to arrive at the following formula that fit the blackbody radiation data:

\[
I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}
\]

Planck’s radiation law

Planck made two modifications to the classical theory:

1) The oscillators (of electromagnetic origin) can only have certain discrete energies determined by \( E_n = nhf \), where \( n \) is an integer, \( f \) is the frequency of the radiation, and \( h \) is called Planck’s constant. \( h = 6.6261 \times 10^{-34} \text{ J} \cdot \text{s} \).

2) The oscillators can absorb or emit energy in discrete multiples of the fundamental quantum of energy given by \( \Delta E = hf \).
Summary: Blackbody radiation

- **Blackbody**: absorbs all radiation striking on it, $(R=0)$, does not reflect.
- Key hole is always black.

- **Empirical laws**: 1879
  - **Wien's Law**: $\lambda_{\text{max}} T = 2.898 \times 10^{-5} \text{ m.k}$
  - **Planck's Radiation Law**:
    
    $I(\lambda, T) = \frac{2\pi c^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$

- **Power/Area**: Stephan-Boltzmann
  - Emissivity, $\varepsilon = 1$ for blackbody
  - $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

- **Theoretical attempt based on classical $\varepsilon$ and $\mu$ (Maxwell Rayleigh-Jeans)**

- **Planck constant**: $\hbar = 6.6 \times 10^{-34} \text{ J.s}$
The Rydberg equation is used to

a. Determine the ratio of the electron charge to its mass
b. Calculate the wavelengths of different spectral lines of hydrogen
c. Measure the mass of the hydrogen atom
d. Calculate the wavelengths of different transitions in energy level of electrons in helium
How did Planck modify the classical theory of blackbody radiation to correctly determine his radiation law?

a. He found that the blackbody model was incorrect for purposes of theory  
b. He accepted the Stefan-Boltzmann law  
c. He assumed light was absorbed and emitted in quanta  
d. He realized that the charge of the electron was not quantized  
e. He proved the necessity of relativistic considerations
Problem 3.33

How many photons are contained in a beam of electromagnetic radiation of total power 180 W, if the source is a radio station of 1 MHz?

\[
\text{Energy/ photon } = hf = (6.626 \times 10^{-34} \text{ J}\cdot\text{s}) (1 \times 10^3 \text{ s}^{-1}) = 7.26 \times 10^{-28} \text{ J}
\]

\[
(180 \text{ J/s}) \left( \frac{1 \text{ photon}}{7.26 \times 10^{-28} \text{ J}} \right) = 2.47 \times 10^{29} \text{ photons/sec}
\]
Dancing on singing sand
3.6: Photoelectric Effect

Methods of electron emission:

- **Thermionic emission**: Application of heat allows electrons to gain enough energy to escape.
- **Secondary emission**: The electron gains enough energy by transfer from another high-speed particle that strikes the material from outside.
- **Field emission**: A strong external electric field pulls the electron out of the material.
- **Photoelectric effect**: Incident light (electromagnetic radiation) shining on the material transfers energy to the electrons, allowing them to escape.

Electromagnetic radiation interacts with electrons within metals and gives the electrons increased kinetic energy. Light can give electrons enough extra kinetic energy to allow them to escape. We call the ejected electrons **photoelectrons**.
Work function = minimum binding energy of the electron

### Table 3.3 Work Functions

<table>
<thead>
<tr>
<th>Element</th>
<th>$\phi$ (eV)</th>
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<tbody>
<tr>
<td>Ag</td>
<td>4.64</td>
<td>K</td>
<td>2.29</td>
<td>Pd</td>
<td>5.22</td>
</tr>
<tr>
<td>Al</td>
<td>4.20</td>
<td>Li</td>
<td>2.93</td>
<td>Pt</td>
<td>5.64</td>
</tr>
<tr>
<td>C</td>
<td>5.0</td>
<td>Na</td>
<td>2.36</td>
<td>W</td>
<td>4.63</td>
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</table>

Experimental Setup

Incident light

Emitter

Collector

Vacuum tube

$e^-$

Ammeter

Power supply

(Voltage $V$)

$I$
Experimental Results

Retarding potential measures maximum kinetic energy

F0 threshold frequency

Same Kinetic energy

Frequency dependence of maximum kinetic energy

Photoelectric current starts in nsec
Experimental Results

1) The kinetic energies of the photoelectrons are independent of the light intensity.

2) The maximum kinetic energy of the photoelectrons, for a given emitting material, depends only on the frequency of the light.

3) The smaller the work function $\varphi$ of the emitter material, the smaller is the threshold frequency of the light that can eject photoelectrons.

4) When the photoelectrons are produced, however, their number is proportional to the intensity of light.

5) The photoelectrons are emitted almost instantly following illumination of the photocathode, independent of the intensity of the light.
Classical theory predicts that the total amount of energy in a light wave increases as the light intensity increases.

The maximum kinetic energy of the photoelectrons depends on the value of the light frequency $f$ and not on the intensity.

The existence of a threshold frequency is completely inexplicable in classical theory.

Classical theory would predict that for extremely low light intensities, a long time would elapse before any one electron could obtain sufficient energy to escape. We observe, however, that the photoelectrons are ejected almost immediately.
Einstein’s Theory

- Einstein suggested that the electromagnetic radiation field is quantized into particles called **photons**. Each photon has the energy quantum:

\[ E = hf \]

where \( f \) is the frequency of the light and \( h \) is Planck’s constant.

- The photon travels at the speed of light in a vacuum, and its wavelength is given by

\[ \lambda f = c \]
Einstein’s Theory

- Conservation of energy yields:

\[
\text{Energy before (photon)} = \text{energy after (electron)}
\]

\[
hf = \phi + \text{K.E. (electron)}
\]

where \( \phi \) is the work function of the metal

Explicitly the energy is

\[
hf = \phi + \frac{1}{2} m v_{\text{max}}^2
\]

- The retarding potentials measured in the photoelectric effect are the opposing potentials needed to stop the most energetic electrons.

\[
eV_0 = \frac{1}{2} m v_{\text{max}}^2
\]
Quantum Interpretation

- The kinetic energy of the electron does not depend on the light intensity at all, but only on the light frequency and the work function of the material.

\[
\frac{1}{2} m v_{\text{max}}^2 = eV_0 = hf - \phi
\]

- Einstein in 1905 predicted that the stopping potential was linearly proportional to the light frequency, with a slope \( h \), the same constant found by Planck.

\[
eV_0 = \frac{1}{2} mv_{\text{max}}^2 = hf - hf_0 = h(f - f_0)
\]

- From this, Einstein concluded that light is a particle with energy:

\[
E = hf = \frac{hc}{\lambda}
\]
For Li

Summary: photoelectric effect

A quantum of light (photon) delivers its entire energy to an electron: \( E = hf \).

Conservation of energy: \( hf = \phi + KE \).

Energy before = energy after (photon) electron.

\[ \frac{1}{2} m v_{\text{max}}^2 = eV_0 = hf - \phi \]

\[ \frac{1}{2} m v_{\text{max}}^2 = eV_0 = hf - \phi \]

Example: What frequency of light is needed to produce electrons of kinetic energy 3eV?

\[ hf = \phi + \frac{1}{2} m v_{\text{max}}^2 = 2.53\, \text{eV} + 3\, \text{eV} = 5.53\, \text{eV} \]

\[ f = \frac{E}{h} = \frac{5.53\, \text{eV}}{6.6 \times 10^{-34}\, \text{J}\cdot\text{s}} = \frac{5.53\, \text{eV}}{1.6 \times 10^{-19}\, \text{J}} = 1.4 \times 10^3 \, \text{eV} \]

\[ f = \frac{5.53\, \text{eV}}{6.6 \times 10^{-34}\, \text{J}\cdot\text{s}} = \frac{5.53\, \text{eV}}{1.6 \times 10^{-19}\, \text{J}} = 1.4 \times 10^3 \, \text{eV} \]

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Light frequency

Intercept = \(-\phi\)

Slope = \( h \)

Retarding potential

Ammeter

Power supply (Voltage \( V \))

Incident light

Collector

Emitter

Vacuum tube

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34. What is the threshold frequency for the photoelectric effect on lithium ($\phi = 2.93$ eV)? What is the stopping potential if the wavelength of the incident light is 380 nm?

See: Fundamental Constance and useful Fundamental Constants

$$f_t = \frac{\phi}{h} = \frac{2.93 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 7.08 \times 10^{14} \text{ Hz} : \quad eV_0 = \frac{hc}{\lambda} - \phi \quad \text{so} \quad V_0 = \frac{1}{e} \left[ \frac{hc}{\lambda} - \phi \right];$$

$$V_0 = \frac{1}{e} \left[ \frac{1240 \text{ eV} \cdot \text{nm}}{380 \text{ nm}} - 2.93 \text{ eV} \right] = 0.333 \text{ V}$$
What is the maximum wavelength of incident light that can produce photoelectrons from silver ($\phi = 4.64$ eV)? What will be the maximum kinetic energy of the photoelectrons if the wavelength is halved?

$$\lambda_t = \frac{hc}{\phi} = \frac{1240 \text{ eV} \cdot \text{nm}}{4.64 \text{ eV}} = 267.2 \text{ nm}.$$ 

If the wavelength is halved (to $\lambda = 133.6$ nm), then

$$K = \frac{hc}{\lambda} - \phi = \frac{1240 \text{ eV} \cdot \text{nm}}{133.6 \text{ nm}} - 4.64 \text{ eV} = 4.64 \text{ eV}$$
Problem 34. What is the threshold frequency for the photo electric effect in lithium with a work function of 2.93 eV? What is the stopping potential if the wavelength of the incident light is 380 nm?

\[ f_t = \frac{\phi}{h} = \frac{2.93 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 7.08 \times 10^{14} \text{ Hz} \]

\[ eV_0 = \frac{hc}{\lambda} - \phi \]

so \( V_0 = \frac{1}{e} \left[ \frac{hc}{\lambda} - \phi \right] \);

\[ V_0 = \frac{1}{e} \left[ \frac{1240 \text{ eV} \cdot \text{nm}}{380 \text{ nm}} - 2.93 \text{ eV} \right] = 0.333 \text{ V} \]
Question from chapter 3 quiz

When you increase only the intensity of the light onto the emitter, you measure
a. A decrease in the necessary stopping voltage
b. An increase in the necessary stopping voltage
c. No change in either current or stopping voltage
d. Either a or c. You cannot determine which from the information given.
e. An increased current
Maxwell classical light wave  Einstein Photon particle

Rayleigh-jeans  Planck quantization  Einstein quantized photons
3.7: X-Ray Production (inverse photoelectric effect)

- An energetic electron passing through matter will radiate photons and lose kinetic energy which is called **bremsstrahlung**, from the German word for “braking radiation.” Since linear momentum must be conserved, the nucleus absorbs very little energy, and it is ignored. The final energy of the electron is determined from the conservation of energy to be
  \[ E_f = E_i - hf \]

- An electron that loses a large amount of energy will produce an X-ray photon. Current passing through a filament produces copious numbers of electrons by thermionic emission. These electrons are focused by the cathode structure into a beam and are accelerated by potential differences of thousands of volts until they impinge on a metal anode surface, producing x rays by bremsstrahlung as they stop in the anode material.

Unlike the photon an electron can give up part of its energy (as bremsstrahlung) and be the same electron
What is the bremsstrahlung process?
   a. The emission of a photon from an electron being accelerated by a nucleus
   b. The emission of an electron from a metal when light is shined on it
   c. Thermal excitation of photons in a substance
   d. The emission of an electron from an inner electron shell and the resulting photon when an electron drops from an outer shell to take its place
   e. Converting power-producing nuclear material to weapons grade
Inverse Photoelectric Effect.

- Conservation of energy requires that the electron kinetic energy equal the maximum photon energy where we neglect the work function because it is normally so small compared to the potential energy of the electron. This yields the Duane-Hunt limit which was first found experimentally. The photon wavelength depends only on the accelerating voltage and is the same for all targets.

\[
e V_0 = h f_{\text{max}} = \frac{hc}{\lambda_{\text{min}}}
\]

\[
\lambda_{\text{min}} = \frac{hc}{e V_0} = \frac{1.240 \times 10^{-6}}{V_0} \text{ V} \cdot \text{m}
\]
Example 3.15  Inverse photoelectric effect

We have a tungsten anode (work function $\Phi = 4.63 \text{eV}$) and an electron acceleration voltage of 35 kV. What is the minimum wavelength of the X-rays?

$$E_f = E_i - h\nu$$

In the usual case of X-ray acceleration, the work function can be neglected compared to the acceleration energy.

$$\lambda_{\text{max}} = \frac{h}{E_f} = \frac{hc}{E_i}$$

Duane-Hunt rule

$$\lambda_{\text{min}} = \frac{hc}{E_0}$$

$$\lambda_{\text{min}} = \frac{1.24 \times 10^{-6} \text{eV} \cdot \text{m}}{35 \times 10^3 \text{V}} = 3.54 \times 10^{-11} \text{m}$$

$E_0 = 35 \text{kV}$

in good agreement with Fig 3.15
3.7: X-Ray Production (inverse photoelectric effect)

- An energetic electron passing through matter will radiate photons and lose kinetic energy which is called **bremsstrahlung**, from the German word for “braking radiation.” Since linear momentum must be conserved, the nucleus absorbs very little energy, and it is ignored. The final energy of the electron is determined from the conservation of energy to be

\[ E_f = E_i - hf \]

- An electron that loses a large amount of energy will produce an X-ray photon. Current passing through a filament produces copious numbers of electrons by thermionic emission. These electrons are focused by the cathode structure into a beam and are accelerated by potential differences of thousands of volts until they impinge on a metal anode surface, producing X rays by bremsstrahlung as they stop in the anode material.

Unlike the photon an electron can give up part of its energy (as bremsstrahlung) and be the same electron.
Bremsstrahlung, from bremsen "to brake" and Strahlung "radiation"; i.e., "braking radiation" or "deceleration radiation", is electromagnetic radiation produced by the deceleration of a charged particle when deflected by another charged particle, typically an electron by an atomic nucleus.

What is the bremsstrahlung process?
   a. The emission of a photon from an electron being accelerated by a nucleus
   b. The emission of an electron from a metal when light is shined on it
   c. Thermal excitation of photons in a substance
   d. The emission of an electron from an inner electron shell and the resulting photon when an electron drops from an outer shell to take its place
   e. Converting power-producing nuclear material to weapons grade
The maximum photon energy (when electron is stopped) is emitted as Bremsstrahlung: in X-ray emission.
3.8: Compton Effect

- When a photon enters matter, it is likely to interact with one of the atomic electrons. The photon is scattered from only one electron, rather than from all the electrons in the material, and the laws of conservation of energy and momentum apply as in any elastic collision between two particles. The momentum of a particle moving at the speed of light is

\[ p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \]

- The electron energy can be written as

\[ E_e^2 = (mc^2)^2 + p_e^2c^2 \]

- This yields the change in wavelength of the scattered photon which is known as the **Compton effect**:

\[ \Delta \lambda = \lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta) \]
Table 3.4 Results of Compton Scattering

<table>
<thead>
<tr>
<th>Energy or Momentum</th>
<th>Initial System</th>
<th>Final System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon energy</td>
<td>$hf$</td>
<td>$hf'$</td>
</tr>
<tr>
<td>Photon momentum in $x$ direction ($p_x$)</td>
<td>$\frac{h}{\lambda}$</td>
<td>$\frac{h}{\lambda'} \cos \theta$</td>
</tr>
<tr>
<td>Photon momentum in $y$ direction ($p_y$)</td>
<td>0</td>
<td>$\frac{h}{\lambda'} \sin \theta$</td>
</tr>
<tr>
<td>Electron energy</td>
<td>$mc^2$</td>
<td>$E_e = mc^2 + $ K.E.</td>
</tr>
<tr>
<td>Electron momentum in $x$ direction ($p_x$)</td>
<td>0</td>
<td>$p_e \cos \phi$</td>
</tr>
<tr>
<td>Electron momentum in $y$ direction ($p_y$)</td>
<td>0</td>
<td>$-p_e \sin \phi$</td>
</tr>
</tbody>
</table>
Thomson scattering = photon scattering from an tightly bound electron (use atom mass)

Compton scattering = photon scattering from a loosely bound electron (use electron mass)
A 650-keV gamma ray Compton-scatters from an electron. Find the energy of the photon scattered at 110°, the kinetic energy of the scattered electron, and the recoil angle of the electron.

\[
\lambda' = \lambda + \lambda_c (1 - \cos \theta) = \frac{hc}{E} + \lambda_c (1 - \cos \theta);
\]

\[
\lambda' = \frac{1240 \text{ eV} \cdot \text{nm}}{650 \times 10^3 \text{ eV}} + (2.43 \times 10^{-3} \text{ nm})(1 - \cos 110°) = 5.17 \text{ pm}
\]

\[
E' = \frac{hc}{\lambda'} = \frac{1240 \text{ eV} \cdot \text{nm}}{5.17 \times 10^{-3} \text{ nm}} = 2.40 \times 10^5 \text{ eV} = 240 \text{ keV}
\]

By conservation of energy we find: \( K_e = E - E' = 650 \text{ keV} - 240 \text{ keV} = 410 \text{ keV} \) which agrees with the \( K \) formula in the previous problem. Also from the previous problem we have:

\[
\cot \phi = \left[ 1 + \frac{hf}{mc^2} \right] \tan \left( \frac{\theta}{2} \right) = \left[ 1 + \frac{650 \text{ keV}}{511 \text{ keV}} \right] \tan \left( \frac{110°}{2} \right) = 3.245 \text{ so}
\]

\[
\phi = 17.1°.
\]
If a photon can create an electron, it must also create a positive charge to balance charge conservation.

In 1932, C. D. Anderson observed a positively charged electron ($e^+$) in cosmic radiation. This particle, called a positron, had been predicted to exist several years earlier by P. A. M. Dirac.

A photon’s energy can be converted entirely into an electron and a positron in a process called pair production.

$$\gamma \rightarrow e^+ + e^-$$
Pair production
A dramatic proof of relativistic change of mass
\[ E = h\nu = pc \]
\[ E^2 = p^2c^2 + m^2c^4 \]

Rest mass of electron
\[ E = m_0c^2 = 9.1 \times 10^{-31} (3 \times 10^8)^2 \text{ kg} \frac{m^2}{s^2} \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ g}} \]
\[ E = 0.51 \text{ MeV} \]

For pair production (birth of positron and electron) as shown in the cloud chamber picture; an energy of at least 2x0.51MeV=1.02MeV is needed. The photon which transforms into particle must have an energy
\[ E = h\nu = pc > 1.02 \text{ MeV} \]

Pair annihilation
Inverse process; where 2 or 3 photons are produced when
\[ e^+ + e^- = nh\nu \]

Conservation of mass energy
Proton – antiproton
Electron – positron
Hydrogen – antihydrogen
Neutron – antineutron
Matter – antimatter
Every type of particle is associated with an antiparticle

Cloud chamber photograph of the first positron ever observed. The thick horizontal line is a lead plate. The positron entered the cloud chamber in the lower left, was slowed down by the lead plane, and curved to the upper left. The curvature of the path is caused by an applied magnetic field that acts perpendicular to the image plane. The higher energy of the entering positron resulted in lower curvature of its path. Carl D. Anderson (1905–1991) - Anderson, Carl D. (1933). "The Positive Electron". *Physical Review* **43** (6): 491–494. DOI:10.1103/PhysRev.43.491
Pair Production in Empty Space

- Conservation of energy for pair production in empty space is

\[ hf = E_+ + E \]

- Considering momentum conservation yields

\[ hf = p_- c \cos \theta_- + p_+ c \cos \theta_+ \]

- This energy exchange has the maximum value \( hf_{\text{max}} = p_- c + p_+ c \)

- Recall that the total energy for a particle can be written as

\[ E_{\pm}^2 = p_{\pm}^2 c^2 + m^2 c^4 \]

However this yields a contradiction: \( hf > p_- c + p_+ c \)
and hence the conversion of energy in empty space is an impossible situation.
Pair Production in Matter

- Since the relations $h f_{\text{max}} = p_- c + p_+ c$ and $h f > p_- c + p_+ c$ contradict each other, a photon can not produce an electron and a positron in empty space.
- In the presence of matter, the nucleus absorbs some energy and momentum.

$$h f = E_+ + E_- + \text{K.E. (nucleus)}$$

- The photon energy required for pair production in the presence of matter is $h f > 2 m_e c^2 = 1.022 \text{ MeV}$

Conservation laws are fulfilled by the energy and momentum absorbed by a nearby nucleus a in matter
**Pair Annihilation**

- A positron passing through matter will likely **annihilate** with an electron. A positron is drawn to an electron by their mutual electric attraction, and the electron and positron then form an atomlike configuration called **positronium**.

- Pair annihilation in empty space will produce two photons to conserve momentum. Annihilation near a nucleus can result in a single photon.

- Conservation of energy: 
  \[ 2m_e c^2 \approx hf_1 + hf_2 \]

- Conservation of momentum: 
  \[ 0 = \frac{hf_1}{c} - \frac{hf_2}{c} \]

- The two photons will be almost identical, so that 
  \[ f_1 = f_2 = f \]

- The two photons from positronium annihilation will move in opposite directions with an energy:

\[ hf = m_e c^2 = 0.511 \text{ MeV} \]
use positron-emitting pharmaceuticals. They accumulate, for instance, in cancer cells, and are transported by blood. When a positron annihilates with an electron, the two $\gamma$-rays indicate the location.

\[ e^+ + e^- = 2\gamma \]

The emission is opposite direction $2\gamma = 1.02\text{MeV}$

\[ E_0 = m_0 c^2 = 9.1 \times 10^{-31} \text{ kg} \quad (3 \times 10^8 \text{ m/s})^2 \frac{1\text{ eV}}{1.6 \times 10^{-19} \text{ J}} = 0.51\text{MeV} \]

Figure 3.24 Positron emission tomography is a useful medical diagnostic tool to study the path and location of a positron-emitting radiopharmaceutical in the human body. (a) Appropriate radiopharmaceuticals are chosen to concentrate by physiological processes in the region to be examined. (b) The positron travels only a few millimeters before annihilation, which produces two photons that can be detected to give the positron position. (c) PET scan of a normal brain. (a) and (b) are after G. L. Brownell et al., Science 215, 619 (1982); (c) National Institutes of Health/SPL/Photo Researchers, Inc.