Comparison between sodium and hydrogen spectrum

Rydberg atoms are highly excited atoms with their outermost electron in a highly excited state= H-like Z=1 shieldingof nuclear charge by inner inner electrons

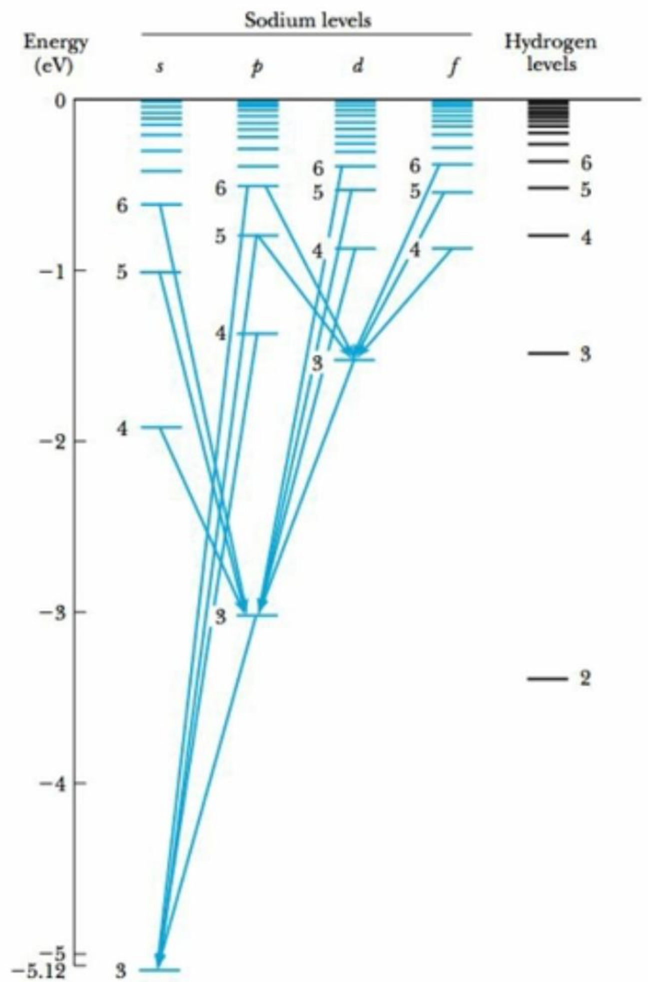
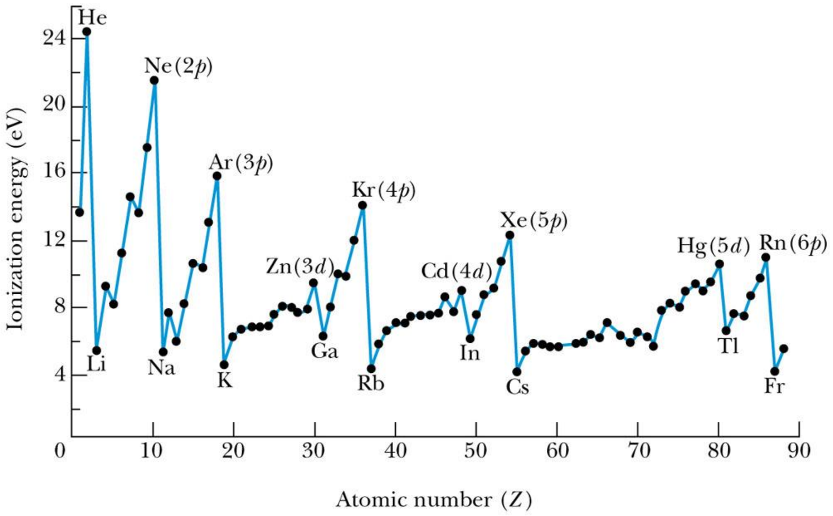
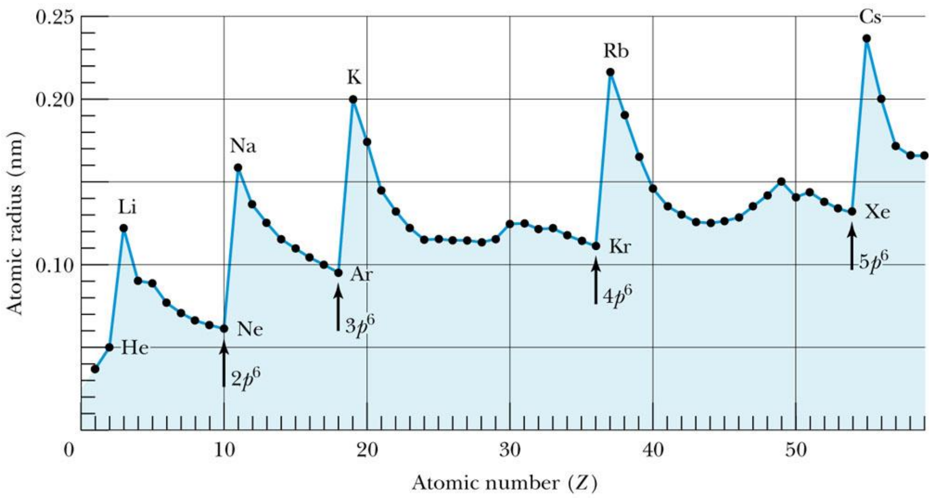


Figure 8.8 The energy-level diagram of sodium (a single electron outside an inert core) is compared to that of hydrogen. Coulomb effects cause the lower ℓ states of sodium to be lower than the corresponding levels of hydrogen. Several allowed transitions are shown for sodium.

Ionization Energies of Elements and Atomic Radii

Some properties of elements are compared by the **ionization energies of elements** and **atomic radii**:





Problem 8.7

Homework 8 requires 1.84e for answer

The 3s state of Na has an energy of -5.14eV. Determine the effective nuclear charge.

1. From Figure 8.4 we see that the radius of Na is about 0.16 nm. We know that for single-electron atoms $E = -\frac{Ze^2}{8\pi\varepsilon_0 r}$. Therefore

$$Ze = -\frac{8\pi\varepsilon_0 rE}{e^2}e = -2\frac{4\pi\varepsilon_0}{e^2}rEe = -\frac{2(0.16\text{ nm})(-5.14\text{ eV})}{1.44\text{ eV}\cdot\text{nm}}e = 1.14e.$$

$$\frac{e^2}{4\pi\epsilon_0} = 1.44 \text{ eV nm}$$

Structure of matter

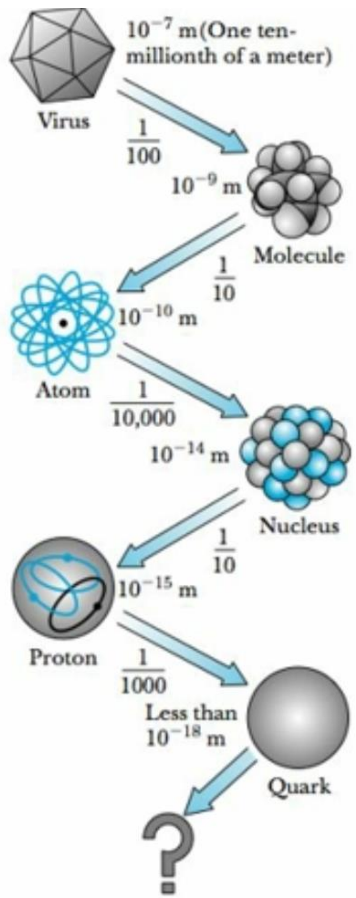
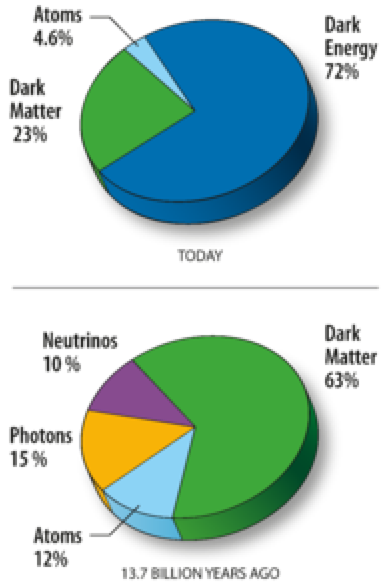
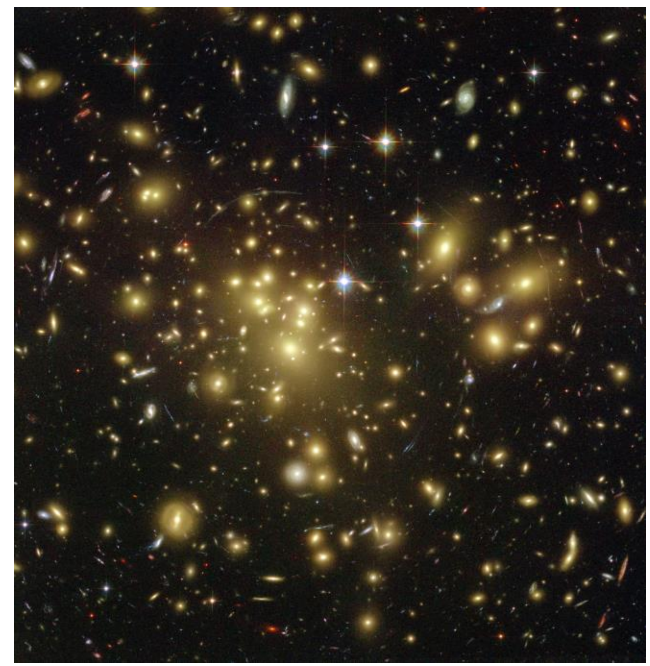


Figure 14.1 Starting from a virus, the structure of matter can be divided into smaller and smaller entities down to the quark and to whatever lies beyond. *Cour-*



Dark matter and **dark energy** are the yin and yang of the cosmos. **Dark matter** produces an attractive force (gravity), while **dark energy** produces a repulsive force (antigravity). ... Astronomers know **dark matter** exists because visible **matter** doesn't have enough gravitational muster to hold galaxies together.



Hierarchy of forces

Table 1.1 Fundamental Forces

Interaction		Relative Strength*	Range
Strong		1	Short, $\sim 10^{-15}$ m
Electroweak	Electromagnetic	10^{-2}	Long, $1/r^2$
	Weak	10^{-9}	Short, $\sim 10^{-15}$ m
Gravitational		10^{-39}	Long, $1/r^2$

*These strengths are quoted for neutrons and/or protons in close proximity.

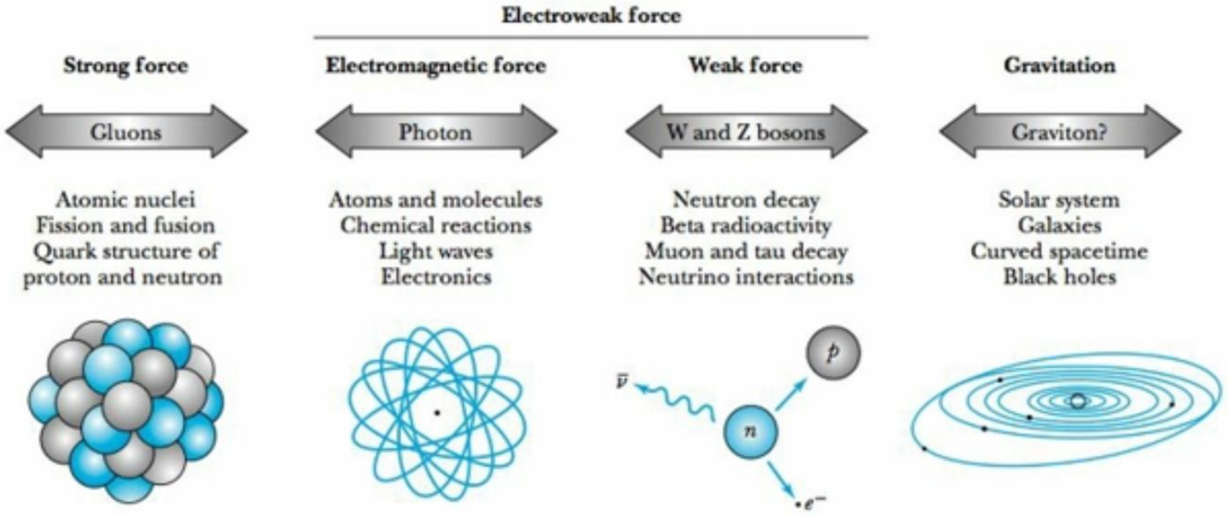


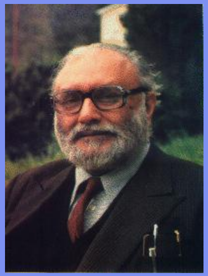
Figure 14.5 Some manifestations of the fundamental forces of nature. The mediating particles are shown, as well as the areas in which the forces are effective. *Courtesy Universities Research Association.*

Table 14.1 The Fundamental Interactions

Interaction	Relative Strength	Range	Mediating Particle
Strong	1	10^{-15} m	Gluons
Electroweak:			
Electromagnetic	10^{-2}	∞	Photons
Weak	10^{-6}	10^{-18} m	$W^{\pm}$, Z bosons
Gravitation	10^{-43}	∞	Graviton

Standard Model tries to unify the forces into one force

Story so far: Unification



Faraday	Glashow,Weinberg,Salam	Georgi,Glashow	Green,Schwarz	Witten
1831	1967	1974	1984	1995

Electricity }
}
} Electromagnetic force }
} }
Magnetism} } Electro-weak force }
} }
Weak nuclear force} } Grand unified force }
} } 5 Different }
Strong nuclear force} } D=10 String } M-theory ?
} Theories }
Gravitational force}
+branes in D=11

The nuclear force can be all of the following EXCEPT:

a. short-range
b. saturable
c. spin dependent
d. charge dependent

CHAPTER 12
The Atomic Nucleus

- 12.1 Discovery of the Neutron
- 12.2 Nuclear Properties
- 12.3 The Deuteron
- 12.4 Nuclear Forces
- 12.5 Nuclear Stability
- 12.6 Radioactive Decay
- 12.7 Alpha, Beta, and Gamma Decay
- 12.8 Radioactive Nuclides

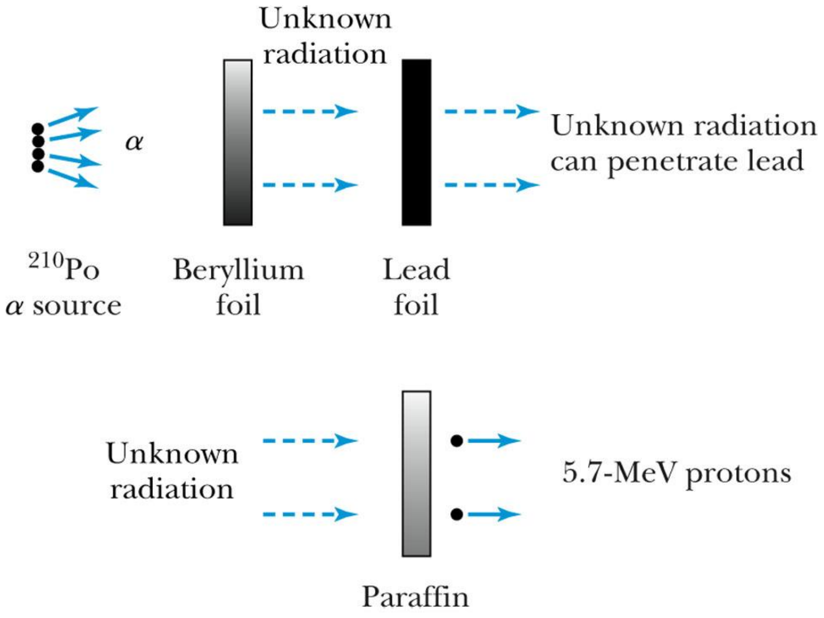
Discovery of the Neutron

3) Nuclear magnetic moment:

The magnetic moment of an electron is over 1000 times larger than that of a proton.

The measured nuclear magnetic moments are on the same order of magnitude as the proton's, so an electron is not a part of the nucleus.

- In 1930 the German physicists Bothe and Becker used a radioactive polonium source that emitted α particles. When these α particles bombarded beryllium, the radiation penetrated several centimeters of lead.



Discovery of the Neutron

- Photons are called *gamma rays* when they originate from the nucleus. They have energies on the order of MeV (as compared to *X-ray photons* due to electron transitions in atoms with energies on the order of KeV.)

- Curie and Joliot performed several measurements to study penetrating high-energy gamma rays.

- In 1932 Chadwick proposed that the new radiation produced by α + Be consisted of neutrons. His experimental data estimated the neutron's mass as somewhere between 1.005 u and 1.008 u, not far from the modern value of 1.0087 u.

12.2: Nuclear Properties

- The nuclear charge is $+e$ times the number (Z) of protons.

- Hydrogen's **isotopes**:
  - **Deuterium**: Heavy hydrogen; has a neutron as well as a proton in its nucleus
  - **Tritium**: Has two neutrons and one proton

- The nuclei of the deuterium and tritium atoms are called *deuterons* and *tritons*.
- Atoms with the same Z, but different mass number A, are called **isotopes**.

Nuclear Properties

- The symbol of an atomic nucleus is ${}^{A}_{Z}X_{N}$.

  where Z = atomic number (number of protons)

  N = neutron number (number of neutrons)

  A = mass number ($Z + N$)

  X = chemical element symbol
- Each nuclear species with a given Z and A is called a **nuclide**.
- Z characterizes a chemical element.
- The dependence of the chemical properties on N is negligible.
- Nuclides with the same neutron number are called *isotones* and the same value of A are called *isobars*.

Nuclear Properties

- Atomic masses are denoted by the symbol u.
- 1 u = 1.66054×10^{-27} kg = 931.49 MeV/c^2

Table 12.1 Some Nucleon and Electron Properties

Particle	Symbol	Rest Energy (MeV)	Charge	Mass (u)	Spin	Magnetic Moment
Proton	p	938.272	$+e$	1.0072765	1/2	$2.79\ \mu_N$
Neutron	n	939.566	0	1.0086649	1/2	$-1.91\ \mu_N$
Electron	e	0.51100	$-e$	5.4858×10^{-4}	1/2	$-1.00116\ \mu_B$

- Both neutrons and protons, collectively called **nucleons**, are constructed of other particles called *quarks*.

Clicker - Questions

Considered the following nuclides and then list the isotones.

$^{14}_{6}C$, $^{16}_{6}C$, $^{16}_{8}O$, $^{17}_{8}O$, $^{17}_{9}F$, $^{15}_{7}N$

a) $^{14}_{6}C$, $^{16}_{8}O$, $^{17}_{9}F$, $^{15}_{7}N$

b) $^{16}_{6}C$, $^{16}_{8}O$, $^{17}_{8}O$, $^{17}_{9}F$

c) $^{16}_{8}O$, $^{17}_{8}O$, $^{17}_{9}F$, $^{15}_{7}N$

Sizes and Shapes of Nuclei

- Rutherford concluded that the range of the nuclear force must be less than about 10^{-14} m.
- Assume that nuclei are spheres of radius *R*.
- Particles (electrons, protons, neutrons, and alphas) scatter when projected close to the nucleus.
- It is not obvious whether the maximum interaction distance refers to the nuclear size (*matter radius*), or whether the nuclear force extends beyond the nuclear matter (*force radius*).

- The nuclear force is often called the **strong** force.

Nuclear force radius ≈ mass radius ≈ charge radius

Sizes and Shapes of Nuclei

- The nuclear radius may be approximated to be $R = r_0 A^{1/3}$ where $r_0 \approx 1.2 \times 10^{-15}$ m.

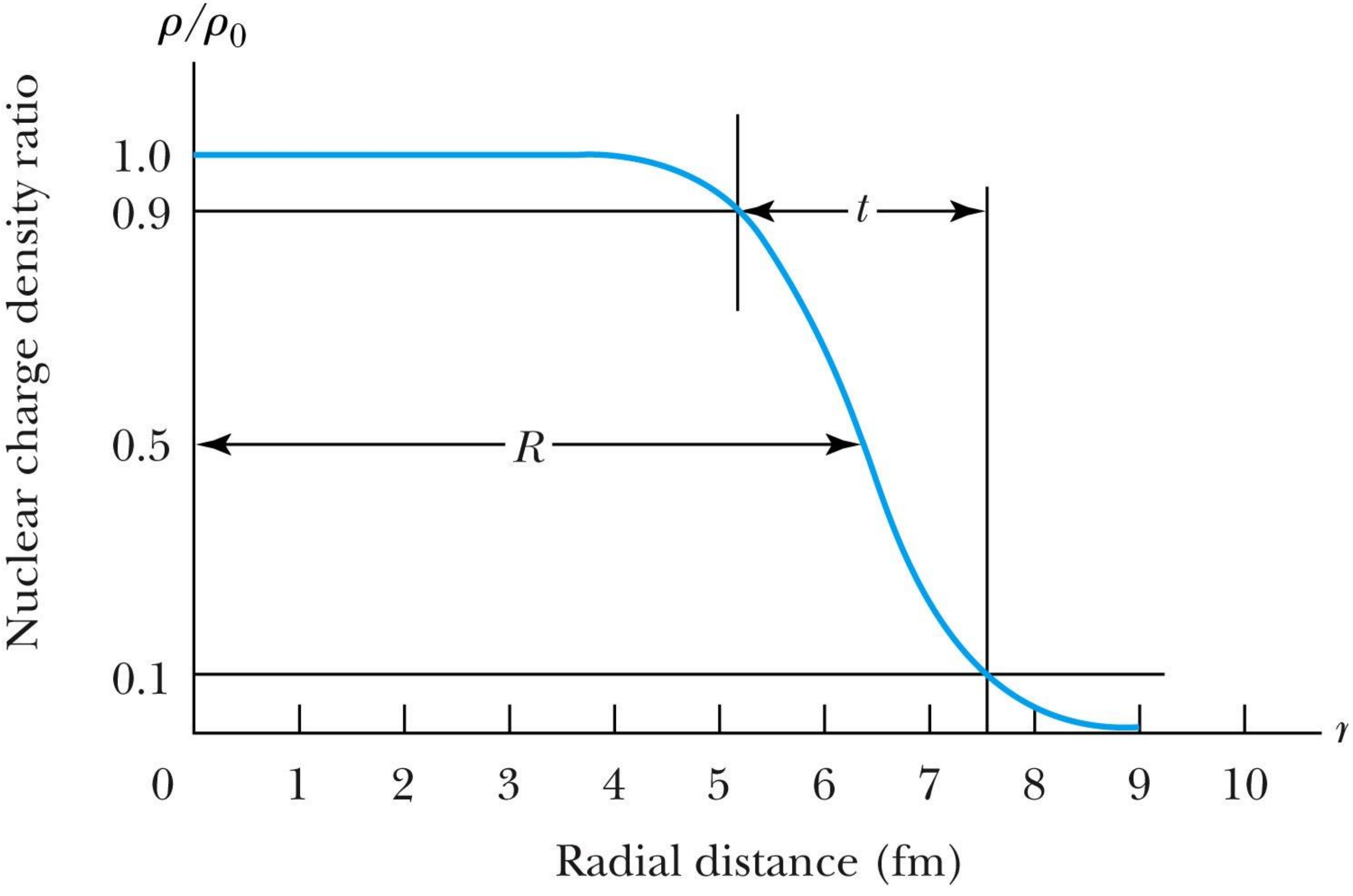
- We use the **femtometer** with 1 fm = 10^{-15} m, or the fermi.

- The lightest nuclei by the Fermi distribution for the nuclear charge density $\rho(r)$ is

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

Sizes and Shapes of Nuclei

The shape of the Fermi distribution



Nuclear Density and Intrinsic Spin

Nuclear Density: If we approximate the nuclear shape as a sphere, then we have: $V = \frac{4}{3}\pi r_0^3 A$ the nuclear mass density (mass/volume) can be determined from (Au/V) to be 2.3 x 10^{17} kg/m^3.

Intrinsic Spin: The neutron and proton are fermions with spin quantum numbers s = ½. The spin quantum numbers are those previously learned for the electron (see Chapter 7).

Intrinsic Magnetic Moment

- The proton's intrinsic magnetic moment points in the same direction as its intrinsic spin angular momentum.
- Nuclear magnetic moments are measured in units of the nuclear magneton μ_N.

$$\mu_N = \frac{e\hbar}{2m_p}$$

- The divisor in calculating μ_N is the proton mass m_p, which makes the nuclear magneton some 1836 times smaller than the Bohr magneton.
- The proton magnetic moment is $\mu_p = 2.79\mu_N$.
- The magnetic moment of the electron is $\mu_e = -1.00116\mu_B$.
- The neutron magnetic moment is $\mu_n = -1.91\mu_N$.
- The *nonzero* neutron magnetic moment implies that the neutron has negative and positive internal charge components at different radii.

→ Complex internal *charge distribution*.

Nuclear Magnetic Resonance (NMR)

- A widely used medical application using the nuclear magnetic moment's response to large applied magnetic fields.
- Although NMR can be applied to other nuclei that have intrinsic spin, proton NMR is used more than any other kind.

Nuclear magnetic moment

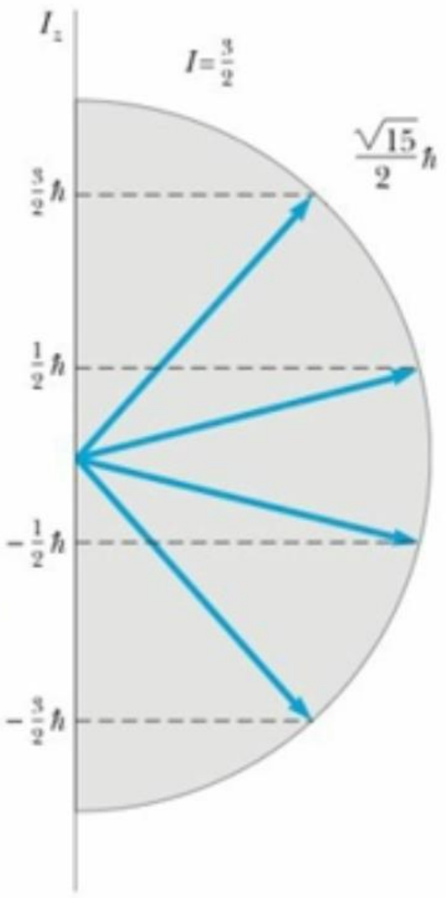


Figure 13.5 The possible orientations of the nuclear angular momentum and its projections along the z-axis for the case $I = \frac{3}{2}$.

$$\mu_n \equiv \frac{e\hbar}{2m_p} = 5.05 \times 10^{-27}\ \text{J/T}$$

Nuclear magnetic resonance and imaging

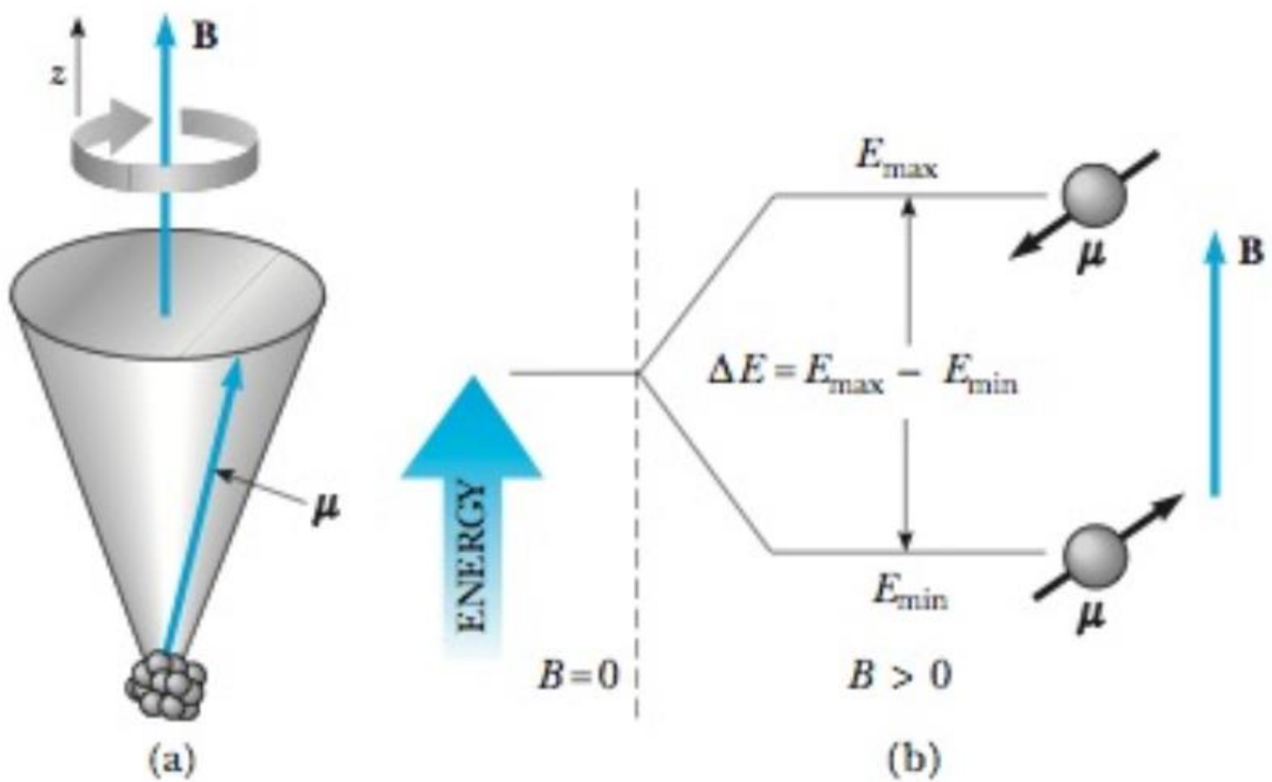
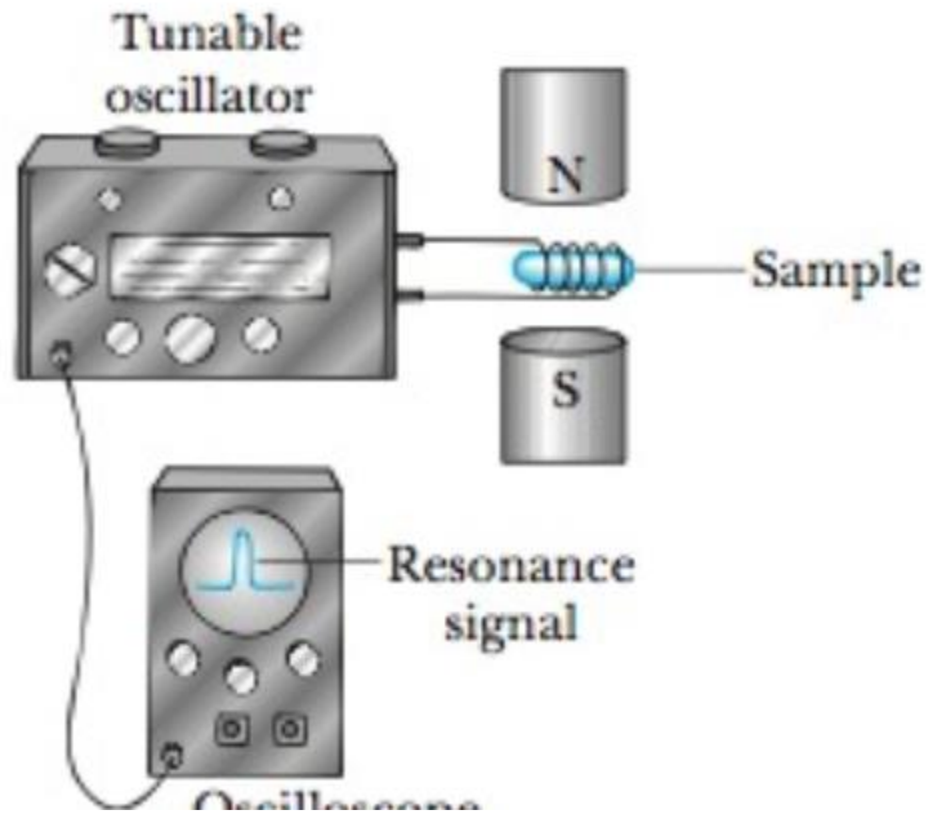


Figure 13.6 (a) When a nucleus is placed in an external magnetic field, **B**, the magnetic moment precesses about the magnetic field with a frequency proportional to the field. (b) A nucleus with spin $\frac{1}{2}$ can occupy one of two energy states when placed in an external magnetic field. The lower energy state E_{min} corresponds to the case where the spin is aligned with the field as much as possible according to quantum mechanics, and the higher energy state E_{max} corresponds to the case where the spin is opposite the field as much as possible.

NMR apparatus



Compare NMR with X-rays

$$\frac{eh/2\pi}{2m_e} = \mu_B = 9.27 \times 10^{-24} \frac{J}{T} \frac{1eV}{1.6 \times 10^{-19} J} = 5.79 \times 10^{-5} \frac{eV}{T}$$

$$u_N = \frac{5.79 \times 10^{-5}}{1836} = 3.15 \times \frac{10^{-8} eV}{T} \qquad \mu_P = 2.79 u_N$$

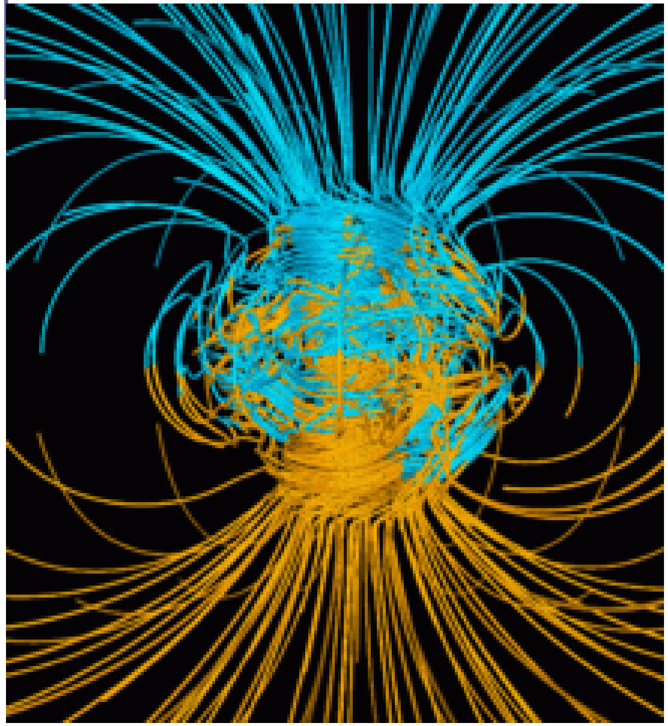
NMR at magnetic field of 2T

Find the energy difference between the two nuclear spin orientation

$$\Delta V_B = 2\mu_P B = 2 \times 2.79 \times 3.15 \times 10^{-8} \frac{eV}{T} \times 2T = 3.5 \times 10^{-7} eV$$

no damage to tissue

X rays =1keV or more is about 10^{10} larger energy and can seriously damage cells



Earth's magnetic field, also known as the geomagnetic field, is the magnetic field that extends from the Earth's interior out into space, where it meets the solar wind, a stream of charged particles emanating from the Sun. Its magnitude at the Earth's surface ranges from 25 to 65 microteslas (0.25 to **0.65 gauss**).

- Earth's magnetic field - Wikipedia

12.3: The Deuteron

- The determination of how the neutron and proton are bound together in a deuteron.
- The deuteron mass = 2.013553 u
- The mass of a deuteron atom = 2.014102 u
- The difference = 0.000549 u; $\longrightarrow$ the mass of an electron
- The deuteron nucleus is bound by a mass-energy B_d
- The mass of a deuteron is

$$m_d = m_p + m_n - B_d / c^2$$

- Add an electron mass to each side of Eq. (12.6)

$$m_d + m_e = m_p + m_n + m_e - B_d / c^2$$

The Deuteron

- $m_d + m_e$ is the atomic deuterium mass $M(^2\text{H})$ and $m_p + m_e$ is the atomic hydrogen mass. Thus Eq.(12.7) becomes

$$M(^2\text{H}) = m_n + M(^1\text{H}) - B_d / c^2$$

- Because the electron masses cancel in almost all nuclear-mass difference calculations, we use atomic masses rather than nuclear masses.

$$m_n = 1.008665 \text{ u} \qquad \text{Neutron mass}$$
$$M(^1\text{H}) = 1.007825 \text{ u} \qquad \text{Atomic hydrogen mass}$$
$$M(^2\text{H}) = 2.014102 \text{ u} \qquad \text{Atomic deuterium mass}$$
$$B_d / c = m_n + M(^1\text{H}) - M(^2\text{H}) = 0.002388 \text{ u}$$

- Convert this to energy using u = 931.5 MeV / c^2

$$B_d = 0.002388\ c^2 \cdot \text{u}\left(\frac{931.5 \text{ MeV}}{c^2 \cdot \text{u}}\right) = 2.224 \text{ MeV}$$

- Even for heavier nuclei we neglect the electron binding energies (13.6 eV) because the nuclear binding energy (2.2 MeV) is almost one million times greater.

The Deuteron

- The binding energy of any nucleus ${}^{A}_{Z}X$ = the energy required to separate the nucleus into free neutrons and protons.

$$B\left({}^{A}_{Z}X\right)=\left[Nm_n+ZM\left({}^{1}\mathrm{H}\right)-M\left({}^{A}_{Z}X\right)\right]c^2$$

Experimental Determination of Nuclear Binding Energies

- Check the 2.22-MeV binding energy by using a nuclear reaction. We scatter gamma rays from deuteron gas and look for the breakup of a deuteron into a neutron and a proton:

$$\gamma+d\rightarrow n+p$$

- This nuclear reaction is called *photodisintegration* or a *photonuclear reaction*.
- The mass-energy relation is

$$hf+M({}^{2}\mathrm{H})c^2=m_nc^2+M({}^{1}\mathrm{H})c^2+K_n+K_p$$

- where *hf* is the incident photon energy.

K_n and K_p are the neutron and proton kinetic energies.

The Deuteron

- The minimum energy required for the photodisintegration:
- Momentum must be conserved in the reaction (K_n, $K_p \neq 0$)

$$hf_{\min} = B_d \left[1 + \frac{B_d}{2M(^2\text{H})c^2} \right]$$

- Experiment shows that a photon of energy less than 2.22 MeV cannot dissociate a deuteron

$$2.79\mu_\text{N} - 1.91\mu_\text{N} = 0.88\mu_\text{N}$$

Deuteron Spin and Magnetic Moment

- Deuteron's nuclear spin quantum number is 1. This indicates the neutron and proton spins are aligned parallel to each other.
- The nuclear magnetic moment of a deuteron is $0.86\mu_\text{N}$ ≈ the sum of the free proton and neutron $2.79\mu_\text{N} - 1.91\mu_\text{N} = 0.88\mu_\text{N}$.

From chapter12 quiz

The mass of the deuteron

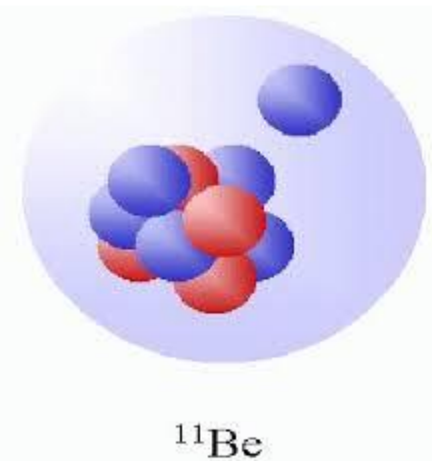
a. is exactly the sum of the neutron and proton mass.

b. is slightly less than the sum of the neutron and proton mass.

c. is exactly 2.000000 u.

d. is exactly the sum of a neutron, proton, and electron mass.

Halo –nucleus (a nuclear hydrogen atom)



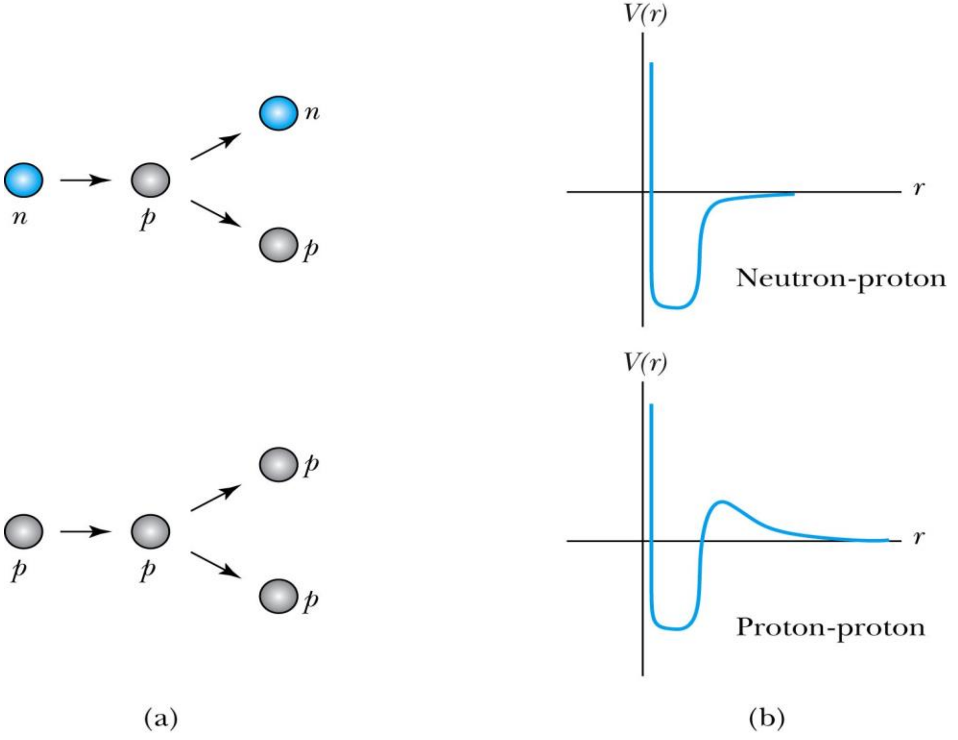
A. Takamine, M. Wada, K. Okada, T. Sonoda, P. Schury, T. Nakamura, Y. Kanai, T. Kubo, I. Katayama, S. Ohtani, H. Wollnik, and H. A. Schuessler
Hyperfine Structure Constant of the Neutron Halo Nucleus 11Be+
Phys. Rev. Lett. 112, 162502 (2014).

The short range strong nuclear force causes the halo neutron to be 7 fm outside the nucleus

half life 13.8 s

12.4: Nuclear Forces

- The angular distribution of neutron classically scattered by protons.
- Neutron + proton (*np*) and proton + proton (*pp*) elastic



The nuclear potential

Nuclear Forces

- The internucleon potential has a “hard core” that prevents the nucleons from approaching each other closer than about 0.4 fm.

- The proton has charge radius up to 1 fm.
- Two nucleons within about 2 fm of each other feel an attractive force.

- The nuclear force (*short range*):
- It falls to zero so abruptly with interparticle separation. $=$ **stable**
- The interior nucleons are completely surrounded by other nucleons with which they interact.

- The only difference between the *np* and *pp* potentials is the Coulomb potential shown for $r \geq 3$ fm for the *pp* force.

Nuclear Forces

- The nuclear force is known to be spin dependent.
- The neutron and proton spins are aligned for the bound state of the deuteron, but there is no bound state with the spins antialigned.

- The *nn* system is more difficult to study because free neutrons are not stable from analyses of experiments.
- The nuclear potential between two nucleons seems independent of their charge (*charge independence of nuclear forces*).

- The term *nucleon* refers to either neutrons or protons because the neutron and proton can be considered different charge states of the same particle.

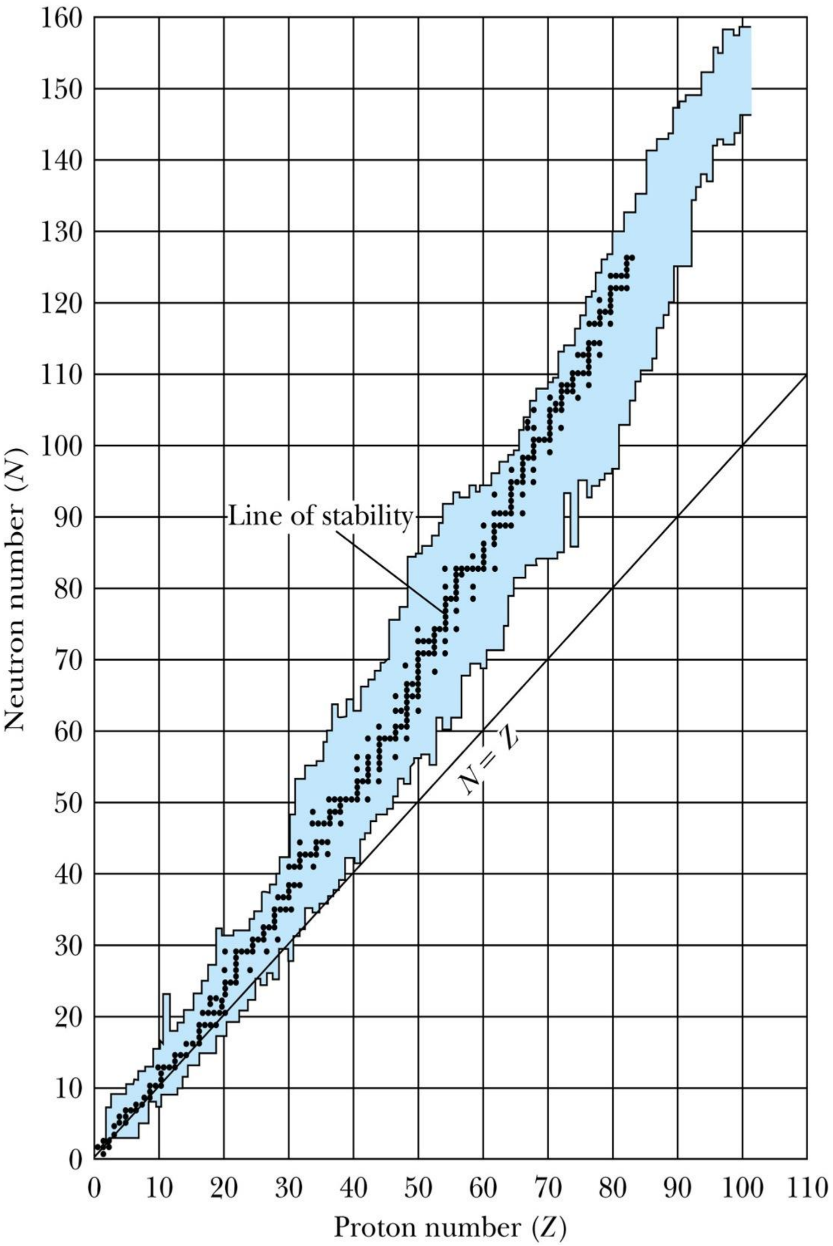
12.5: Nuclear Stability

- The binding energy of a nucleus against dissociation into any other possible combination of nucleons. Ex. nuclei *R* and *S*.

$$B = \left[M(R) + M(S) - M\left({}^{A}_{Z}X \right) \right] c^2$$

- Proton (or neutron) *separation energy*:
  - The energy required to remove one proton (or neutron) from a nuclide.

- All stable and unstable nuclei that are long-lived enough to be observed.



Nuclear Stability

- The line representing the stable nuclides is the **line of stability**.
- It appears that for $A \leq 40$, nature prefers the number of protons and neutrons in the nucleus to be about the same $Z \approx N$.

  However, for $A \geq 40$, there is a decided preference for $N > Z$ because the nuclear force is independent of whether the particles are *nn*, *np*, or *pp*.

- As the number of protons increases, the Coulomb force between all the protons becomes stronger until it eventually affects the binding significantly.

- The work required to bring the charge inside the sphere from infinity is

$$\Delta E_{\text{Coul}} = \frac{3}{5}\frac{(Ze)^2}{4\pi\varepsilon_0 R}$$

Nuclear Stability

- For a single proton,

$$\Delta E_{\text{Coul}} = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R}$$

- The total Coulomb repulsion energy in a nucleus is

$$\Delta E_{\text{Coul}} = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

- For heavy nuclei, the nucleus will have a preference for fewer protons than neutrons because of the large Coulomb repulsion energy.

- Most stable nuclides have both even *Z* and even *N* (even-even nuclides).
- Only four stable nuclides have odd *Z* and odd *N* (odd-odd nuclides).

$^{2}_{1}\text{H}$, $^{6}_{3}\text{Li}$, $^{10}_{5}\text{B}$, and $^{14}_{7}\text{N}$.

From chapter12 quiz

Which of the following statements best describes the line of stability?

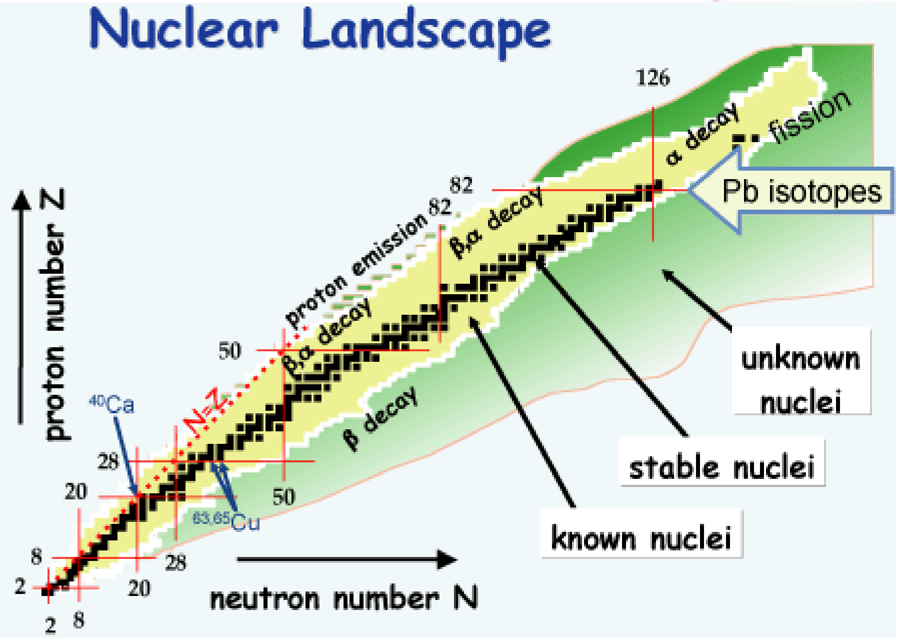
a. It has N = Z when A = 240.

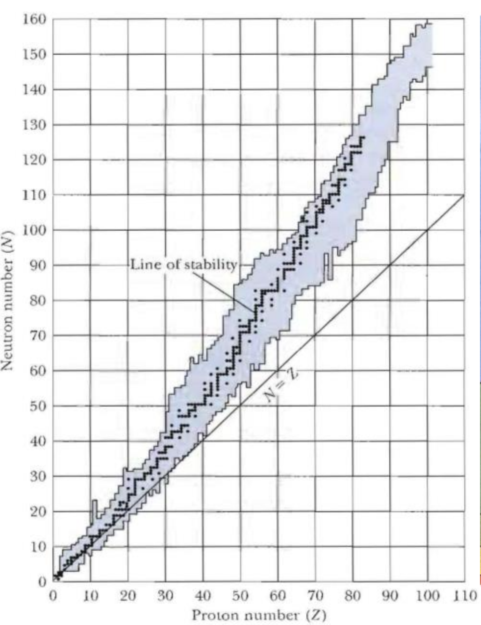
b. It has Z > N at A = 240

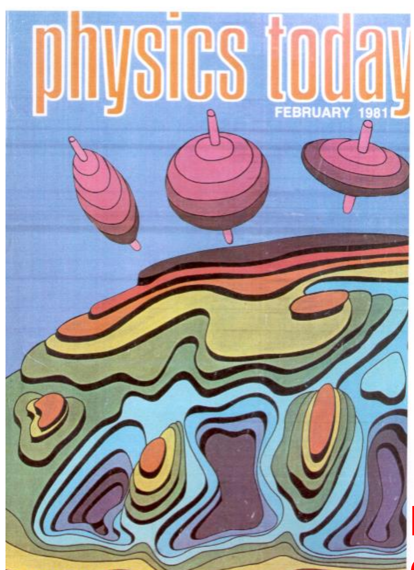
c. N always tends to be greater than Z.

d. N tends to be greater than Z, especially for masses greater than calcium.

The chart of nuclei







Consider the line of stability indicated by solid points

note: (1) for low nucleon numbers up to about $A \leq 40$, nature prefers the number of protons to be about the same $Z \simeq N$

(2) for $A \geq 40$ there is a decided preference for $N > Z$

In the region described by (1) the nuclear force gives most of the attraction and the Coulomb repulsion between the few protons is minimal

In region describe by (2) in addition to the attractive nuclear force also the small Coulomb repulsion between the many protons must be considered

Magic numbers(high stability nuclei) show shell structur
Goeppert-Mayer,Jensen(1963 Nobel price)
N or Z=2,8,20,28,50,82,126

The Liquid Drop Model

- Treats the nucleus as a collection of interacting particles in a liquid drop.
- The total binding energy, the semi-empirical mass formula is

$$B\left({}^{A}_{Z}X\right) = a_V A - a_A A^{2/3} - \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 r} - a_S \frac{(N-Z)^2}{A} + \delta$$

- The volume term (a_v) indicates that the binding energy is approximately the sum of all the interactions between the nucleons.
- The second term is called the *surface effect* because the nucleons on the nuclear surface are not completely surrounded by other nucleons.
- The third term is the Coulomb energy in Eq. (12.17) and Eq. (12.18)

The Liquid Drop Model

- The fourth term is due to the symmetry energy. In the absence of Coulomb forces, the nucleus prefers to have $N \approx Z$ and has a quantum-mechanical origin, depending on the exclusion principle.
- The last term is due to the pairing energy and reflects the fact that the nucleus is more stable for even-even nuclides. Use values given by Fermi to determine this term.

$$a_V = 14 \text{ MeV} \qquad \text{Volume}$$

$$a_A = 13 \text{ MeV} \qquad \text{Surface}$$

$$a_S = 19 \text{ MeV} \qquad \text{Symmetry}$$

$$\text{Pairing } \delta = \begin{cases} +\Delta & \text{for even-even nuclei} \\ 0 & \text{for odd-}A\text{ (even-odd, odd-even) nuclei} \\ -\Delta & \text{for odd-odd nuclei} \end{cases}$$

where $\Delta = 33 \text{ MeV}\cdot A^{-3/4}$

- No nuclide heavier than $^{238}_{92}\text{U}$ has been found in nature. If they ever existed, they must have decayed so quickly that quantities sufficient to measure no longer exist.

From chapter12 quiz

The "liquid drop model" of the nucleus allowed von Weizsaecker to propose his equation for a semi-empirical mass formula. This formula includes all of the following EXCEPT:

a. A correction for nuclear surface interactions being different than interior saturated interactions.

b. A term providing for the repulsion of protons in the nucleus.

c. A term proportional to the total number of nucleons.

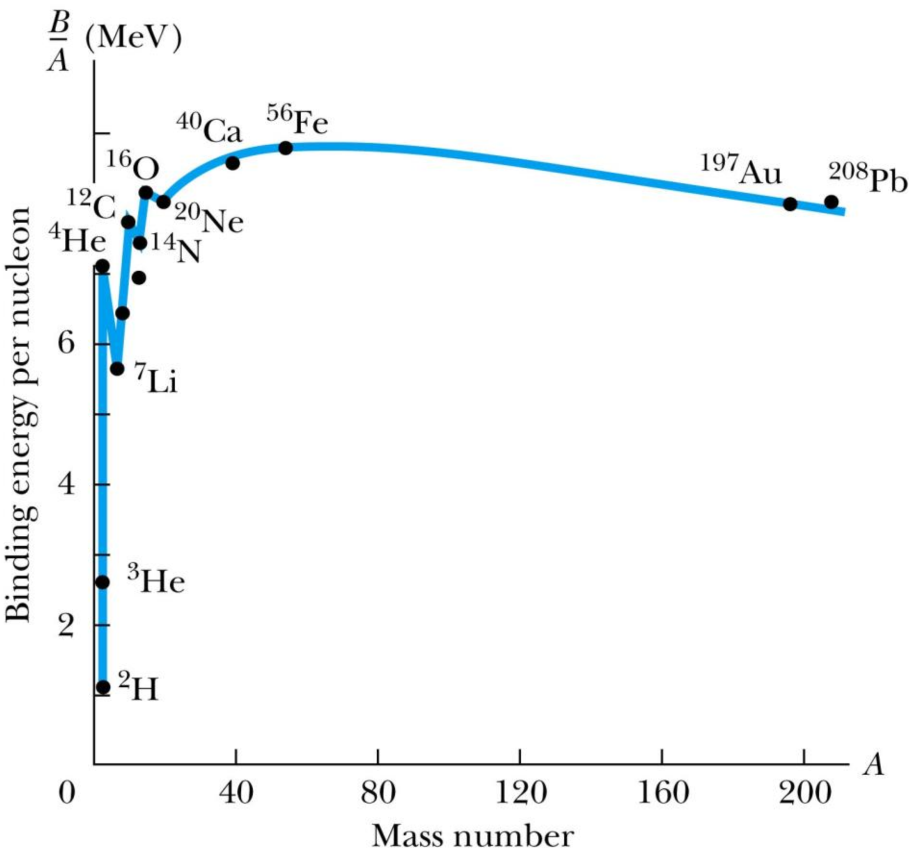
d. A term for the energy associated with the fact that most stable nuclei prefer to have N approximately equal to Z.

e. A term incorporating the instability of protons within the nucleus.

Binding Energy Per Nucleon

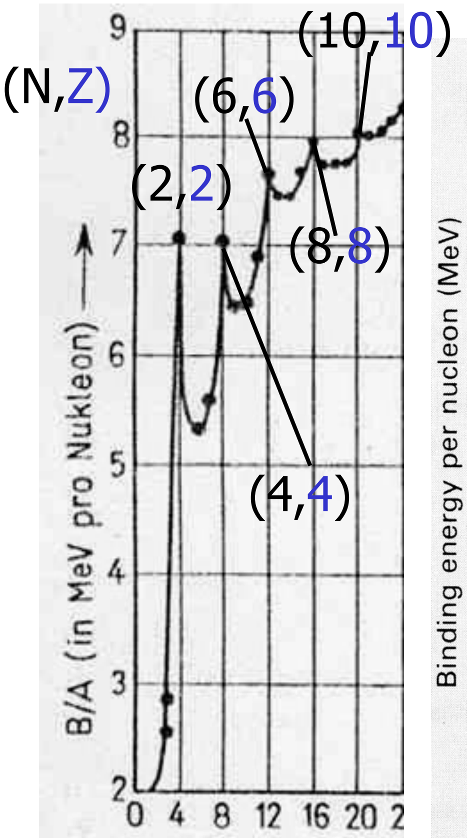
- Use this to compare the relative stability of different nuclides
- It peaks near A = 56
- The curve increases rapidly, demonstrating the saturation effect of nuclear force
- Sharp peaks for the even-even nuclides ^{4}He, ^{12}C, and ^{16}O

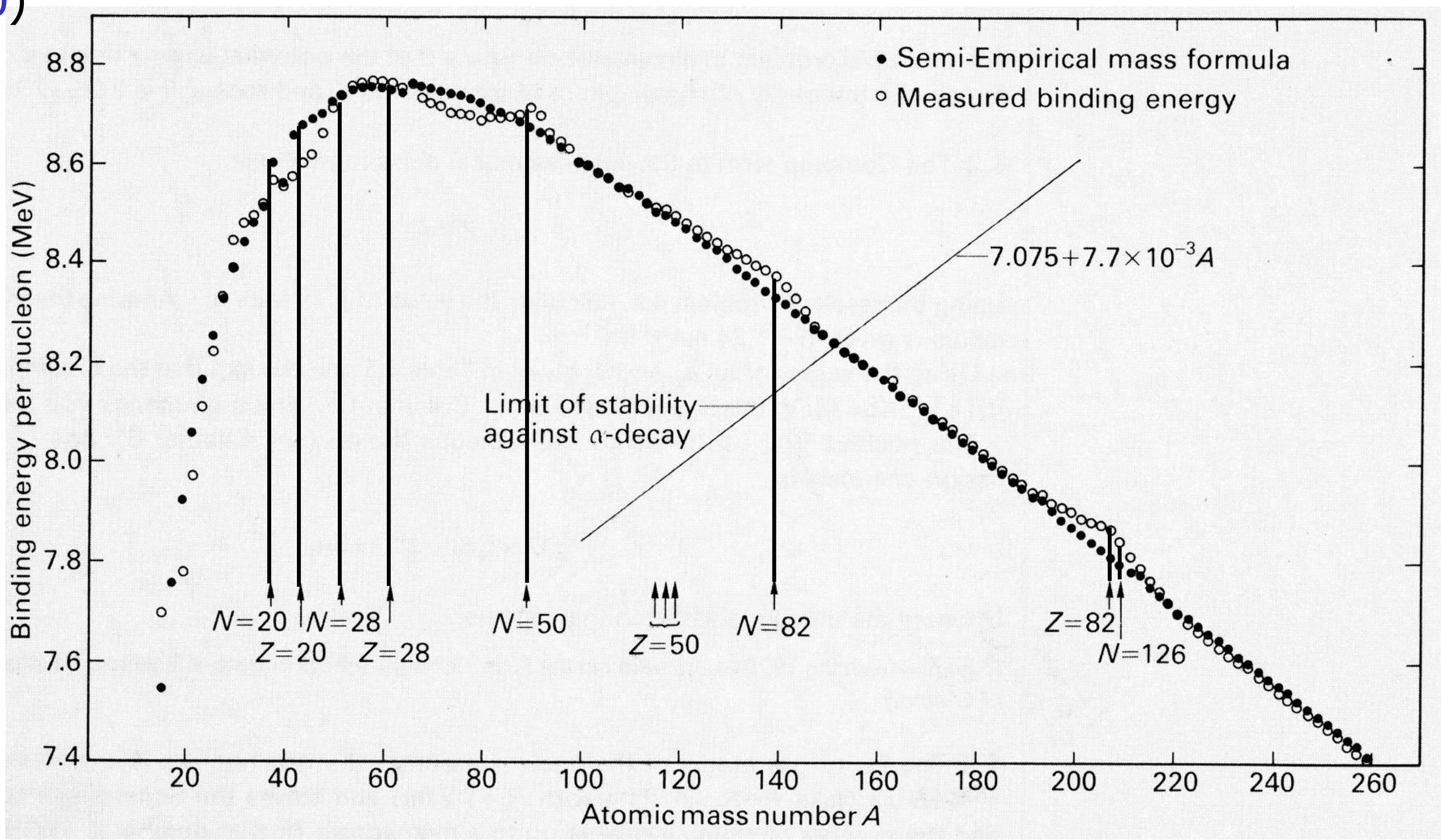
  $=$ tightly bound



Shortcomings of the Liquid Drop Model

- Simple model does not show the peaks of particular stable nuclei for A < 20





Example 12.7

Calculate the binding energy per nucleon for $^{20}_{10}\text{Ne}$, $^{56}_{26}\text{Fe}$, and $^{238}_{92}\text{U}$.

Strategy We first find the binding energy of each of these nuclides using Equation (12.10) and then divide by the mass number to obtain the binding energy per nucleon.

Solution

$$B(^{20}_{10}\text{Ne}) = [10m_n + 10M(^1\text{H}) - M(^{20}_{10}\text{Ne})]c^2$$
$$= [10(1.008665\text{ u}) + 10(1.007825\text{ u})$$
$$- 19.992440\text{ u}]c^2\left(\frac{931.5\text{ MeV}}{c^2\cdot\text{u}}\right)$$
$$= 161\text{ MeV}$$

$$\frac{B(^{20}_{10}\text{Ne})}{20\text{ nucleons}} = 8.03\text{ MeV/nucleon}$$

$$B(^{56}_{26}\text{Fe}) = [30m_n + 26M(^1\text{H}) - M(^{56}_{26}\text{Fe})]c^2$$
$$= [30(1.008665\text{ u}) + 26(1.007825\text{ u})$$
$$- 55.934942\text{ u}]c^2\left(\frac{931.5\text{ MeV}}{c^2\cdot\text{u}}\right)$$
$$= 492\text{ MeV}$$

$$\frac{B(^{56}_{26}\text{Fe})}{56\text{ nucleons}} = 8.79\text{ MeV/nucleon}$$

$$B(^{238}_{92}\text{U}) = [146m_n + 92M(^1\text{H}) - M(^{238}_{92}\text{U})]c^2$$
$$= [146(1.008665\text{ u}) + 92(1.007825\text{ u})$$
$$- 238.050783\text{ u}]c^2\left(\frac{931.5\text{ MeV}}{c^2\cdot\text{u}}\right)$$
$$= 1800\text{ MeV}$$

$$\frac{B(^{238}_{92}\text{U})}{238\text{ nucleons}} = 7.57\text{ MeV/nucleon}$$

All three nuclides have a binding energy per nucleon near 8 MeV, with ^{56}Fe having the largest binding energy per nucleon, as shown in Figure 12.6.

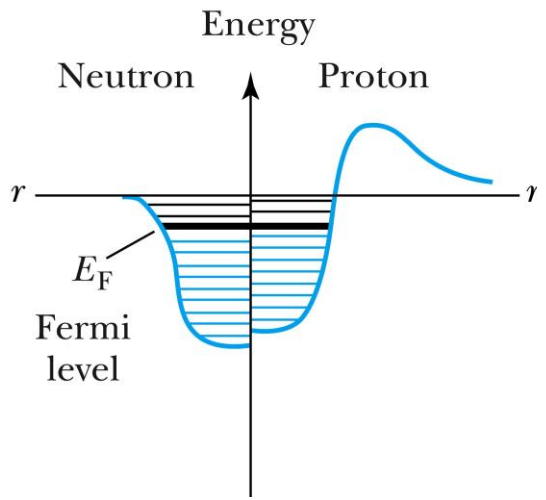
Nuclear Models

- Current research focuses on the constituent quarks and physicists have relied on a multitude of models to explain nuclear force behavior.

1) **Independent-particle models**:
   The nucleons move nearly independently in a common nuclear potential. The shell model has been the most successful of these.
2) **Strong-interaction models**:
   The nucleons are strongly coupled together. The liquid drop model has been successful in explaining nuclear masses as well as nuclear fission.

Nuclear Shell Model

The nuclear potential felt by the neutron and the proton



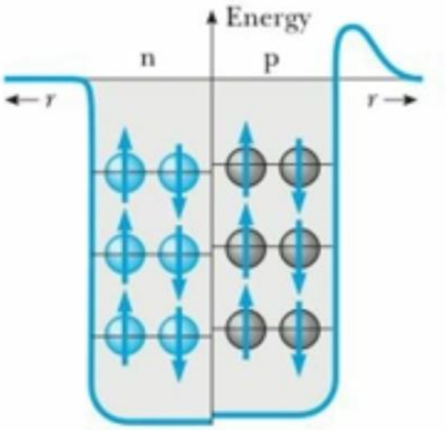
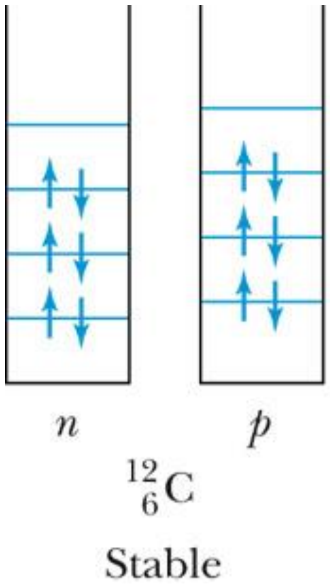


Figure 13.13 A square-well potential containing 12 nucleons. The gray circles represent protons, and the colored circles represent neutrons. The energy levels for the protons are slightly higher than those for the neutrons because of the Coulomb potential in the case of the protons. The difference in the levels increases as Z increases. Note that only two nucleons with opposite spin can occupy a given level, as required by the Pauli exclusion principle.

- The difference of the shape between the proton and the neutron potentials are due to the Coulomb interaction on the proton.
- Nuclei have a Fermi energy level which is the highest energy level filled in the nucleus.
- In the ground state of a nucleus, all the energy levels below the Fermi level are filled.

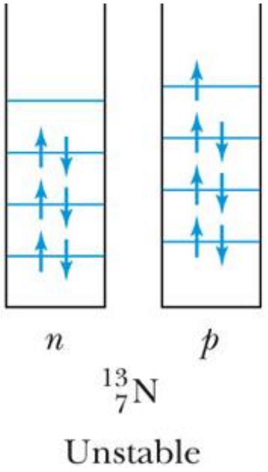
Nuclear Models

- Energy-level diagrams for ^{12}C and ^{16}O.



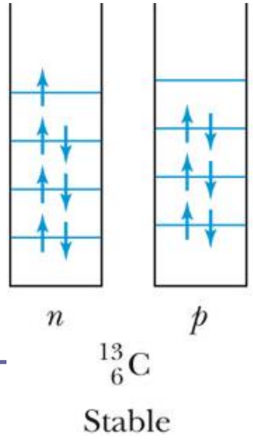
- Both are stable because they are even-even.

Case 1: If we add one proton to ^{12}C to make $^{13}_{7}N$



unstable

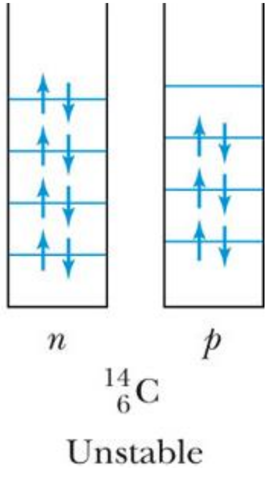
Case 2: If we add one neutron to ^{12}C to make ^{13}C:



stable

Nuclear Shell Model

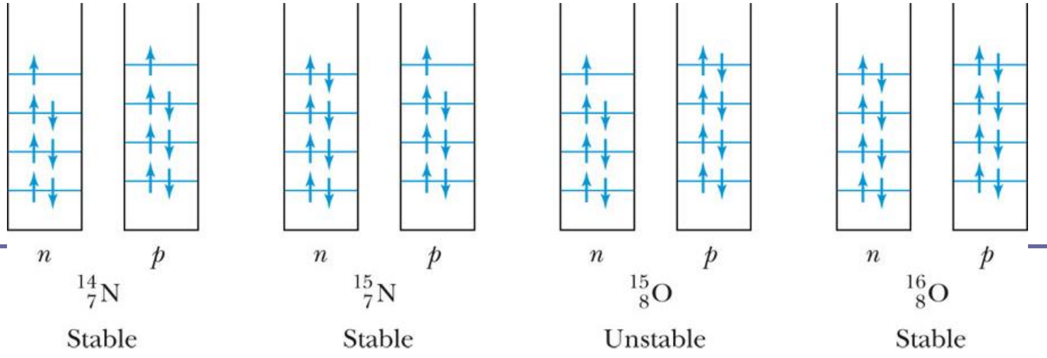
- Even when we add another neutron to produce ^{14}C, we find it is barely unstable.



- In this mass region, nature prefers the number of neutrons and protons to be $N \approx Z$, but it doesn't want $N < Z$.

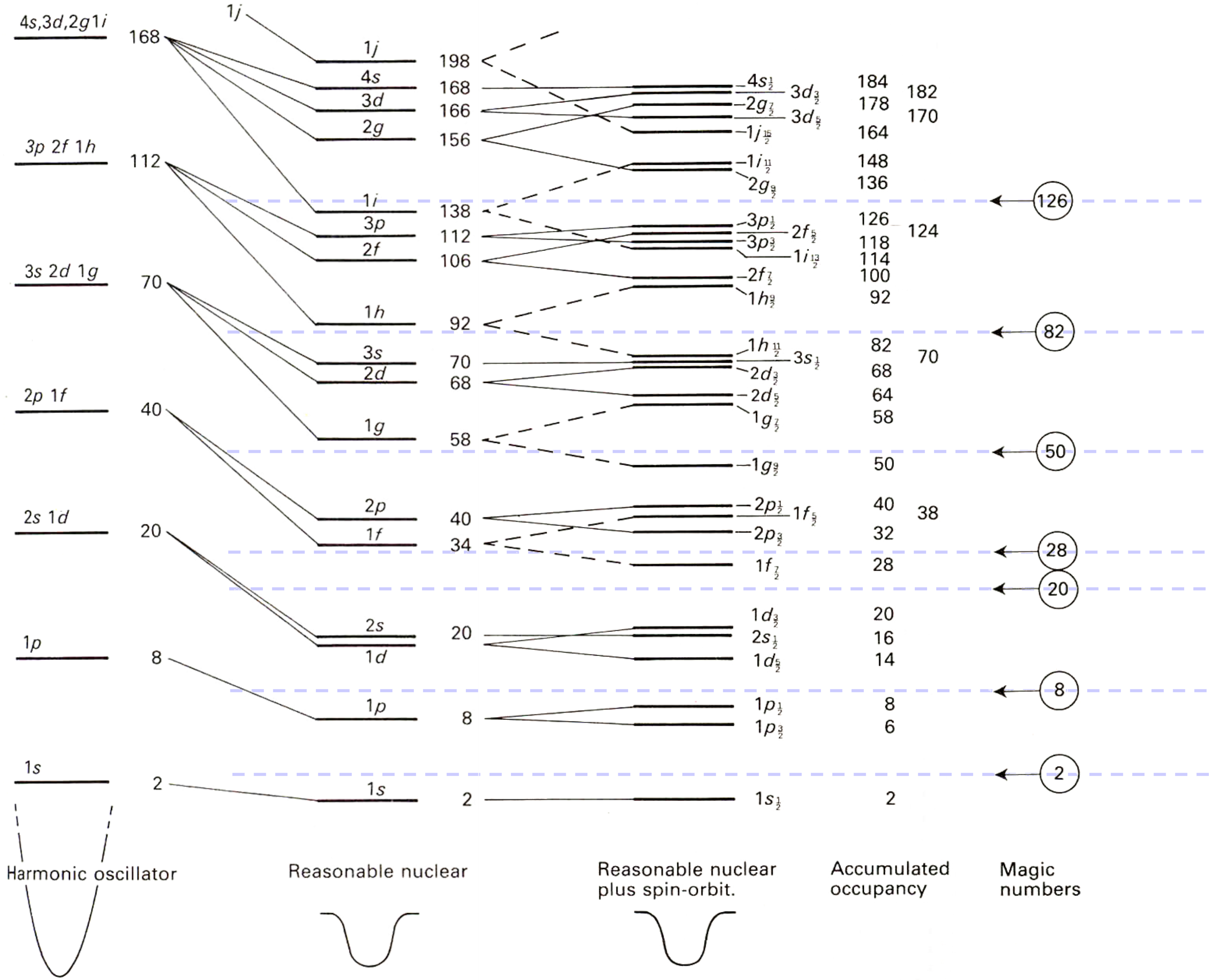
This helps explain why ^{13}C is stable, but not ^{13}N

- Indicating neutron energy levels to be lower in energy than the corresponding proton ones.



Nuclear shell model with well defined orbital states

(each nucleon moves in the average field of all other nucleons)



The Nobel Prize in Physics 1963.

Nuclear Shell Model

Magic numbers(high stability nuclei) show shell structure Goeppert-Mayer,Jensen(1963 Nobel price)
N or Z=2,8,20,28,50,82,126

Maria **Goeppert Mayer**

J. **Hans D. Jensen**

12.6: Radioactive Decay

- The discoverers of radioactivity were Wilhelm Röntgen, Henri Becquerel, Marie Curie and her husband Pierre.
- Marie Curie and her husband Pierre discovered polonium and radium in 1898.
  - The simplest decay form is that of a gamma ray, which represents the nucleus changing from an excited state to lower energy state.
  - Other modes of decay include emission of α particles, *β* particles, protons, neutrons, and fission.

- The disintegrations or decays per unit time (**activity**):

$$\text{Activity} = -\frac{dN}{dt} = R$$

where *dN* / *dt* is negative because total number *N* decreases with time.

Radioactive Decay

- SI unit of activity is the becquerel: 1 Bq = 1 decay / s
- Recent use is the Curie (Ci) 3.7×10^{10} decays / s

- If N(t) is the number of radioactive nuclei in a sample at time *t*, and λ (**decay constant**) is the probability per unit time that any given nucleus will decay:

$$R = \lambda N(t)$$

$$dN(t) = -R\,dt = -\lambda N(t)\,dt$$

$$\int \frac{dN}{N} = -\int \lambda\,dt$$

$$\ln N = -\lambda t + \text{constant}$$

$$N(t) = e^{-\lambda t + \text{constant}}$$

- If we let $N(t = 0) \equiv N_0$

$$N(t) = N_0 e^{-\lambda t}$$ ----- **radioactive decay law**

Radioactive Decay

- The activity R is

$$R = \lambda N(t) = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

where R_0 is the initial activity at $t = 0$

- It is common to refer to the half-life $t_{1/2}$ or the mean lifetime τ rather than its decay constant.

$$N(t_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

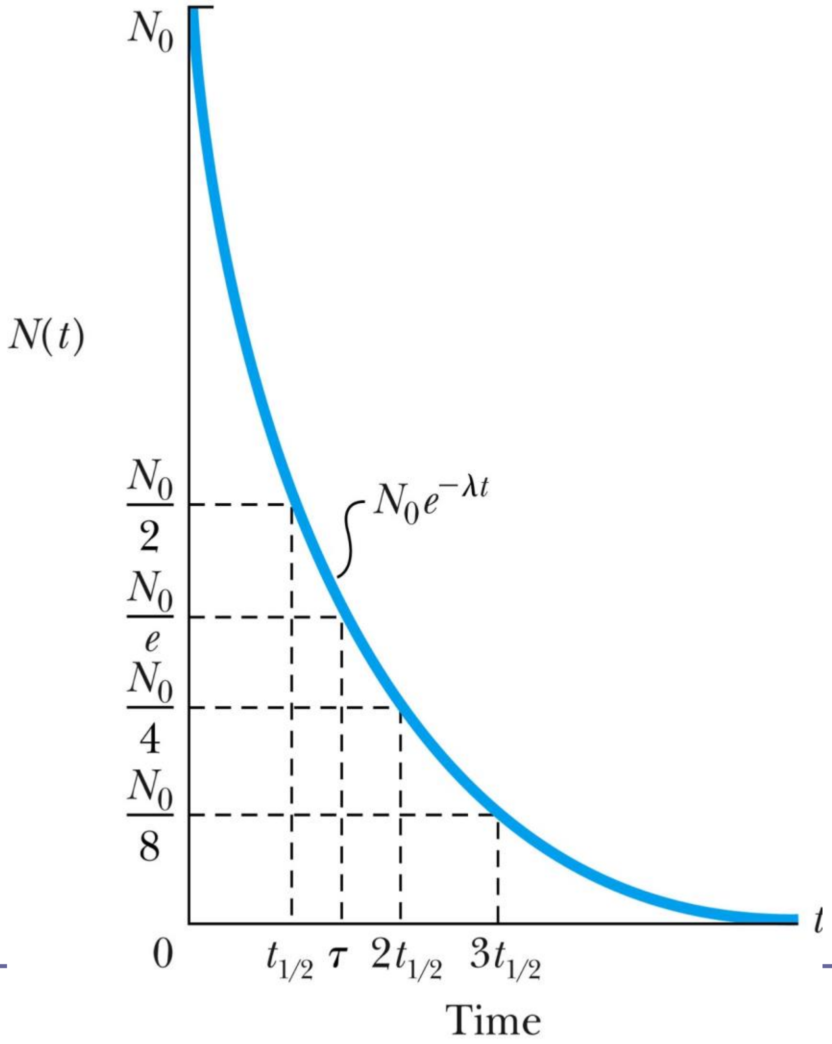
$$\ln\left(\tfrac{1}{2}\right) = \ln(e^{-\lambda t_{1/2}}) = -\lambda t_{1/2}$$

- The half-life is $t_{1/2} = \dfrac{-\ln(1/2)}{\lambda} = \dfrac{\ln(2)}{\lambda} = \dfrac{0.693}{\lambda}$

- The mean lifetime is $\tau = \dfrac{1}{\lambda} = \dfrac{t_{1/2}}{\ln(2)}$

Radioactive Decay

- The number of radioactive nuclei as a function of time



Euler's number
e=2.71828..

12.7: Alpha, Beta, and Gamma Decay

When a nucleus decays, all the conservation laws must be observed:

- Mass-energy
- Linear momentum
- Angular momentum
- Electric charge
- **Conservation of nucleons**
  - The total number of nucleons (*A*, the mass number) must be conserved in a low-energy nuclear reaction or decay.

Alpha, Beta, and Gamma Decay

- Let the radioactive nucleus ${}^{A}_{Z}X$ be called the parent and have the mass $M\left({}^{A}_{Z}X\right)$
- Two or more products can be produced in the decay.
- Let the lighter one be M_y and the mass of the heavier one (*daughter*) be M_D.
- The conservation of energy is $M\left({}^{A}_{Z}X\right) = M_D + M_y + Q/c^2$

  where Q is the energy released (**disintegration energy**) and equal to the total kinetic energy of the reaction products(note:Q(disintegration) is the negative of B(binding)
- If $B > 0$, a nuclide is bound and stable; $Q = \left[M\left({}^{A}_{Z}X\right) - M_D - M_y\right]c^2$
- If $Q > 0$, a nuclide is unbound, unstable, and may decay
- If $Q < 0$, decay emitting nucleons do not occur

Alpha Decay

- The nucleus ^{4}He has a binding energy of 28.3 MeV.
- If the last two protons and two neutrons in a nucleus are bound by less than 28.3 MeV, then the emission of an alpha particle (alpha decay) is possible.

$$^{230}_{92}\mathrm{U} \rightarrow \alpha + ^{226}_{90}\mathrm{Th}$$

$$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}D + \alpha$$

- If Q > 0, alpha decay is possible

EX. $$Q = \left[M\left(^{A}_{Z}X\right) - M\left(^{A-4}_{Z-2}D\right) - M(^4\mathrm{He})\right]c^2$$

The appropriate masses are

$$M\left(^{230}_{92}\mathrm{U}\right) = 230.033927\ \mathrm{u};\ M(^4\mathrm{He}) = 4.002603\ \mathrm{u};\ M(^{226}_{90}\mathrm{Th}) = 226.024891\ \mathrm{u}$$

Q= 6 MeV and alpha decay is possible

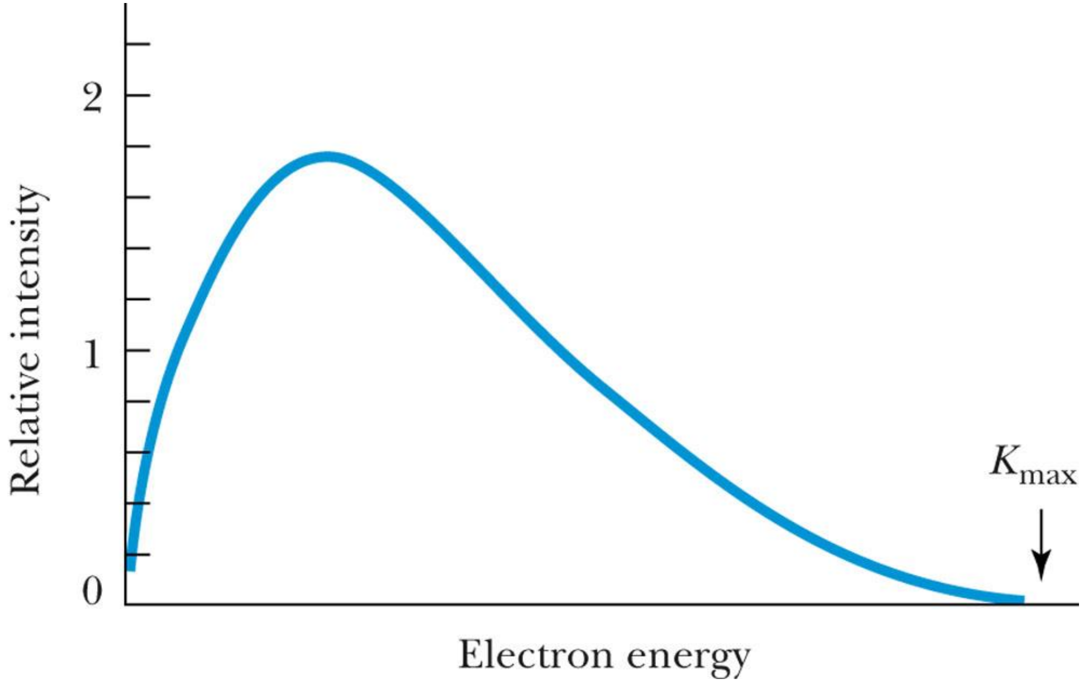
Beta Decay

- Unstable nuclei may move closer to the line of stability by undergoing beta decay.
- The decay of a free neutron is

$$n \rightarrow p + \beta^-$$

- The beta decay of ^{14}C (unstable) to form ^{14}N, a stable nucleus, can be written as

$$^{14}_{6}C \rightarrow ^{14}_{7}N + \beta^-$$



The electron energy spectrum from the beta decay

Beta Decay

- There was a problem in neutron decay, the spin ½ neutron cannot decay to two spin ½ particles, a proton and an electron. ^{14}C has spin 0, ^{14}N has spin 1, and the electron has spin ½.

⟶ we cannot combine spin ½ & 1 to obtain a spin 0.

- Wolfgang Pauli suggested a **neutrino** ν that must be produced in beta decay. It has spin quantum number ½, charge 0, and carries away the additional energy missing in Fig. (12.15).

Beta Decay

- An occasional electron is detected with the kinetic energy K_{max} required to conserve energy, but in most cases the electron's kinetic energy is less than K_{max}.

→ the neutrino has little or no mass, and its energy may be all kinetic

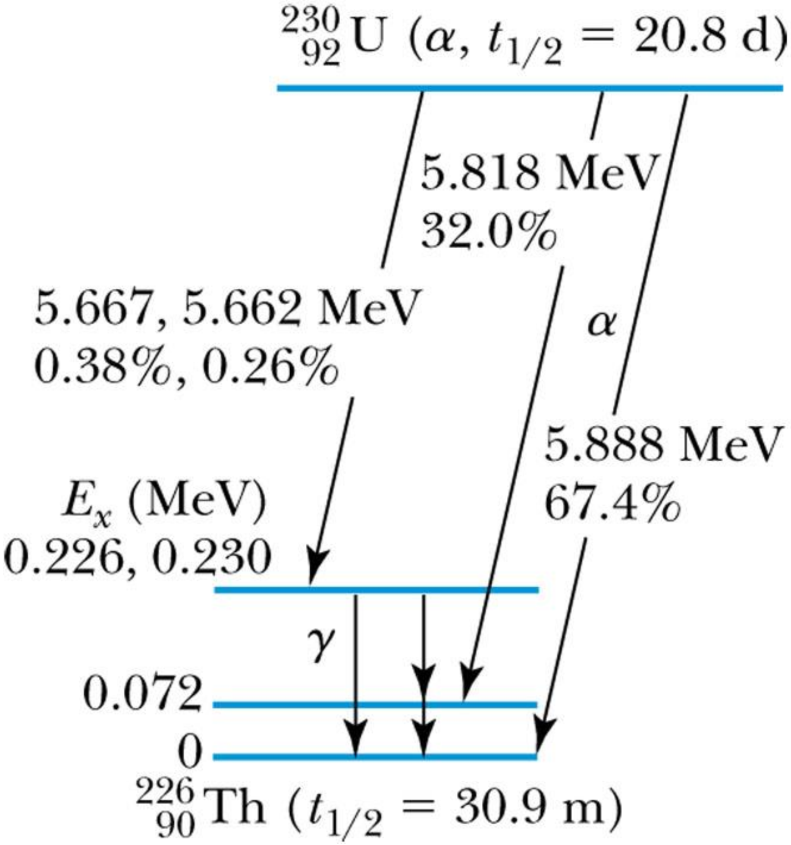
- Neutrinos have no charge and do not interact *electromagnetically*.
- They are not affected by the *strong* force of the nucleus.
- They are the *weak* interaction.
- The electromagnetic and weak forces are the *electroweak* force.

Gamma Decay

- If the decay proceeds to an excited state of energy E_x rather than to the ground state, then Q for the transition to the excited state can be determined with respect to the transition to the ground state. The disintegration energy Q to the ground state Q_0.

- Q for a transition to the excited state E_x is

$$Q = Q_0 - E_x$$



Gamma Decay

- The excitation energies tend to be much larger, many keV or even MeV.
- The possibilities for the nucleus to rid itself of this extra energy is to emit a photon (gamma ray).
- The gamma-ray energy *hf* is given by the difference of the higher energy state $E_>$ and lower one $E_<$.

$$hf = E_> - E_<$$

- The decay of an excited state of $^AX^*$ (where * is an excited state) to its ground state is

$$^AX^* \rightarrow {}^AX + \gamma$$

- A transition between two nuclear excited states $E_>$ and $E_<$ is

$$^AX^*(E_>) \rightarrow {}^AX^*(E_<) + \gamma$$

12.8: Radioactive Nuclides

- The unstable nuclei found in nature exhibit natural radioactivity.

Table 12.2 Some Naturally Occurring Radioactive Nuclides

Nuclide	$t_{1/2}$ (y)	Natural Abundance
$^{40}_{19}K$	1.28×10^{9}	0.01%
$^{87}_{37}Rb$	4.8×10^{10}	27.8%
$^{113}_{48}Cd$	9×10^{15}	12.2%
$^{115}_{49}In$	4.4×10^{14}	95.7%
$^{128}_{52}Te$	7.7×10^{24}	31.7%
$^{130}_{52}Te$	2.7×10^{21}	33.8%
$^{138}_{57}La$	1.1×10^{11}	0.09%
$^{144}_{60}Nd$	2.3×10^{15}	23.8%
$^{147}_{62}Sm$	1.1×10^{11}	15.0%
$^{148}_{62}Sm$	7×10^{15}	11.3%

Big Bang was 13.7 billion years ago

3.154^+7 s/y

Radioactive Nuclides

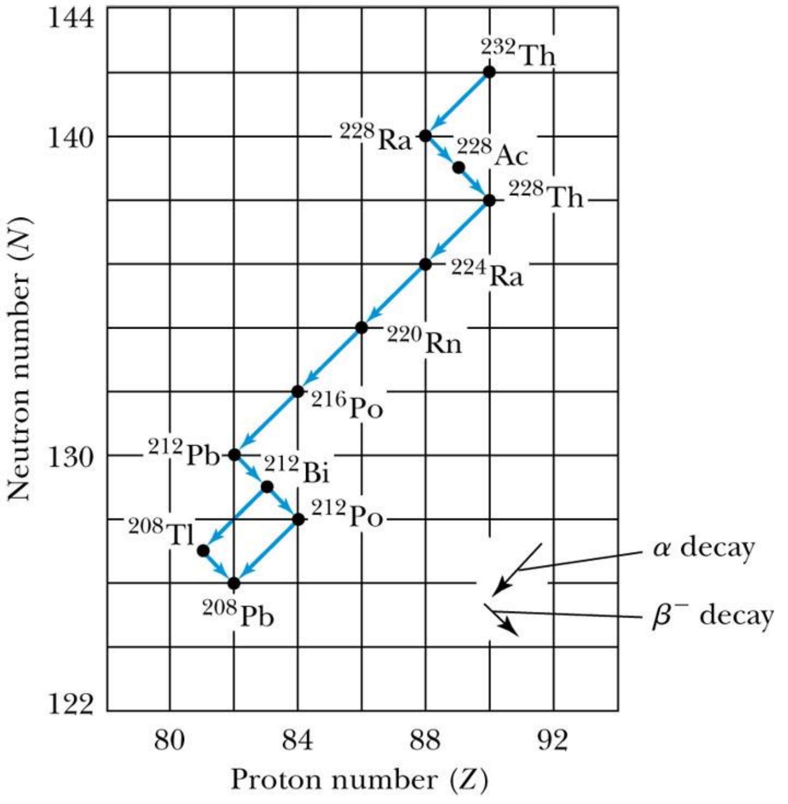
- The radioactive nuclides made in the laboratory exhibit artificial radioactivity.
- Heavy radioactive nuclides can change their mass number only by alpha decay ($^{A}X \rightarrow {}^{A-4}D$) but can change their charge number Z by either alpha or beta decay.
- There are only four paths that the heavy naturally occurring radioactive nuclides may take as they decay.
- Mass numbers expressed by either:
  - $4n$
  - $4n + 1$
  - $4n + 2$
  - $4n + 3$

Table 12.3 The Four Radioactive Series

Mass Numbers	Series Name	Parent	$t_{1/2}$ (y)	End Product
$4n$	Thorium	$^{232}_{90}Th$	1.40×10^{10}	$^{208}_{82}Pb$
$4n + 1$	Neptunium	$^{237}_{93}Np$	2.14×10^{6}	$^{209}_{83}Bi$
$4n + 2$	Uranium	$^{238}_{92}U$	4.47×10^{9}	$^{206}_{82}Pb$
$4n + 3$	Actinium	$^{235}_{92}U$	7.04×10^{8}	$^{207}_{82}Pb$

Radioactive Nuclides

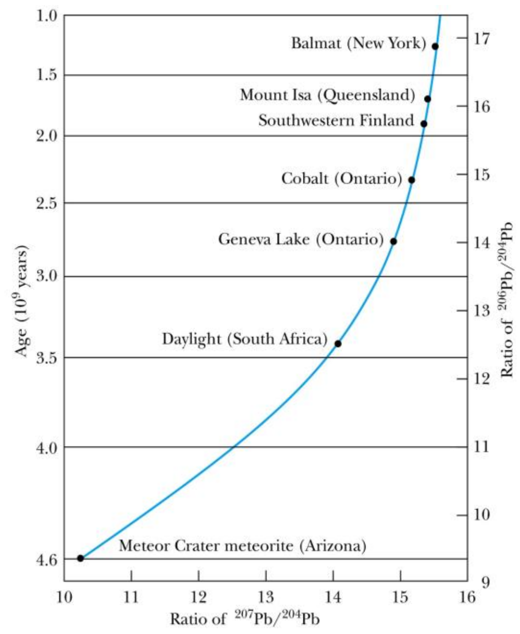
- The sequence of one of the radioactive series ^{232}Th



- ^{212}Bi can decay by either alpha or beta decay (*branching*).

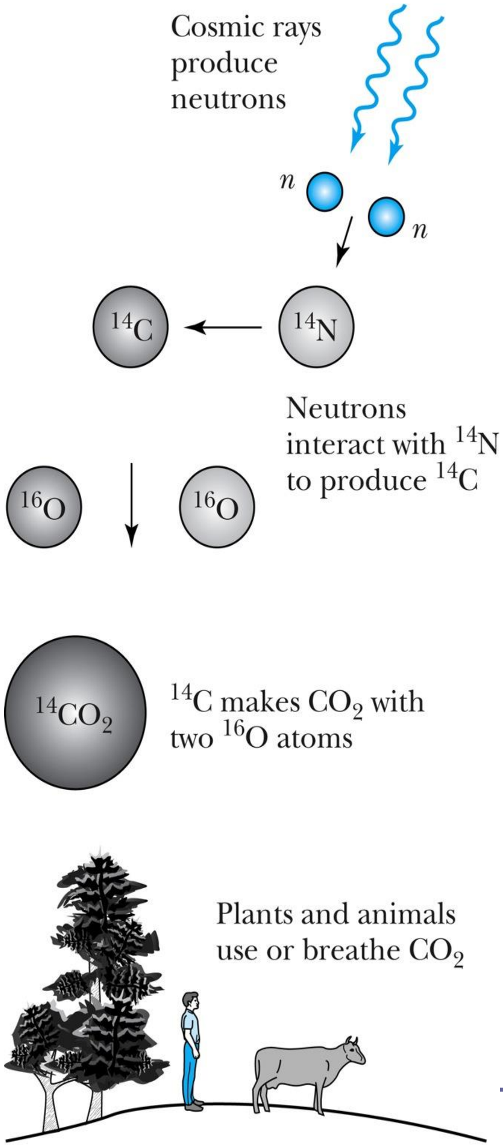
Time Dating Using Lead Isotopes

- A plot of the abundance ratio of ^{206}Pb / ^{204}Pb versus ^{207}Pb / ^{204}Pb can be a sensitive indicator of the age of lead ores. Such techniques have been used to show that meteorites, believed to be left over from the formation of the solar system, are 4.55 billion years old.
- The growth curve for lead ores from various deposits:



The age of the specimens can be obtained from the abundance ratio of $^{206}Pb/^{204}Pb$ versus $^{207}Pb/^{204}Pb$.

Radioactive Carbon Dating



When an organism dies, the ratio of $^{14}C/^{12}C$ decreases.

- Radioactive ^{14}C is produced in our atmosphere by the bombardment of ^{14}N by neutrons produced by cosmic rays.

$$n + {}^{14}N \rightarrow {}^{14}C + p$$

- When living organisms die, their intake of ^{14}C ceases, and the ratio of ^{14}C / ^{12}C (= R) decreases as ^{14}C decays. The period just before 9000 years ago had a higher ^{14}C / ^{12}C ratio by factor of about 1.5 than it does today.
- Because the half-life of ^{14}C is 5730 years, it is convenient to use the ^{14}C / ^{12}C ratio to determine the age of objects over a range up to 45,000 years ago.

Calculate the binding energies of the most loosely bound neutron in the following nuclei

17. (a) Think of the nucleus as the composite of ${}^{A-1}_{Z}X$ and ${}^{1}n$, so that

$$B = \left[M\left({}^{A-1}_{Z}X\right) + m_n - M\left({}^{A}_{Z}X\right) \right] c^2$$

(b) Details for ${}^{6}\mathrm{Li}$ are shown. The other examples are similar. Use the atomic masses from Appendix 8:

$$B = \left[M\left({}^{5}\mathrm{Li}\right) + m_n - M\left({}^{6}\mathrm{Li}\right) \right] c^2$$

$$= (5.012540\ \mathrm{u} + 1.008665\ \mathrm{u} - 6.015122\ \mathrm{u})c^2\left(931.49\ \mathrm{MeV}/\left(\mathrm{u}\cdot c^2\right)\right) = 5.67\ \mathrm{MeV}$$

${}^{16}\mathrm{O}$: $B = \left[M\left({}^{15}\mathrm{O}\right) + m_n - M\left({}^{16}\mathrm{O}\right) \right] c^2 = 15.7\ \mathrm{MeV}$

${}^{207}\mathrm{Pb}$: $B = \left[M\left({}^{206}\mathrm{Pb}\right) + m_n - M\left({}^{207}\mathrm{Pb}\right) \right] c^2 = 6.74\ \mathrm{MeV}$

What is the energy released when three alpha particles combine to form 12C?

19. The energy release comes from the mass difference:

$$\Delta E = \Delta mc^2 = \left[3\,M\left({}^{4}\text{He}\right) - M\left({}^{12}\text{C}\right)\right]c^2$$

$$= \left[3\left(4.002603\ \text{u}\right) - 12.000\ \text{u}\right]c^2\left(931.49\ \text{MeV}/\left(\text{u}\cdot c^2\right)\right) = 7.27\ \text{MeV}$$

A radioactive sample of ^{60}Co ($t_{1/2} = 5.271$ y) has a β^- activity of 4.4×10^7 Bq. How many grams of ^{60}Co are present?

27. $$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(5.271\ \text{y})(3.156\times10^7\ \text{s/y})} = 4.167\times10^{-9}\ \text{s}^{-1}$$

$$N = \frac{R}{\lambda} = \frac{4.4\times10^7\ \text{s}^{-1}}{4.167\times10^{-9}\ \text{s}^{-1}} = 1.06\times10^{16};$$

$$m = (1.06\times10^{16})\frac{1\ \text{mol}}{6.022\times10^{23}}\left(\frac{60\ \text{g}}{\text{mol}}\right) = 1.05\ \mu\text{g}$$

An unknown radioactive sample is observed to decrease in activity by a factor of five in a one-hour period. What is its half-life?

28. $R = R_0 e^{-\lambda t} = \dfrac{R_0}{5}$ at $t = T = 3600\ \text{s}$; $\lambda = \dfrac{\ln 5}{T}$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\ln 5} T = \frac{\ln 2}{\ln 5}(3600\ \text{s}) = 1550\ \text{s} \approx 26\,\text{minutes}$$

From chapter12 quiz

Which of the following reasons explains why the neutrino must exist?

A. The neutrino is a product of gamma ray decay.

B. The neutrino is necessary to allow for the correct spin angular momentum conservation in a nuclear disintegration.

C. The neutrino is necessary to carry away a charge in a nuclear disintegration.

D. The neutrino is the force carrier that holds together quarks within protons and neutrons.

E. The neutrino decays into electrons and protons in an unstable nucleus.

EXAMPLE 12.18

Assume that all the ^{206}Pb found in a given sample of uranium ore resulted from decay of ^{238}U and that the ratio of $^{206}Pb/^{238}U$ is 0.60. How old is the ore?

Strategy Let N_0 be the original number of ^{238}U nuclei that existed. The ^{238}U nuclei eventually decay to ^{206}Pb, and the longest time in the radioactive decay chain $^{238}U \rightarrow {}^{206}Pb$ is the half-life of ^{238}U, $t_{1/2} = 4.47 \times 10^9$ y. The numbers of nuclei for ^{238}U and ^{206}Pb are then

$$N(^{238}U) = N_0 e^{-\lambda t}$$

$$N(^{206}Pb) = N_0 - N(^{238}U) = N_0(1 - e^{-\lambda t})$$

The abundance ratio is

$$R' = \frac{N(^{206}Pb)}{N(^{238}U)} = \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} = e^{\lambda t} - 1 \qquad (12.50)$$

We can solve Equation (12.50) for t, because we know experimentally the ratio R' and the decay constant λ for ^{238}U.

Solution The result for t from Equation (12.50) is

$$t = \frac{1}{\lambda}\ln(R' + 1) = \frac{t_{1/2}}{\ln(2)}\ln(R' + 1)$$

$$= \frac{4.47 \times 10^9 \text{ y}}{\ln(2)}\ln(1.60) = 3.0 \times 10^9 \text{ y}$$

If the age of the Earth is 4.5 billion years, what should the ratio of N^206 (Pb)/(N ^238 (U)) in a uranium-bearing rock as old as the Earth?

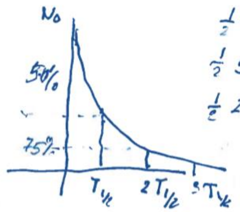
52. $R' = \frac{N\left({}^{206}\text{Pb}\right)}{N\left({}^{238}\text{U}\right)} = e^{\lambda t} - 1 = e^{\ln 2} - 1 = 1$ where the substitution for λ occurs since the time given in the problem almost exactly matches the half-life of U-238. Thus $\lambda t = \left(\ln(2)/t_{1/2}\right)t = \ln(2)$. A more exact answer would be $\lambda t = \left(\ln(2)/4.47\times10^9\right)4.6\times10^9 = 1.03\left[\ln(2)\right]$ and thus the ratio would be $R' = \frac{N\left({}^{206}\text{Pb}\right)}{N\left({}^{238}\text{U}\right)} = e^{\lambda t} - 1 = e^{(1.03)\ln 2} - 1 = 1.04$ revealing a slightly higher amount of lead.

$1\,Bq = 1\,\frac{decay}{s}$ $\quad 1\,Ci = 3.7\times 10^{10}\,\frac{decay}{s}$

modified example 12.9

mass 1.0 mg
$T_{1/2} = 1\,h$ } 50% will decay in 1 h

(a) What fraction decays in 3 h?



$\frac{1}{2}\,100\% = 50\%$

$\frac{1}{2}\,50\% = 25\%$

$\frac{1}{2}\,25\% = 12.5\%$

87.5% or $1\,mg \times 0.875 = 0.875\,mg$

(b) How much is left? ~~1 − 0.875 = 0.125 mg~~

---

^{210}Po has $T_{1/2} = 138$ day, you observe 2000 Bq/s

(a) what is the activity in μCi

$$2000\,\frac{decay}{s}\left(\frac{1\,Ci}{3.7\times10^{10}\,\frac{decay}{s}}\right) = 0.054\times10^{-6}\,Ci \;(= 0.054\,\mu Ci)$$

(b) What is the mass of Po-sample

$R = \lambda N(t)$

$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2}$ } $N = \frac{R}{\lambda} = \frac{2000\,\frac{decays}{s}}{\ln 2}\,138\,d\,\frac{24\,h}{1\,day}\,\frac{3600\,s}{1\,h}$

$= 3.44\times10^{10}$ [nuclei = atoms]

use Avogadro $6\times10^{23}\,\frac{atoms}{mol}$

$$mass = 3.44\times10^{10}\,atoms\,\frac{1\,mol}{6\times10^{23}\,atoms}\cdot\frac{0.210\,kg}{1\,mol} = 1.2\times10^{-14}\,kg$$

12.16 Consider two protons in the ^{27}Al nucleus with their centers located 2.4 fm apart. How strong must the nuclear force be to overcome the Coulomb repulsion?

at least

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{(8.988\times10^9\,Nm^2/C^2)(1.602\times10^{-19}C)^2}{(2.4\times10^{-15}m)} = \boxed{40N}$$

12.28 An unknown nuclear sample is observed to decrease its activity by a factor of 5 in a 1 hour period. What is its activity?

$$R = R_0\, e^{-\lambda t} = \frac{R_0}{5} \quad \text{at } t = 3600s \quad \lambda = \frac{\ln 5}{t}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\ln 5}\,3600s = 1550s \approx \boxed{26\ min}$$

12.33 The halflive of tritium is $t_{1/2} = 12.33y$ $N_0 = 75kg = M_0$
How much remained after 7 years?

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{12.33y} = 5.622\times10^{-2}/y$$

$$N = N_0\, e^{-\lambda t} = 75\,kg\; e^{-\frac{5.622\times10^{-2}}{y}\cdot 7y} = \boxed{51\,kg}$$

12.49 Two rocks are found to have different ratios R' of ^{238}U to ^{206}Pb $R' = 0.76$ and ~~3.1~~ . 3.1
What are the ages of the rocks. Did they have the same origin?

$$R' = \frac{N(^{206}Pb)}{N(^{238}U)} = e^{\lambda t} - 1 \quad \text{rearranging} \quad t = \frac{t_{1/2}}{\ln 2}(\ln R' + 1)$$

$$R = 0.76 \quad t = \frac{4.47 \times 10^9 y}{\ln 2} \ln(1.76) = 3.65 \times 10^9 y$$

$$R = 3.1 \quad t = \frac{4.47 \times 10^9 y}{\ln 2} \ln 4.1 = 9.1 \times 10^9 y$$

not the same origin
values too different

12.53 Use only Z and A values to calculate the number of α and β particles produced by the decay of $^{235}_{92}U$ to its stable product $^{207}_{82}Pb$

change in A: 235 − 207 = 28 $\left.\begin{array}{l}\text{change in A}\\ \text{4 α-particle}\end{array}\right\}$ ⤳ $\frac{28}{4} = 7\alpha$

change in Z 92 − 82 = 10 ;
7 α change in Z = 14 } → 4β to bring the difference to 10