The Foucault pendulum in Aggieland

What does it show?? Seeing is believing

T=10 sec   how long is it??

https://sibor.physics.tamu.edu/home/courses/physics-222-modern-physics/

CHAPTER 2
Special Theory of Relativity

- 2.1 The Apparent Need for Ether
- 2.2 The Michelson-Morley Experiment
- 2.3 Einstein's Postulates
- 2.4 The Lorentz Transformation
- 2.5 Time Dilation and Length Contraction
- 2.6 Addition of Velocities
- 2.7 Experimental Verification
- 2.8 Twin Paradox
- 2.9 Space-time
- 2.10 Doppler Effect
- 2.11 Relativistic Momentum
- 2.12 Relativistic Energy
- 2.13 Computations in Modern Physics
- 2.14 Electromagnetism and Relativity

It was found that there was no displacement of the interference fringes, so that the result of the experiment was negative and would, therefore, show that there is still a difficulty in the theory itself…

- Albert Michelson, 1907

Newtonian (Classical) Relativity

Assumption

- It is assumed that Newton's laws of motion must be measured with respect to (relative to) some reference frame.

Inertial Reference Frame

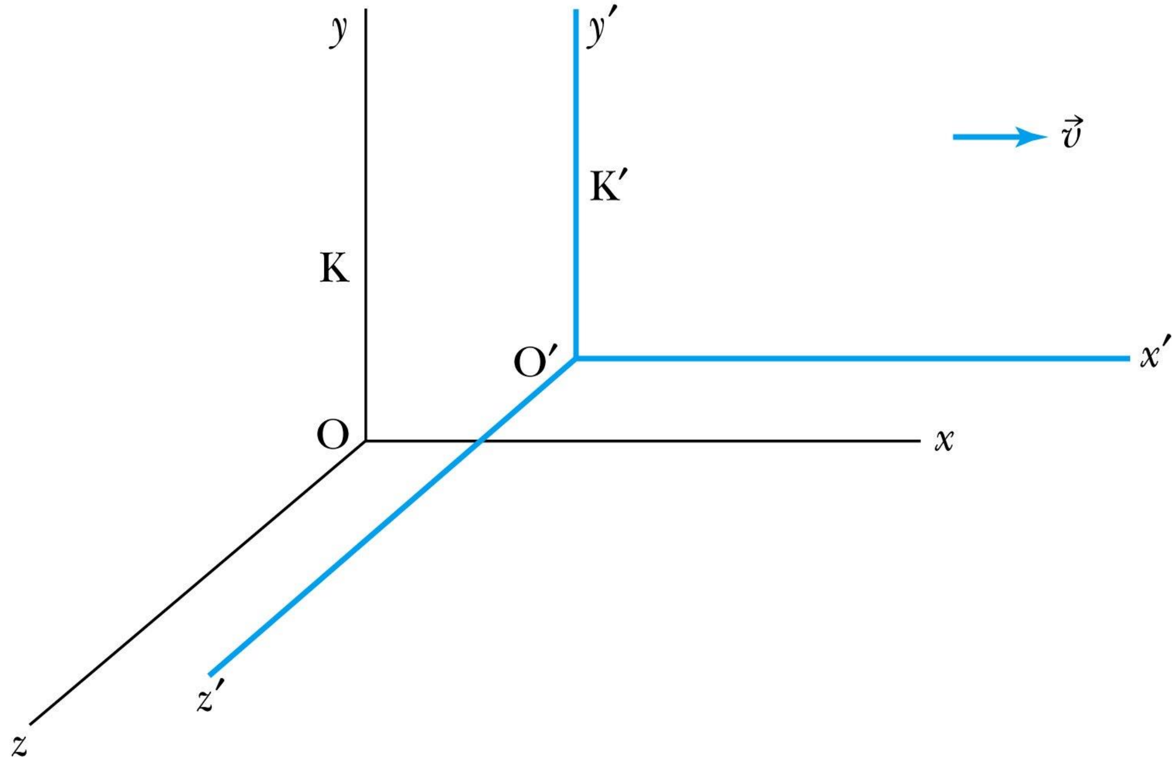
- A reference frame is called an **inertial frame** if Newton laws are valid in that frame.
- Such a frame is established when a body, not subjected to net external forces, is observed to move in rectilinear (along a straight line) motion at constant velocity.

Newtonian Principle of Relativity or Galilean Invariance

- If Newton's laws are valid in one reference frame, then they are also valid in another reference frame moving at a uniform velocity relative to the first system.

- This is referred to as the **Newtonian principle of relativity** or **Galilean invariance.**

Inertial Frames K and K'

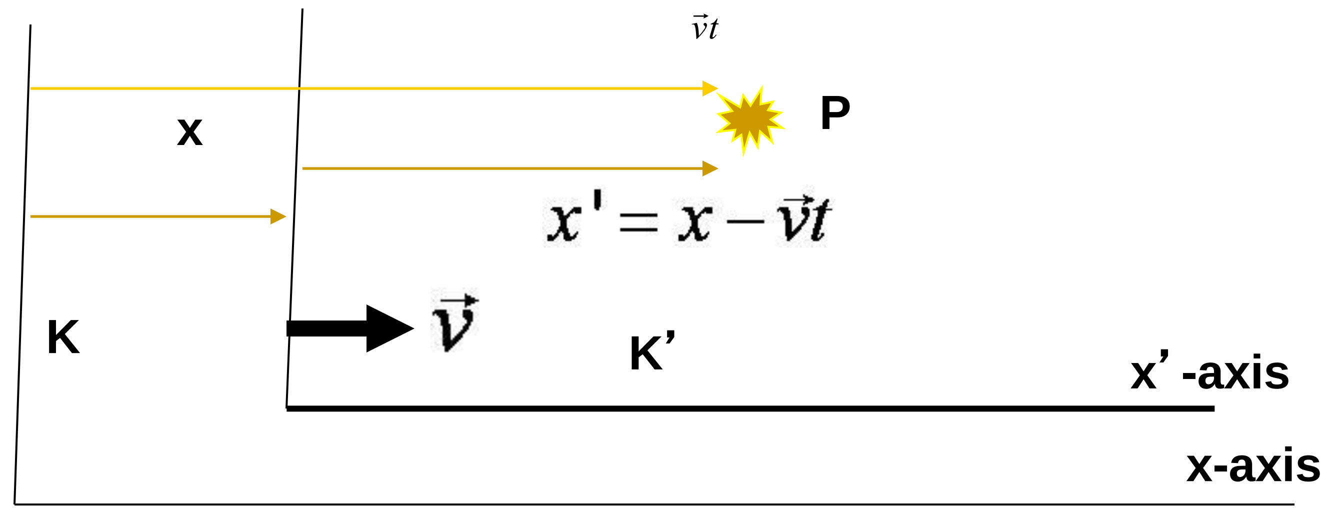


- K is at rest and K' is moving with velocity $\vec{v}$
- Axes are parallel
- K and K' are said to be *INERTIAL COORDINATE SYSTEMS*

The Galilean Transformation

For a point P

- In system K: P = (x, y, z, t)
- In system K': P = (x', y', z', t')



Conditions of the Galilean Transformation

- Parallel axes
- K' has a constant relative velocity in the *x*-direction with respect to K

$$x' = x - \vec{v}t$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

- **Time** (*t*) for all observers is a *Fundamental invariant*, i.e., the same for all inertial observers

The Inverse Relations

Step 1. Replace $\vec{v}$ with $-\vec{v}$

Step 2. Replace "primed" quantities with "unprimed" and "unprimed" with "primed"

$$x = x' + \vec{v}t$$

$$y = y'$$

$$z = z'$$

$$t = t'$$

The Transition to Modern Relativity

- Although Newton's laws of motion had the same form under the Galilean transformation, Maxwell's equations did not.
- In 1905, Albert Einstein proposed a fundamental connection between space and time and that Newton's laws are only an approximation.

Historical remark: The year 1905 was *annus mirabilis* (Latin: the year of wonders), as Albert Einstein made important discoveries concerning the photoelectric effect, Brownian motion special theory of relativity.

2.1: The Apparent Need for Ether

- The wave nature of light suggested that there existed a propagation medium called the ***luminiferous ether*** or just ***ether***.
  - Ether had to have such a low density that the planets could move through it without loss of energy
  - It also had to have an elasticity to support the high velocity of light waves

Maxwell's Equations

- In Maxwell's theory the speed of light, in terms of the permeability and permittivity of free space, was given by

$$v = c = 1/\sqrt{\mu_0 \varepsilon_0}$$

- Thus, the velocity of light must be a constant.

An Absolute Reference System

- Ether was proposed as an absolute reference system in which the speed of light was this constant and from which other measurements could be made.
- The Michelson-Morley experiment was an attempt to show the existence of ether.

Similarity between the Michelson-Morley interferometer and the race between two swimmers between floats anchored in the river bed.

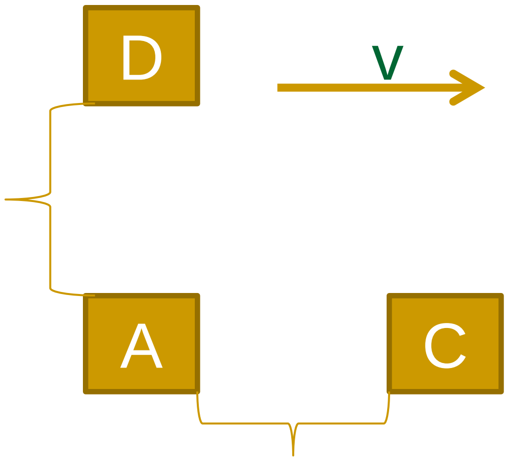
Assumptions

Equally fast swimmers

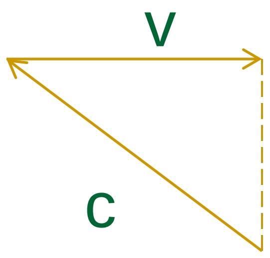
Speed of each swimmer = c

Water velocity or drift of the ether with respect to the earth = v

Equal distance $l_1 = l_2$ between floats



l_1, l_2



$$v_{\perp} = \sqrt{c^2 - v^2}$$

$$t_{\perp} = \frac{2l}{\sqrt{c^2 - v^2}} \qquad t_{ll} = \frac{2lc}{c^2 - v^2} \qquad \frac{t_{ll}}{t_{\perp}} = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad t_{ll} \geq t_{\perp}$$

The perpendicular swimmer wins!

2.2: The Michelson-Morley Experiment

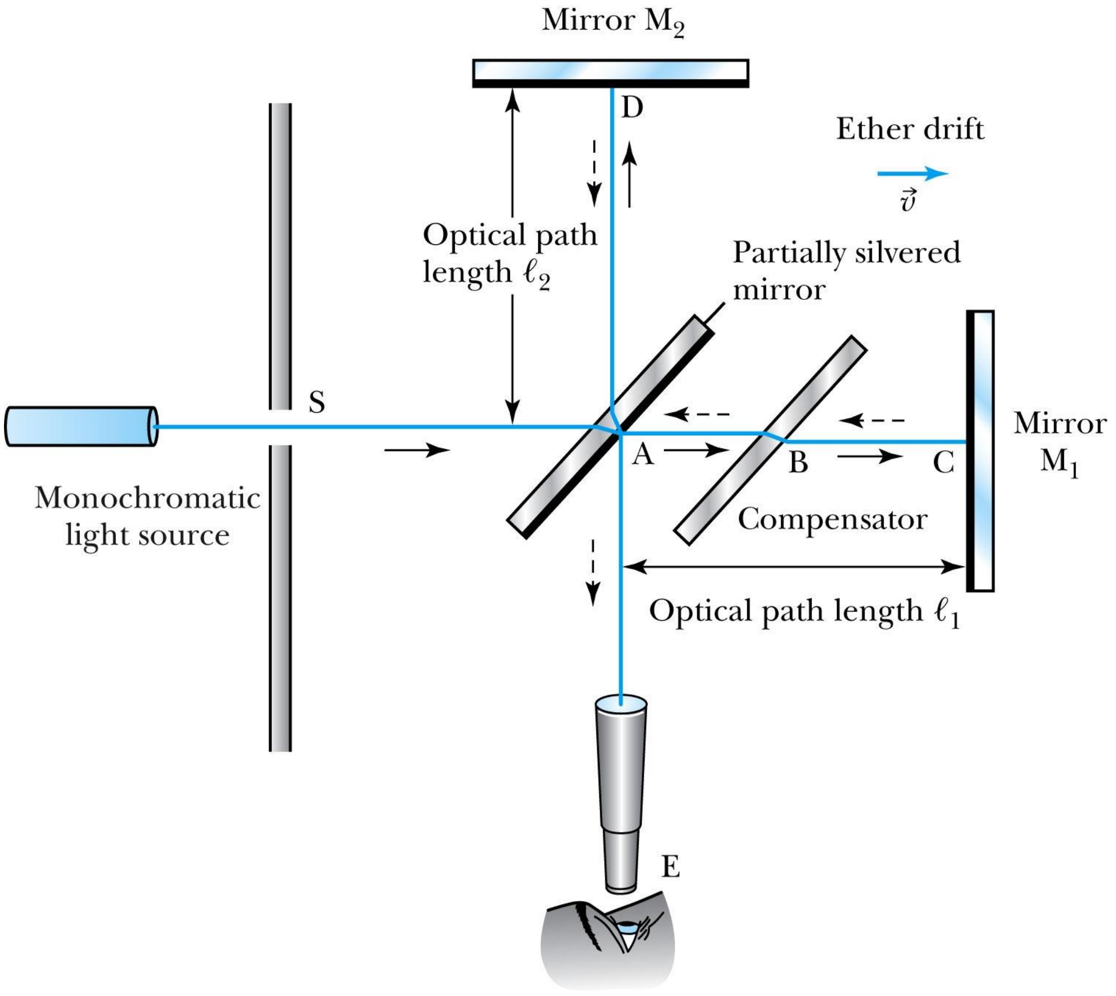
- Albert Michelson (1852–1931) was the first U.S. citizen to receive the Nobel Prize for Physics (1907), and built an extremely precise device called an *interferometer* to measure the minute phase difference between two light waves traveling in mutually orthogonal directions.

The Michelson Interferometer

1. AC is parallel to the motion of the Earth inducing an "ether wind"

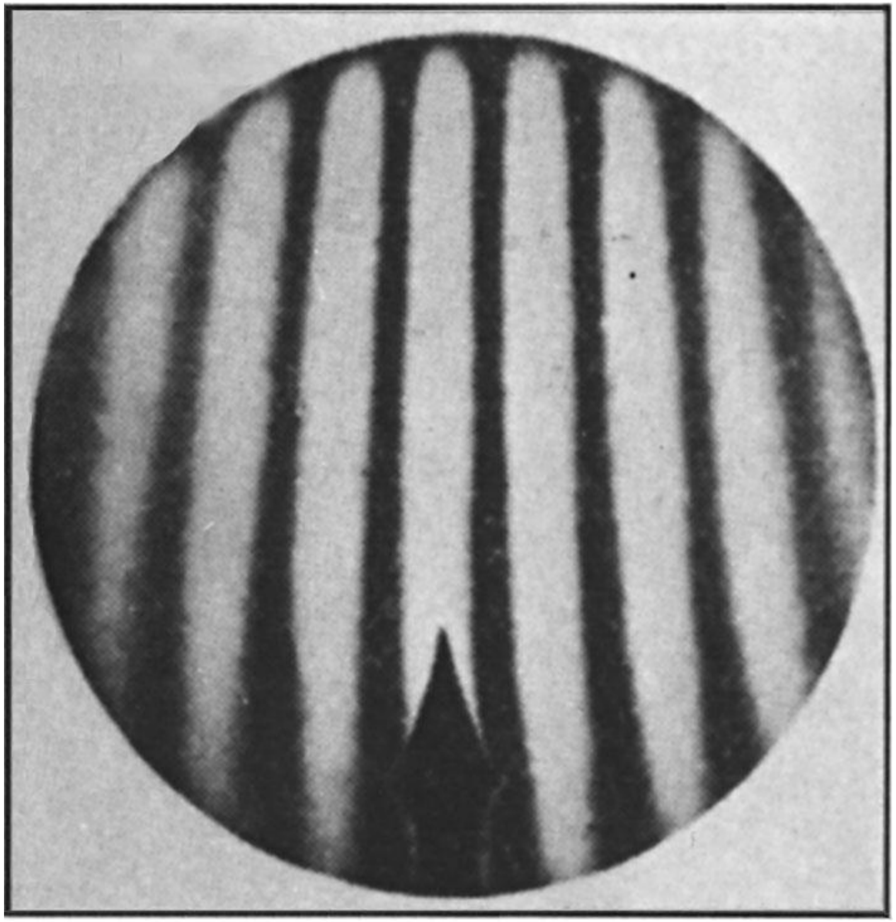
2. Light from source S is split by mirror A and travels to mirrors C and D in mutually perpendicular directions

3. After reflection the beams recombine at A slightly out of phase due to the "ether wind" as viewed by telescope E.



The system was set on a rotatable platform

Typical interferometer fringe pattern, which is expected to shift when the system is rotated



The Analysis

Assuming the Galilean Transformation

Time t_1 from A to C and back on parallel course:

$$t_1 = \frac{\ell_1}{c+v} + \frac{\ell_1}{c-v} = \frac{2c\ell_1}{c^2 - v^2} = \frac{2\ell_1}{c}\left(\frac{1}{1 - v^2/c^2}\right)$$

Time t_2 from A to D and back on perpendicular course:

$$t_2 = \frac{2\ell_2}{\sqrt{c^2 - v^2}} = \frac{2\ell_2}{c} \cdot \frac{1}{\sqrt{1 - v^2/c^2}}$$

So that the change in time is:

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{\ell_2}{\sqrt{1 - v^2/c^2}} - \frac{\ell_1}{1 - v^2/c^2}\right)$$

The Analysis (continued)

Upon rotating the apparatus, the optical path lengths ℓ_1 and ℓ_2 are interchanged producing a different change in time: (note the change in denominators)

$$\Delta t' = t'_2 - t'_1 = \frac{2}{c}\left(\frac{\ell_2}{1 - v^2/c^2} - \frac{\ell_1}{\sqrt{1 - v^2/c^2}}\right)$$

The Analysis (continued)

Thus a time difference between rotations is given by:

$$\Delta t' - \Delta t = \frac{2}{c}\left(\frac{\ell_1 + \ell_2}{1 - v^2/c^2} - \frac{\ell_1 + \ell_2}{\sqrt{1 - v^2/c^2}}\right)$$

and upon a binomial expansion, assuming *v*/*c* << 1, this reduces to

$$\Delta t' - \Delta t \approx v^2(\ell_1 + \ell_2)/c^3$$

$$(1+x)^{-1} = 1 - x + x^2 \ldots\ldots$$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{1(3)}{2(4)}x^2 \ldots.$$

Results

- Using the Earth's orbital speed as:

$$V = 3 \times 10^4 \text{ m/s}$$

  together with

$$\ell_1 \approx \ell_2 = 1.2 \text{ m}$$

  So that the time difference becomes

$$\Delta t' - \Delta t \approx v^2(\ell_1 + \ell_2)/c^3 = 8 \times 10^{-17} \text{ s}$$

The light period this is about T=λ/c~600nm/(3×10^8 m/s)=2 ×10^{-15} s, thus ($\Delta t' - \Delta t$) /T~0.04 (λ is a wavelength of light wave).

- Although a very small number, it was within the experimental range of measurement for light waves.

Michelson's Conclusion

- Michelson noted that he should be able to detect a phase shift of light due to the time difference between path lengths but found none.
- He thus concluded that the hypothesis of the stationary ether must be incorrect.
- After several repeats and refinements with assistance from Edward Morley (1893-1923), again *a null result.*
- ***Thus, ether does not seem to exist!***

Most famous "failed" experiment, but great conclusive results!

Possible Explanations

- Many explanations were proposed but the most popular was the *ether drag* hypothesis.
  - This hypothesis suggested that the Earth somehow "dragged" the ether along as it rotates on its axis and revolves about the sun.
  - This was contradicted by *stellar aberration* wherein telescopes had to be tilted to observe starlight due to the Earth's motion. If ether was dragged along, this tilting would not exist.

The Lorentz-FitzGerald Contraction

- Another hypothesis proposed independently by both H. A. Lorentz and G. F. FitzGerald suggested that the length ℓ_1, in the direction of the motion was *contracted* by a factor of

$$\sqrt{1 - v^2 / c^2}$$

thus making the path lengths equal to account for the zero phase shift, which is seen from the equation

$$\Delta t' - \Delta t = \frac{2}{c}\left(\frac{\ell_1 + \ell_2}{1 - v^2/c^2} - \frac{\ell_1 + \ell_2}{1 - v^2/c^2}\right)$$

This, however, was an ad hoc assumption that could not be experimentally tested.

Section 2.1, problem 6

6. In the 1887 experiment by Michelson and Morley, the length of each arm was 11 m. The experimental limit for the fringe shift was 0.005 fringes. If sodium light was used with the interferometer ($\lambda = 589$ nm), what upper limit did the null experiment place on the speed of the Earth through the expected ether?

The shift for the light in two arms is $\Delta d = c(\Delta t - \Delta t')$. From (2.5)

$$\Delta t - \Delta t' = \frac{v^2(\ell_1 + \ell_2)}{c^3}$$

The number of fringes shifted is

$$n = \frac{\Delta d}{\lambda} = \frac{c(\Delta t - \Delta t')}{\lambda} = \frac{c\, v^2(\ell_1+\ell_2)}{\lambda c^3} =$$

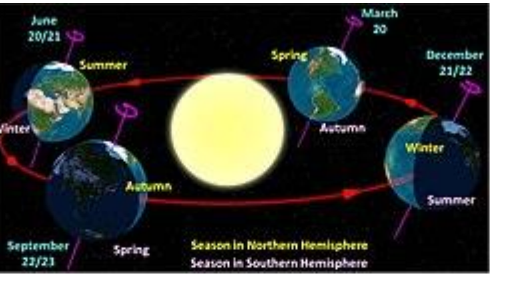
$$= \frac{v^2(\ell_1+\ell_2)}{\lambda c^2} = 0.005.$$

Thus,

$$v = \sqrt{\frac{n \lambda c^2}{\ell_1 + \ell_2}} = \left(3.00 \cdot 10^8 \frac{m}{s}\right)\sqrt{\frac{0.005 \cdot 589 \cdot 10^{-9} m}{22 m}} =$$

=3.47 km/s

Earth's orbital speed averages 29.78 km/s



2.3: Einstein's Postulates

- Albert Einstein (1879–1955) was only two years old when Michelson reported his first null measurement for the existence of the ether.
- At the age of 16 Einstein began thinking about the form of Maxwell's equations in moving inertial systems.
- In 1905, at the age of 26, he published his startling proposal about the **principle of relativity**, which he believed to be fundamental.

Einstein's Two Postulates

With the belief that Maxwell's equations must be valid in all inertial frames, Einstein proposes the following postulates:

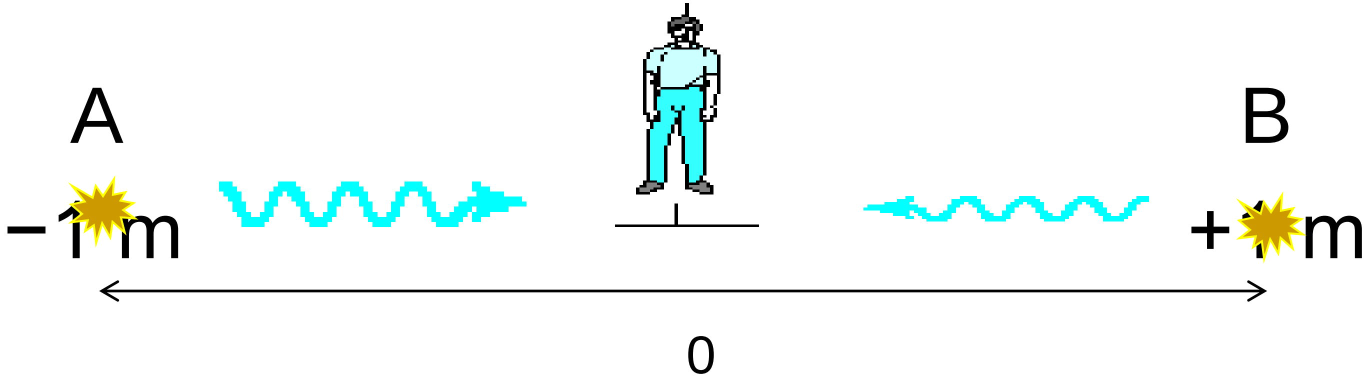
1) **The principle of relativity**: The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.
2) **The constancy of the speed of light**: Observers in all inertial systems measure the same value for the speed of light in a vacuum.

Revisiting Inertial Frames and the Re-evaluation of Time

- In Newtonian physics we previously assumed that $t = t'$
  - *Thus with "synchronized" clocks, events in K and K' can be considered simultaneous*
- Einstein realized that each system must have its own observers with their own clocks and meter sticks
  - Thus, events considered simultaneous in K may not be simultaneous in K'.

The Problem of Simultaneity: "Gedanken" (German) (i.e. thought) experiment

Frank at rest is equidistant from events A and B:

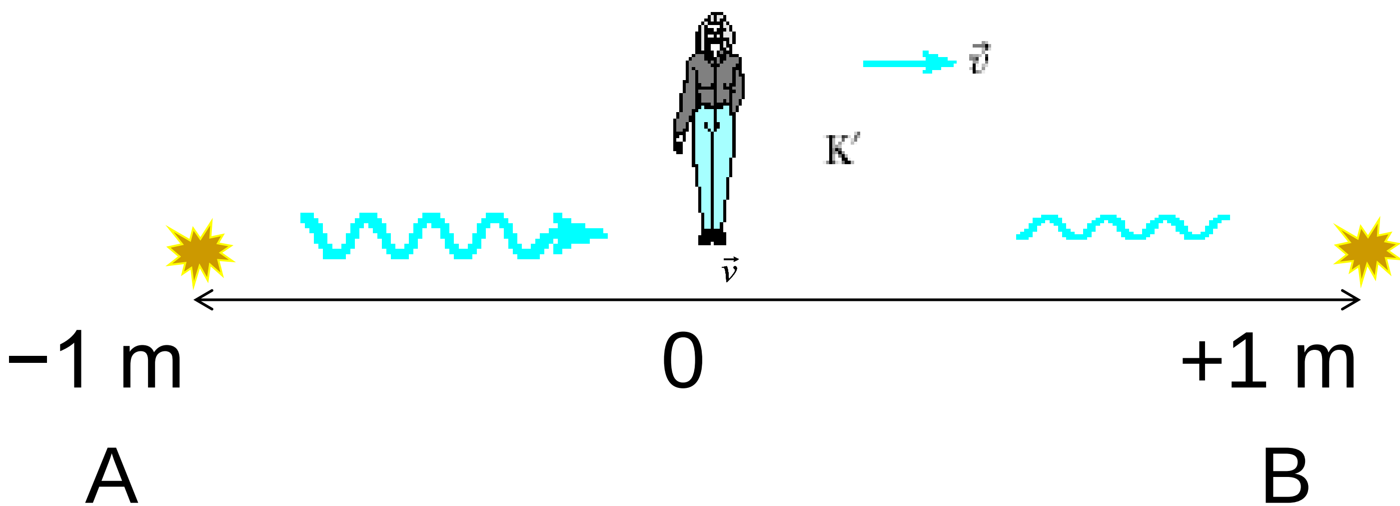


Frank "sees" both flashbulbs go off simultaneously.

The Problem of Simultaneity

Mary, moving to the right with speed $\vec{v}$

is at the same 0 position when flashbulbs go off, but she sees event B and then event A.



Thus, the order of events in *K’ can be different!*

We thus observe…

- *Two events that are simultaneous in one reference frame (K) are not necessarily simultaneous in another reference frame (K’) moving with respect to the first frame.*

- This suggests that each coordinate system must have its own observers with “clocks” that are synchronized…

Synchronization of Clocks

Step 1: Place observers with clocks throughout a given system

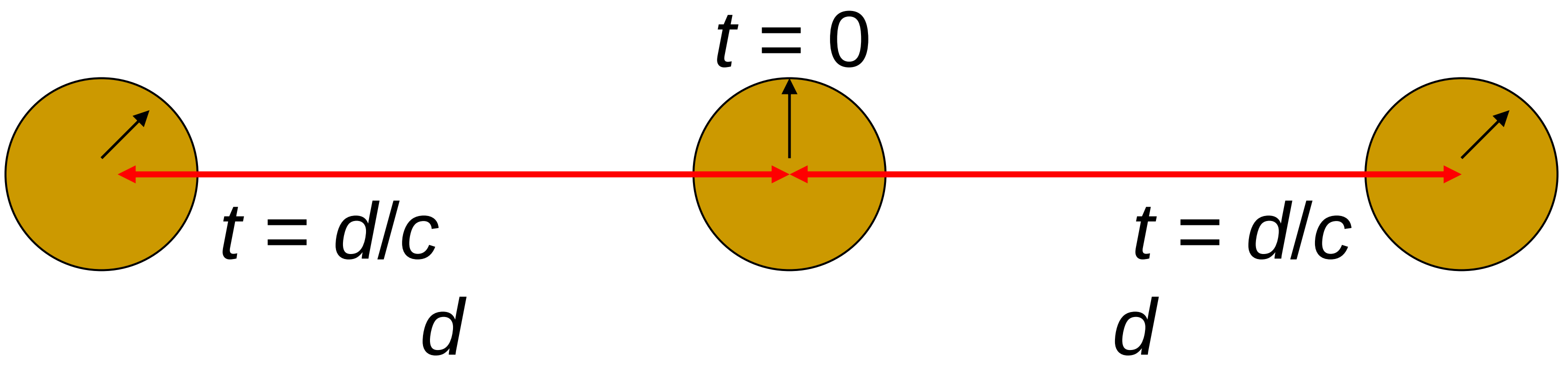
Step 2: In that system bring all the clocks together at one location

Step 3: Compare the clock readings

- If all of the clocks agree, then the clocks are said to be synchronized

A method to synchronize…

- One way is to have one clock at the origin set to $t = 0$ and advance each clock by a time (d/c) with d being the distance of the clock from the origin.
  - Allow each of these clocks to begin timing when a light signal arrives from the origin.



The Lorentz Transformations

The special set of linear transformations that:

1) preserve the constancy of the speed of light (c) between inertial observers;

and,

2) account for the problem of simultaneity between these observers

known as the **Lorentz transformation equations**

Lorentz Transformation Equations

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - (vx/c^2)}{\sqrt{1 - v^2/c^2}}$$

Lorentz Transformation Equations

A more symmetric form:

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$

$$x' = \gamma(x - \beta ct)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \beta x / c)$$

Section 2.4, problem 17

17. A light signal is sent from the origin of a system K at $t = 0$ to the point $x = 3$ m, $y = 5$ m, $z = 10$ m. (a) At what time t is the signal received? (b) Find (x', y', z', t') for the receipt of the signal in a frame K′ that is moving along the x axis of K at a speed of $0.8c$. (c) From your results in (b) verify that the light traveled with a speed c as measured in the K′ frame.

(a) $t = \frac{\sqrt{x^2+y^2+z^2}}{c} = \frac{\sqrt{(3m)^2+(5m)^2+(10m)^2}}{3.00 \cdot 10^8 \, m/s} =$

$= 3.86 \cdot 10^{-8} s$

(b) We have $y' = y = 5m$, $z' = z = 10m$

For $v = 0.8c$, $\gamma = \frac{1}{\sqrt{1-(0.8)^2}} = \frac{1}{0.6} = \frac{5}{3}$

$x' = \gamma(x - vt) = \frac{5}{3}\left[3m - \left(2.40 \cdot 10^8 \frac{m}{s}\right)\left(3.86 \cdot 10^{-8} s\right)\right] =$

$= -10.4m$

$t' = \gamma(t - vx/c^2) = \frac{5}{3}\left[\left(3.86 \cdot 10^{-8} s\right) - \left(2.40 \cdot 10^8 \frac{m}{s}\right) \cdot 3m / \left(3.00 \cdot 10^8 \frac{m}{s}\right)^2\right] = 5.1 \cdot 10^{-8} s = 51 \, ns$

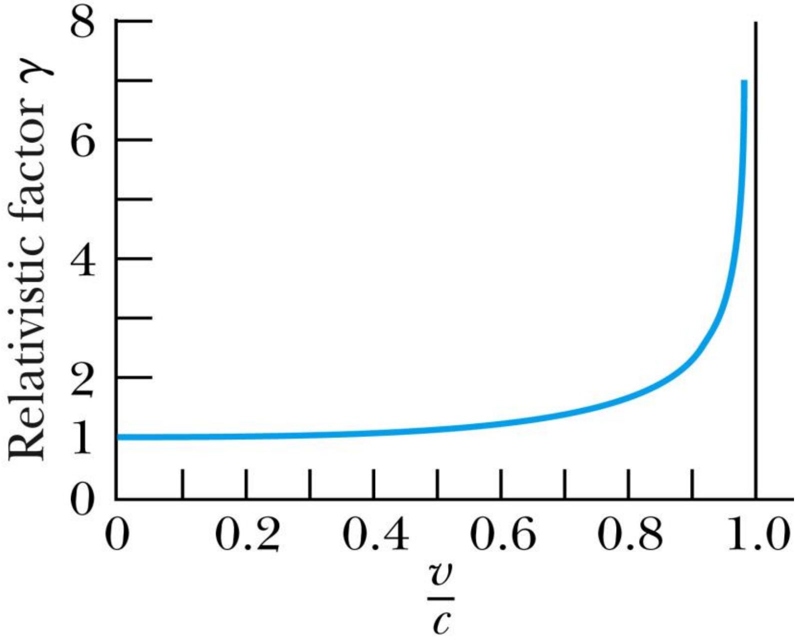
(c) $\frac{\sqrt{x'^2+y'^2+z'^2}}{t'} = \frac{\sqrt{(10.4m)^2+(5m)^2+(10m)^2}}{5.1 \cdot 10^{-8} s} =$

$= 2.99 \cdot 10^8 \frac{m}{s} = c$

Properties of γ

Recall $\beta = v/c < 1$ for all observers

1) $\gamma \geq 1$ equals 1 only when $v = 0$
2) Graph of β:
   (note $v \neq c$)



Thank you for your attention!