Key

1) According to relativistic mechanics, when you double the speed of a particle, the magnitude of its momentum increases by...
   a. a factor of 2.
   b. a factor greater than 2.
   c. a factor between 1 and 2 that depends on the mass of the particle.
   d. a factor of 1.
   e. a factor of less than 1.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \leftarrow \text{quadratic}$$

$$P = \gamma m v$$

2) Newton's laws are a good approximation of relativistic laws at low velocities...
   a. in inertial reference frames
   b. always
   c. in non-inertial frames
   d. only within our solar system
   e. never

3) Acceleration due to gravity is...
   a. different depending on location.
   b. a universal constant.
   c. a constant determined by Cavendish.
   d. dependent only on the distance from the Sun.
   e. unknowable.

$$F = G\frac{Mm}{r^2} = ma$$

$\underset{\leftarrow \text{location dependent}}{}$

4) As a proton approaches the speed of light the *increase* in its speed is...
   a. Less and less.
   b. Larger and larger.
   c. Stays the same.
   d. Unpredictable.
   e. Unknowable.

$\leftarrow$ asymptotic approach, Change in v decreases.

5) Galilean Invariance means...
   a. that if Newton's laws are valid in one reference frame, then they are also valid in another reference frame moving at a constant velocity relative to the first system.
   b. the same as Newtonian Principle of Relativity.
   c. that the Newton's laws are invariant in inertial reference frames.
   d. (a) and (b)
   e. (a), (b), and (c)

6) The relativistic factor γ provides that...
   a. if two frames are aligned at t = t' = 0 but one frame moves at a constant velocity, an interval of time t in the proper frame will be measured as a time γt in the reference frame.
   b. an object of length L measured in a frame at rest is measured to be γL when that object is moving with respect to the rest frame.
   c. increased velocities decrease the value of γ.
   d. a particle moving faster than the speed of light gives a real relativistic factor.
   e. an object at rest gives a relativistic factor of 0.

dilation
$t' = \gamma t \Rightarrow$
$L' = \frac{L}{\gamma}, \neq>$
Contraction

7) Why can a particle with mass never reach the exact speed of light?
   a. A particle moving at the speed of light would have an infinite amount of energy and momentum, breaking both quantities' respective conservation laws.
   b. A particle moving at the speed of light would have a gamma factor greater than 1.
   c. A particle moving at the speed of light would have infinite mass, breaking the conservation of mass law.
   d. A particle moving at the speed of light would be in violation of all known speed limit signs in the state of Texas.
   e. A particle moving at the speed of light is possible and has been experimentally confirmed.

$\lim\limits_{v \to c} \gamma \approx \frac{1}{\sqrt{1-1}} = \frac{1}{0}$

8) A police officer aims his radar-gun at an oncoming car. How large is the relativistic Doppler-shift for the return frequency?

①

blue shift 1

②
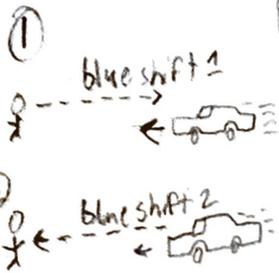
blue shift 2

a. The frequency *shift* will be approximately doubled, due to blue shifting on both the outgoing and return path.

b. The frequency will stay approximately the same since red shift from a stationary source on the outgoing path will cancel with the blue shift from the moving source on the return path.

c. The frequency *shift* will be approximately halved since in both cases there is a stationary component, causing red shifting on both the outgoing and return path.

d. The frequency will be approximately doubled, due to blue shifting on both the outgoing and return path.

e. The frequency will be approximately halved since in both cases there is a stationary component, causing red shifting on both the outgoing and return path.

9) This information relates to the next four questions. Suppose the astronauts traveled from the earth to a far planet and back a total distance of 4 light years, and the trip took 7 years as measured on earth. Assuming (incorrectly) that they moved with a constant velocity. (Hint: 1 light year is the distance that light travels in 1 year.)

What was the relativistic factor γ?

$d = 4 \text{ ly}, \ t_E = 7y \rightarrow \boxed{\beta = \frac{v}{c} = \frac{d/t_E}{c} = \frac{4/7}{c} = 0.57}$

a. 1.22
b. 1.32
c. 1.42
d. 1.52
e. 1.12

$\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.22}$

10) What was the proper time for the trip? (Proper time here is the time as measured by the spaceship)

a. 5.74y
b. 5.00y
c. 4.72y
d. 6.01y
e. 2.02y

$\boxed{t_s = \frac{t_E}{\gamma} = \frac{7y}{1.22} = 5.74y}$

11) What was the time difference for the trip between the clocks on earth and their clocks?

a. 1.26y
b. 0.83y
c. 1.1y
d. 1.4y
e. 0y, there was no measurable difference in time.

$\boxed{\Delta t = (t_E - t_s) = (7 - 5.74) = 1.26}$

12) What is the total energy of a proton having a momentum of 77 GeV /c? $P = 77 \frac{GeV}{c}, \ m_p = 0.938 \frac{GeV}{c^2}$

a. 77.0057 GeV
b. 40.011 GeV
c. 40.938 GeV
d. 77.931 GeV
e. 72.249 GeV

$\boxed{E = \sqrt{p^2c^2 + m^2c^4} = \sqrt{(77)^2 + (0.938)^2} = 77.0057 \, GeV}$

13) The kinetic energy of a proton is equal to (1/2) of its rest energy. What is the speed of a proton as a fraction of c?

a. 0.75
b. 0.82
c. 0.58
d. 0.63
e. 0.49

$E = \frac{1}{2}E_0 + E_0 = \frac{3}{2}E_0$

$E = \gamma mc^2 \gg \gamma = \frac{E}{E_0} = \frac{\frac{3}{2}E_0}{E_0} = \frac{3}{2}$

$E_0 = mc^2$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$ check lecture notes for derivation of this.

$\boxed{\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.745}$

14) The kinetic energy of a proton is equal to (1/2) of its rest energy. What is the momentum of the proton (expressed in MeV/c)?

a. 1049
b. 813
c. 976.3
d. 368.5
e. 1251

$m_p = 938 \left(\frac{MeV}{c^2}\right)$

$\boxed{P = \gamma mv = \left(\frac{3}{2}\right)(938) \cdot (0.75) = 1049 \left(\frac{MeV}{c}\right)}$

15) A particle initially has a speed of 0.5 c. At what speed does its momentum increase by 50%?

a. 0.667c

$U_0 = 0.5c, \ P = \gamma_0 m U_0 = \frac{1}{\sqrt{1-(0.5)^2}} \cdot m \cdot (0.5c) = m(0.577c)$

shown in lecture notes

$P_1 = 1.5 P_0 \Rightarrow \gamma_1 m v_1 = (1.5)(0.577c)\gamma \Rightarrow \gamma_1 v_1 = 0.866c \rightarrow \text{find } v_1$

b. 0.803c

$$V_1 = \frac{0.866c}{\sqrt{1 + 0.866)^2}} = \boxed{0.655c}$$

c. 0.550c
d. 0.750c
e. 0.500c

16) A spacecraft traveling away from us out of the solar system at a speed of 0.84c sends back information at a rate of 700Hz. At what rate do we receive the information?

   a. 206 Hz
   b. 103 Hz $f = 700\,Hz, \quad V = (-) 0.84c$
   c. 309 Hz
   d. 407 Hz $f' = (700\,Hz)\sqrt{\dfrac{1 + -0.84}{1 - -0.84}} = 206\,Hz$
   e. 813 Hz

17) Will a meter stick manufactured on earth work still be able to measure a meter on a spaceship moving with a speed of 0.8c?

   a. Yes, it will not change its length in the frame of the space ship.
   b. No, it will be longer in the frame of the space ship.
   c. Yes, time dilation will balance length contraction.
   d. No, it will be contracted in the frame of spaceship.
   e. It depends on the material it is made of.

18) A star outputs orange light with a wavelength of 607nm. It is moving towards the earth direction at a speed of 0.032c. What is the wavelength of light as seen from an observer on earth?

   a. yellow light (588nm)
   b. Orange-red (627nm) $\beta = 0.032, \quad \lambda = 607\,nm$
   c. purple (435nm)
   d. blue (485nm) $\boxed{\lambda' = \lambda\sqrt{\dfrac{1-\beta}{1+\beta}}} = 607\sqrt{\dfrac{1 - 0.032}{1 + 0.032}} \approx \boxed{588\,nm}$
   e. red (678nm)

19) By how much would the *rest* energy of a proton change if it was accelerated to c?

   a. It would not change.
   b. It would grow to infinity. $E_0 = mc^2$
   c. It would change by gamma. $\uparrow$
   d. It would drop to zero.
   e. It would become undefined.   no velocity term, E_0 is independent of movement.

20) Why is fusion of Deuterium (^2H) and Tritium (^3H) a clean and efficient energy source?

   a. The reaction products (^4He and n) are not radioactive, and the release binding energy is quite large at 17.6 MeV.
   b. Deuterium and Tritium are both easily obtained.
   c. The nuclear reaction is easy to perform, and not much containment is required.
   d. The reaction products is simply hydrogen and energy.
   e. Fusion of Deuterium and Tritium is not clean and efficient.