L6bCH2 09-15-2021

Total Energy and Rest Energy, Mass-energy Equivalence

Binding Energy

- The equivalence of mass and energy becomes apparent when we study the binding energy of systems like atoms and nuclei that are formed from individual particles.

- The potential energy associated with the force keeping the system together is called the **binding energy** E_B.

Total Energy and Rest Energy, Mass-energy Equivalence

We rewrite the energy equation in the form

$$\gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = K + mc^2 \qquad (2.63)$$

The term mc^2 is called the rest energy and is denoted by E_0.

$$E_0 = mc^2 \qquad (2.64)$$

This leaves the sum of the kinetic energy and rest energy to be interpreted as the total energy of the particle. The total energy is denoted by E and is given by

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = \frac{E_0}{\sqrt{1-u^2/c^2}} = K + E_0 \qquad (2.65)$$

Relationship of Energy and Momentum

$$p = \gamma m u = \frac{mu}{\sqrt{1 - u^2/c^2}}$$

We square this result, multiply by c^2, and rearrange the result.

$$p^2c^2 = \gamma^2 m^2 u^2 c^2$$

$$= \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2}\right) = \gamma^2 m^2 c^4 \beta^2$$

We use the equation for γ to express β^2 and find

Expressing β through γ

$$\gamma^2 = 1/(1-\beta^2) \quad \beta^2 = (\gamma^2 - 1)/\gamma^2 = 1 - 1/\gamma^2$$

$$p^2c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right)$$

$$= \gamma^2 m^2 c^4 - m^2 c^4$$

Energy and Momentum

The first term on the right-hand side is just E^2, and the second term is E_0^2. The last equation becomes

$$p^2c^2 = E^2 - E_0^2$$

We rearrange this last equation to find the result we are seeking, a relation between energy and momentum.

$$E^2 = p^2c^2 + E_0^2 \qquad (2.70)$$

or

$$E^2 = p^2c^2 + m^2c^4 \qquad (2.71)$$

Equation (2.70) is a useful result to relate the total energy of a particle with its momentum. The quantities ($E^2 - p^2c^2$) and m are invariant quantities. Note that when a particle's velocity is zero and it has no momentum, Equation (2.70) correctly gives E_0 as the particle's total energy.

Useful formulas

$\beta = pc / E$ from $p = \gamma m u$ and $E = \gamma m c^2$

$$p = \frac{1}{c}(E^2 - E_0{}^2)^{1/2} = \frac{E}{c}\left[1 - \left(\frac{E_0}{E}\right)^{1/2}\right]$$

$$\beta = \left[1 - \left(\frac{E_0}{E}\right)^2\right]^{1/2} = \left(1 - \frac{1}{\gamma^2}\right)^{1/2}$$

2.13: Computations in Modern Physics

- We were taught in introductory physics that the international system of units is preferable when doing calculations in science and engineering (“everyday” scales).
- In modern physics a somewhat different set of units is often used, which is more convenient for problems considered in modern physics.
- The smallness of quantities often used in modern physics suggests the need for some new units more practical for smaller scales .

Units of Work and Energy

- Recall that the work done in accelerating a charge through a potential difference is given by $W = qV$.
- For a proton, with the charge $e = 1.602 \times 10^{-19}$ C being accelerated across a potential difference of 1 V, the work done on the particle is

  $$W = (1.602 \times 10^{-19}\,\mathrm{C})(1\ \mathrm{V}) = 1.602 \times 10^{-19}\ \mathrm{J}$$

The Electron Volt (eV)

- The work done to accelerate the proton across a potential difference of 1 V could also be written as

$$W = (1\ e)(1\ \text{V}) = 1\ \text{eV}$$

- Thus eV, pronounced "electron volt," is also a unit of energy. It is related to the SI (*Système International*) unit joule by the 2 previous equations.

$$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$$

Other Units

1) Rest energy of a particle:
   Example: E_0 (proton)

$$E_0(\text{proton}) = (1.67\times10^{-27}\ \text{kg})(3\times10^{8}\ \text{m/s})^2 = 1.50\times10^{-10}\ \text{J}$$

$$= 1.50\times10^{-10}\ \text{J}\ \frac{1\,\text{eV}}{1.602\times10^{-19}\ \text{J}} = 9.38\times10^{8}\ \text{eV}$$

2) **Atomic mass unit** (amu):
   Example: carbon-12

$$\text{Mass } (^{12}\text{C atom}) = \frac{12\ \text{g/mol}}{6.02\times10^{23}\ \text{atoms/mol}}$$

$$= 1.99\times10^{-23}\ \text{g/atom}$$

$$\text{Mass } (^{12}\text{C atom}) = 1.99\times10^{-26}\ \text{kg} = 12\ \text{u/atom}$$

1 u=1.66x10-27 kg=931.5 MeV/c^2

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What is the kinetic energy of (a) an electron having a momentum of 40 GeV/c ? (b) a proton having a momentum of 40 GeV/c

(a) $E^2 = p^2c^2 + E_0^2$

$E = \sqrt{p^2c^2 + E_0^2} = \sqrt{\left(\frac{40\text{ GeV}}{c}\right)^2 c^2 + (511\text{ keV})^2}$

$= \sqrt{(40\text{ GeV})^2 + (511\text{ keV})^2} \approx 40.0\text{ GeV}$

$K = E - E_0 = 40.0\text{ GeV}$

(b) $E = \sqrt{p^2c^2 + E_0^2} = \sqrt{(40\text{ GeV})^2 + (0.938\text{ GeV})^2} = 40.011\text{ GeV}$

$K = E - E_0 = (40.011 - 0.938)\text{ GeV}$

$K = 39.07\text{ GeV}$

94. An electron has a total energy that is 200 times its rest energy. Determine (a) its kinetic energy (b) its speed (c) its momentum

(a) $K = E - E_0$

$K = \gamma m_0 c^2 - m_0 c^2$

↑ total energy ↑ rest energy

$K = (200-1)E_0 = 199 E_0 = 199\,(511\,keV) = \boxed{102\,MeV}$

(b) $\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.9999875$ $\quad \boxed{v = 0.9999875\,c}$

(c) $p^2c^2 = E^2 - E_0^2 \qquad E^2 = p^2c^2 + E_0^2 \qquad p = \frac{\sqrt{E^2 - E_0^2}}{c}$

$$p = \frac{\sqrt{(200 \times 511\,keV)^2 - (511\,keV)^2}}{c} = \boxed{102\,\frac{MeV}{c}}$$

Binding Energy

The binding energy is *the difference between the rest energy of the individual particles and the rest energy of the combined bound system*.

The binding energy is the work required to pull the particles out of the bound system into separate, free particles at rest

Conservation of energy

$$M_{\text{bound system}}c^2 + E_B = \sum_i m_i c^2$$

$$E_B = \sum_i m_i c^2 - M_{\text{bound system}}c^2$$

Example:

deuteron 

proton $E_0 = 1.007276\,c^2 u = 1.007276\,c^2 u \times \frac{931.494\ \text{MeV}}{c^2 \cdot u}$ (factor of unity)

$= 938.7\ \text{MeV}$

What is the deuteron's binding energy?

$$\boxed{E_B = \sum m_i c^2 - M_{bound} c^2}$$

$$E_B(^2H) = 938.27\ \text{MeV} + 939.57\ \text{MeV} - 1875.61\ \text{MeV} = 2.23\ \text{MeV}$$

~~Exam~~

neutron $E_0 = 1.008665\,c^2 u \cdot \frac{931.494/c^2}{1u} = 939.57\ \text{MeV}$

deuteron $E_0 = 2.01355\,c^2 u\ \frac{931.494/c^2}{1u} = 1875.61\ \text{MeV}$

Why can we neglect the 13.6 eV binding energy of the electron in the atomic mass (^1H = hydrogen) when working out as done above the binding energy of the deuteron?

Answer $13.6\ \text{eV} \leq 939\ \text{MeV}$
$10^1 \leq 10^9$ } 8 orders of magnitude difference

13.6: Fusion

- Similar to the energy emitted by stars, if two light nuclei fuse together, they also form a nucleus with a larger binding energy per nucleon and energy is released. This reaction is called **nuclear fusion**.

- The most energy is released if two isotopes of hydrogen fuse together in the reaction.

$$^{2}\mathrm{H} + {}^{3}\mathrm{H} \rightarrow n + {}^{4}\mathrm{He} \qquad Q = 17.6\ \mathrm{MeV}$$

Problem 85,Ch2

The reaction $^2H + {^3H} \rightarrow n + {^4He}$ is one of the reactions useful for producing energy through nuclear fusion. (a) Assume the deuterium (^2H) and tritium (^3H) nuclei are at rest and use the atomic mass units of the masses in Appendix 8 to calculate the mass-energy imbalance in this reaction. This amount of energy is given up when this nuclear reaction occurs. (b) What percentage of the initial rest energy is given up?

Fusion is a clean and efficient energy source

Problem 85,Ch2 (solution)

$$E_B = \sum_i m_i c^2 - M_{bound system} c^2$$ Binding energy

$^2H + {}^3H \rightarrow n + {}^4He$ produces energy through fusion

85 (a) The mass energy imbalance occurs because ^4He is more tightly bound than deuterium (^2H) and tritium (^3H) nuclei

$$\Delta E = \{[m(^2H) + m(^3H)] - [m_n + m(^4He)]\} c^2$$

$$= [(2.014102\,u + 3.016029\,u) - (1.008665 + 4.002603)] c^2 \frac{931.5\,MeV}{c^2\,u}$$

$$= 17.6\ MeV$$

(b) The initial rest energy is $[m(^2H) + m(^3H)] c^2 = (5.030131\,u) c^2$ ~~931.5~~ $\times \frac{931.5\,MeV}{c^2\,u}$

$$= 4686\,MeV$$

4686 MeV — 100%
17.6 MeV — ?

$$\frac{100 \times 17.6}{4686} = \boxed{0.37\%}$$

Two high energy protons hit each other headon

Solution (a) We use $K = 2.00$ GeV and the proton rest energy, 938 MeV, to find the total energy from Equation (2.65),

$$E = K + E_0 = 2.00 \text{ GeV} + 938 \text{ MeV} = 2.938 \text{ GeV}$$

The momentum is determined from Equation (2.70).

$$p^2c^2 = E^2 - E_0^2 = (2.938 \text{ GeV})^2 - (0.938 \text{ GeV})^2$$
$$= 7.75 \text{ GeV}^2$$

The momentum is calculated to be

$$p = \sqrt{7.75(\text{GeV}/c)^2} = 2.78 \text{ GeV}/c$$

Notice how naturally the unit of GeV/c arises in our calculation.

In order to find β we first find the relativistic factor γ. There are several ways to determine γ; one is to compare the rest energy with the total energy. From Equation (2.65) we have

$$E = \gamma E_0 = \frac{E_0}{\sqrt{1 - u^2/c^2}}$$

$$\gamma = \frac{E}{E_0} = \frac{2.938 \text{ GeV}}{0.938 \text{ GeV}} = 3.13$$

We use Equation (2.62) to determine β.

$$\beta = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} = \sqrt{\frac{3.13^2 - 1}{3.13^2}} = 0.948$$

The speed of a 2.00-GeV proton is $0.95c$ or 2.8×10^8 m/s.

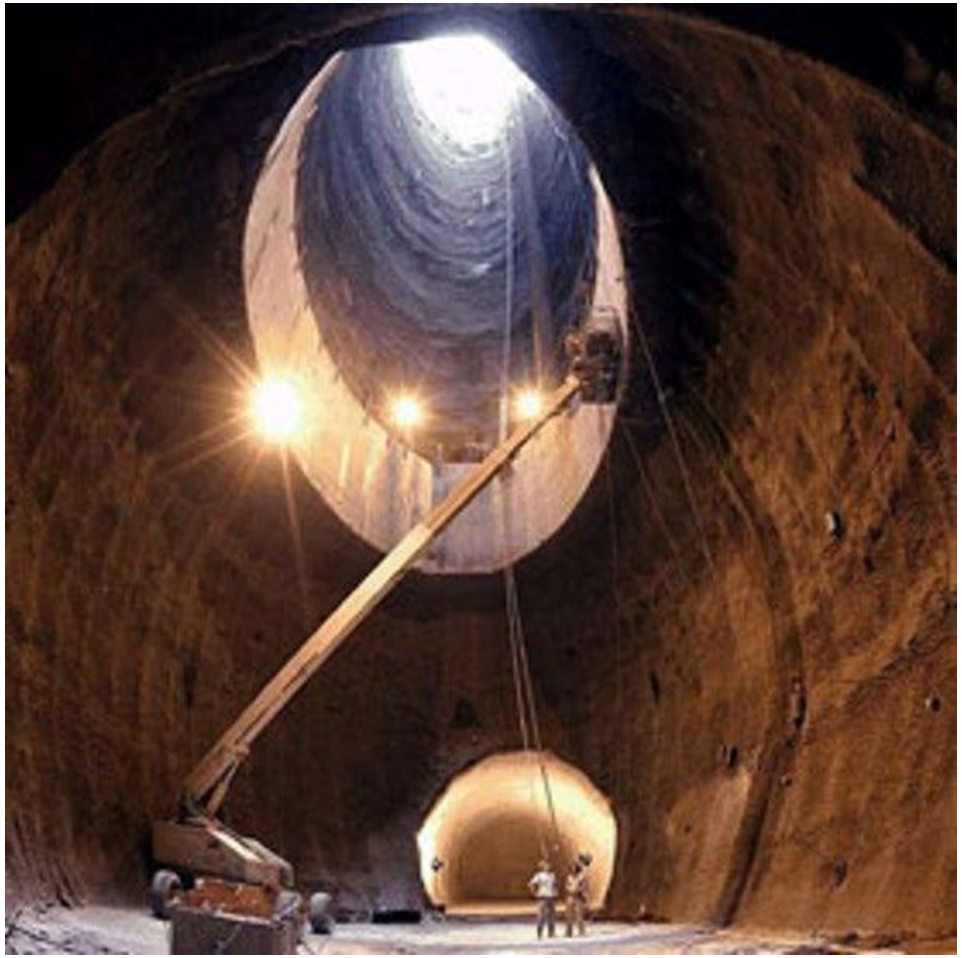
(b) When the two protons collide head-on, the situation is similar to the case when the two blocks of wood collided head-on with one important exception. The time for the two protons to interact is less than 10^{-20} s. If the two protons did momentarily stop at rest, then the two-proton system would have its mass increased by an amount given by Equation (2.68), $2K/c^2$ or 4.00 GeV/c^2. The result would be a highly excited system. In fact, the collision between the protons happens very quickly, and there are several possible outcomes. The two protons may either remain or disappear, and new additional particles may be created. Two of the possibilities are

$$p + p \rightarrow p + p + p + \overline{p} \qquad (2.78)$$

$$p + p \rightarrow \pi^+ + d \qquad (2.79)$$

where the symbols are p (proton), $\overline{p}$ (antiproton), π (pion), and d (deuteron). We will learn more about the possibilities later when we study nuclear and particle physics. Whatever happens must be consistent with the conservation laws of charge, energy, and momentum, as well as with other conservation laws to be learned. Such experiments are routinely done in particle physics. In the analysis of these experiments, the equivalence of mass and energy is taken for granted.

The Texas Supercollider That never was Waxahachie



The aborted Superconducting Super Collider (SSC) in Texas would have had a circumference of 87 km. Construction was started in 1991, but abandoned in 1993

Supercollider at CERN

Aerial photograph representing the Large Hadron Collider, with the border between France and Switzerland indicated by a dashed line. The 27 km LHC tunnel cannot be seen, because it is 45 to 175 meters underground, but it is represented by the large circle