The Lorentz Velocity Transformations

defining velocities as: $u_x = dx/dt$, $u_y = dy/dt$,
$u'_x = dx'/dt'$, etc. it is easily shown that:

$$u_x = \frac{dx}{dt} = \frac{\gamma(dx' + v\,dt')}{\gamma[dt' + (v/c^2)\,dx']} = \frac{u'_x + v}{1 + (v/c^2)u'_x}$$

With similar relations for u_y and u_z:

$$u_y = \frac{u'_y}{\gamma\left[1 + (v/c^2)u_x'\right]} \qquad u_z = \frac{u'_z}{\gamma\left[1 + (v/c^2)u_x'\right]}$$

The Lorentz Velocity Transformations

In addition to the previous relations, the **Lorentz velocity transformations** for u'_x, u'_y, and u'_z can be obtained by switching primed and unprimed and changing v to $-v$:

$$u'_x = \frac{u_x - v}{1 - (v/c^2)u_x}$$

$$u'_y = \frac{u_y}{\gamma\left[1 - (v/c^2)u_x\right]}$$

$$u'_z = \frac{u_z}{\gamma\left[1 - (v/c^2)u_x\right]}$$

The Lorentz Velocity Transformations: an object moves with the speed of light

$u'_x = c$ (*light or, if* neutrinos are massless, they must travel at the speed of light)

$$u_x = \frac{u'_x + v}{1 + {vu_x}/{c^2}} = \frac{c + v}{1 + {vc}/{c^2}} = \frac{c(1 + {v}/{c})}{1 + {v}/{c}} = c$$

Problems: Time dilation,length contraction

chapt 2 _ Problem 20

A planet is orbiting a star that is 20 ly away (a) How fast must a rocket space ship go, if the round trip is not going to take longer than 40 years in time for the astronauts aboard. (b) How much time will the trip take as measured on earth?

(a) Roundtrip distance $d = 40\,ly$. Assume constant speed for space ship $v = \beta c$. Space ship is moving frame so distance is contracted

$$d' = \frac{d}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}} \qquad d' = d\sqrt{1-\beta^2}$$

$$v = \frac{\text{distance}}{\text{time}} = \frac{d'}{40y} = \frac{40ly\sqrt{1-\beta^2}}{40y} = c\sqrt{1-\beta^2} \qquad \beta = \frac{v}{c} = \sqrt{1-\beta^2}$$

$1\,ly = 1c\,y$

solving $\beta = \sqrt{0.5}$

$v = \sqrt{0.5}\,c = 0.71c$

(b) on earth $t' = 40y$ ↑ proper time $\qquad t = \gamma t' = \frac{1}{\sqrt{1-\beta^2}} 40y = 56.6\,y$ ↑ dilated time

Chapter 2 – 21

A muon has a mean lifetime of 2.2 μs and makes a track 9.5 cm long in a cloud chamber before decaying into an electron and two neutrinos. What was the speed of the muon?

$T' = \gamma T_0$ ← proper time

$$9.5\text{ cm} = vT' = v\gamma T_0 = \beta c \gamma T_0, \quad \text{since } v = \beta c$$

$$\beta = \frac{9.5\text{ cm}\sqrt{1-\beta^2}}{cT_0} = \frac{9.5\text{ cm}\sqrt{1-\beta^2}}{c\,(2.2\,\mu s)}$$

solving for β with $c = 3\times10^{10}$ cm/s

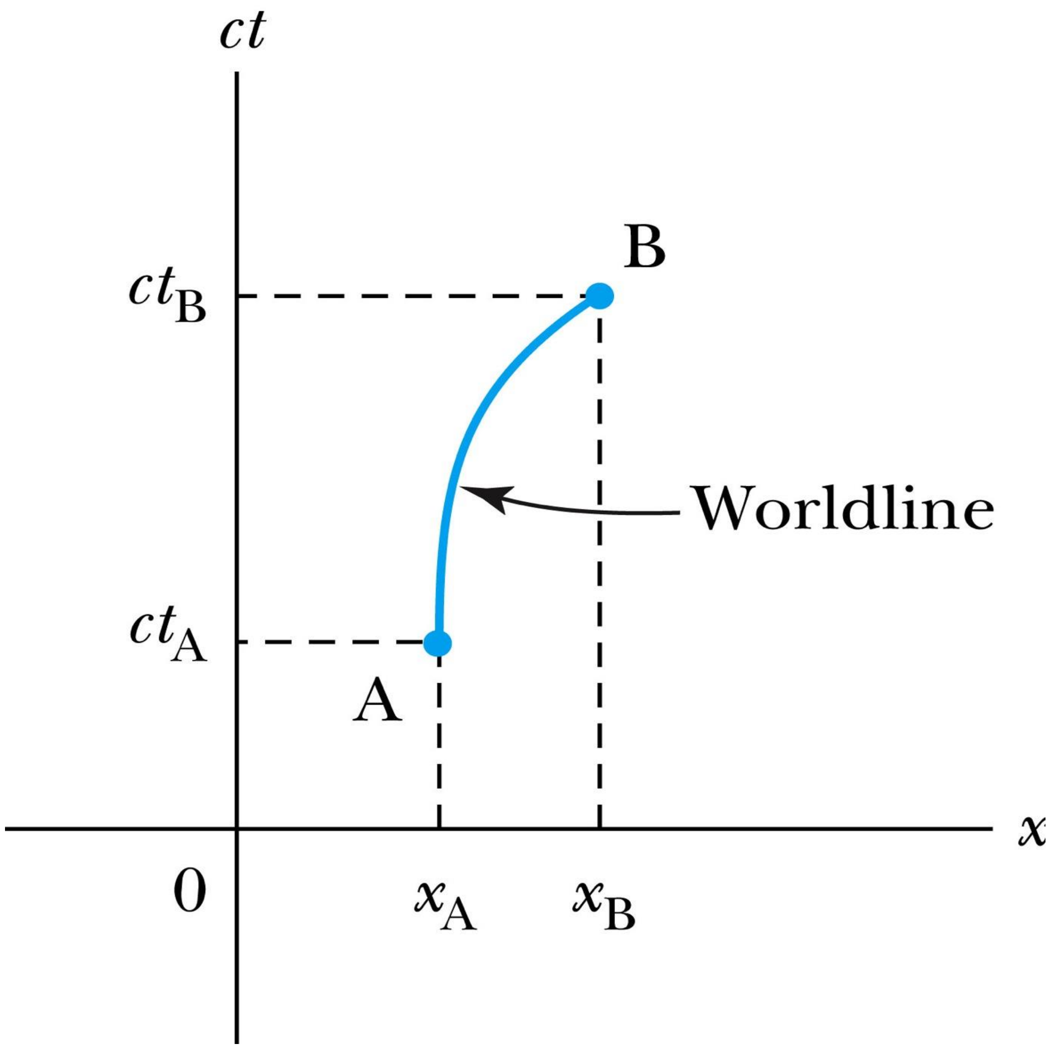
$$\beta = 1.4\times10^{-4} \rightarrow \boxed{v = 1.4\times10^{-4}\,c}$$

2.9: Spacetime

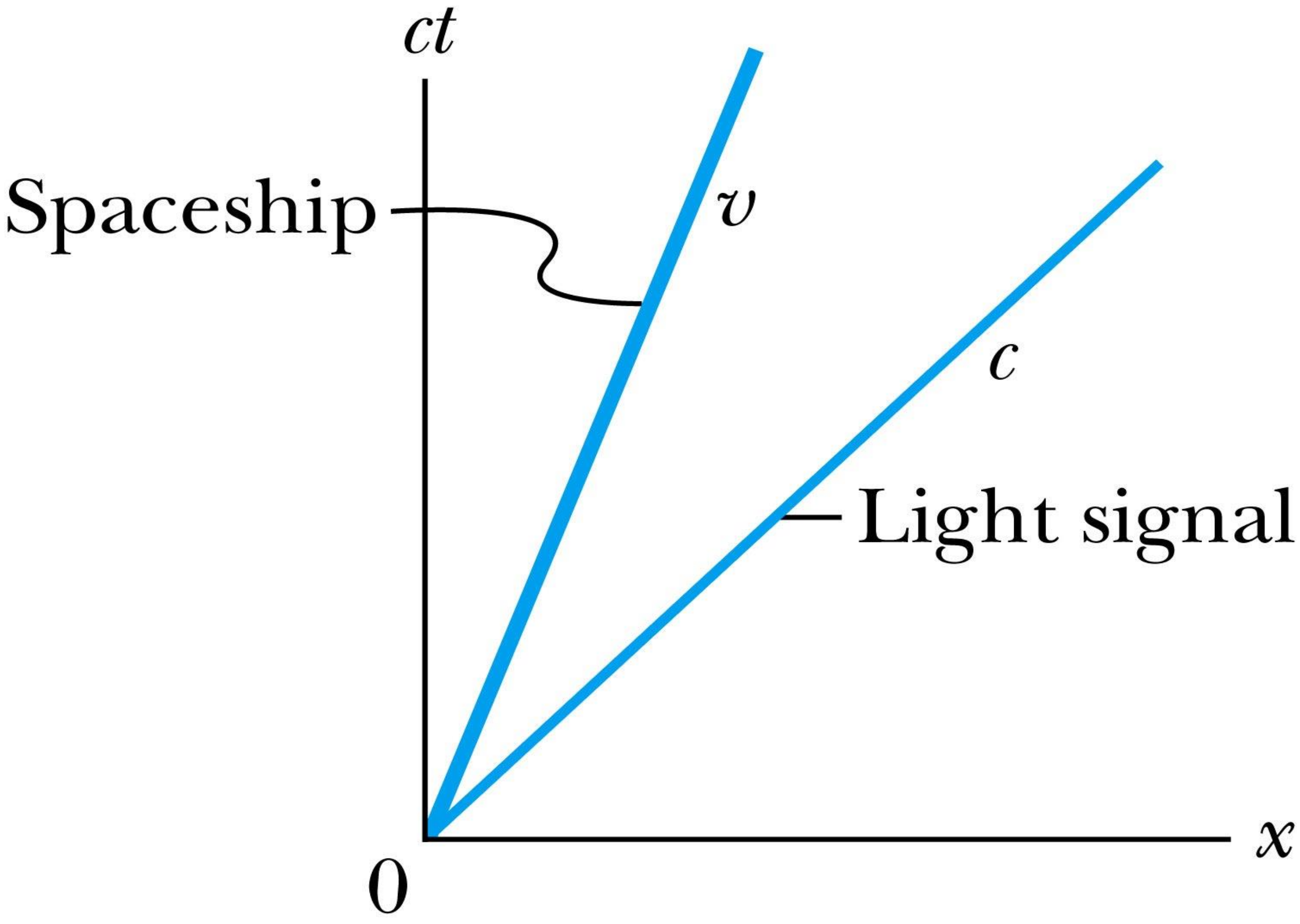
- When describing events in relativity, it is convenient to represent events on a **spacetime** diagram.
- In this diagram one spatial coordinate *x*, to specify position, is used and instead of time *t*, *ct* is used as the other coordinate so that both coordinates will have dimensions of length.
- Spacetime diagrams were first used by H. Minkowski in 1908 and are often called **Minkowski diagrams**. Paths in Minkowski spacetime are called **worldlines**.

Spacetime Diagram

It takes 4 dimensions to specify an event

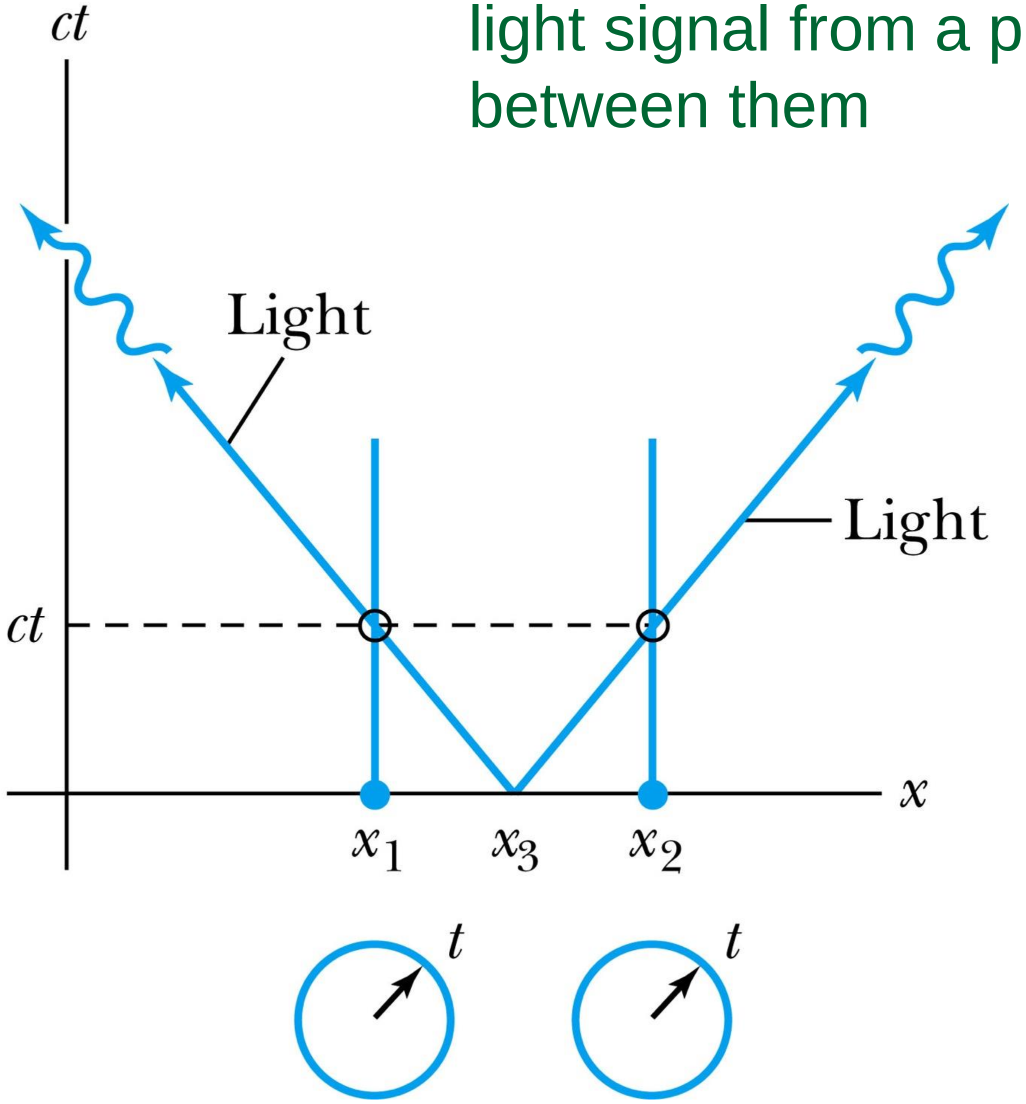


Particular Worldlines



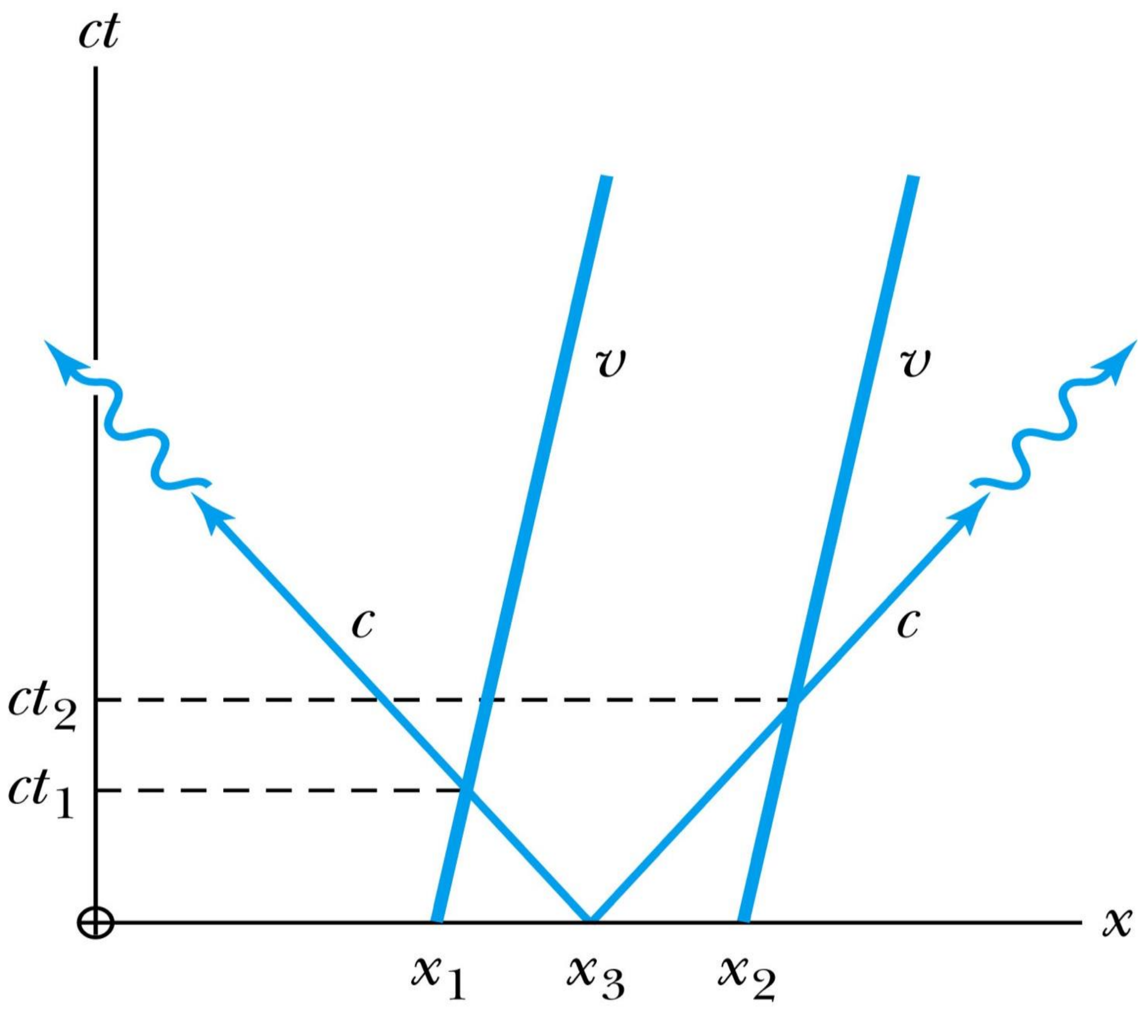
Worldlines and Time

Synchronize clocks by sending a light signal from a position halfway between them

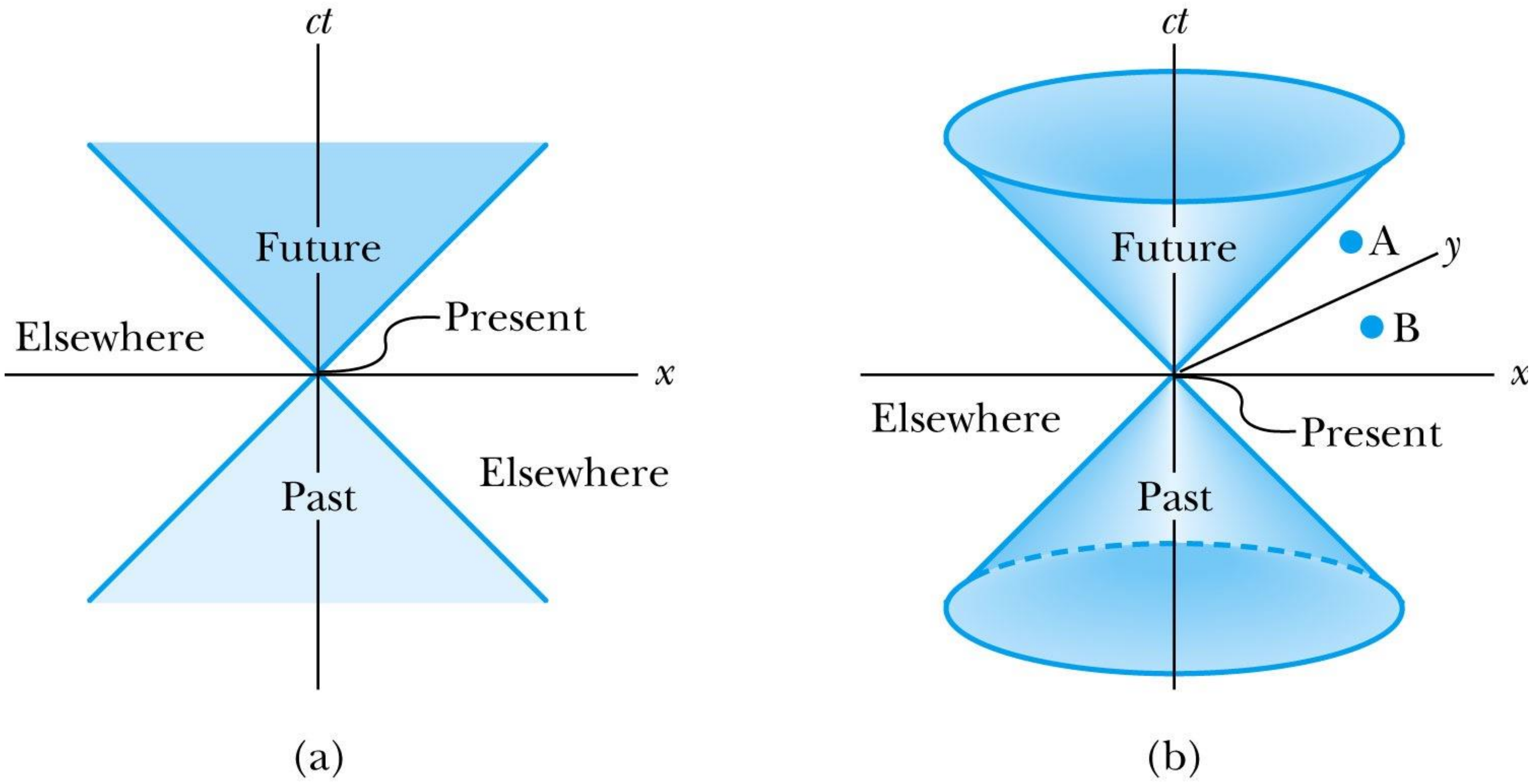


Moving Clocks

If the corresponding x' position where the light flashes is on K' ,then the arrival times will not be simultaneous in K

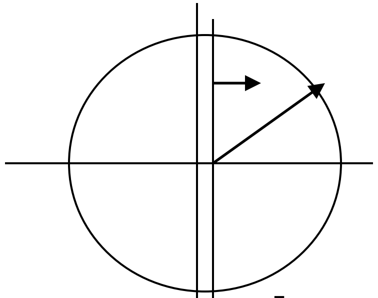


The Light Cone



Invariant Quantities: The Spacetime Interval

Since all observers "see" the same speed of light, then all observers, regardless of their velocities, must see spherical wave fronts.

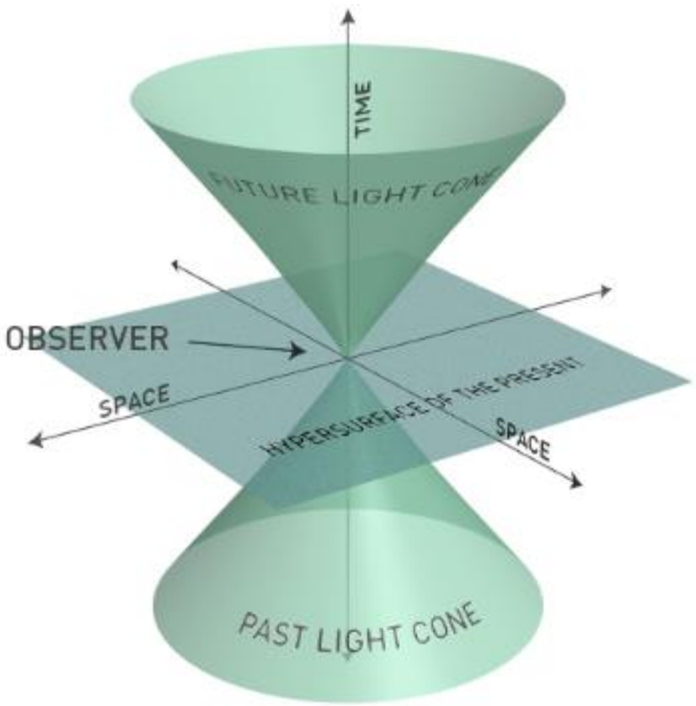


$$s^2 = x^2 - c^2t^2 = (x')^2 - c^2(t')^2 = (s')^2$$

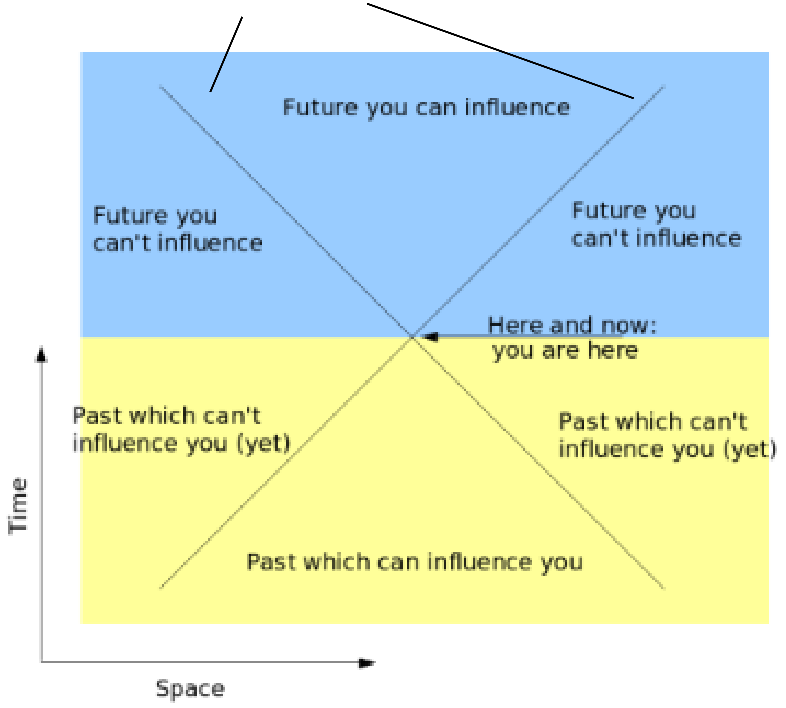
Spacetime Invariants

- If we consider two events, we can determine the quantity Δs^2 between the two events, and we find that it is **invariant** in any inertial frame. The quantity Δ**s** is known as **the spacetime interval** between two events.

Spacetime diagrams



lightlines



Causality: cause-and-effect relations

Invariant: $s^2 = x^2 + y^2 + z^2 - (ct)^2$

For two events

$(x_1, t_1), (x_2, t_2); \Delta x = x_2 - x_1; \Delta t = t_2 - t_1$

Spacetime interval

$$\Delta s^2 = \Delta x^2 - (c\Delta t)^2$$

=

$\Delta s^2 = 0$ lightlike

$\Delta s^2 > 0$ spacelike

$\Delta s^2 < 0$ timelike

Courtesy: Wikimedia Commons and John Walker

Spacetime Invariants

There are three possibilities for the invariant quantity Δs^2:

1) $\Delta s^2 = 0$: $\Delta x^2 = c^2 \Delta t^2$, and the two events can be connected only by a light signal. The events are said to have a **lightlike** separation.

2) $\Delta s^2 > 0$: $\Delta x^2 > c^2 \Delta t^2$, and no signal can travel fast enough to connect the two events. The events are not causally connected and are said to have a **spacelike** separation.

3) $\Delta s^2 < 0$: $\Delta x^2 < c^2 \Delta t^2$, and the two events can be causally connected. The interval is said to be **timelike**.

chapter 2 - 27

Two events occur in an inertial system K at the same time but 4 km apart. What is the time difference measured in a system K' moving parallel to these two events when the distance separation of the events is measured to be 5 km in K'?

space time invariant: $c^2\Delta t'^2 - \Delta x'^2 = c^2\Delta t^2 - \Delta x^2$ (with $\Delta t = 0$)

$$\Delta t'^2 = \frac{\Delta x'^2 - \Delta x^2}{c^2} = \frac{(5000\,m)^2 - (4000\,m)^2}{(3\times10^8\,m/s)^2} = 1\times10^{-10}\,s^2$$

$$\boxed{\Delta t' = 1.0\times10^{-5}\,s}$$

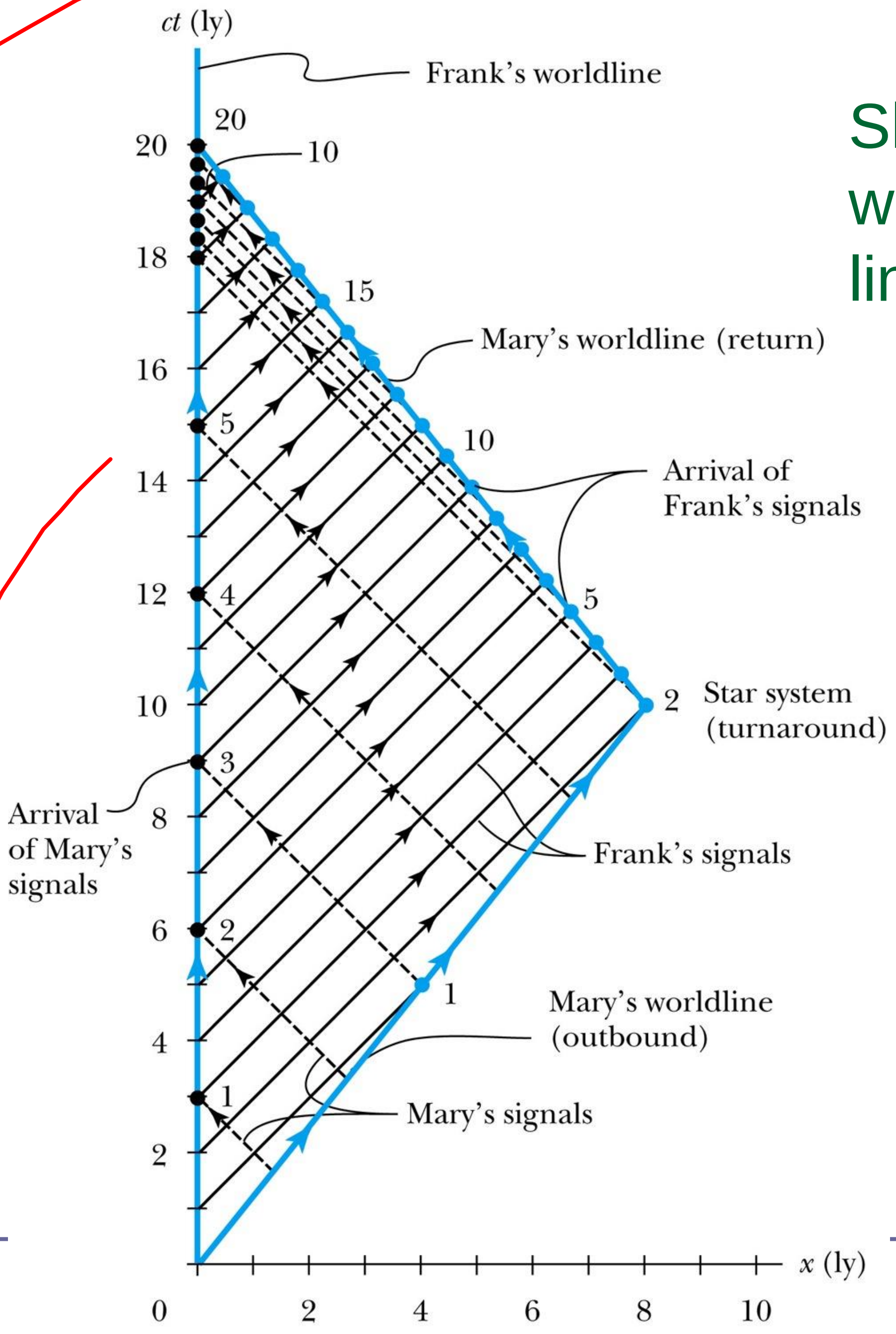
The Twin Paradox in Space-Time

Spaceship's speed=0.8c

Slope of world lines=c/0.8 c

EXAMPLE 2.7

The twins Frank and Mary sent signals at annual intervals and count the number of signals from each other.
Frank sends 20 signals, Mary sends 12 since her clock runs slower due to time dilation.
Mary is 8years younger as Frank at return



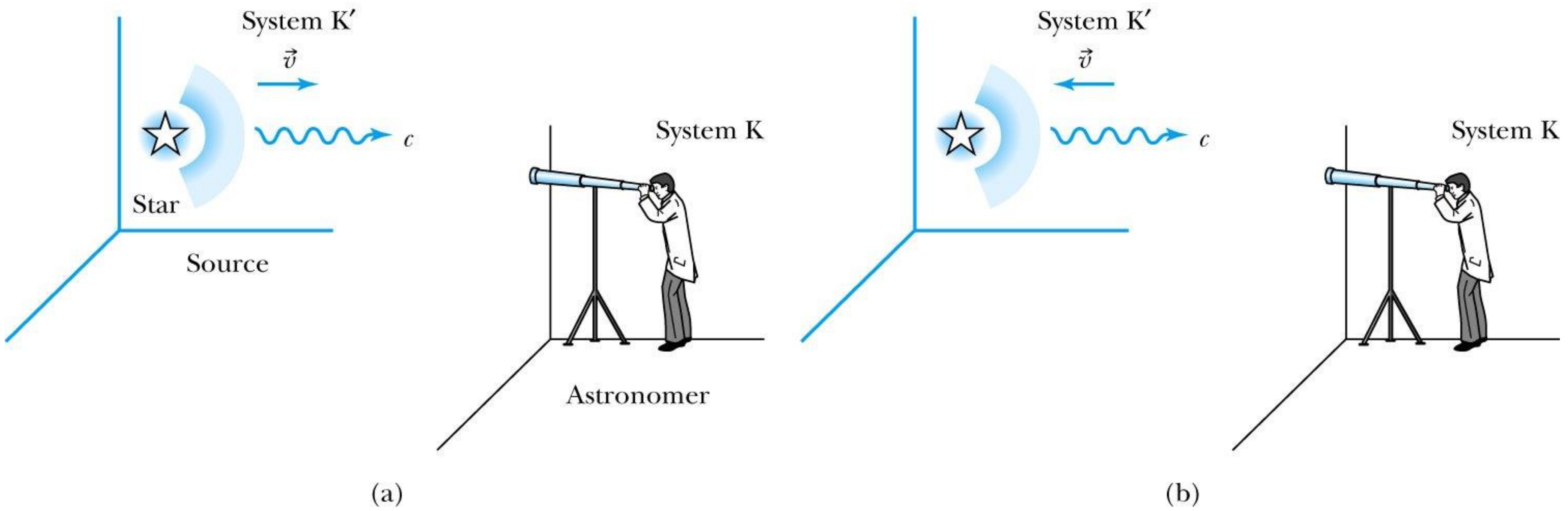
2.10: The Doppler Effect

- The Doppler effect of <u>sound</u> in introductory physics is represented by an *increased frequency* of sound as a source such as a train (with whistle blowing) approaches a receiver (our eardrum) and a *decreased frequency* as the source recedes.

- Also, the same change in sound frequency occurs when the source is fixed and the receiver is moving. The change in frequency of the sound wave depends on whether the source or receiver is moving.

- On first thought it seems that the Doppler effect in sound violates the principle of relativity, until we realize that there is in fact a special frame for sound waves. Sound waves depend on media such as air, water, or a steel plate in order to propagate; however, light does not!

Recall the Doppler Effect



The Relativistic Doppler Effect

Consider a source of light (for example, a star) and a receiver (an astronomer) approaching one another with a relative velocity *v*.

1) Consider the receiver in system K and the light source in system K' moving toward the receiver with velocity *v*.
2) The source emits *n* waves during the time interval *T*.
3) Because the speed of light is always *c* and the source is moving with velocity *v*, the total distance between the front and rear of the wave transmitted during the time interval *T* is:

$$\text{Length of wave train} = cT - vT$$

The Relativistic Doppler Effect (con't)

Because there are *n* waves, the wavelength is given by

$$\lambda = \frac{cT - vT}{n}$$

And the resulting frequency is

$$f = \frac{cn}{cT - vT}$$

The Relativistic Doppler Effect (con’t)

In this frame: $f_0 = n / T'_0$ and

$$T'_0 = \frac{T}{\gamma}$$

Thus:

$$f = \frac{c f_0 T / \gamma}{cT - vT}$$

$$= \frac{1}{1 - v/c} \frac{f_0}{\gamma} = \frac{\sqrt{1 - v^2 / c^2}}{1 - v/c} f_0$$

Source and Receiver Approaching

With $\beta = v / c$ the resulting frequency is given by:

$$f = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0$$

(source and receiver approaching)

Source and Receiver Receding

In a similar manner, it is found that:

$$f = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} f_0$$ (source and receiver receding)

The Relativistic Doppler Effect (con't)

Equations (2.32) and (2.33) can be combined into one equation if we agree to use a + sign for β ($+v/c$) when the source and receiver are approaching each other and a – sign for β ($-v/c$) when they are receding. The final equation becomes

$$f = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0 \qquad \text{Relativistic Doppler effect} \quad (2.34)$$

Chapt. 2 — 51

A space craft travelling out of the solar system at a speed of 0.95c sends back information at a rate of 1400 kHz. At what rate do we receive the information?

$$f = f_0 \sqrt{\frac{1-\beta}{1+\beta}} = 1400\,\text{kHz} \sqrt{\frac{1-0.95c}{1+0.95c}} = 224\,\text{kHz}$$

source and receiver receding! should be lower!

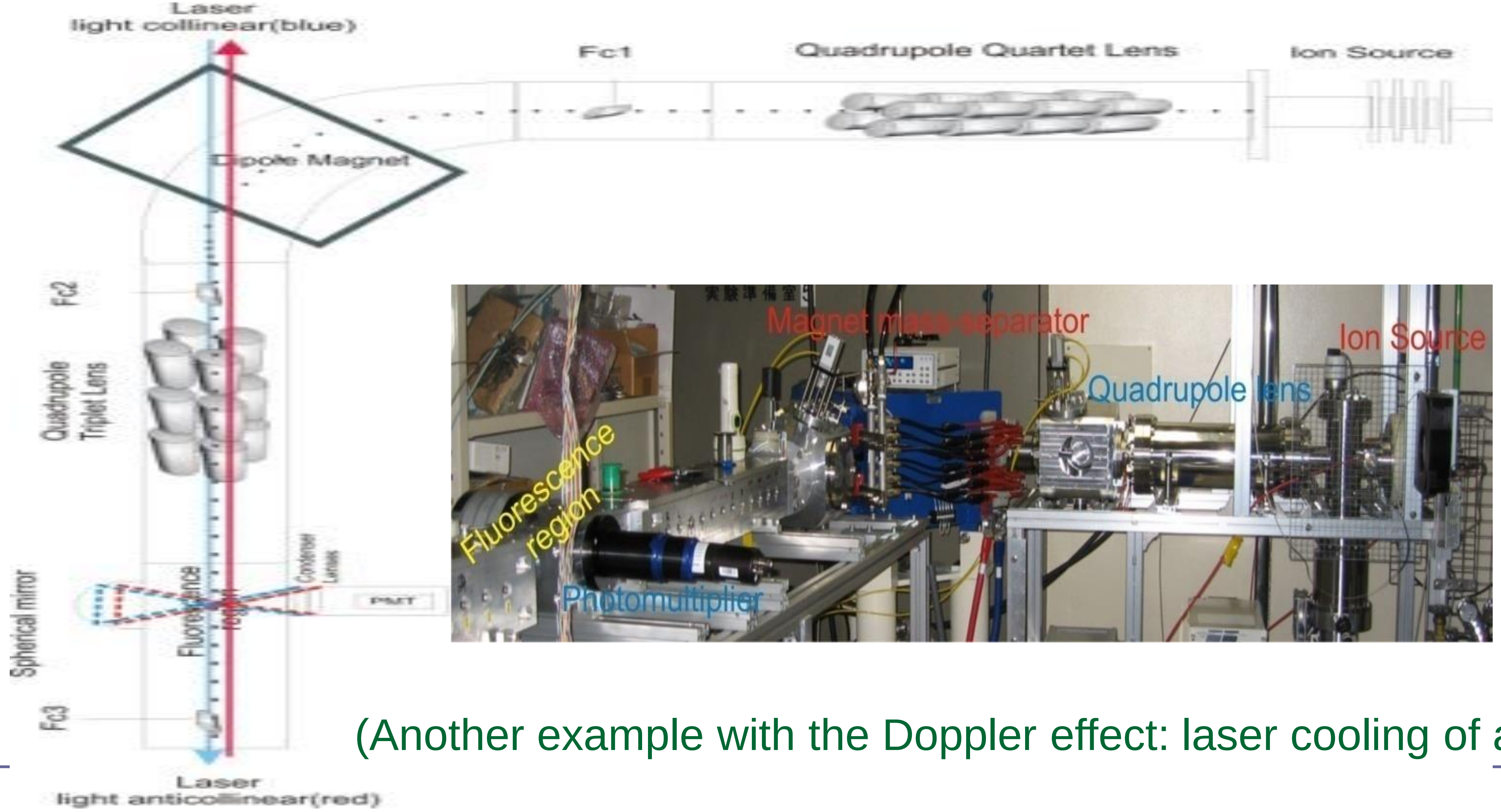
chapter 2 – 55

A particle having a speed of ~~0.5c~~ 0.92c has a momentum p of 10^{-16} kgm/s. What is its mass?

$$p = \gamma m v \quad \text{with } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.92^2}} = 2.5516$$

$$m = \frac{p}{\gamma v} = \frac{10^{-16}\,\text{kg m/s}}{(2.5516)(0.92)\,3\times 10^8\,\frac{m}{s}} = \boxed{1.42 \times 10^{-25}\,\text{kg}}$$

EXAMPLE: DOPPLER EFFECT IN FAST ION BEAM PRECISION LASER SPECTROSCOPY IN COLLINEAR AND ANTI-COLLINEAR GEOMETRIES



(Another example with the Doppler effect: laser cooling of atoms)

Exclusion of relativistic frequency shifts by combining collinear and anticollinear measurements

Frequency of light perceived by a moving ion $\nu' = \frac{\nu\,[1-\beta\cos(\theta)]}{\sqrt{1-\beta^2}}$ $\beta = v/c$

Laser tuned to resonance; the perceived frequency equals the transition frequency $\nu' = \nu_{tr}$

Thus, the resonance frequencies for collinear geometry $(\theta = 0^\circ)$ $\nu_c = \sqrt{\frac{1+\beta}{1-\beta}}\,\nu_{tr}$

and for anticollinear geometry $(\theta = 180^\circ)$ $\nu_{ac} = \sqrt{\frac{1-\beta}{1+\beta}}\,\nu_{tr}$

To obtain the transition frequency we take the product
and this is an exact relativistic formula!

$$\nu_{tr} = \sqrt{\nu_c \nu_{ac}}$$

Another example with Doppler effect: laser cooling of atoms

Ion trap basics

My teachers
Nobel price 1989

- Ions move in a time-averaged harmonic pseudo potential, created by AC electric fields

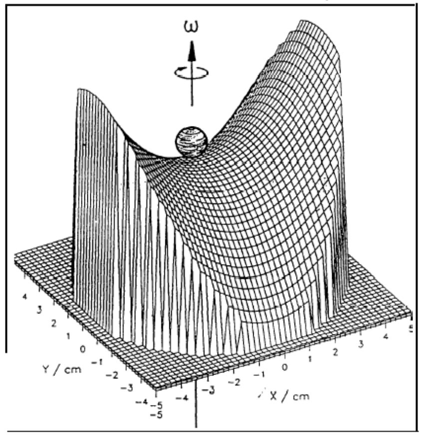
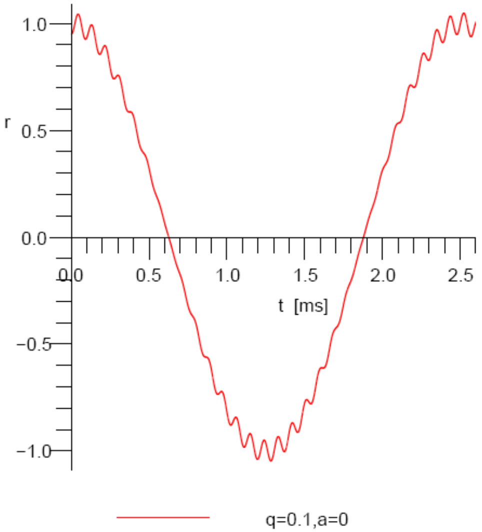


Figure 8. Mechanical analogue model for the ion trap with steelball as "particle"

$$\rho_i(t) = \rho_{0i}\cos(\omega_{\sec,i}t + \phi_i)\left(1 + \frac{q_i}{2}\cos\Omega_{RF}t\right)$$

$$\omega_{\sec,i} = \frac{1}{2}\Omega_{RF}\sqrt{a_i + \frac{1}{2}q_i^2} \qquad q_i := \frac{Q}{m}\frac{2U_{RF}}{\Omega_{RF}^2 r_0^2 L_i}, \qquad i \in \{x, y\}$$

- Secular motion: Characteristic oscillation in the trap potential
- Micro motion: Oscillation with driving RF frequency



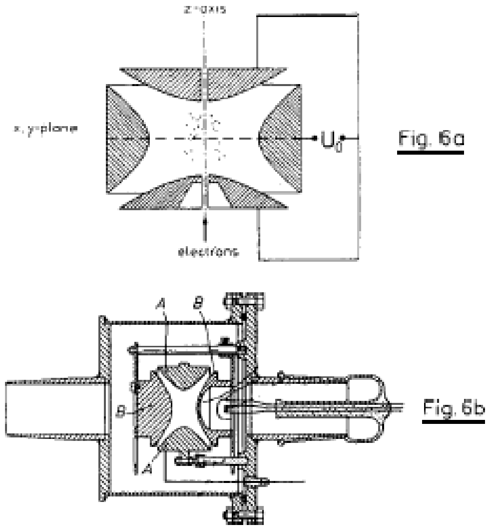
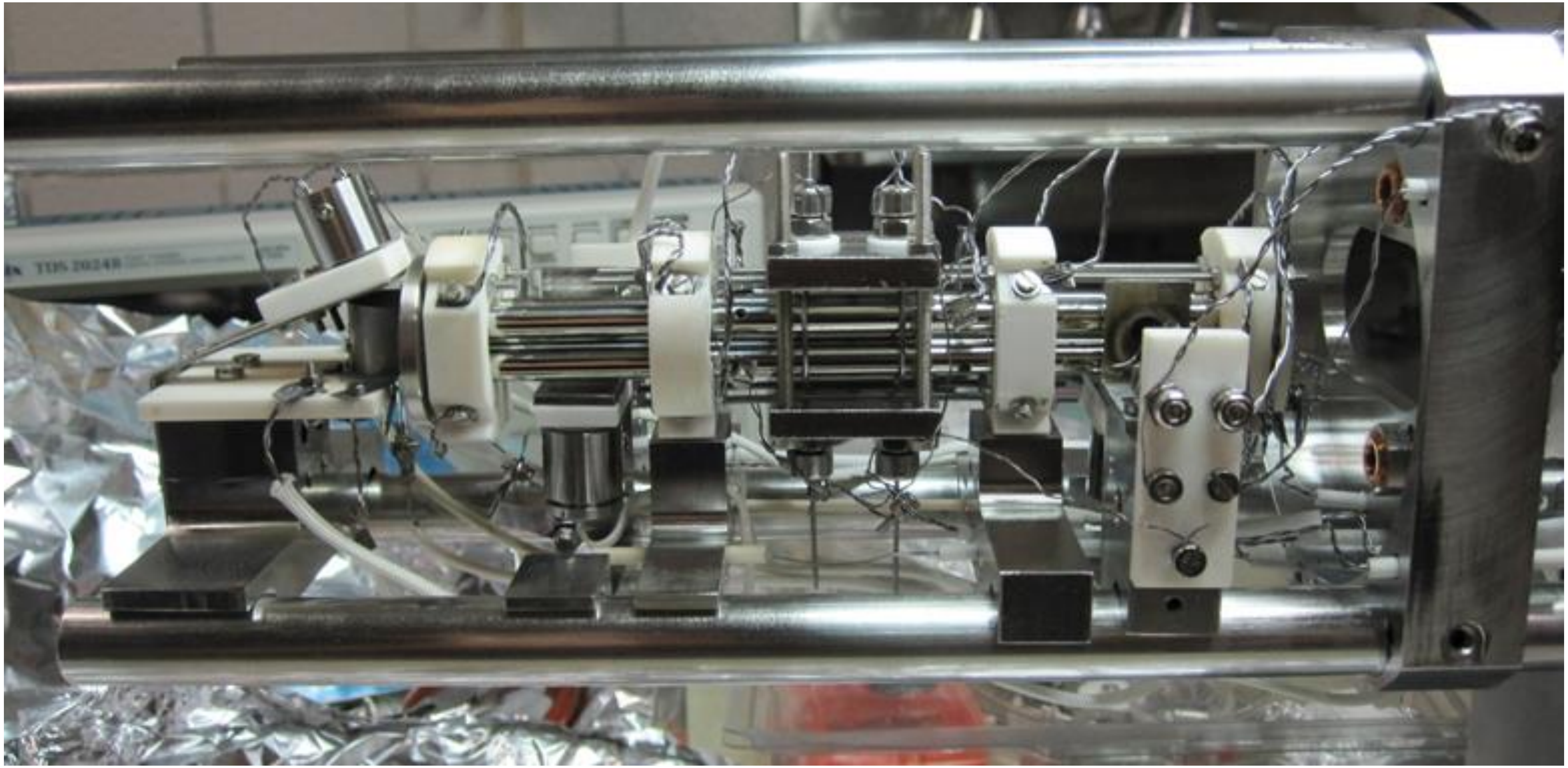
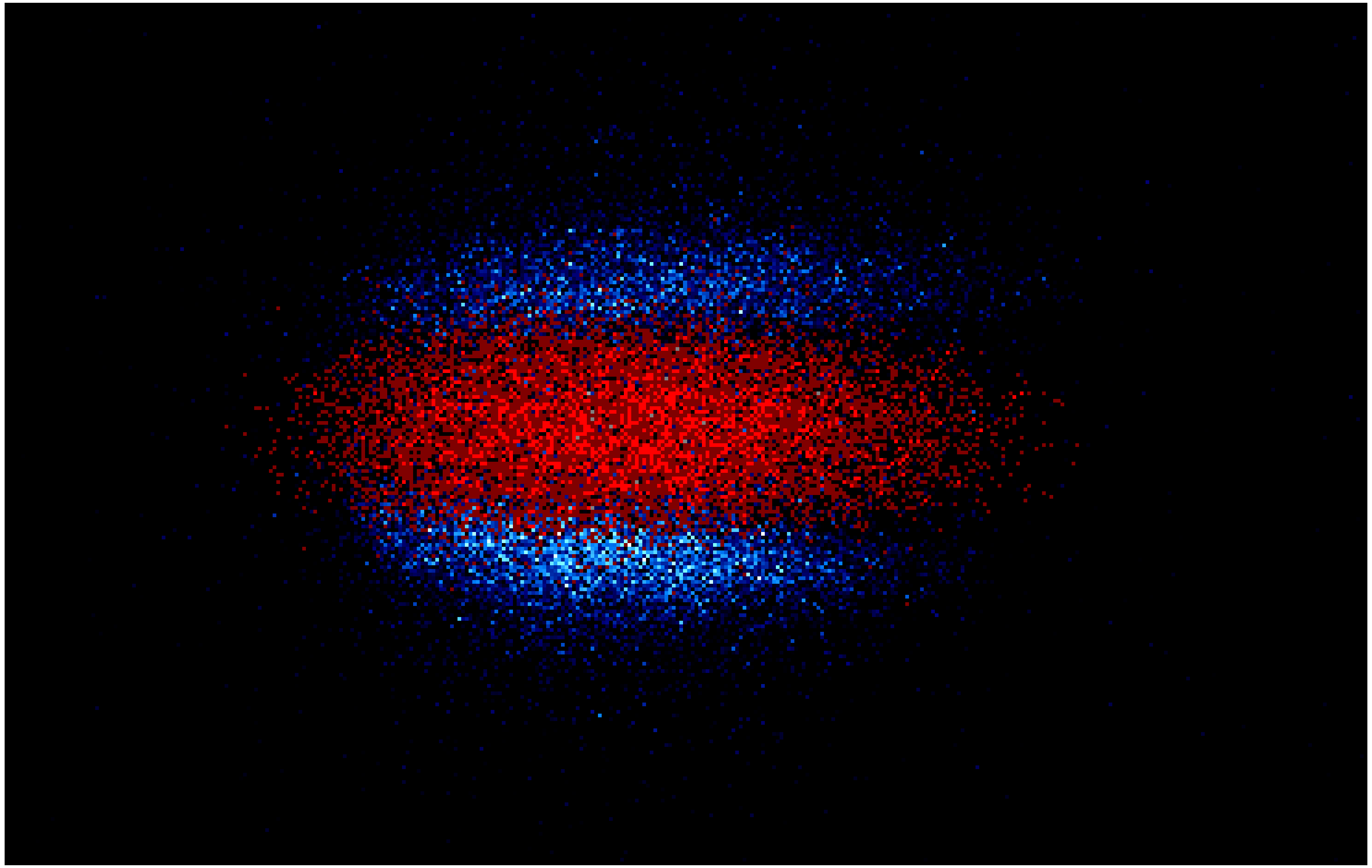


Figure 6. a) Schematic view of the ion trap. b) Cross section of the first trap (1955).

Trapping ions: ion trap picture

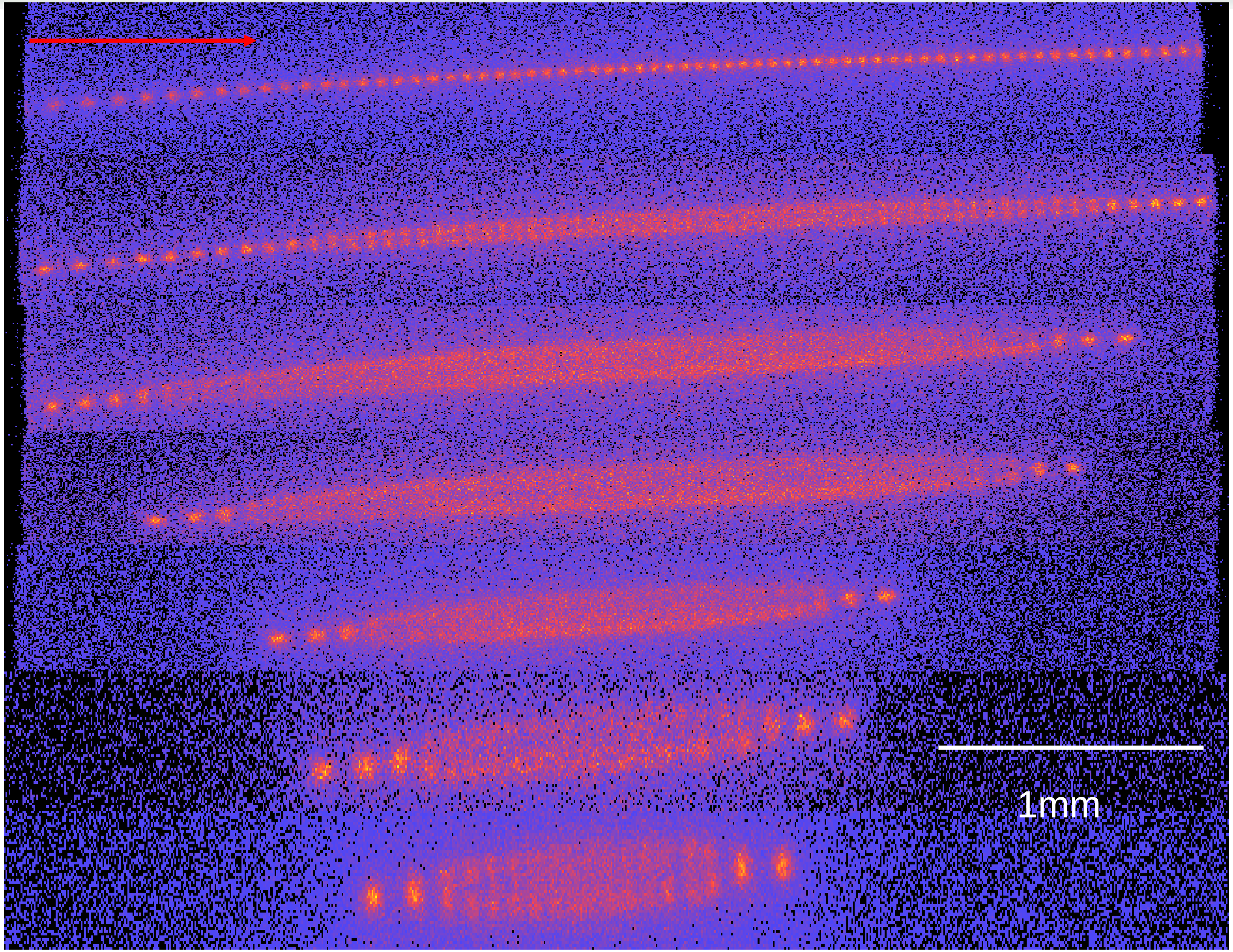


Tamu



Large $^{24}Mg^{+}$ - $^{26}Mg^{+}$ ion crystal ($N \sim 10^{4}$)

Structure phase transition (N~70)



α~ 0.001

α~ 0.004

α~ 0.02

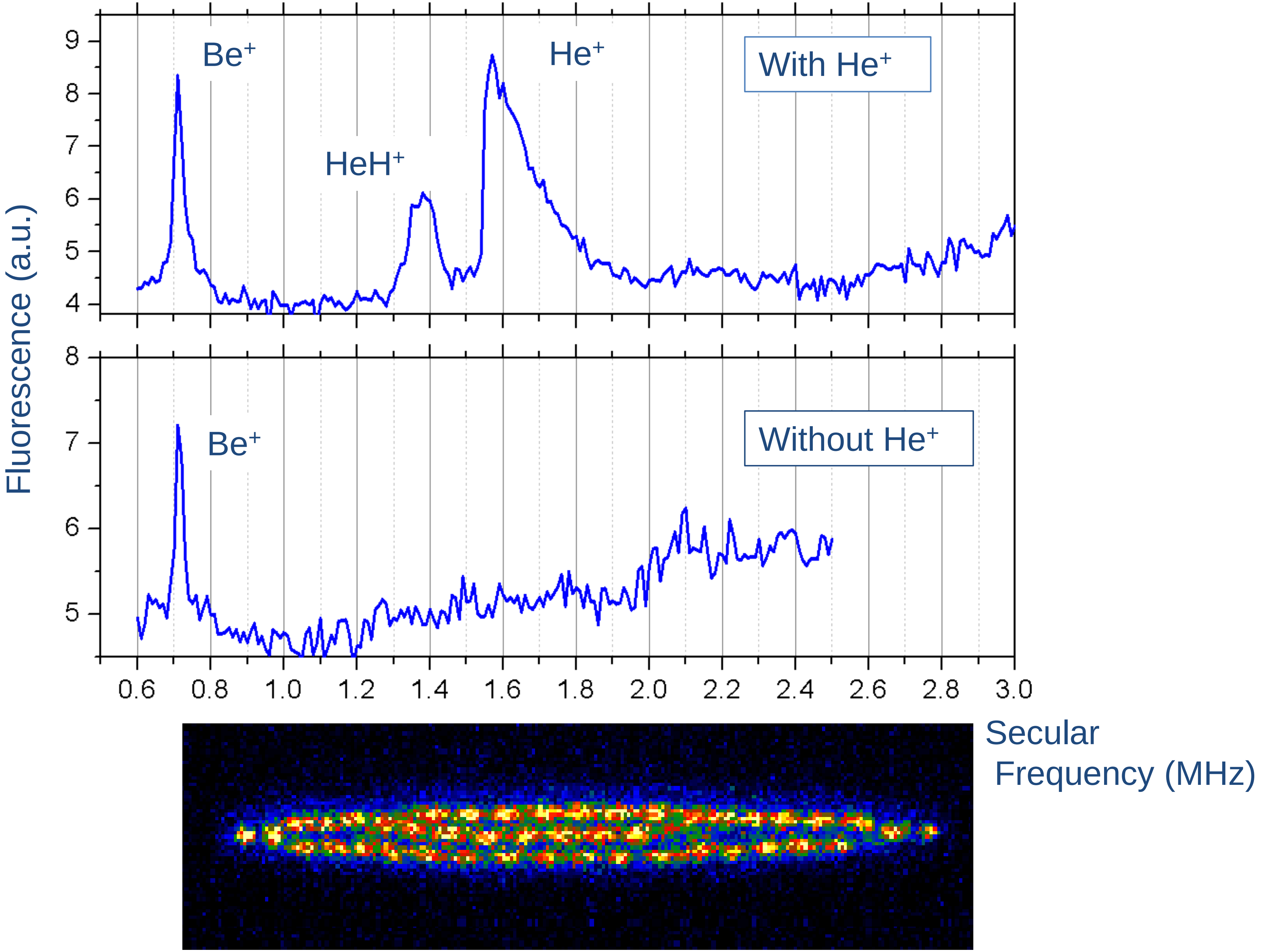
α~ 0.08

α~ 0.12

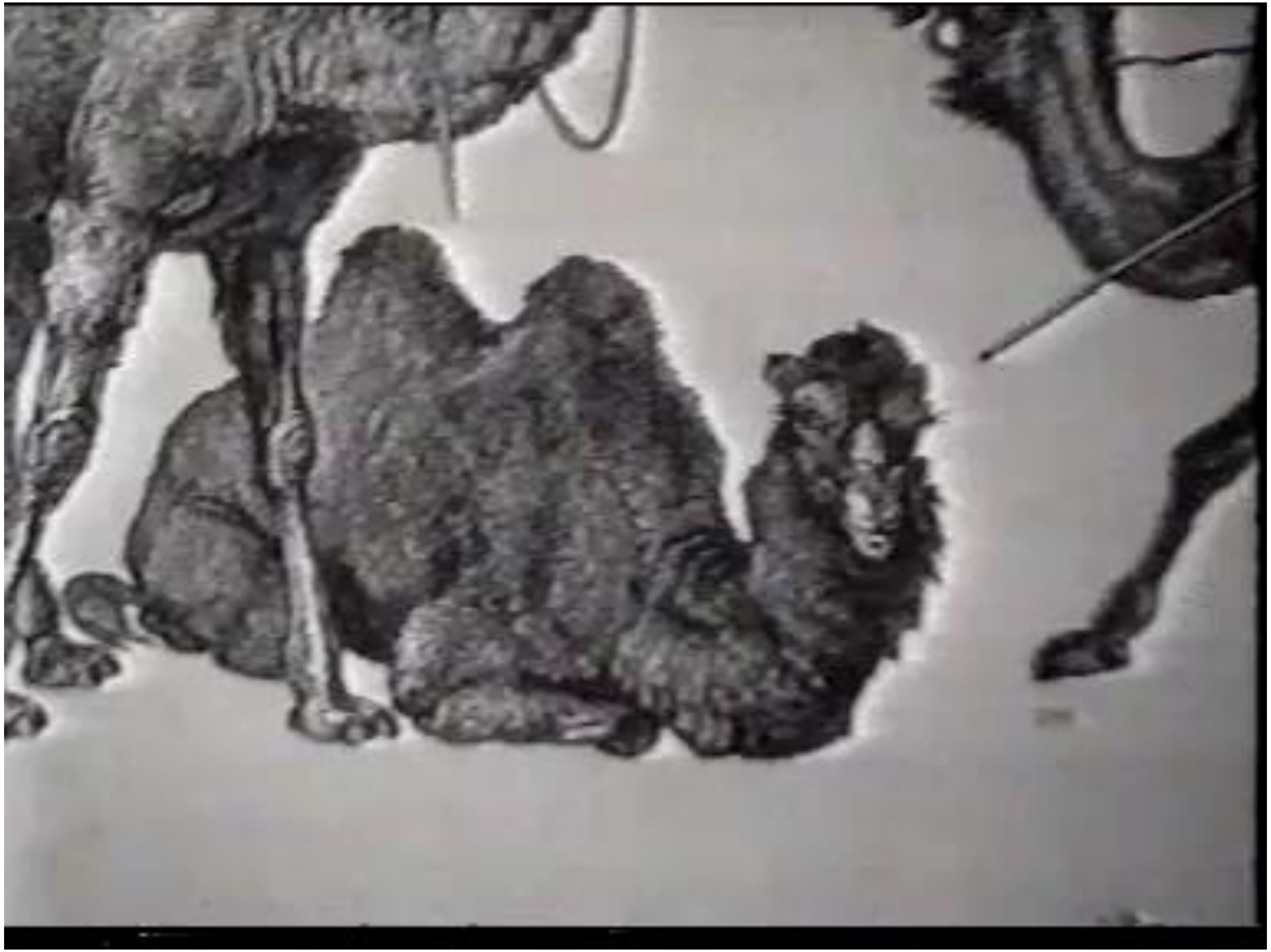
α~ 0.20

α~ 0.24

Be^+-He^+ mixed crystal: Secular excitation

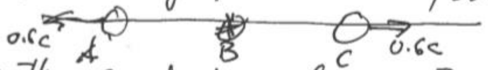


Ion trap video

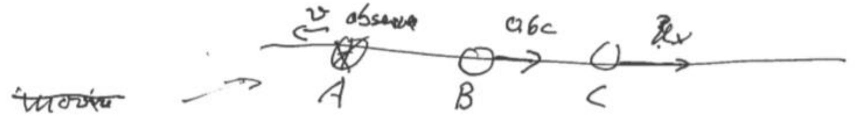


Problems: velocity addition

Three galaxies are aligned along an axis in the order A, B, C. An observer in B is in the middle and sees that galaxies A and C are moving in opposite directions away from him, both with speeds 0.6c



What is the speed of galaxies B and C as observed by some one in A?



speed of B is $v = 0.6c$

To find the speed of C use $u_x = 0.6c$ and $v = -0.6c$

$$u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}} = \frac{0.6c - (-0.6c)}{1 - \frac{(0.6c)(0.6c)}{c^2}} = \boxed{0.88c}$$

2.6
#31

A spaceship is moving at a speed of $0.84c$ away from an observer at rest. A boy in the spaceship shoots a proton gun with protons having a speed of $0.62c$. What is the speed of the protons measured by the observer at rest when the gun is shot (a) away from the observer and (b) toward the observer?

31. $$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}$$

(a) $$u_x = \frac{0.62c + 0.84c}{1 + \frac{(0.62c)(0.84c)}{c^2}} = \frac{1.46c}{1.52} = \boxed{0.96c}$$

(b) $$u_x = \frac{-0.62c + 0.84c}{1 + \frac{(-0.62c)(0.84c)}{c^2}} = \frac{0.22c}{0.48c} = \boxed{0.46c}$$

2.6
#32

A proton and an antiproton are moving toward each other in a head-on collision. If each has a speed of 0.8c with respect to the collision point, how fast are they moving with respect to each other?

Solution: use velocity addition $u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$ with $v = -0.8c$, $u_x = 0.8c$

$$u_x' = \frac{0.8c - (-0.8c)}{1 - \frac{(-0.8c)(0.8c)}{c^2}} = \frac{1.6c}{1.64} = \boxed{0.976c}$$

Thank you for your attention!