Chapter 13 Fluid Mechanics

Fluid statics = equilibrium situations
Fluid dynamics = fluids in motion

$$\boxed{\text{Density} = \rho = \frac{m}{V} = \frac{mass}{volume}\ [\text{kg/m}^3]}$$

<u>The density ρ has a wide range</u>

Osmium = $22.5\text{x}10^3$ kg/m^3
Gold = $19.3\text{x}10^3$ kg/m^3
Lead = $11.3\text{x}10^3$ kg/m^3
Air (gases) = 1.2 kg/m^3
Water = 1.0 kg/m^3
Ice = 0.92 kg/m^3

Chapter 13 Fluid Mechanics

- 13.1 To understand density.
- 13.2 To understand pressure in a fluid.
- 13.3 To apply Archimedes principle of buoyancy.

Sections not covered:

- 13.4 To understand surface tension and capillary action.
- 13.5 To understand fluid flow
- 13.6 To Bernoulli’s equation.
- 13.7 The application of Bernoulli’s equation.
- 13.8 To undersand turbulence and viscosity in real fluids.

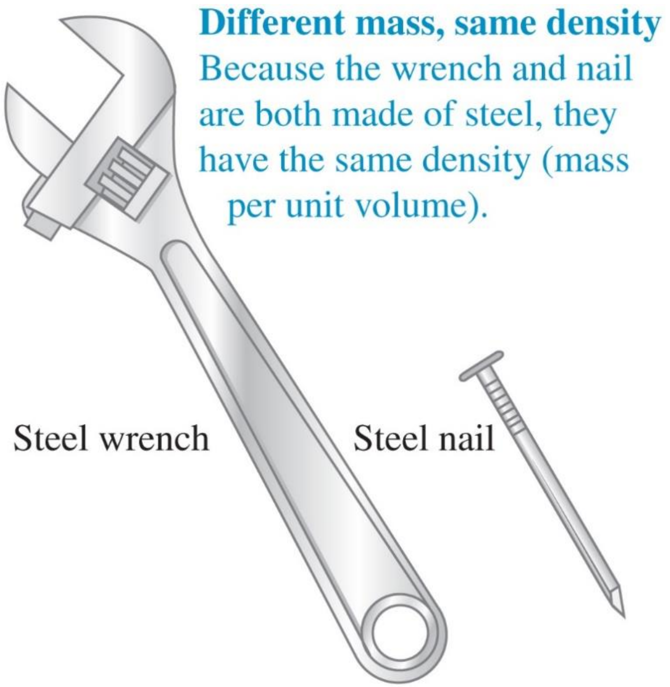
Courtesy of Wenhao Wu

Goals for Chapter 13

- To study density and pressure in a fluid withPascal's Law
- To apply Archimedes principle of buoyancy.

13.1 Density

- Mass is an extensive quantity.
- Density: mass per unit volume
  - Density $\rho = \frac{m}{V}$ (kg/m^3)
  - Density is an intensive quantity



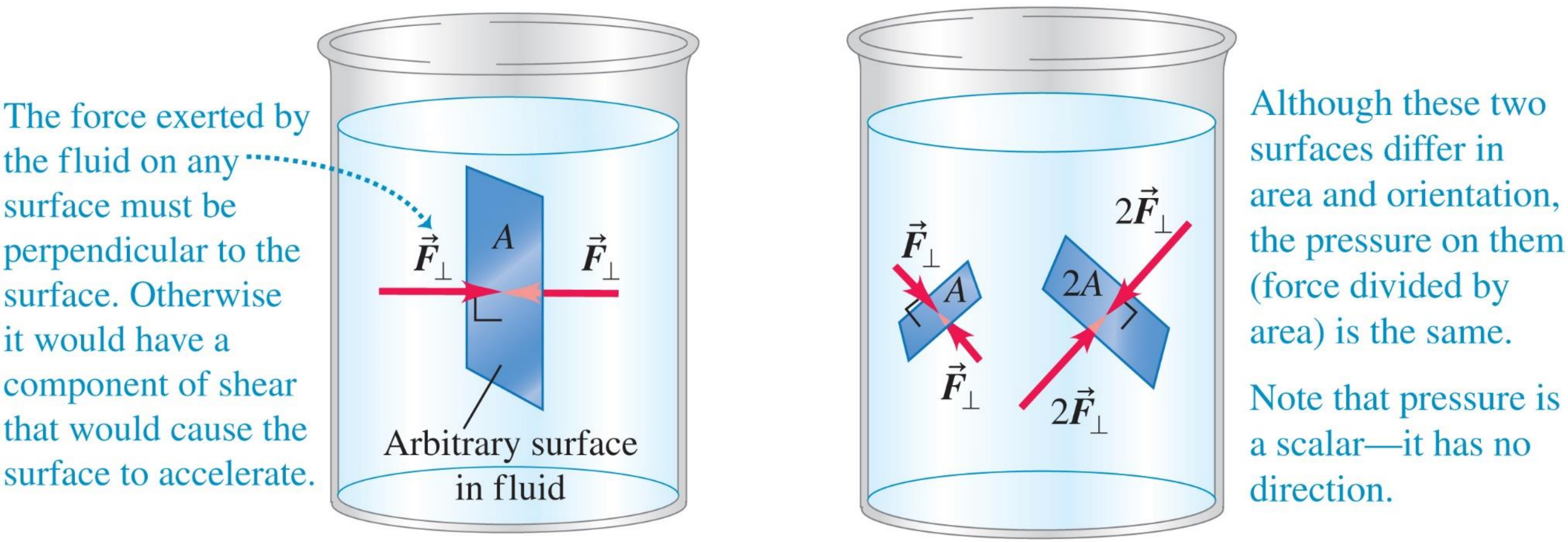
Courtesy of Wenhao Wu

TABLE 13.1 Densities of some common substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Gas		Concrete	2.0×10^3
Air (1 atm, 20°C)	1.20	Aluminum	2.7×10^3
		Iron, steel	7.8×10^3
Liquids		Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ethanol	0.81×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Gold	19.3×10^3
Blood	1.06×10^3	Platinum	21.4×10^3
Mercury	13.6×10^3		
		Astrophysical	
Solids		White-dwarf star	10^{10}
Glycerin	1.26×10^3	Neutron star	10^{18}
Ice	0.92×10^3		

*To obtain the densities in grams per centimeter, simply divide by 10^3.

13.2 Pressure in a Fluid



(a) Why forces due to fluid pressure must act perpendicular to a surface

(b) Pressure equals force divided by area.

Courtesy of Wenhao Wu

13.2 Pressure in a Fluid

- Pressure: The perpendicular component of the force acting on a surface divided of the area. Or, the perpendicular force per unit area.

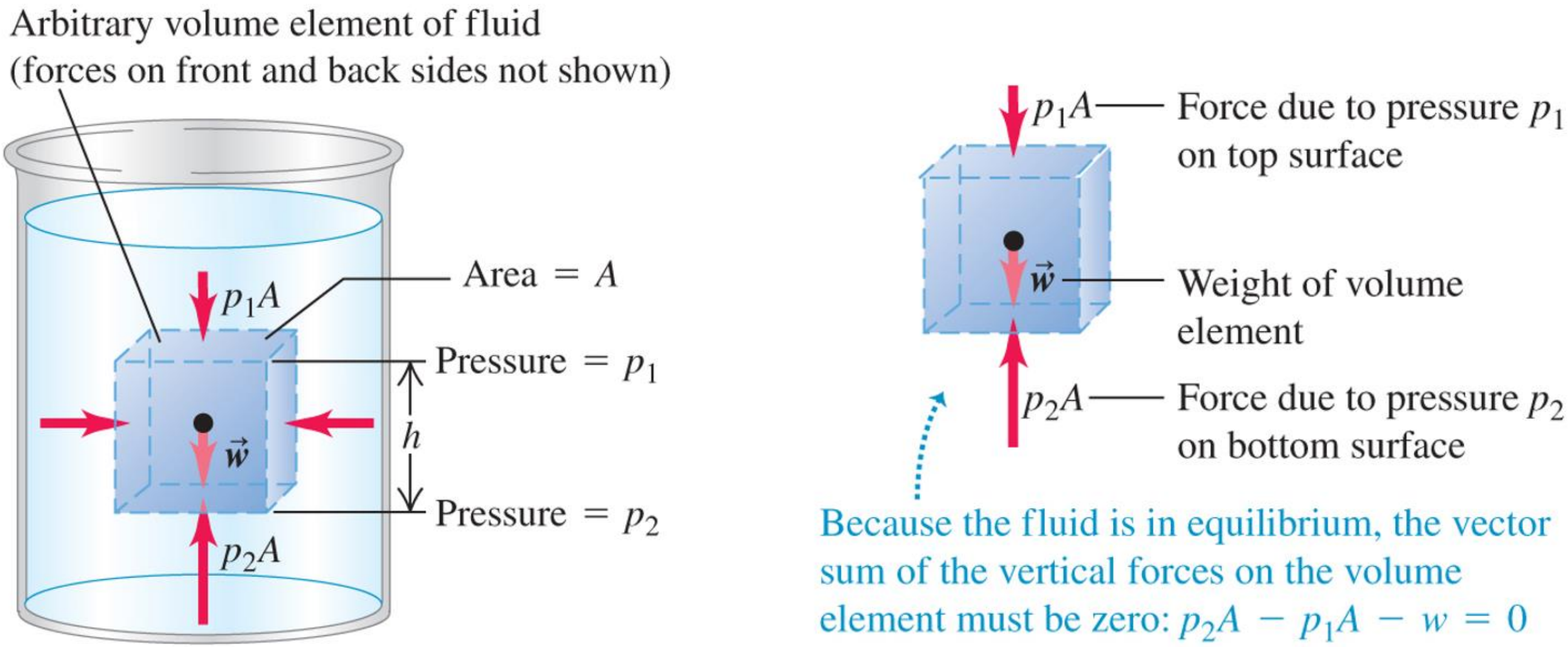
$$p = \frac{F_\perp}{A} \qquad (\mathrm{N/m^2 \ or \ Pa})$$

- Force is a vector quantity.
- Pressure is a scalar quantity.
- Atmosphere pressure

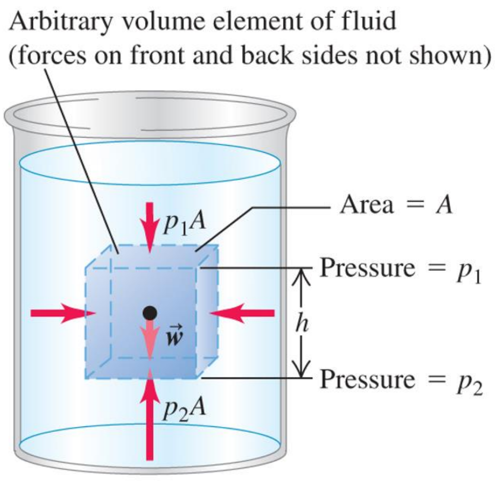
p_{atm} = 1.013×10^5 Pa = 14.7 psi = 1.013 bars = 1013 millibars (mbar) = 760 mm Hg = 76 cm Hg

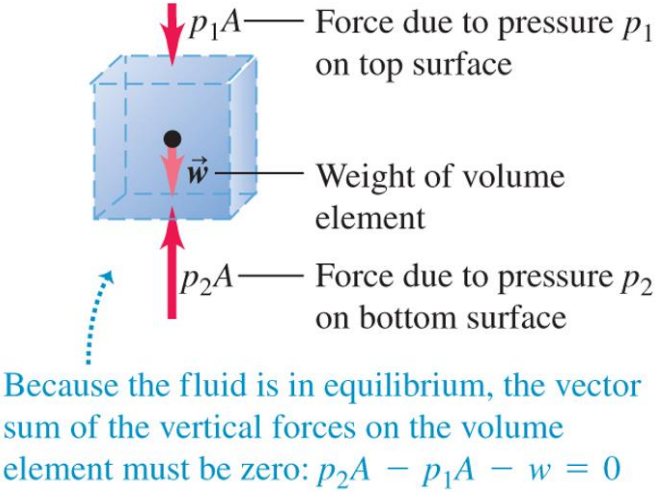
Courtesy of Wenhao Wu

Question: How does the pressure vary in a fluid?



Question: How does the pressure vary in a fluid?





In equilibrium:

$$F_2 - F_1 - mg = 0$$

$$p_2A - p_1A - \rho ghA = 0$$

$$p_2 - p_1 - \rho gh = 0$$

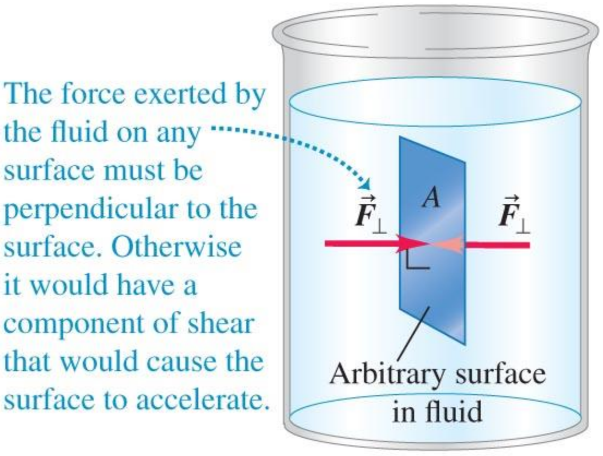
Variation of pressure with depth in a fluid

$$p_2 = p_1 + \rho gh$$

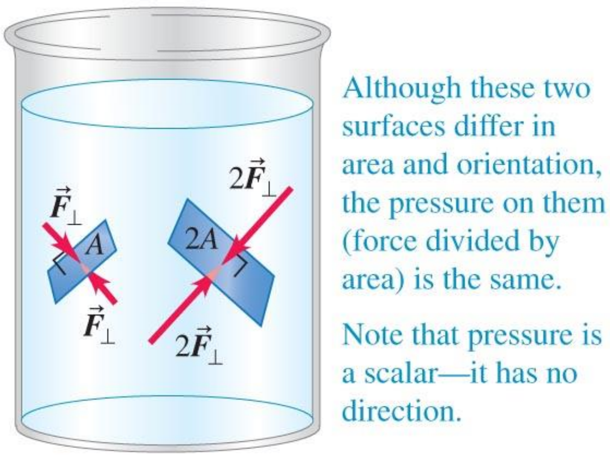
ρ is the density of the fluid;
h is the depth below the fluid level p_1 is measured;
p is the pressure which is always positive.

Pressure In a Fluid

- $P = F/A$
- The pressure is equal to force (in N) per unit area (in m^2).
- A new derived unit N/m^2 = 1 Pascal = 1 Pa
- Atmospheric pressure is 1 atm = 760 mm Hg = 14.7lb/in^2 = 101325 Pa = 1.013 bars
- 1mm Hg=1 torr=133.3Pa



(a) Why forces due to fluid pressure must act perpendicular to a surface



(b) Pressure equals force divided by area.

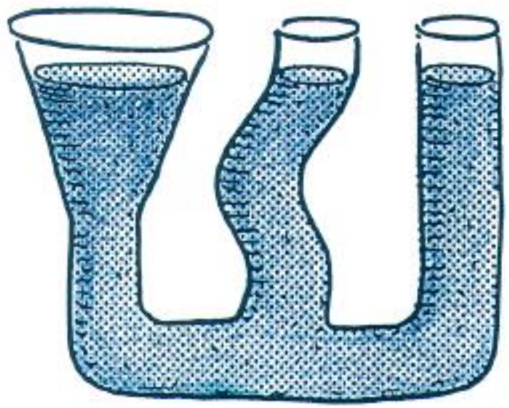
Pressure in a fluid open to air

Variation of pressure with depth in a fluid open to the air at the top

$$p = p_{atm} + \rho g h$$

p_{atm} is the atmospheric pressure at the surface of the fluid;
ρ is the density of the fluid;
h is the depth below the surface of the fluid;
p is the pressure at depth h below the surface. p is always positive.

Clicker - Questions



Everybody knows that "water seeks its own level," but very few people know **why** water seeks its own level. The reason has most to do with

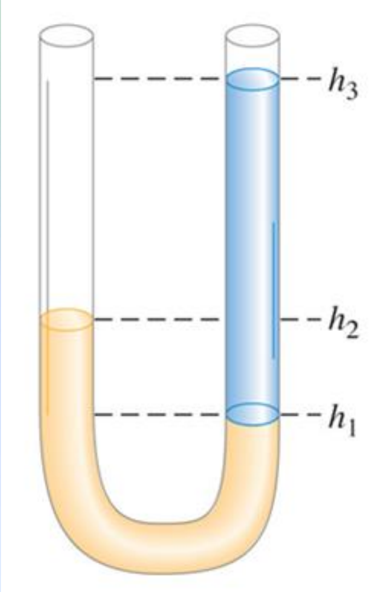
a) atmospheric pressure.

b) water pressure depending on depth.

c) water's density.

Clicker - Questions

The U tube in the figure contains two liquids in equilibrium. At which level(s) must the pressure be the same in both sides of the tube?



A) h_1

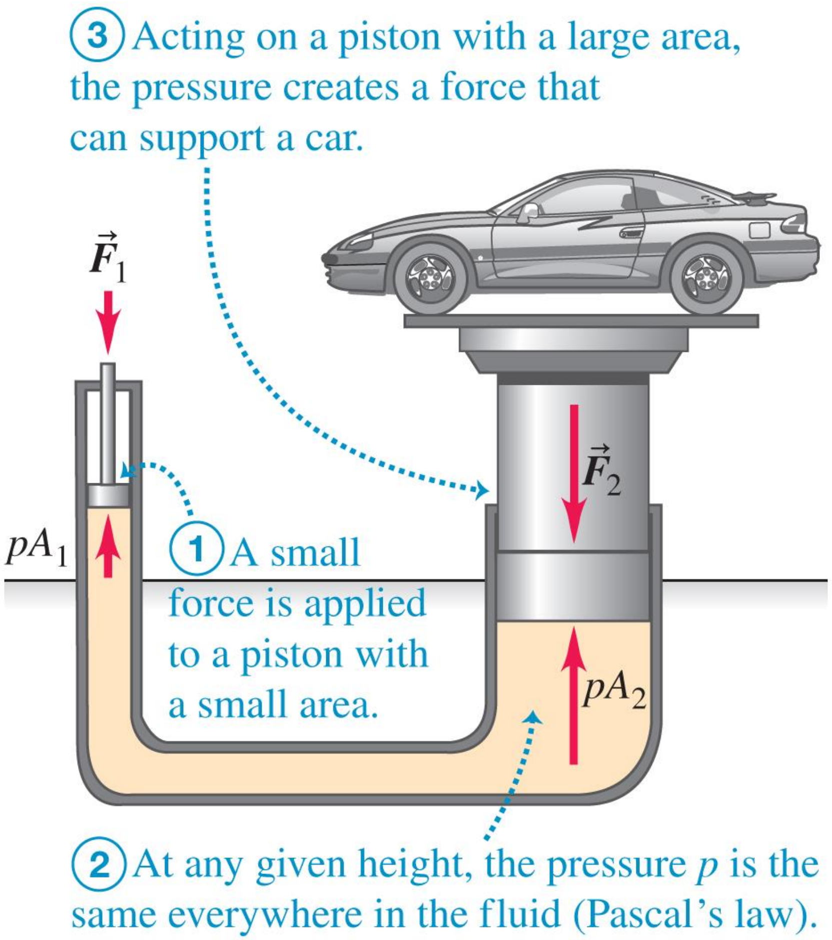
B) h_2

C) h3

D) The pressure at h_1 equals that at h_2.

Pascal’s Law

Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel. The pressure depends only on the depth; the shape of the container does not matter.



Hydraulic Lift

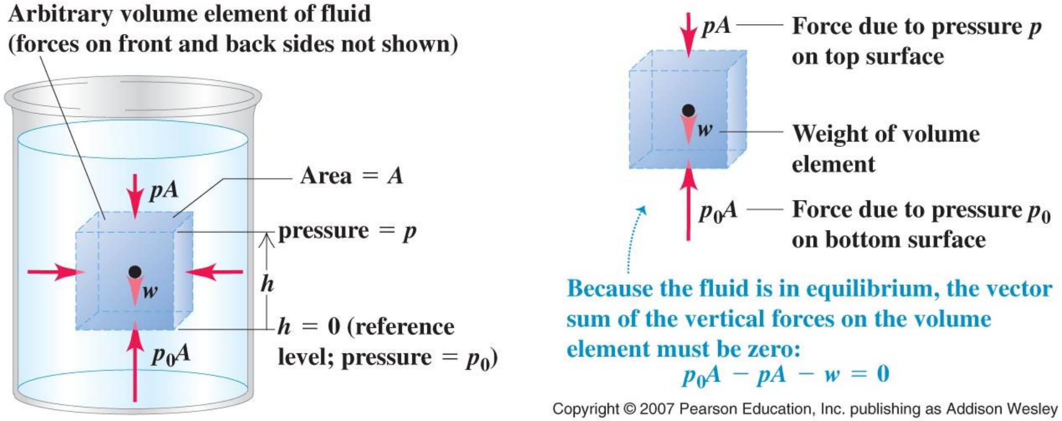
Neglect the pressure variation due to the weight or the fluid.

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Therefore

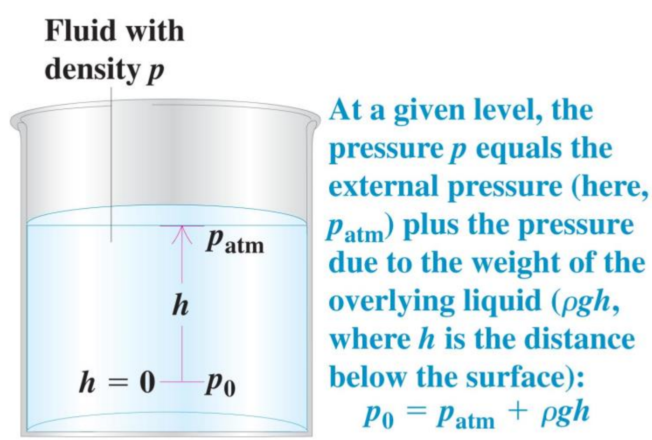
$$F_2 = F_1 \frac{A_2}{A_1}$$

Pascal’s Law

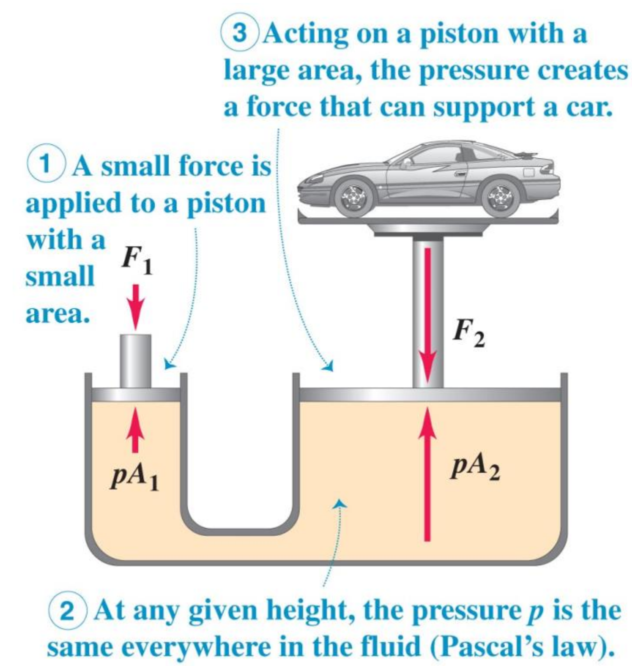


Communicating tubes:
Fluid has the same height at every height of the tubes, where the pressure is the same. $\boldsymbol{P = P_{atm} + \rho g h}$

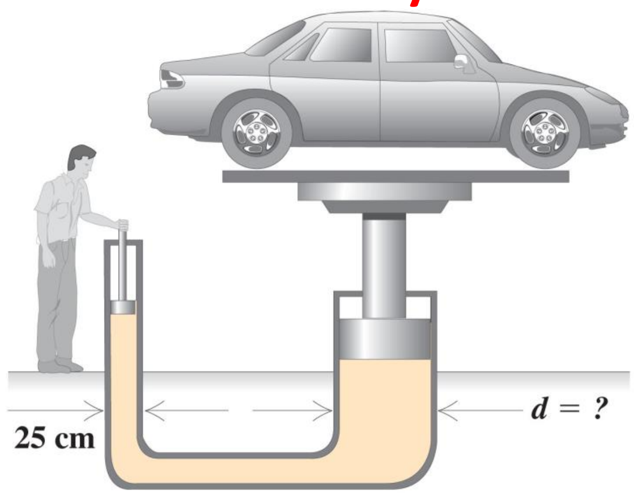
Pascal law:
Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.



$$\boldsymbol{P = \frac{F_1}{A_1} = \frac{F_2}{A_2}}$$
$$\boldsymbol{\rightarrow F_2 = \frac{A_2}{A_1} F_1}$$



Hydraulic lift



Problem 13-27

Design a lift which can handle cars up to 3000kg, plus the 500kg platform. The worker should need to exert 100N.

a) What is the diameter of the pipe under the platform?
b) If the worker pushes down with a stroke 50cm long, by how much will he raise the car?

Use: Pascal law → $\frac{F_1}{A_1} = \frac{F_2}{A_2}$: pressure is the same everywhere in the fluid.

Note: $F_1 = 100N$ and $F_2 = 3500kg * 9.8\frac{m}{s^2} = 3.43x10^4 N$ volume displaced at each piston, when they move by d_1 and d_2 is the same. (Area) $A = \pi r^2$

a) $\frac{F_1}{r_1^2} = \frac{F_2}{r_2^2}$; $r_2 = r_1\sqrt{\frac{F_2}{F_1}} = 0.125m\sqrt{\frac{3.43x10^4}{100}} = 2.32m$ → diameter (d_2)=4.64m

b) $d_1 A_1 = d_2 A_2$; $d_2 = d_1\frac{A_1}{A_2} = d_1\frac{\pi r_1^2}{\pi r_2^2} = 50cm(\frac{0.125m}{2.32m})^2 = 1.45mm$

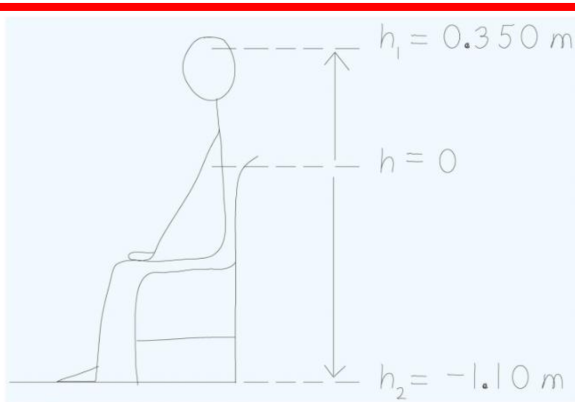
Reflect: Work done by worker and on car must be the same.

$$F_1 d_1 = 100N * 0.5m = 50\,J$$
$$F_2 d_2 = 3.43x10^4 N * 1.45x10^{-3} m = 50\,J$$

Medical instruments measure pressure

← Eye Heart →

A tonometer measures pressure within the eyeball to diagnose glaucoma.

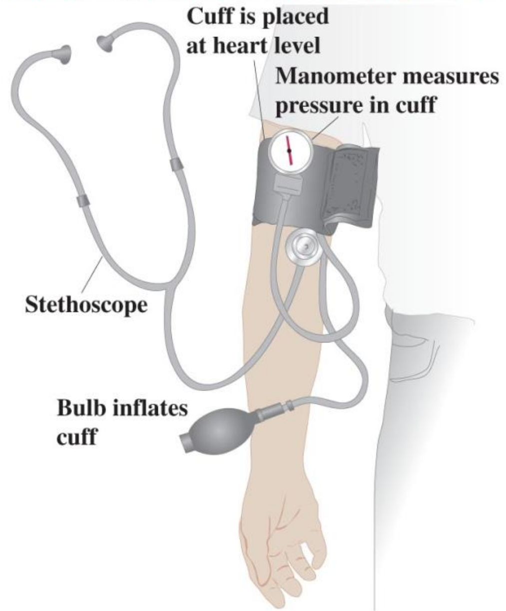


<u>Blood pressure varies with height:</u>

At the height of the heart $P_o = 1.3x10^4 Pa$, and $\rho_{blood} = 1.06x10^3 \frac{kg}{m^3}$

Use: $P = P_o - \rho g h$

To measure blood pressure, you listen through the stethoscope for arterial flow while letting the cuff deflate. Flow starts when the pressure in the deflating cuff reaches the arterial pressure.



Healthy blood pressure is 120/80mm Hg.

Pressure is measured at the height of the heart

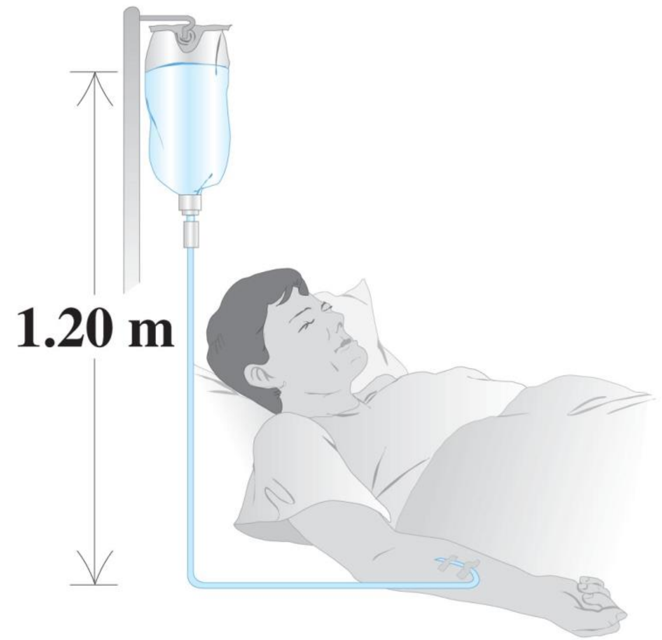
<u>Systolic pressure:</u>
Blood starts to spread discontinuously through the compressed vessel.

<u>Distolic pressure:</u>
Blood flows continuously

a) Pressure at the level of the head:
$P_1 = P_o - \rho g h = 1.30x10^4 - 1.6x10^3 * 9.8 * 0.35 = 9.4x10^3\ Pa$
(light - headed)

b) Pressure at the level of the foot
$P_2 = 1.30x10^4 - 1.6x10^3 * 9.8 * (-1.1) = 2.44x10^4\ Pa$

Intravenous feeding



Problem 13.17:

The liquid has a density of 1060 kg/m^3. The container hangs 1.2m above the patients arm.

What is the pressure this fluid exerts on the patient's vein in millimeters of mercury.

$$P = \rho g h = 1060\frac{kg}{m^3} * 9.8\frac{m}{s^2} * 1.2\ m = 1.25x10^4\ Pa$$

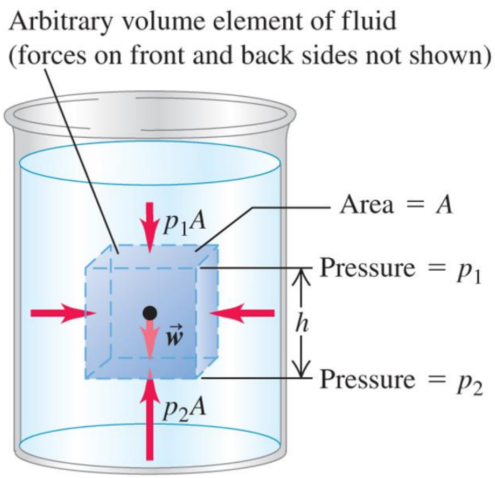
$$\therefore\ 1\ mm\ Hg = 133.3\ Pa$$

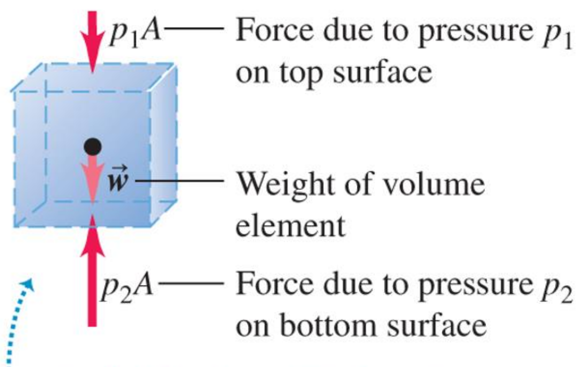
So; $\frac{1.25x10^4\ Pa}{133.3\ Pa} * 1\ mm\ Hg = 93.5\ mm\ Hg$

13.3 Archimedes Principle: Buoyancy

- When an object is completely or partially immersed in a fluid, the fluid exerts an upward force on the object equal to the weight of the fluid that is displaced by the object.

Question: How does the pressure vary in a fluid?





Because the fluid is in equilibrium, the vector sum of the vertical forces on the volume element must be zero: $p_2 A - p_1 A - w = 0$

In equilibrium:

$$F_2 - F_1 - mg = 0$$

$$p_2 A - p_1 A - \rho g h A = 0$$

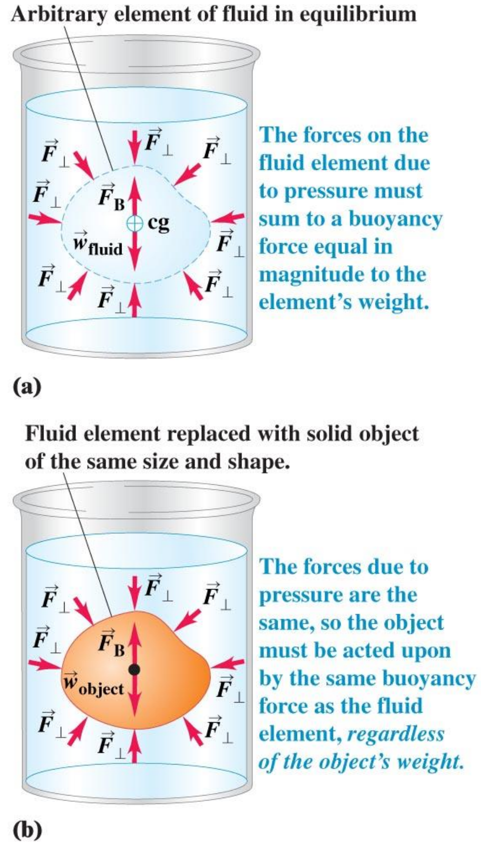
$$p_2 - p_1 - \rho g h = 0$$

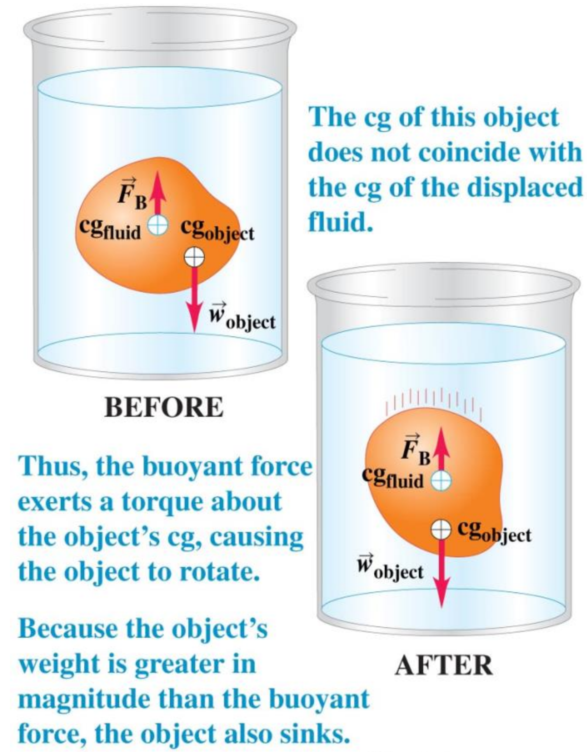
$$p_2 A - p_1 A - \rho g h A = 0$$

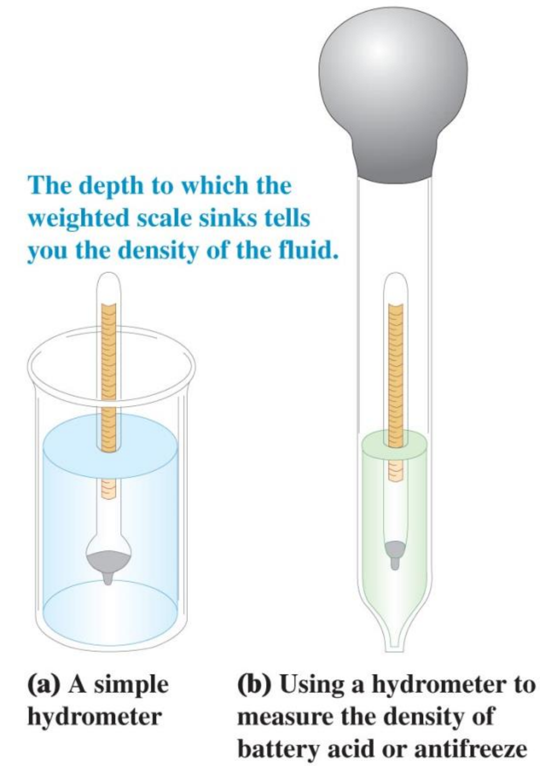
$$F_2 = F_1 + mg$$

$$F_2 = F_1 + \text{weight of the fluid displaced}$$

Archimedes’s Principle and Buoyancy







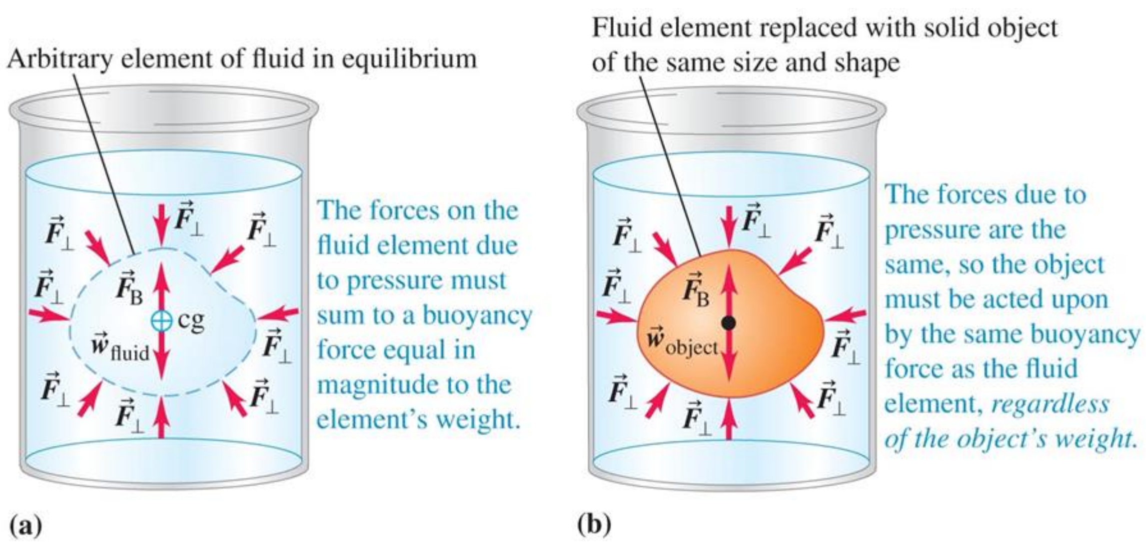
Note: hydrometer floats higher in denser fluids

Archimedes' principle:
When an object is immersed into a fluid, the fluid exerts an upward force on the object equal to the weight of the displaced fluid.

Note: The upward force is labeled the buoyant force

Archimedes's Principle – Figure 13.15

- An object submersed in a fluid experiences buoyant force equal to the mass of any fluid it displaces.
- An object can experience buoyant force greater than its mass and float. Even if it sinks, it would weigh measurably less.
- Refer to Example 13.7.



Clicker - Questions

Compare with an empty ship. Will a ship loaded with a cargo of Styrofoam float

a)lower in water?   b) higher in water?

Clicker - Questions

Buoyant force is greater on a empty steel barge when it is;

A) Floating on the surface

B) Capsized and sitting on the bottom

C) Same either way.

Buoyant force is greater on a submarine when it is;

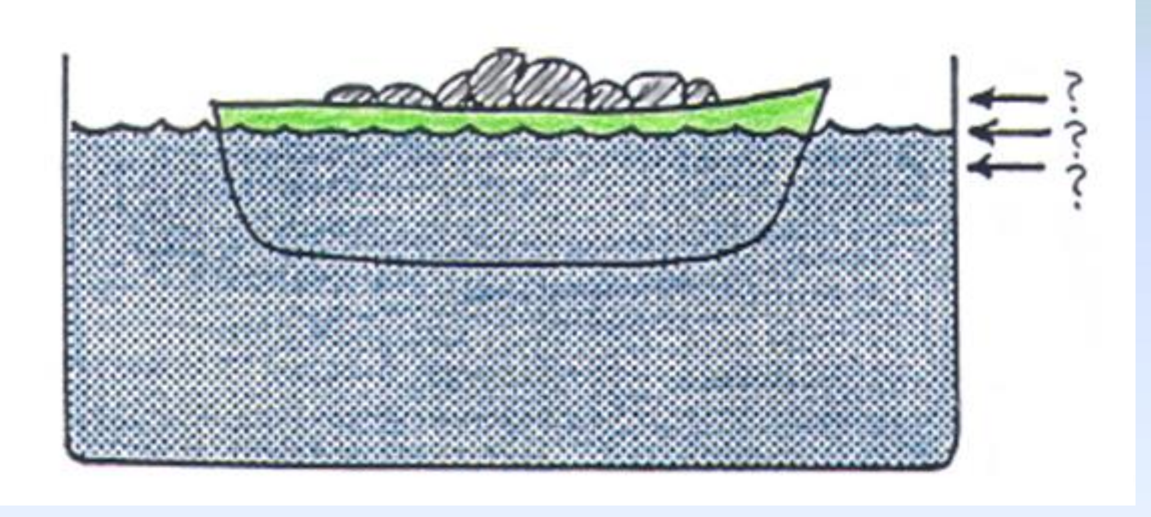
D) Floating

E) Submerged

D) Same either way.

Clicker - Questions

Consider a boat loaded with scrap iron in a swimming pool. If the iron is thrown overboard into the pool, will the water level at the edge of the pool a) rise, b)fall or c) remain unchanged?



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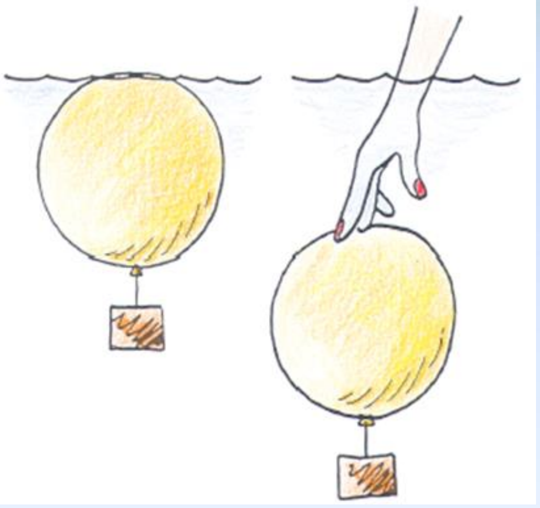
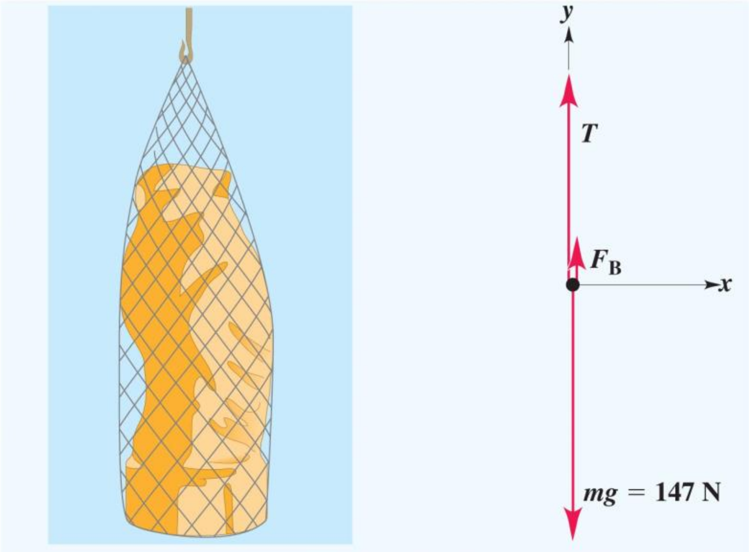
Consider an air-filled balloon weighted so that it is on the verge of sinking – that is, its overall density just equals that of water.

Now if you push it beneath the surface. It will;

a) Sink
b) Return to the surface
c) Stay at the depth to which it is pushed.





(a) Immersed statue in equilibrium (b) Free-body diagram of statue

Note: Weighing in water decreases the scale reading

A 15kg gold statue is raised from a sunken ship:

a) Find the tension in the hoisting cable while the statue is submerged

$$\text{(Statue) } V = \frac{m}{\rho} = \frac{15\ kg}{19.3x10^3 \frac{kg}{m^3}} = 7.77x10^{-4} m^3$$

Weight of equal volume of water is the buoyant force.

$$F_B = \rho_{water} * V_{water} * g = 1x10^3 * 7.77x10^{-4} * 9.8 = 7.61\ N$$

$$T + F_B - W = 0 \rightarrow T + 7.61 - 15 * 9.8 = 0 \quad \therefore T = 139\ N$$

b) Find the tension when the statue is out of the water

$$T = F_g = 15\ kg * 9.8 = 147\ N$$

Archimedes buoyancy

32. II An ore sample weighs 17.50 N in air. When the sample is suspended by a light cord and totally immersed in water, the tension in the cord is 11.20 N. Find the total volume and the density of the sample.

13.32. Set Up: $F_B = \rho_{water} V_{obj} g$. $w = mg = 17.50\text{ N}$ and $m = 1.79\text{ kg}$.

Solve: $T + F_B - mg = 0$. $F_B = mg - T = 17.50\text{ N} - 11.20\text{ N} = 6.30\text{ N}$.

$$V_{obj} = \frac{F_B}{\rho_{water} g} = \frac{6.30\text{ N}}{(1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 6.43\times10^{-4}\text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{1.79\text{ kg}}{6.43\times10^{-4}\text{ m}^3} = 2.78\times10^3\text{ kg/m}^3$$

Archimedes buoyancy

!3.35 A hollow plastic sphere is held below the surface of a lake by a cord anchored to the bottom of a lake. The sphere has a volume of 0.650 m^3 and the tension in the cord is is 900 N .

a) calculate the buoyant force.

B) what is the mass of the sphere?

C) The cord breaks and the sphere rises to the surface. When the sphere comes to rest, what fraction of its volume will be submerged??

Set Up: $F_B = \rho_{water} V_{obj} g$. The net force on the sphere is zero.

Solve: **(a)** $F_B = (1000 \text{ kg/m}^3)(0.650 \text{ m}^3)(9.80 \text{ m/s}^2) = 6.37 \times 10^3 \text{ N}$

(b) $F_B = T + mg$ and

$$m = \frac{F_B - T}{g} = \frac{6.37 \times 10^3 \text{ N} - 900 \text{ N}}{9.80 \text{ m/s}^2} = 558 \text{ kg}.$$

(c) Now $F_B = \rho_{water} V_{sub} g$, where V_{sub} is the volume of the sphere that is submerged. $F_B = mg$. $\rho_{water} V_{sub} = mg$ and

$$V_{sub} = \frac{m}{\rho_{water}} = \frac{558 \text{ kg}}{1000 \text{ kg/m}^3} = 0.558 \text{ m}^3$$

$$\frac{V_{sub}}{V_{obj}} = \frac{0.558 \text{ m}^3}{0.650 \text{ m}^3} = 0.858 = 85.8\%$$

Reflect: When the sphere is totally submerged, the buoyant force on it is greater than its weight. When it is floating, it needs to be only partially submerged in order to produce a buoyant force equal to its weight.

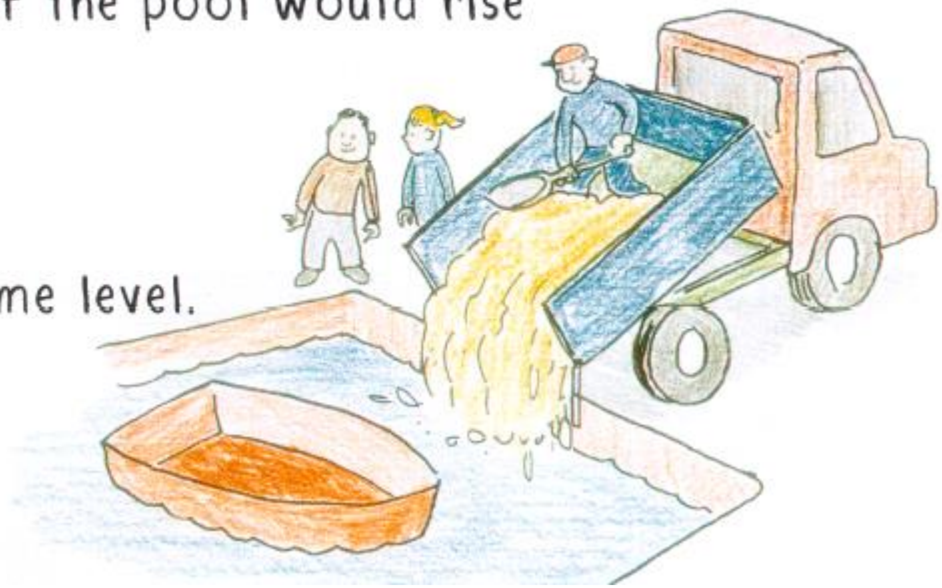
Clicker - Questions

A load of sand is poured into a pool to give it a new sandy bottom. It also raises the water level of the pool. If the sand were instead poured into a boat floating in the pool, the water level of the pool would rise

a) less.

b) more.

c) to the same level.



Archimedes buoyancy

A solid aluminum ingot weights 89 N in air. a) What is its volume? b)The ingot is suspended by a rope and totally submerged in water what is the tension(the apparent weight) in the rope??

***13.29. Set Up:** The density of aluminum is $2.7 \times 10^3\ \text{kg/m}^3$. The density of water is $1.00 \times 10^3\ \text{kg/m}^3$. $\rho = m/V$. The buoyant force is $F_B = \rho_{\text{water}} V_{\text{obj}} g$.

Solve: **(a)** $T = mg = 89\ \text{N}$ so $m = 9.08\ \text{kg}$. $V = \dfrac{m}{\rho} = \dfrac{9.08\ \text{kg}}{2.7 \times 10^3\ \text{kg/m}^3} = 3.36 \times 10^{-3}\ \text{m}^3 = 3.4\ \text{L}$.

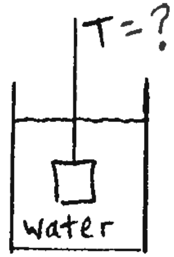
(b) When the ingot is totally immersed in the water while suspended, $T + F_B - mg = 0$.

$$F_B = \rho_{\text{water}} V_{\text{obj}} g = (1.00 \times 10^3\ \text{kg/m}^3)(3.36 \times 10^{-3}\ \text{m}^3)(9.80\ \text{m/s}^2) = 32.9\ \text{N}$$

$$T = mg - F_B = 89\ \text{N} - 32.9\ \text{N} = 56\ \text{N}$$

Reflect: The buoyant force is equal to the difference between the apparent weight when the object is submerged in the fluid and the actual gravity force on the object.

(5 pts) 3. A block of metal with density $4000\ kg/m^3$ is suspended from a light string. When the block is in air, the tension in the string is 39.2 N. What is the tension in the string when the block is totally immersed below the surface of water that is in a bucket? The block is not touching the bucket. The density of water is $1000\ kg/m^3$.



(a) 4.9 N
(b) 9,8 N
d (c) 18.6 N
(d) 29.4 N
(e) none of these answers

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