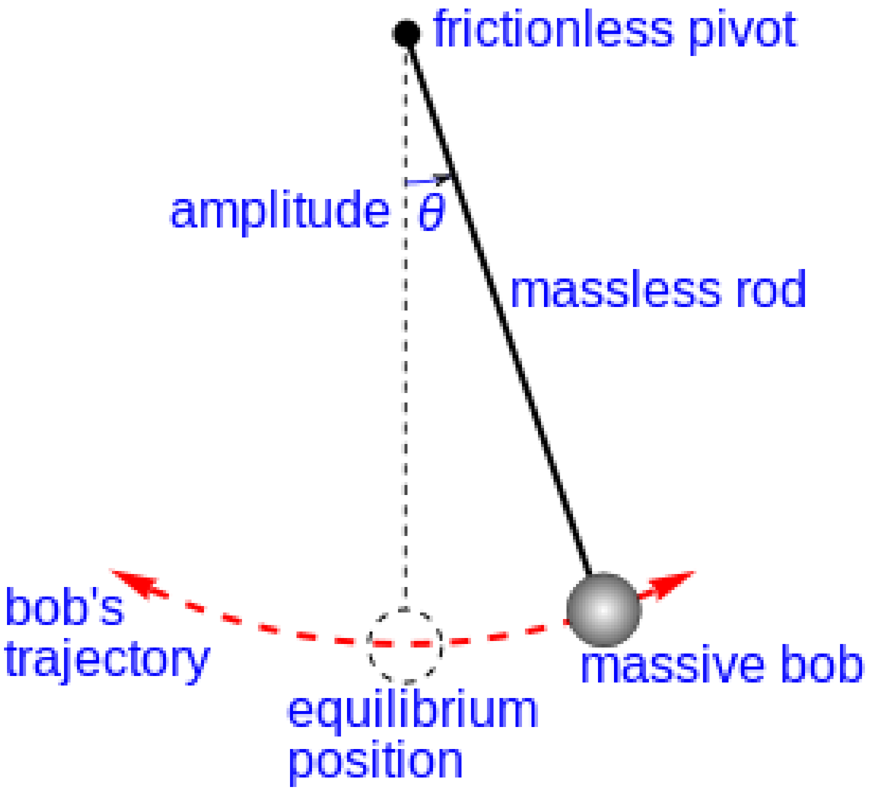
Foucault pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

T = period
π = pi
L = pendulum length
g = acceleration due to gravity



CHAPTER 3
The Experimental Basis of Quantum Physics

- 3.1 Discovery of the X Ray and the Electron
- 3.2 Determination of Electron Charge
- 3.3 Line Spectra
- 3.4 Quantization
- 3.5 Blackbody Radiation
- 3.6 Photoelectric Effect
- 3.7 X-Ray Production
- 3.9 Pair Production and Annihilation

3.1: Discovery of the X Ray and the Electron

- X rays were discovered by Wilhelm Röntgen in 1895.
  - Observed x rays emitted by cathode rays bombarding glass

- Electrons were discovered by J. J. Thomson.
  - Observed that cathode rays were charged particles

Cathode Ray Experiments

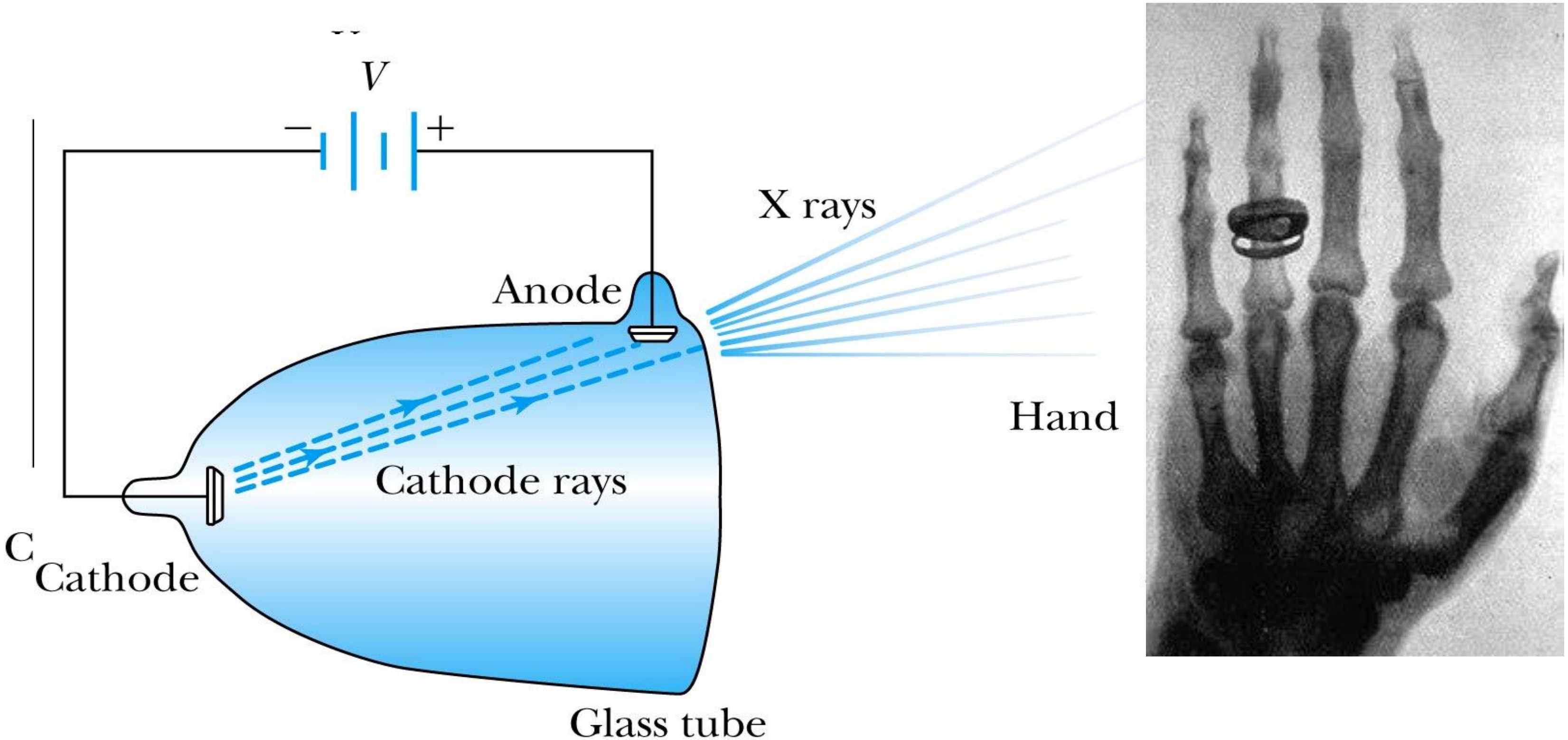
- In the 1890s scientists and engineers were familiar with "cathode rays". These rays were generated from one of the metal plates in an evacuated tube across which a large electric potential had been established.
- It was surmised that cathode rays had something to do with atoms.
- It was known that cathode rays could penetrate matter and their properties were under intense investigation during the 1890s.

Observation of X Rays

- Wilhelm Röntgen studied the effects of cathode rays passing through various materials. He noticed that a phosphorescent screen near the tube glowed during some of these experiments. These rays were unaffected by magnetic fields and penetrated materials more than cathode rays.
- He called them **x rays** and deduced that they were produced by the cathode rays bombarding the glass walls of his vacuum tube.

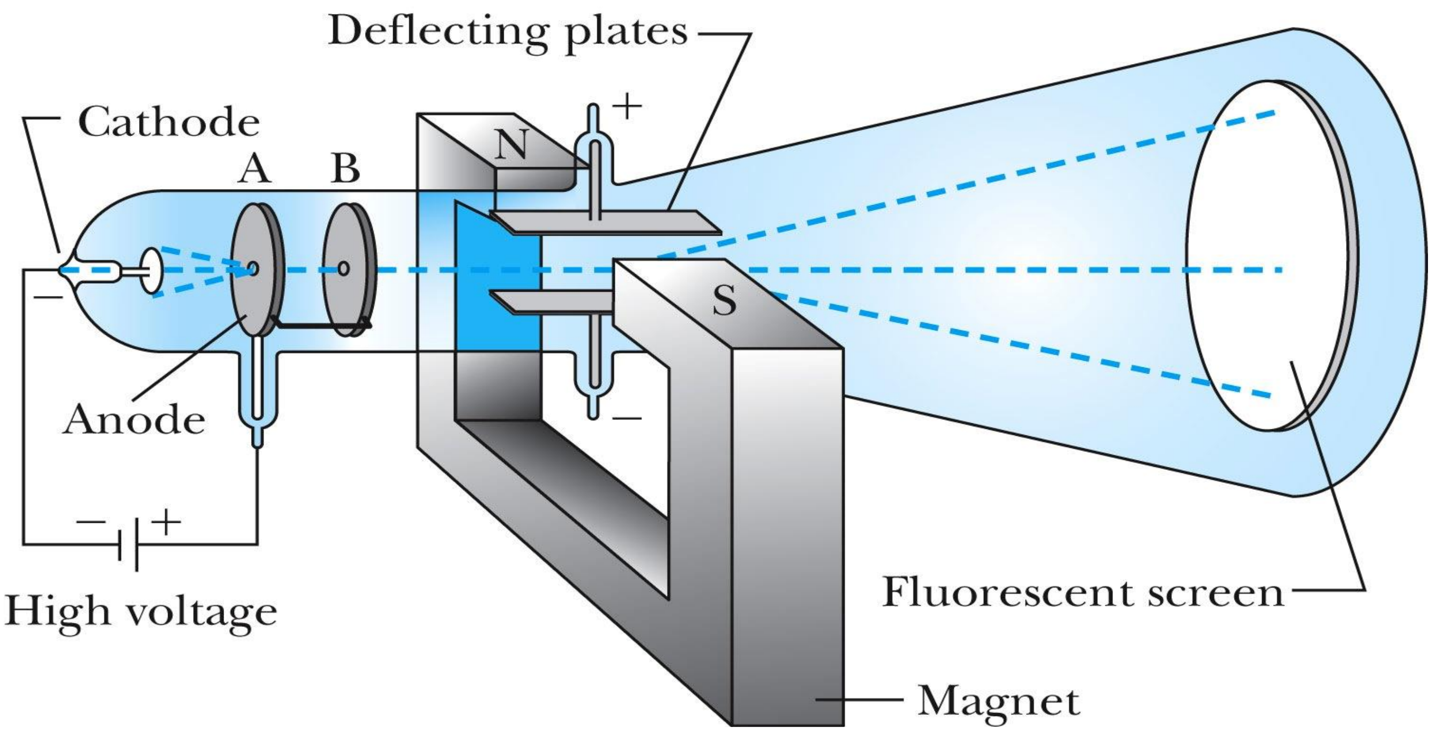
Röntgen's X Ray Tube

- Röntgen constructed an x-ray tube by allowing cathode rays to impact the glass wall of the tube and produced x rays. He used x rays to image the bones of a hand on a phosphorescent screen.



Apparatus of Thomson's Cathode-Ray Experiment

- Thomson used an evacuated cathode-ray tube to show that the cathode rays were negatively charged particles (electrons) by deflecting them in electric and magnetic fields.



12. How did Thomson measure the charge to mass ratio of the electron?

a. He shot helium nuclei into gold foil to measure the nuclei scattering against electrons within.

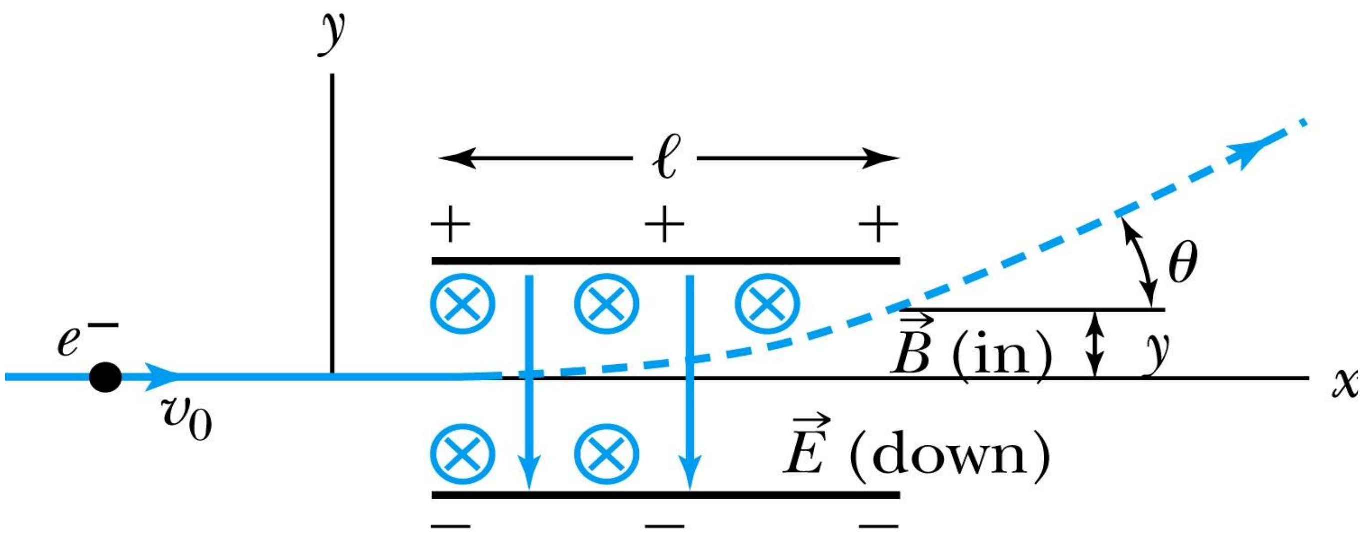
b. He passed cathode rays through a magnetic field and measured the deflection.

c. He suspended a drop of oil between electrodes to measure the electric field from the electrons.

d. He measured very precisely a known quantity of hydrogen atoms and calculated the reduced mass ratio within each atom.

Thomson's Experiment

- Thomson's method of measuring the ratio of the electron's charge to mass was to send electrons through a region containing a magnetic field perpendicular to an electric field.



Experimental Basis of Quantum Theory

X-ray, e/m and e

X-ray	e/m	e
Wilhelm Röntgen	Thomson	Millikan
1845–1923	1856–1940	1868–1953

(a) $t = \ell / v_0$ time to transverse the E-field, $B = 0$

$$\left.\begin{array}{l} \tan\theta = v_y / v_x = a_y t / v_x \\ F_y = m a_y = qE \end{array}\right\} \tan\theta = \frac{qE}{m}\frac{\ell}{v_0^2}$$

(b) zero deflection $F = qE + qvB = 0$ $\quad v_x = v_0 = \frac{E}{B}$

measure v_0

then use v_0 in (a)

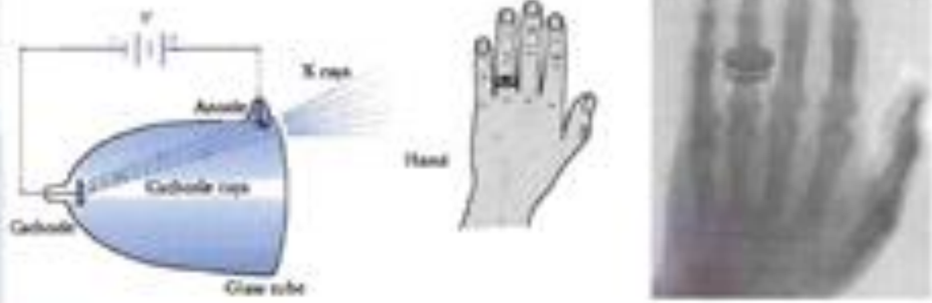


Figure 3.1 In Röntgen's experiments, "x rays" were produced by cathode rays (electrons) hitting the glass near the anode. He studied the penetration of the x rays through several substances and even noted that if the hand was held between the glass tube and a screen, the darker shadow of the bones could be discriminated from the less dark shadow of the hand. Photo courtesy of Bettmann/Corbis.

Figure 3.3 Thomson's method of measuring the ratio of the electron's charge to mass was to send electrons through a region containing a magnetic field ($\vec{B}$ into paper) perpendicular to an electric field ($\vec{E}$ down). The electrons having $v = E/B$ go through undeflected. Then, using electrons of the same energy, the magnetic field is turned off and the electric field deflects the electrons, which exit at angle θ. The ratio of e/m can be determined from B, E, θ, and ℓ, where ℓ is the length of the field distance and θ is the emerging angle. See Equation (3.5).

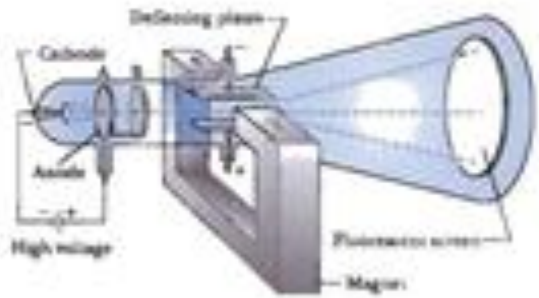


Figure 3.2 Apparatus of Thomson's cathode-ray experiment. Thomson proved that the rays emitted from the cathode were negatively charged particles (electrons) by deflecting them in electric and magnetic fields. The key to the experiment was to evacuate the glass tube.

$$\frac{q}{m} = \frac{v_0^2 \tan\theta}{E\ell} = \frac{E \tan\theta}{B^2 \ell}$$

$$\boxed{\frac{q}{m} = \frac{e}{m} = 1.76 \times 10^{11}\ C/kg}$$

Calculation of *e/m* Examble 3.1

- An electron moving through the electric field is accelerated by a force: $F_y = ma_y = qE$
- Electron angle of deflection: $\tan\theta = \frac{v_y}{v_x} = \frac{a_y t}{v_0} = \frac{qE}{m}\frac{\ell}{v_0^2}$
- The magnetic field deflects the electron against the electric field force. $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} = 0$
- The magnetic field is adjusted until the net force is zero.

$$\vec{E} = -\vec{v} \times \vec{B} \qquad \left|\vec{E}\right| = \left|v_x\right|\left|\vec{B}\right| \qquad v_x = \left|\frac{\vec{E}}{\vec{B}}\right| = v_0$$

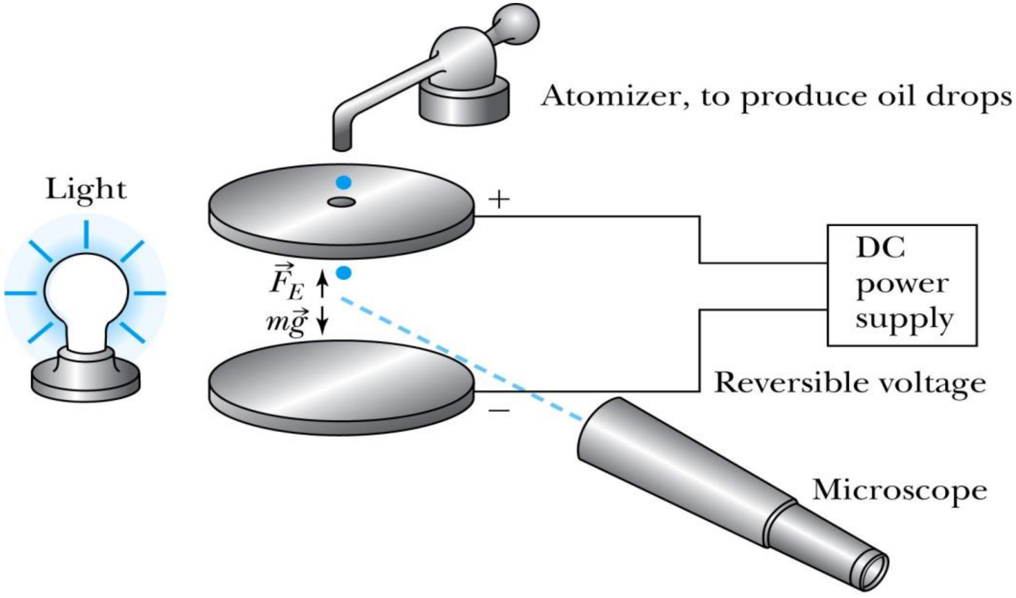
- Charge to mass ratio:

$$\frac{q}{m} = \frac{v_0^2 \tan\theta}{E\ell} = \frac{E\tan\theta}{B^2\ell}$$

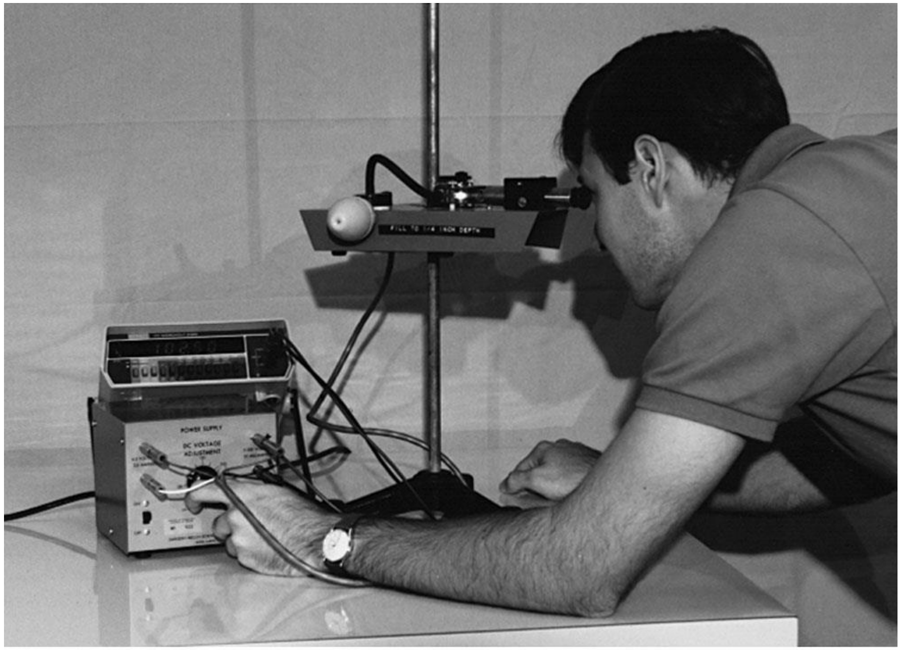
q/m= 1,8x 10 ^11 C/kg

3.2: Determination of Electron Charge

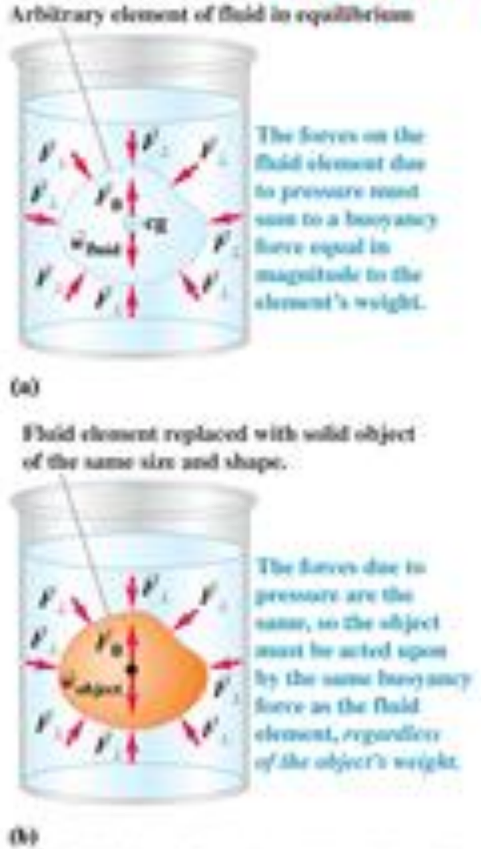
Millikan oil drop experiment



(a)



Archimedes's Principle and Buoyancy



note: hydrometer floats higher in denser fluids

Archimedes's principle: When an object is immersed into a fluid, the fluid exerts an upward force on the object equal to the weight of the dispaced fluid

note the upward force is labeled the buoyant force

Next-Time Question

Buoyant force is greater on an empty steel barge when it is

a) floating on the surface.

b) capsized and sitting on the bottom.

c) same either way.

Buoyant force is greater on a submarine when it is

d) floating.

e) submerged.

f) same either way.

Hewitt Drew it!

Next-Time Question

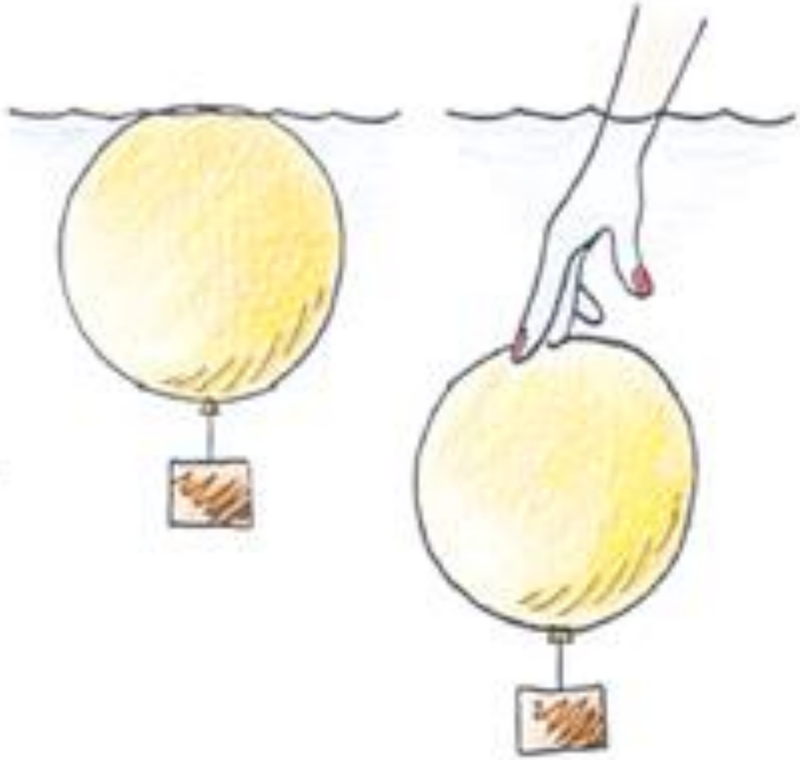
Consider an air-filled balloon weighted so that it is on the verge of sinking — that is, its overall density just equals that of water.

Now if you push it beneath the surface, it will

a) sink.

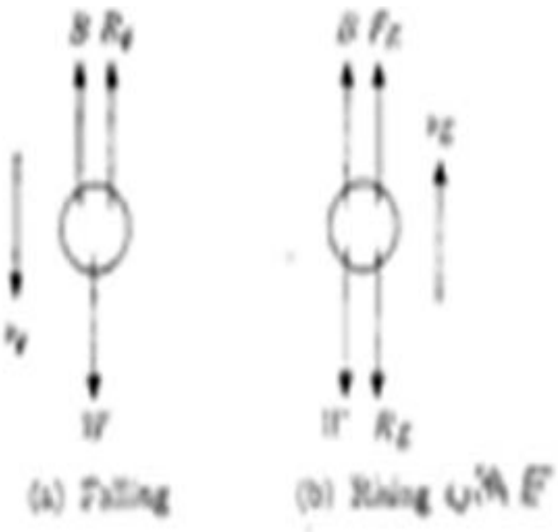
b) return to the surface.

c) stay at the depth to which it is pushed.



Millikan (1909)

e and $\boldsymbol{m_e}$ are incredibly small. Resolution of experiment must be such that the change of one charge can be resolved. Observe charged oil drops on which electric and gravitational forces act.



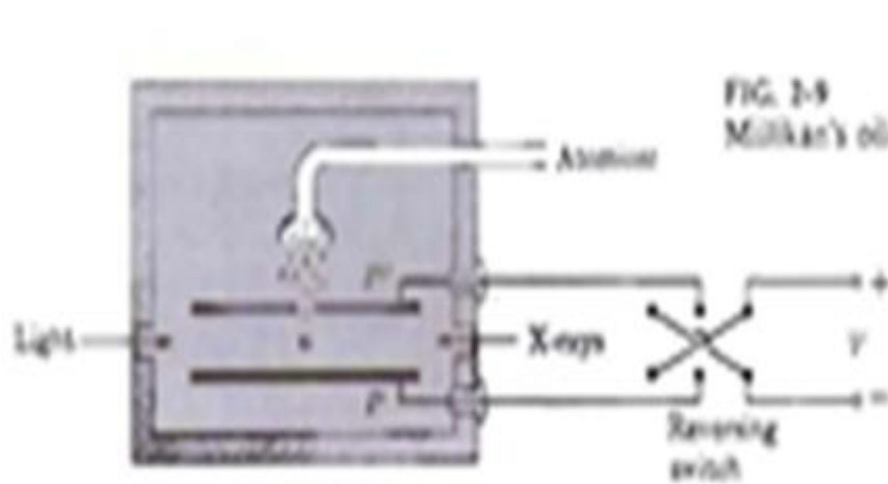


FIG. 2-9
Millikan's oil-drop experiment

Drop falls under gravity and ***E*** electric field yielding F_e and buoyancy force B and R, which is the hindrance by friction in the air against oil drop.

Friction force: $\mathrm{R} = kv$ and $k = 6\pi\eta r$ (Stokes law)

B = buoyant force; F_e = electric force; w = weight force

Millikan (1909)

Oil drop is in motion, either falling without E, or rising with E

(a) Falling oil drop with charge q,but no ***E*** so $F = w - B - kv$; where $w - B = kv_g$ at $F = 0$ when drop falls with constant terminal velocity v_g

$$w = \frac{4}{3}\pi r^3 \rho g; \text{ and } B = \frac{4}{3}\pi r^3 \rho_a g \qquad g = 9.8\, ms^{-2}$$

Where ρ = density of oil and ρ_a= density of air

Now; $\frac{4}{3}\pi r^3(\rho - \rho_a)g = 6\pi\eta r v_g$; solve for $r = \frac{3}{2}\sqrt{(\frac{2\eta v_g}{(\rho - \rho_a)g})}$

$$k = 6\pi\eta r = 18\sqrt{\frac{\eta^3 v_g}{2g(\rho - \rho_a)}}$$

(a) Rising oil drop, with charge and ***E***

$F = qE + B - w - kv$ when $F = 0$ and drop rises with constant terminal velocity v_e

At zero field case $w - b = kv_g$ and $q = \frac{k}{E}(v_g + v_e)$

The q's found are integer multiples n of $q, namely\ nq\ from\ which\ \frac{q}{n} = e = 1.602x10^{-19}C$

Calculation of the oil drop charge(at rest)

- Used an electric field and gravity to suspend a charged oil drop

$$\vec{F}_E = q\vec{E} = -m\vec{g}$$

- Magnitude of the charge on the oil drop

$$q = \frac{mgd}{V}$$

- Mass is determined from Stokes's relationship of the terminal velocity to the radius and density

$$m = \frac{4}{3}\pi r^3 \rho$$

- Thousands of experiments showed that there is a basic quantized electron charge

$$q = 1.602 \times 10^{-19}\ \text{C}$$

3.3: Line Spectra

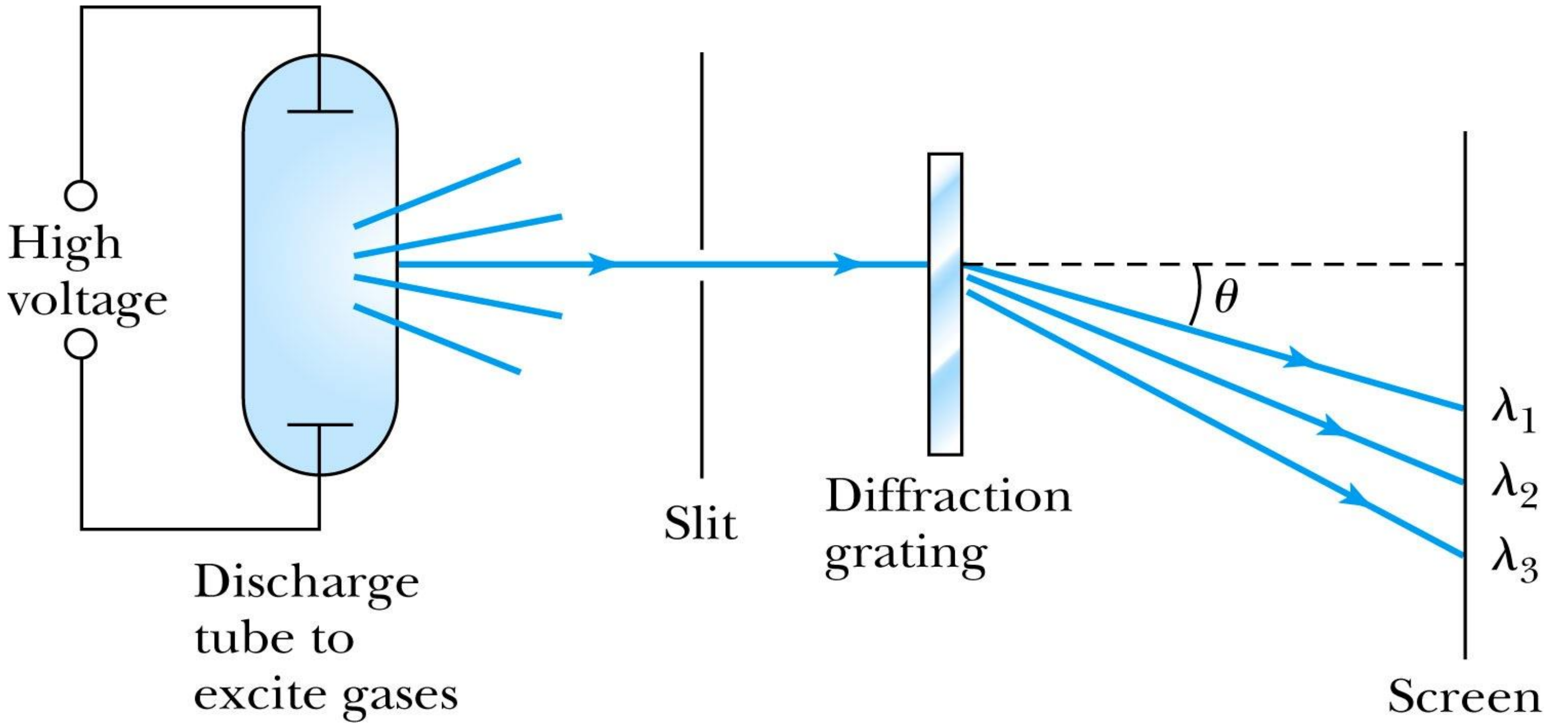
- Chemical elements were observed to produce unique wavelengths of light when burned or excited in an electrical discharge.

- Collimated light is passed through a diffraction grating with thousands of ruling lines per centimeter.
  - The diffracted light is separated at an angle θ according to its wavelength λ by the equation:

$$d \sin\theta = n\lambda$$

where *d* is the distance between rulings and *n* is an integer called the order number

Optical Spectrometer



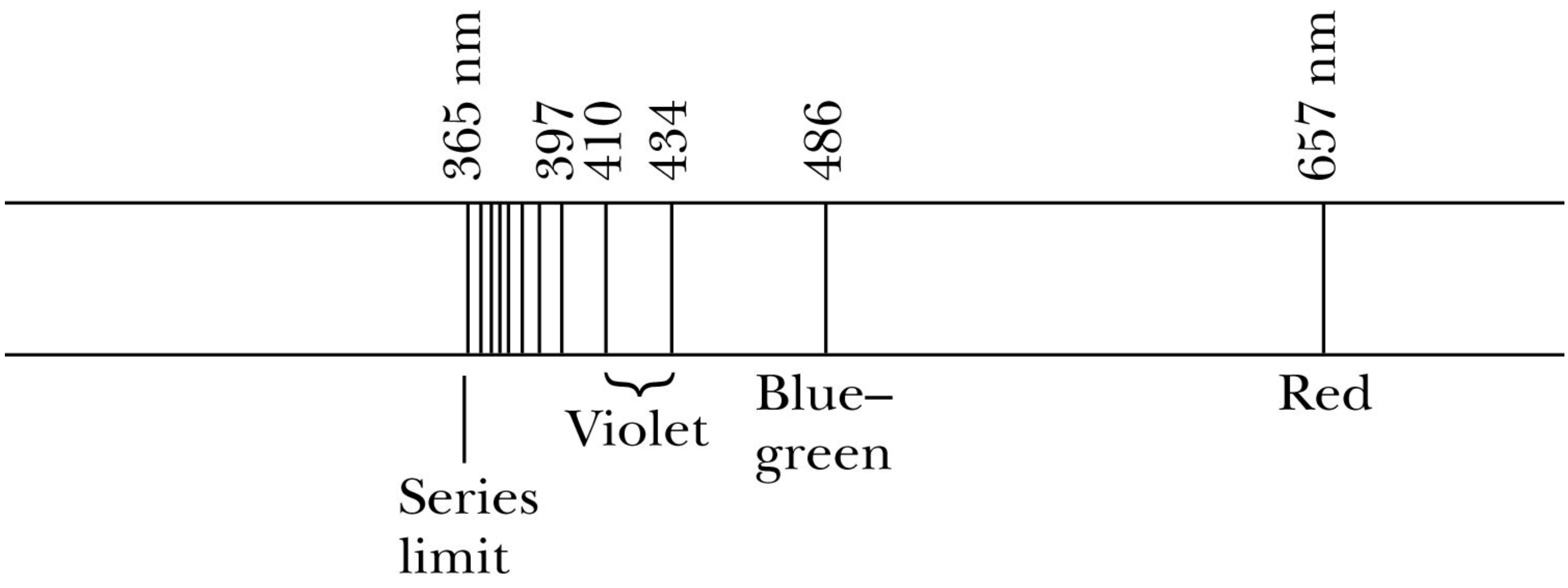
- Diffraction creates a *line spectrum* pattern of light bands and dark areas on the screen.
- Wavelengths of these line spectra allow identification of the chemical elements and the composition of materials.

a
l

Balmer Series

- In 1885, Johann Balmer found an empirical formula for wavelength of the visible hydrogen line spectra in nm:

$$\lambda = 364.56 \frac{k^2}{k^2 - 4} \text{ nm} \quad \text{(where } k = 3,4,5\ldots \text{ and } k > 2)$$



Rydberg Equation

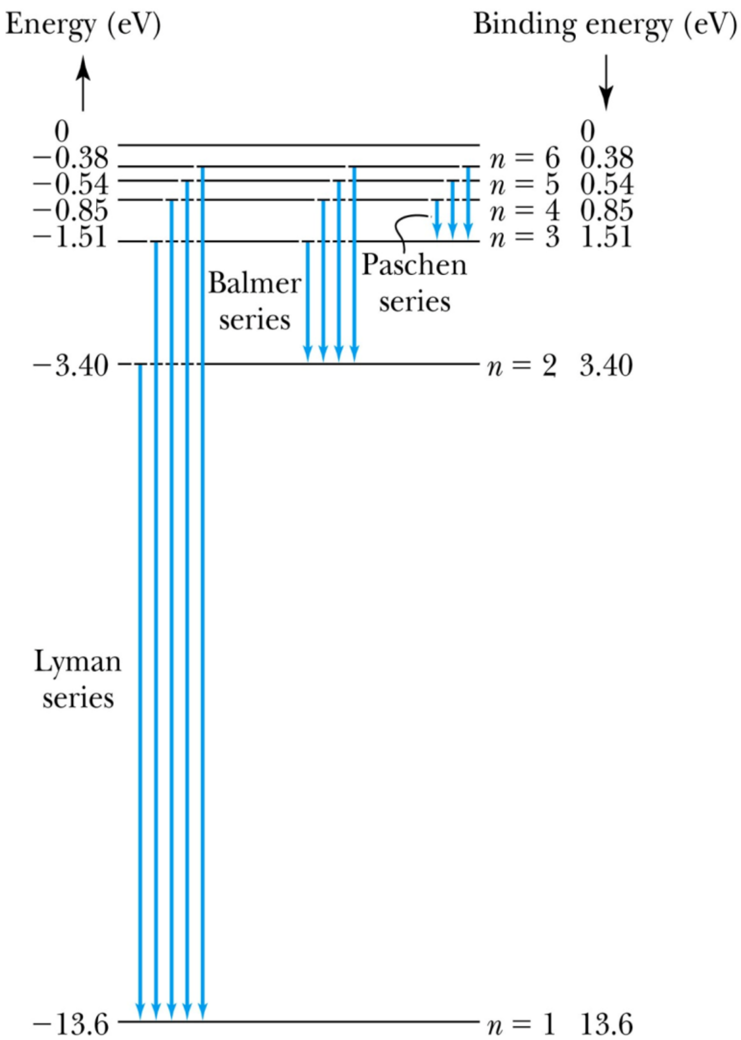
- As more scientists discovered emission lines at infrared and ultraviolet wavelengths, the Balmer series equation was extended to the Rydberg equation:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \qquad R_H = 1.096776\times 10^7\ \mathrm{m}^{-1}$$ (n =1, 2,3…)

Table 3.2 Hydrogen Series of Spectral Lines

Discoverer (year)	Wavelength	n	k
Lyman (1916)	Ultraviolet	1	>1
Balmer (1885)	Visible, ultraviolet	2	>2
Paschen (1908)	Infrared	3	>3
Brackett (1922)	Infrared	4	>4
Pfund (1924)	Infrared	5	>5

Transitions in the Hydrogen Atom



Lyman series

The atom will remain in the excited state for a short time before emitting a photon and returning to a lower stationary state. All hydrogen atoms exist in $n = 1$ (invisible).

Balmer series

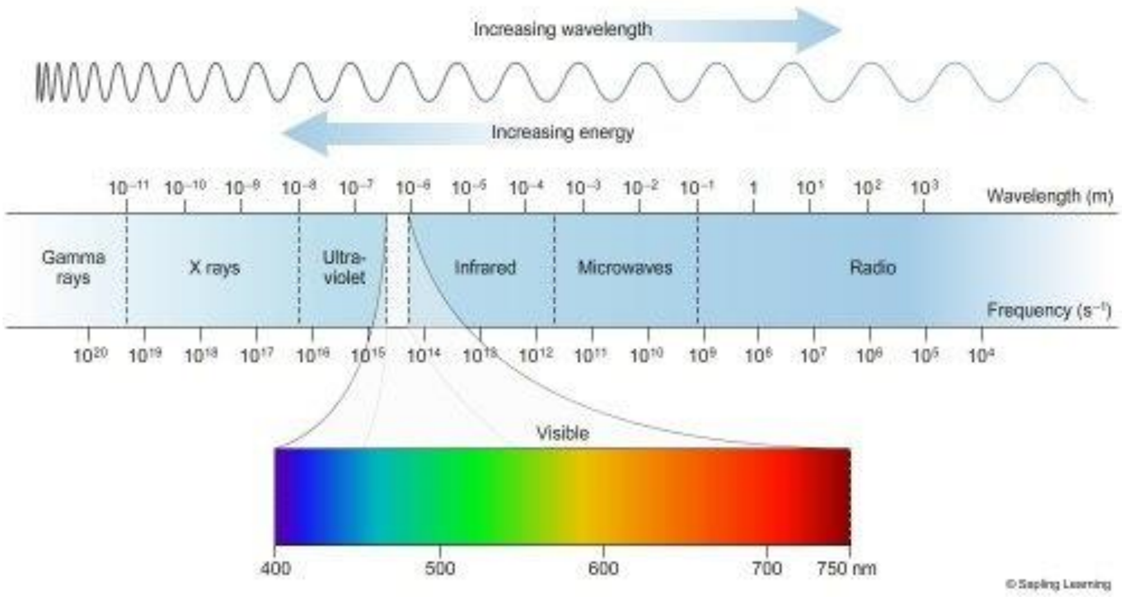
When sunlight passes through the atmosphere, hydrogen atoms in water vapor absorb the wavelengths (visible).

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \qquad R_H = 1.096776 \times 10^7 \text{ m}^{-1}$$

3.4: Quantization

- Current theories predict that charges are quantized in units (**quarks**) of $\pm e/3$ and $\pm 2e/3$, but quarks are not directly observed experimentally. The charges of particles that have been directly observed are quantized in units of $\pm e$.
- The measured atomic weights are not continuous—they have only discrete values, which are close to integral multiples of a unit mass.

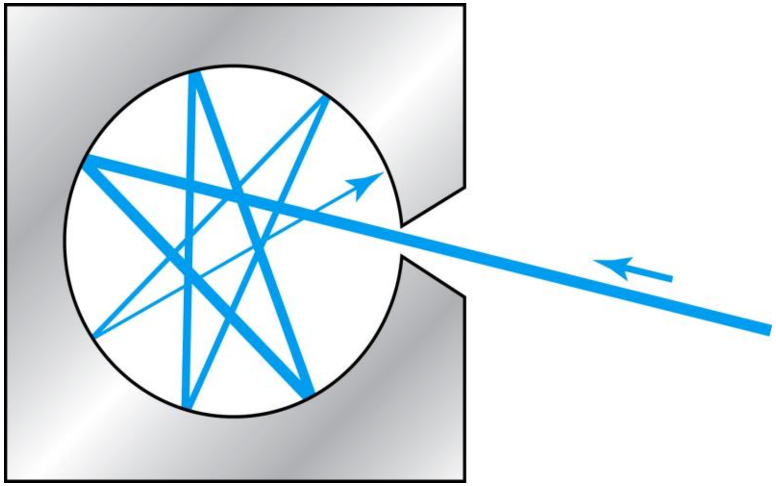
Molecules consist of integral number of atoms. Modes in a cavity are discrete



Light=electromagnetic radiation

3.5: Blackbody Radiation

- When matter is heated, it emits radiation.
- A blackbody is a cavity in a material that only emits thermal radiation. Incoming radiation is absorbed in the cavity.

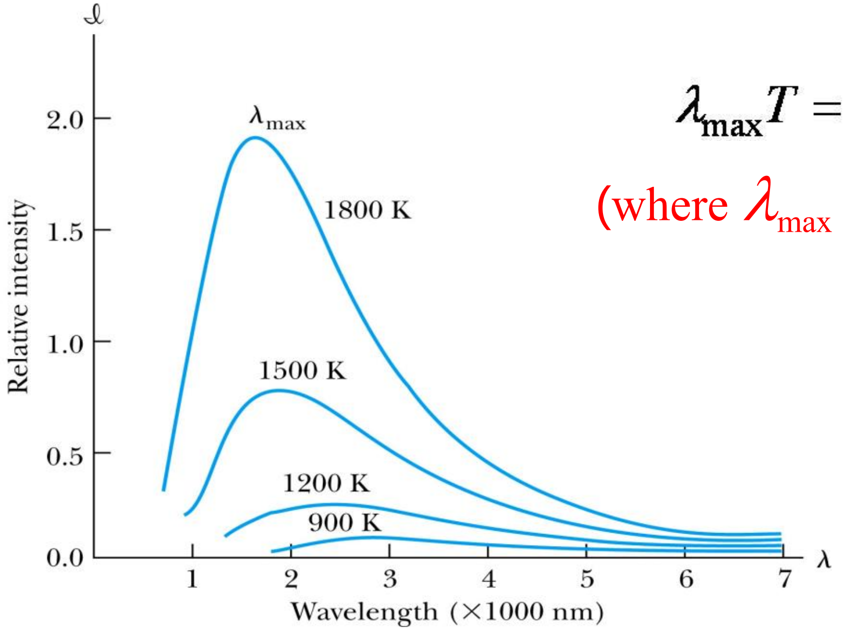


- Blackbody radiation is theoretically interesting because the radiation properties of the blackbody are independent of the particular material. Physicists can study the properties of intensity versus wavelength at fixed temperatures.

A key hole is always black and a black body

Wien's Displacement Law

- The intensity $\mathcal{I}(\lambda, T)$ is the total power radiated per unit area per unit wavelength at a given temperature.
- **Wien's displacement law**: The maximum of the distribution shifts to smaller wavelengths as the temperature is increased.



$$\lambda_{max} T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}$$

(where λ_{max} = wavelength of the peak)

Blacksmith formining a horse shoe

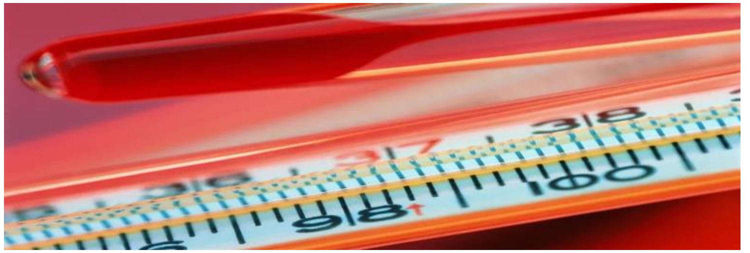
Problem 21. Calculate the maximum wavelength for blackbody radiation(a) liquid helium at 4.2 K (b)room temperature at 293 K,(c)a steel furnace at 2500 K,(d) a blue star at 9000 K

1. (a) $$\lambda_{max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{4.2\ \text{K}} = 0.69\ \text{mm}$$

(b) $$\lambda_{max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{293\ \text{K}} = 9.89\ \mu\text{m}$$

(c) $$\lambda_{max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{2500\ \text{K}} = 1.16\ \mu\text{m}$$

(d) $$\lambda_{max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{9000\ \text{K}} = 0.322\ \mu\text{m}$$



When do you have fever?

Celsius to Fahrenheit	° F = 9/5 (° C) + 32
Kelvin to Fahrenheit	° F = 9/5 (K - 273) + 32
Fahrenheit to Celsius	° C = 5/9 (° F - 32)
Celsius to Kelvin	K = ° C + 273
Kelvin to Celsius	° C = K - 273
Fahrenheit to Kelvin	K = 5/9 (° F - 32) + 273

for body temperature of 100 F (37.8 C)


MOULTRIE
84°F
05/25/11
10:16PM
HAN

Stefan-Boltzmann Law

- The total power radiated increases with the temperature:

$$R(T) = \int_0^\infty \mathscr{I}(\lambda, T)\, d\lambda = \epsilon\, \sigma T^4$$

- This is known as the **Stefan-Boltzmann law**, with the constant σ experimentally measured to be 5.6705×10^{-8} W / ($m^2 \cdot K^4$).

- The **emissivity** ϵ ($\epsilon = 1$ for an idealized blackbody) is simply the ratio of the emissive power of an object to that of an ideal blackbody and is always less than 1.

19. (a) A blackbody's temperature is increased from 900 K to 2300 K. By what factor does the total power radiated per unit area increase? (b) If the original temperature is again 900 K, what final temperature is required to double the power output?

(a) $\frac{P_1}{P_0} = \frac{\sigma T_1^4}{\sigma T_0^4}$; so $P_1 = P_0 \frac{T_1^4}{T_0^4} = \left(\frac{2300\text{ K}}{900\text{ K}}\right)^4 P_0 = 42.7\ P_0$; The power increases by a factor of 42.7.

(b) To double the power output, the ratio of temperatures to the fourth power must equal 2. $\left(\frac{T_1}{900\text{ K}}\right)^4 = 2$. Solving we find T1=900x2^1/4=1070 K

Problem 20 (a) At what wavelength will the human body radiate the maximum radiation? (b) Estimate the total power radiated by a person of medium build (assume an area given by a cylinder of 175-cm height and 13-cm radius). (c) Using your answer to (b), compare the energy radiated by a person in one day with the en- ergy intake of a 2000-kcal diet

1. (a) $\lambda_{max} = \dfrac{2.898\times10^{-3}\ \text{m·K}}{310\text{K}} = 9.35\ \mu\text{m}$

(b) At this temperature the power per unit area is

$R = \sigma T^4 = \left(5.67\times10^{-8}\,\text{W·m}^{-2}\cdot\text{K}^4\right)\left(310\text{K}\right)^4 = 524\text{W/m}^2$. The total surface area of a cylinder is

$2\pi r\left(r+h\right) = 2\pi\left(0.13\ \text{m}\right)\left(1.75\ \text{m}+0.13\ \text{m}\right) = 1.54\ \text{m}^2$ so the total power is

$P = \left(524\ \text{W/m}^2\right)\left(1.54\ \text{m}^2\right) = 807\ \text{W}$.

(c) The total energy radiated in one day is the power multiplied by the time;

$E = P\cdot t = \left(807\ \text{W}\right)\cdot\left(86400\ \text{s}\right) = 6.97\times10^7\ \text{J}$.

$2000\ \text{kcal} = \left(2\times10^6\ \text{cal}\right)\cdot\left(4.186\,\text{J/cal}\right) = 8.37\times10^6\ \text{J}$.

There is more energy radiated away than consumed by eating

There are several assumptions. First, a cylinder may overestimate the total surface area; second, radiation is minimized by hair covering and clothing.

273 +37 (body temperature in C) = 310

Planck's Radiation Law

- Planck assumed that the radiation in the cavity was emitted (and absorbed) by some sort of "oscillators" that were contained in the walls. He used Boltzman's statistical methods to arrive at the following formula that fit the blackbody radiation data

$$\mathcal{I}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Planck's radiation law

- Planck made two modifications to the classical theory:
  1) The oscillators (of electromagnetic origin) can only have certain discrete energies determined by $E_n = nhf$, where n is an integer, f is the frequencyof the radiation, and h is called Planck's constant. $h = 6.6261 \times 10^{-34}$ J·s.
  2) The oscillators can absorb or emit energy in discrete multiples of the fundamental quantum of energy given by

$$\Delta E = hf$$

EXAMPLE 3.6

Show that Wien's displacement law follows from Planck's radiation law.

Strategy Wien's law, Equation (3.14), refers to the wavelength for which $\mathcal{I}(\lambda, T)$ is a maximum for a given temperature. From calculus we know we can find the maximum value of a function for a certain parameter by taking the derivative of the function with respect to the parameter, set the derivative to zero, and solve for the parameter.

Solution Therefore, to find the value of the Planck radiation law for a given wavelength we set $d\mathcal{I}/d\lambda = 0$ and solve for λ.

$$\frac{d\mathcal{I}(\lambda, T)}{d\lambda} = 0 \quad \text{for } \lambda = \lambda_{\text{max}}$$

$$2\pi c^2 h \frac{d}{d\lambda}[\lambda^{-5}(e^{hc/\lambda kT} - 1)^{-1}]\bigg|_{\lambda_{\text{max}}} = 0$$

$$-5\lambda_{\text{max}}^{-6}(e^{hc/\lambda_{\text{max}}kT} - 1)^{-1} - \lambda_{\text{max}}^{-5}(e^{hc/\lambda_{\text{max}}kT} - 1)^{-2} \times \left(\frac{-hc}{kT\lambda_{\text{max}}^2}\right)e^{hc/\lambda_{\text{max}}kT} = 0$$

Multiplying by $\lambda_{\text{max}}^6(e^{hc/\lambda_{\text{max}}kT} - 1)$ results in

$$-5 + \frac{hc}{\lambda_{\text{max}}kT}\left(\frac{e^{hc/\lambda_{\text{max}}kT}}{e^{hc/\lambda_{\text{max}}kT} - 1}\right) = 0$$

Let

$$x = \frac{hc}{\lambda_{\text{max}}kT}$$

Then

$$-5 + \frac{xe^x}{e^x - 1} = 0$$

and

$$xe^x = 5(e^x - 1)$$

This is a transcendental equation and can be solved numerically (try it!) with the result $x \approx 4.966$, and therefore

$$\frac{hc}{\lambda_{\text{max}}kT} = 4.966$$

$$\lambda_{\text{max}}T = \frac{hc}{4.966\,k} = \frac{1240\text{ eV}\cdot\text{nm}}{4.966\left(8.617\times10^{-5}\,\frac{\text{eV}}{\text{K}}\right)}\,\frac{10^{-9}\text{ m}}{\text{nm}}$$

and finally,

$$\lambda_{\text{max}}T = 2.898\times10^{-3}\text{ m}\cdot\text{K}$$

which is the empirically determined Wien's displacement law.

EXAMPLE 3.7

Use Planck's radiation law to find the Stefan-Boltzmann law.

Strategy We determine $R(T)$ by integrating $\mathcal{I}(\lambda, T)$ over all wavelengths.

Solution

$$R(T) = \int_0^\infty \mathcal{I}(\lambda, T)\, d\lambda$$

$$= 2\pi c^2 h \int_0^\infty \frac{1}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}\, d\lambda$$

Let

$$x = \frac{hc}{\lambda kT}$$

Then

$$dx = -\frac{hc}{kT}\frac{d\lambda}{\lambda^2}$$

Now we have

$$R(T) = -2\pi c^2 h \int_\infty^0 \left(\frac{kT}{hc}\right)^6 x^5 \frac{1}{e^x - 1}\frac{1}{x^2}\left(\frac{hc}{kT}\right)^2 dx$$

$$= +2\pi c^2 h \left(\frac{kT}{hc}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1}\, dx$$

We look up this integral in Appendix 7 and find it to be $\pi^4/15$.

$$R(T) = 2\pi c^2 h \left(\frac{kT}{hc}\right)^4 \frac{\pi^4}{15}$$

$$R(T) = \frac{2\pi^5 k^4}{15 h^3 c^2} T^4$$

Putting in the values for the constants k, h, and c results in

$$R(T) = 5.67 \times 10^{-8}\, T^4 \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}$$

EXAMPLE 3.8

Show that the Planck radiation law agrees with the Rayleigh-Jeans formula for large wavelengths.

Strategy We use Equation (3.23) for the Planck radiation law, let $\lambda \to \infty$ for the term involving the exponential, and see whether the result agrees with Equation (3.22).

Solution We follow the strategy and find the result for the term involving the exponential.

$$\frac{1}{e^{hc/\lambda kT} - 1} = \frac{1}{\left[1 + \frac{hc}{\lambda kT} + \left(\frac{hc}{\lambda kT}\right)^2 \frac{1}{2} + \cdots\right] - 1} \to \frac{\lambda kT}{hc}$$

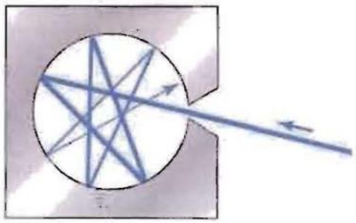
for large λ

Equation (3.23) now becomes

$$\mathscr{I}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{\lambda kT}{hc} = \frac{2\pi ckT}{\lambda^4}$$

which is the same as the Rayleigh-Jeans result in Equation (3.22).

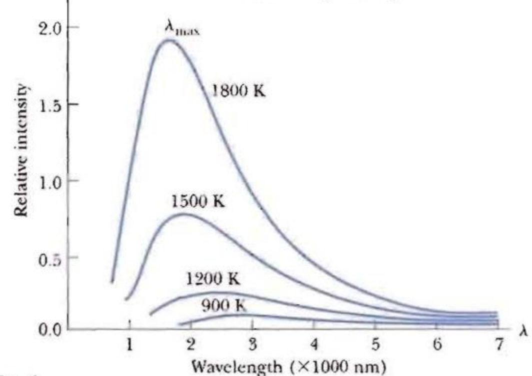
Summary: Blackbody radiation



black body ≡ absorbs all radiation falling on it (R=0)
↑ doesn't reflect

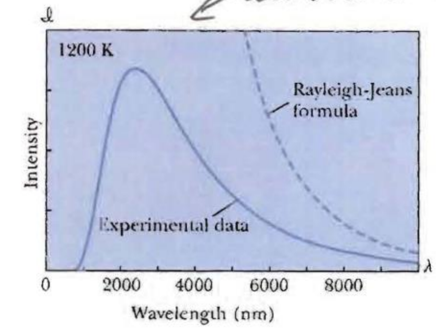
key hole is always black

$I = f(\lambda, T)$



note: maximum shifts to lower wave lengths and total power increases

ultra violet cathastrophy



empirical laws: 1879

Wien $\lambda_{max} T = 2.898 \times 10^{-3}\, m \cdot K$

power/area : Stephan Bolzmann

$$R(T) = \int_0^\infty I(\lambda, T)\, d\lambda = \epsilon \sigma T^4$$

emissivity

$\epsilon = 1$ for black body $\sigma = 5.67 \times 10^{-8}\, W/m^2K^4$

theoretical attempt based on classical E and M (Maxwell)
Rayleigh-Jeans

$$I(\lambda, T) = \frac{2\pi c k T}{\lambda^4}$$

Planck's radiation law

$$I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda k T} - 1}$$

oscillators emit discrete energy bundles = quanta

$E_n = n h f$
↑ frequency

Planck constant $h = 6.6 \times 10^{-34}\, J \cdot s$

Question form chapter 3 quiz

The Rydberg equation is used to

a. Determine the ratio of the electron charge to its mass
b. Calculate the wavelengths of different spectral lines of hydrogen
c. Measure the mass of the hydrogen atom
d. Calculate the wavelengths of different transitions in energy level of electrons in helium

How did Planck modify the classical theory of blackbody radiation to correctly determine his radiation law?

a. He found that the blackbody model was incorrect for purposes of theory

b. He accepted the Stefan-Boltzmann law

c. He assumed light was absorbed and emitted in quanta

d. He realized that the charge of the electron was not quantized

e. He proved the necessity of relativistic considerations problem 3.33

How many photons are contained in a beam of electromagnetic radiation of total power 180 W, if the source is a radio station of 1100 kHz?

$$\text{Energy/photon} = hf = (6.626 \times 10^{-34}\ J \cdot s)(1100 \times 10^{3}\ s^{-1}) = 7.29 \times 10^{-28}\ J$$

$$(180\ J/s)\ \frac{1\ \text{photon}}{7.29 \times 10^{-28}\ J} = 2.47 \times 10^{29}\ \text{photons/sec}$$

3.6: Photoelectric Effect

Methods of electron emission:

- Thermionic emission: Application of heat allows electrons to gain enough energy to escape.
- Secondary emission: The electron gains enough energy by transfer from another high-speed particle that strikes the material from outside.
- Field emission: A strong external electric field pulls the electron out of the material.
- Photoelectric effect: Incident light (electromagnetic radiation) shining on the material transfers energy to the electrons, allowing them to escape.

Electromagnetic radiation interacts with electrons within metals and gives the electrons increased kinetic energy. Light can give electrons enough extra kinetic energy to allow them to escape. We call the ejected electrons **photoelectrons**.

Work function=minimum binding energy of the electron

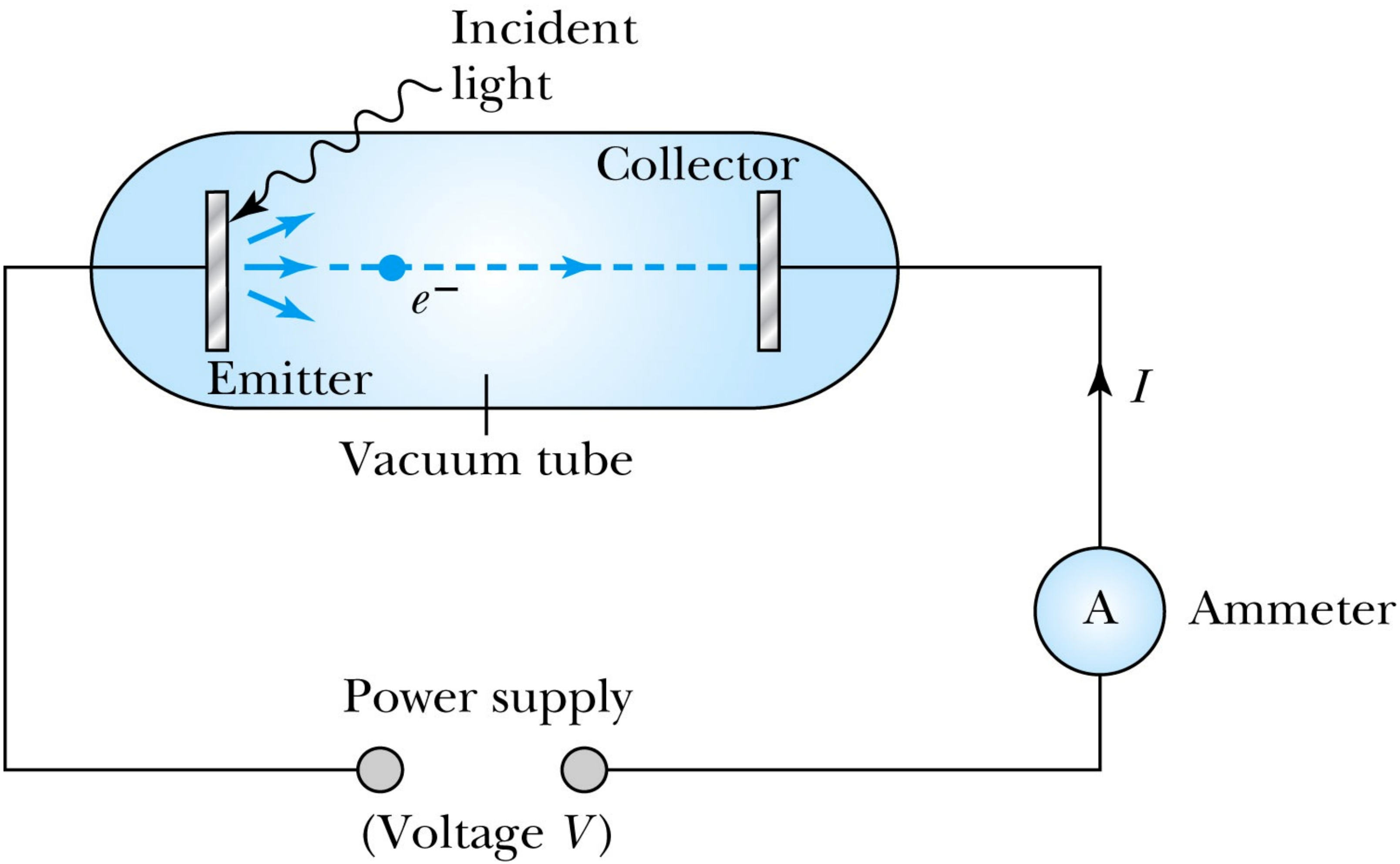
Table 3.3 Work Functions

Element	ϕ (eV)	Element	ϕ (eV)	Element	ϕ (eV)
Ag	4.64	K	2.29	Pd	5.22
Al	4.20	Li	2.93	Pt	5.64
C	5.0	Na	2.36	W	4.63
Cs	1.95	Nd	3.2	Zr	4.05
Cu	4.48	Ni	5.22		
Fe	4.67	Pb	4.25		

From *Handbook of Chemistry and Physics*, 90th ed. Boca Raton, Fla.: CRC Press (2009–10), pp. 12-114.

In metals electrons are weakly bound in the conduction band

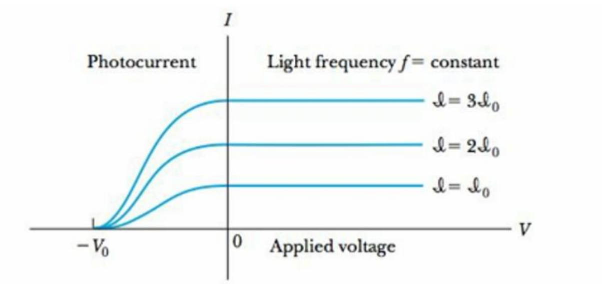
Experimental Setup

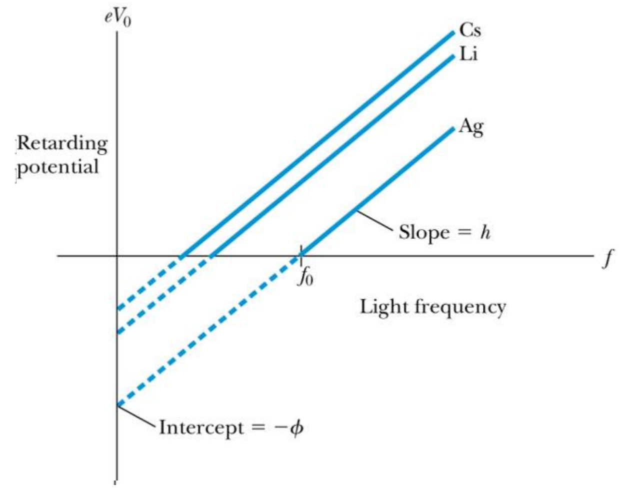


Experimental Results

Stopping potential measures maximum kinetic energy

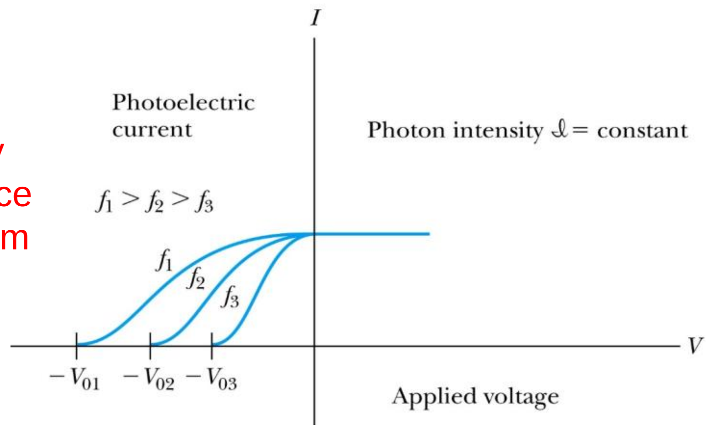
Same Kinetic energy

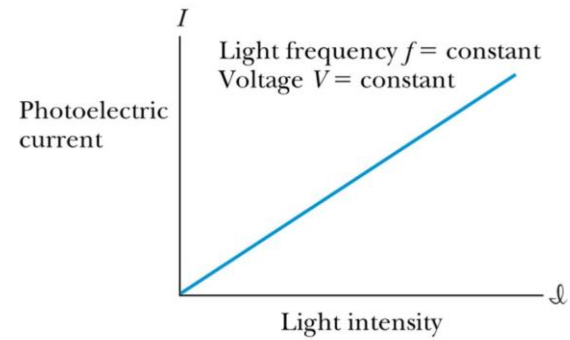




threshold frequency

Frequency dependence of maximum kinetic energy





Photoelectric current starts in nsec

Experimental Results

1) The kinetic energies of the photoelectrons are independent of the light intensity.
2) The maximum kinetic energy of the photoelectrons, for a given emitting material, depends only on the frequency of the light.
3) The smaller the work function φ of the emitter material, the smaller is the threshold frequency of the light that can eject photoelectrons.
4) When the photoelectrons are produced, however, their number is proportional to the intensity of light.
5) The photoelectrons are emitted almost instantly following illumination of the photocathode, independent of the intensity of the light.

Classical Interpretation Is not possible

- Classical theory predicts that the total amount of energy in a light wave increases as the light intensity increases.
- The maximum kinetic energy of the photoelectrons depends on the value of the light frequency f and not on the intensity.
- The existence of a threshold frequency is completely inexplicable in classical theory.
- Classical theory would predict that for extremely low light intensities, a long time would elapse before any one electron could obtain sufficient energy to escape. We observe, however, that the photoelectrons are ejected almost immediately.

Einstein's Theory

- Einstein suggested that the electromagnetic radiation field is quantized into particles called **photons**. Each photon has the energy quantum:

$$E = hf$$

  where *f* is the frequency of the light and *h* is Planck's constant.

- The photon travels at the speed of light in a vacuum, and its wavelength is given by

$$\lambda f = c$$

Einstein's Theory

- Conservation of energy yields:

$$\text{Energy before (photon)} = \text{energy after (electron)}$$

$$hf = \phi + \text{K.E. (electron)}$$

where ϕ is the work function of the metal

Explicitly the energy is

$$hf = \phi + \frac{1}{2}mv_{\max}^2$$

- The retarding potentials measured in the photoelectric effect are the opposing potentials needed to stop the most energetic electrons.

$$eV_0 = \frac{1}{2}mv_{\max}^2$$

Quantum Interpretation

- The kinetic energy of the electron does not depend on the light intensity at all, but only on the light frequency and the work function of the material.

$$\frac{1}{2}mv_{max}^2 = eV_0 = hf - \phi$$

- Einstein in 1905 predicted that the stopping potential was linearly proportional to the light frequency, with a slope *h*, the same constant found by Planck.

$$eV_0 = \frac{1}{2}mv_{max}^2 = hf - hf_0 = h(f - f_0)$$

- From this, Einstein concluded that light is a particle with energy:

$$E = hf = \frac{hc}{\lambda}$$

Summary: photoelectric effect

A quantum of light (photon) delivers his entire energy to an electron: $E = hf$

Conservation of energy $hf = \phi + K.E$

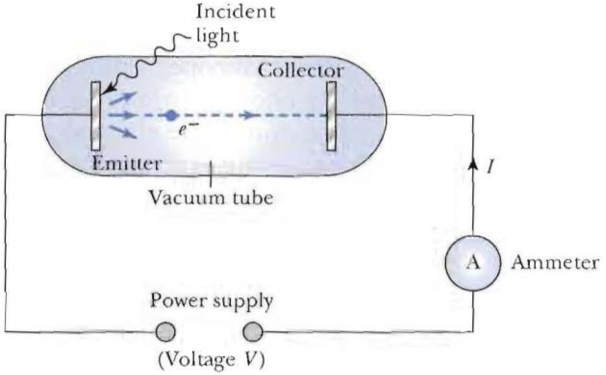
energy before (photon) = energy after electron

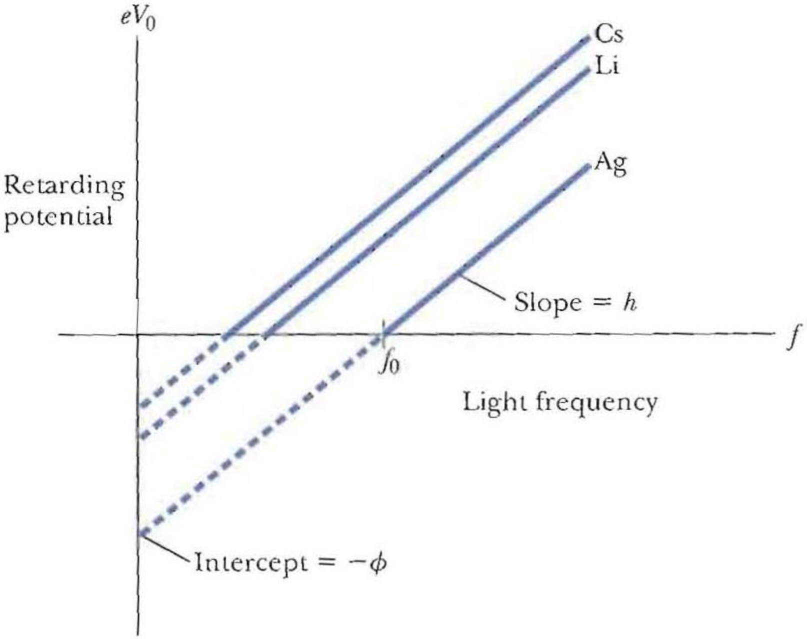
$\frac{1}{2} m v_{max}^2 = eV_0 = hf - \phi$

↑ stopping potential ↑ work function

Example For Li what frequency of light is needed to produce electrons of kinetic energy 3eV ?

$hf = \phi + \frac{1}{2} m v_{max}^2 = 2.93\,eV + 3\,eV = 5.9\,eV$ $f = \frac{E}{h} = \frac{5.9\,eV}{6.6\times10^{-34}\,J\,s} \cdot \frac{1.6\times10^{-19}\,J}{1\,eV} = 1.4\times10^{15}\,\frac{1}{s}$





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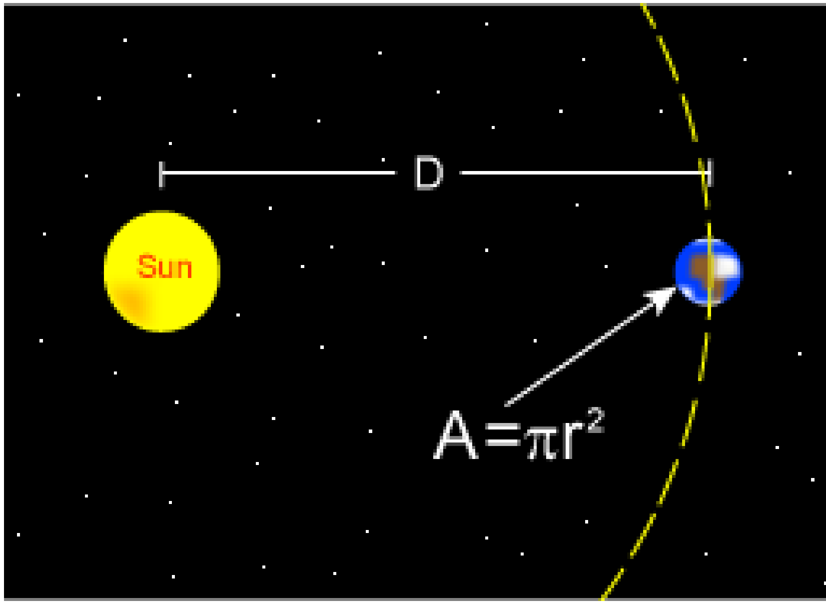
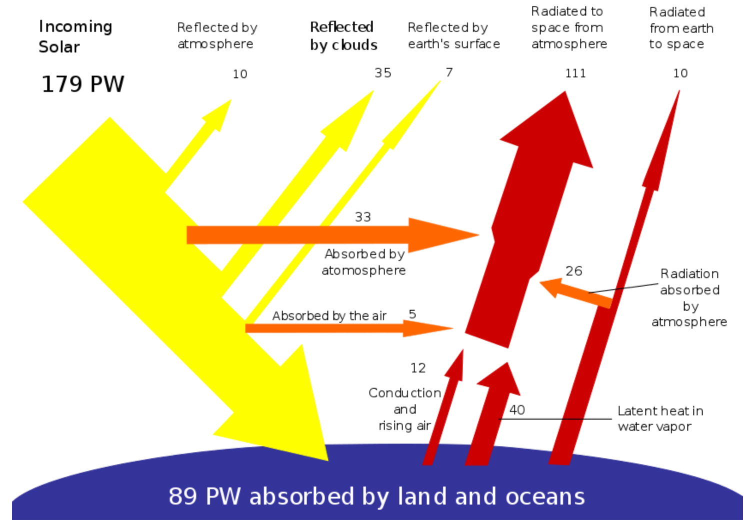

D
Sun
A=πr²


Incoming Solar
179 PW
Reflected by atmosphere
10
Reflected by clouds
35
Reflected by earth's surface
7
Radiated to space from atmosphere
111
Radiated from earth to space
10
33
Absorbed by atomosphere
26
Radiation absorbed by atmosphere
Absorbed by the air
5
12
Conduction and rising air
40
Latent heat in water vapor
89 PW absorbed by land and oceans

An argument for solar power

The wavelength of maximum intensity of the sun's radiation is observed to be near 500 nm. Assume the sun to be a blackbody and calculate (a) the sun's surface temperature, (b) the power per unit area $R(T)$ emitted from the sun's surface, and (c) the energy received by the Earth each day from the sun's radiation.

Strategy (a) We use Equation (3.14) with λ_{max} to determine the sun's surface temperature. (b) We assume the sun is a blackbody. We use the temperature T with Equation (3.16) to determine the power per unit area $R(T)$. (c) Because we know $R(T)$, we can determine the amount of the sun's energy intercepted by the Earth each day.

Solution (a) From Equation (3.14) we calculate the sun's surface temperature with $\lambda_{max} = 500$ nm.

$$(500\text{ nm})T_{sun} = 2.898 \times 10^{-3}\text{ m}\cdot\text{K}\,\frac{10^9\text{ nm}}{\text{m}}$$

$$T_{sun} = \frac{2.898 \times 10^6}{500}\text{ K} = 5800\text{ K} \qquad (3.17)$$

(b) The power per unit area $R(T)$ radiated by the sun is found to be

$$R(T) = \sigma T^4 = 5.67 \times 10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}(5800\text{ K})^4$$

$$= 6.42 \times 10^7\text{ W/m}^2 \qquad (3.18)$$

(c) Because this is the power per unit surface area, we need to multiply it by $4\pi r^2$, the surface area of the sun. The radius of the sun is 6.96×10^5 km.

$$\text{Surface area (sun)} = 4\pi(6.96 \times 10^8\text{ m})^2 = 6.09 \times 10^{18}\text{ m}^2$$

Thus the total power, P_{sun}, radiated from the sun's surface is

$$P_{sun} = 6.42 \times 10^7\frac{\text{W}}{\text{m}^2}(6.09 \times 10^{18}\text{ m}^2)$$

$$= 3.91 \times 10^{26}\text{ W} \qquad (3.19)$$

The fraction F of the sun's radiation received by Earth is given by the fraction of the total area over which the radiation is spread.

$$F = \frac{\pi r_E^2}{4\pi R_{Es}^2}$$

where r_E = radius of Earth = 6.37×10^6 m, and R_{Es} = mean Earth-sun distance = 1.49×10^{11} m. Then

$$F = \frac{\pi r_E^2}{4\pi R_{Es}^2} = \frac{(6.37 \times 10^6\text{ m})^2}{4(1.49 \times 10^{11}\text{ m})^2} = 4.57 \times 10^{-10}$$

Thus the radiation received by the Earth from the sun is

$$P_{Earth}(\text{received}) = (4.57 \times 10^{-10})(3.91 \times 10^{26}\text{ W})$$

$$= 1.79 \times 10^{17}\text{ W}$$

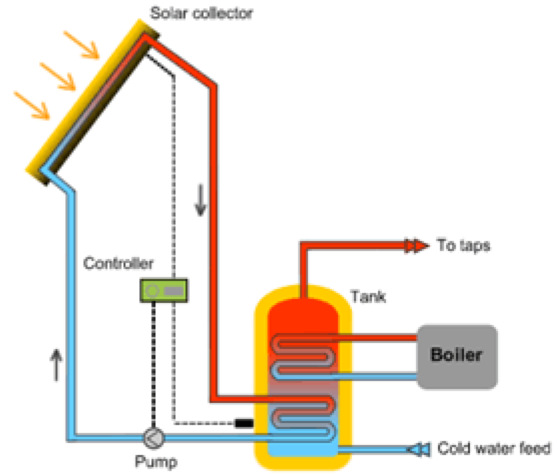
and in one day the Earth receives

$$U_{Earth} = 1.79 \times 10^{17}\frac{\text{J}}{\text{s}}\,\frac{60\text{ s}}{\text{min}}\,\frac{60\text{ min}}{\text{h}}\,\frac{24\text{ h}}{\text{day}}$$

$$= 1.55 \times 10^{22}\text{ J} \qquad (3.20)$$

The power received by the Earth per unit of exposed area is

$$R_{Earth} = \frac{1.79 \times 10^{17}\text{ W}}{\pi(6.37 \times 10^6\text{ m})^2} = 1400\text{ W/m}^2 \qquad (3.21)$$



Our sun acts as a black body
Measurements of the sun's radiation outside the earth's atmosphere gives $R \approx 1400\text{ W/m}^2$

34. What is the threshold frequency for the photoelectric effect on lithium ($\phi = 2.93$ eV)? What is the stopping potential if the wavelength of the incident light is 380 nm?

$$f_t = \frac{\phi}{h} = \frac{2.93 \text{ eV}}{4.136\times10^{-15} \text{ eV}\cdot\text{s}} = 7.08\times10^{14} \text{ Hz}: \; eV_0 = \frac{hc}{\lambda} - \phi \text{ so } V_0 = \frac{1}{e}\left[\frac{hc}{\lambda} - \phi\right];$$

$$V_0 = \frac{1}{e}\left[\frac{1240 \text{ eV}\cdot\text{nm}}{380 \text{ nm}} - 2.93 \text{ eV}\right] = 0.333 \text{ V}$$

35. What is the maximum wavelength of incident light that can produce photoelectrons from silver (ϕ = 4.64 eV)? What will be the maximum kinetic energy of the photoelectrons if the wavelength is halved?

$$\lambda_t = \frac{hc}{\phi} = \frac{1240\ \text{eV}\cdot\text{nm}}{4.64\ \text{eV}} = 267.2\ \text{nm}\,.$$

If the wavelength is halved (to $\lambda = 133.6\,\text{nm}$), then

$$K = \frac{hc}{\lambda} - \phi = \frac{1240\ \text{eV}\cdot\text{nm}}{133.6\ \text{nm}} - 4.64\ \text{eV} \ = 4.64\ \text{eV}$$

Problem 34. What is the threshold frequency for the photo electric effect in lithium with a work function of 2.93eV?What is the stopping potential if the wavelength of the incident light is 380 nm

$$f_t = \frac{\phi}{h} = \frac{2.93\text{ eV}}{4.136\times10^{-15}\text{ eV}\cdot\text{s}} = 7.08\times10^{14}\text{ Hz}: \quad eV_0 = \frac{hc}{\lambda} - \phi \text{ so } V_0 = \frac{1}{e}\left[\frac{hc}{\lambda} - \phi\right];$$

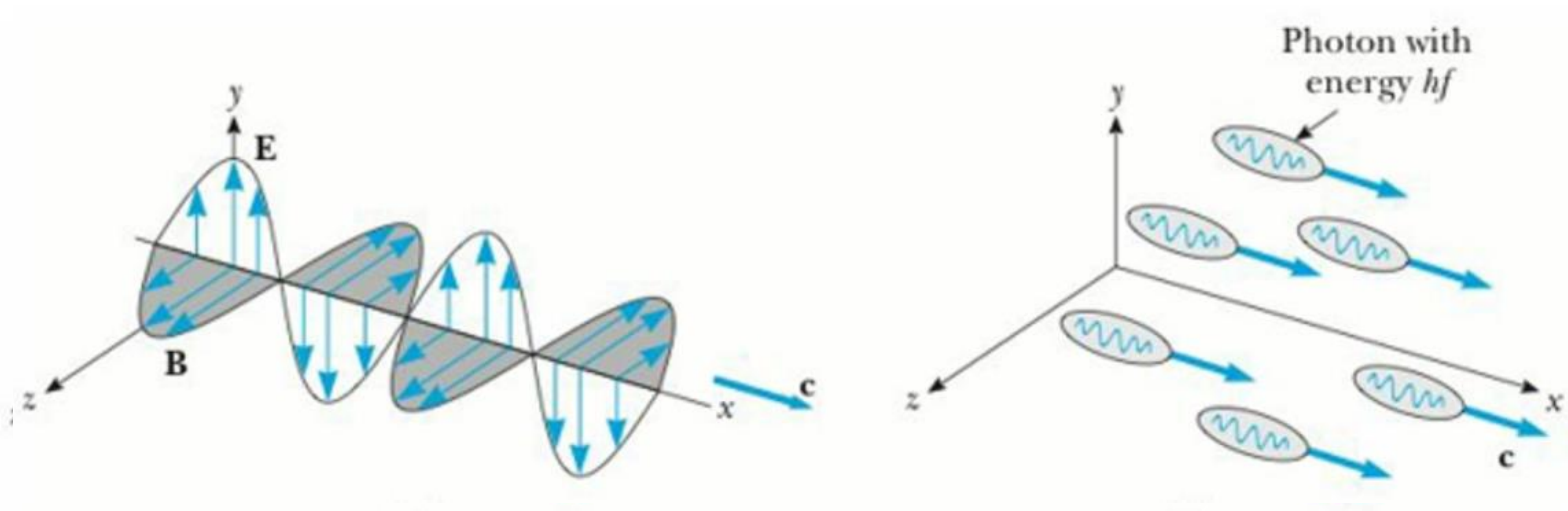
$$V_0 = \frac{1}{e}\left[\frac{1240\text{ eV}\cdot\text{nm}}{380\text{ nm}} - 2.93\text{ eV}\right] = 0.333\text{ V}$$

Question from chapter3 quiz

When you increase only the intensity of the light onto the emitter, you measure

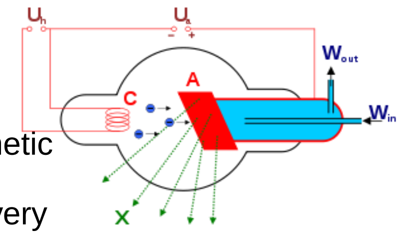
a. A decrease in the necessary stopping voltage
b. An increase in the necessary stopping voltage
c. No change in either current or stopping voltage
d. Either a or c. You cannot determine which from the information given.
e. An increased current

Maxwell classical light wave   Einstein Photon particle

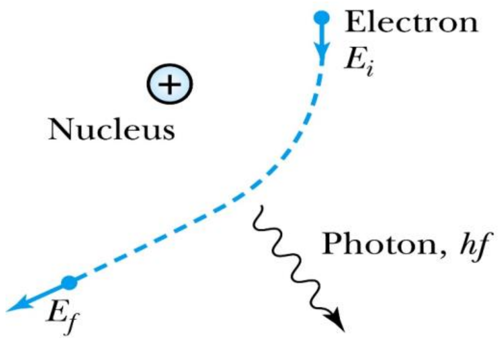


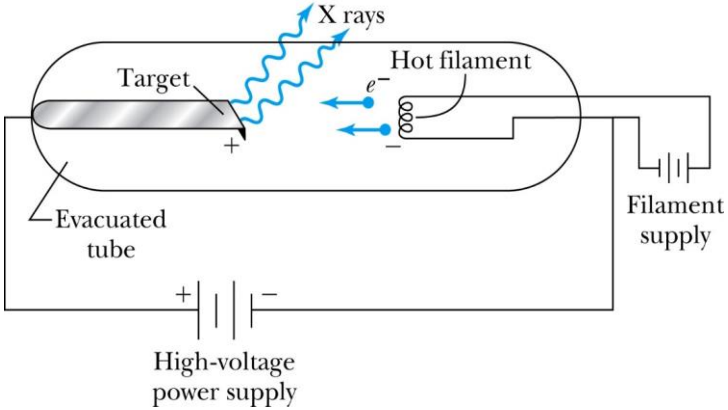
Rayleigh-jeans   Planck quantization   Einstein quantized photons

3.7: X-Ray Production(inverse photoelectric effect)



- An energetic electron passing through matter will radiate photons and lose kinetic energy which is called **bremsstrahlung**, from the German word for "braking radiation." Since linear momentum must be conserved, the nucleus absorbs very little energy, and it is ignored. The final energy of the electron is determined from the conservation of energy to be $E_f = E_i - hf$
- An electron that loses a large amount of energy will produce an X-ray photon. Current passing through a filament produces copious numbers of electrons by thermionic emission. These electrons are focused by the cathode structure into a beam and are accelerated by potential differences of thousands of volts until they impinge on a metal anode surface, producing x rays by bremsstrahlung as they stop in the anode material.





Unlike the photon an electron can give up part of its energy(as bremsstrahlung) and be the same electron

bremsstrahlung, from the German word for "braking radiation "

What is the bremsstrahlung process?

a. The emission of a photon from an electron being accelerated by a nucleus

b. The emission of an electron from a metal when light is shined on it

c. Thermal excitation of photons in a substance

d. The emission of an electron from an inner electron shell and the resulting photon when an electron drops from an outer shell to take its place

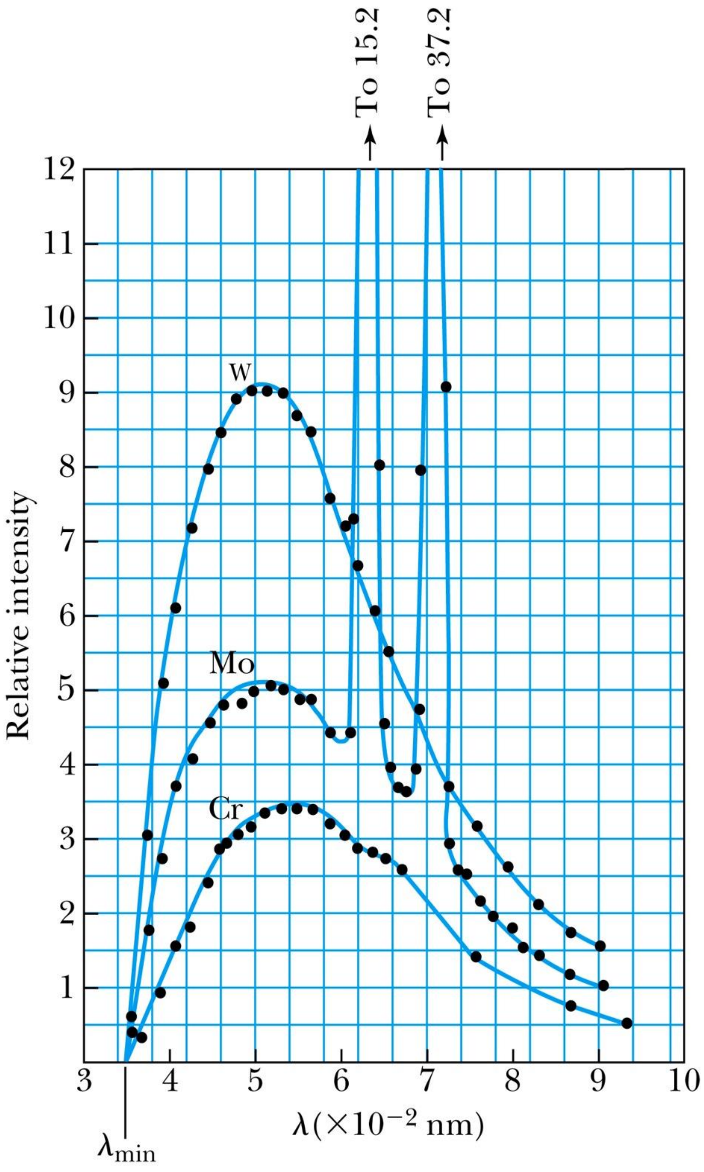
e. Converting power-producing nuclear material to weapons grade

Inverse Photoelectric Effect.

- Conservation of energy requires that the electron kinetic energy equal the maximum photon energy where we neglect the work function because it is normally so small compared to the potential energy of the electron. This yields the **Duane-Hunt limit** which was first found experimentally. The photon wavelength depends only on the accelerating voltage and is the same for all targets.

$$eV_0 = hf_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{e}\frac{1}{V_0} = \frac{1.240\times10^{-6}\ \text{V}\cdot\text{m}}{V_0}$$



Example 3.15 Inverse photoelectric effect

We have a tungsten anode (work function $\phi = 4.63 eV$) and an electron acceleration voltage of 35 kV. What is the minimum wavelength of the x-rays?

$$E_f = E_i - hf - \phi$$

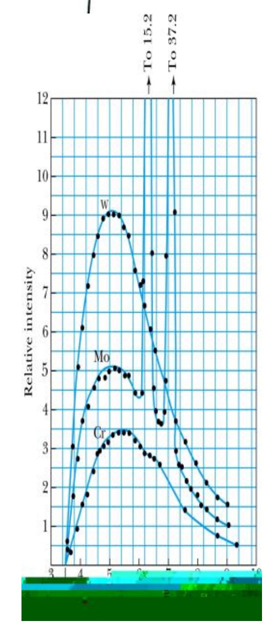
(ϕ) can be neglected compared to the usually kV acceleration

maximum energy } $eV_0 = hf_{max} = \frac{hc}{\lambda_{min}}$

Duane-Hunt rule $\boxed{\lambda_{min} = \frac{hc}{eV_0}} = \frac{1.24 \times 10^{-6} V \cdot m}{V_0} = \frac{1.24 \times 10^{-6} Vm}{35 \times 10^3 V} = 3.54 \times 10^{-11} m$

$V_0 = 35 kV$

in good agreement with Fig 3.15



A comment

Light is described by wavelengths in nm. It is useful to calculate the energy in terms of λ for a photon

$$E = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ J\cdot s)(3\times10^{8}\ m/s)}{} \ \frac{}{1.6\times10^{-19}\ J/eV} \ \frac{1\ nm}{10^{-9}\ m}$$

$$\boxed{E = \frac{1.240\times10^{3}\ eV\ nm}{\lambda}}$$

Bremsstrahlung:
in X-ray emission

$$1.240\times10^{3}\ eV\ nm\ \frac{10^{-9}\ m}{1\ nm}$$

$$1.240\times10^{-6}\ eV\ m$$

$$\lambda_{min} = \frac{hc}{eV_0} = \frac{1.240\times10^{-6}\ eV\cdot m}{e\,V_0}$$

V_0 ↑ acceleration voltage

3.8: Compton Effect

- When a photon enters matter, it is likely to interact with one of the atomic electrons. The photon is scattered from only one electron, rather than from all the electrons in the material, and the laws of conservation of energy and momentum apply as in any elastic collision between two particles. The momentum of a particle moving at the speed of light is

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

- The electron energy can be written as

$$E_e^2 = (mc^2)^2 + p_e^2 c^2$$

- This yields the change in wavelength of the scattered photon which is known as the **Compton effect**:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

Compton wavelength = h/mc = 2.426x 10^-3 nm

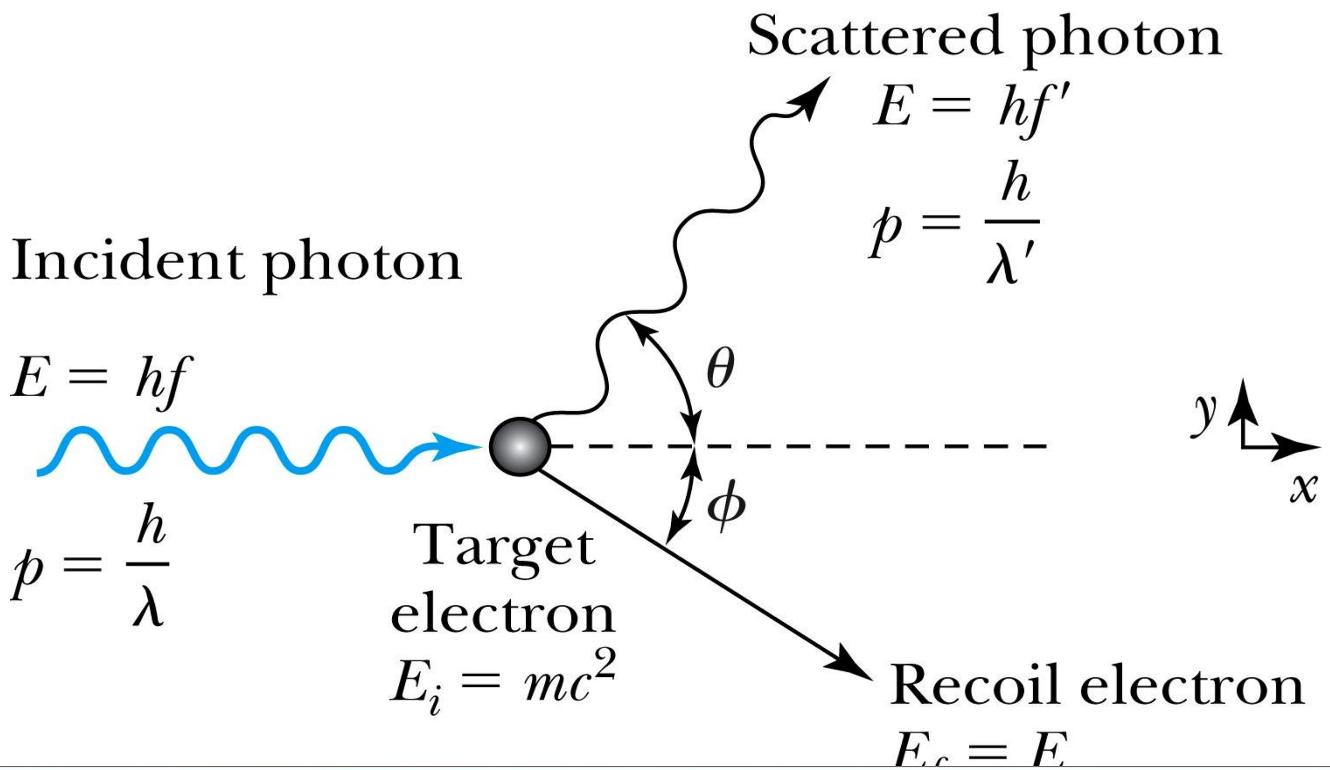
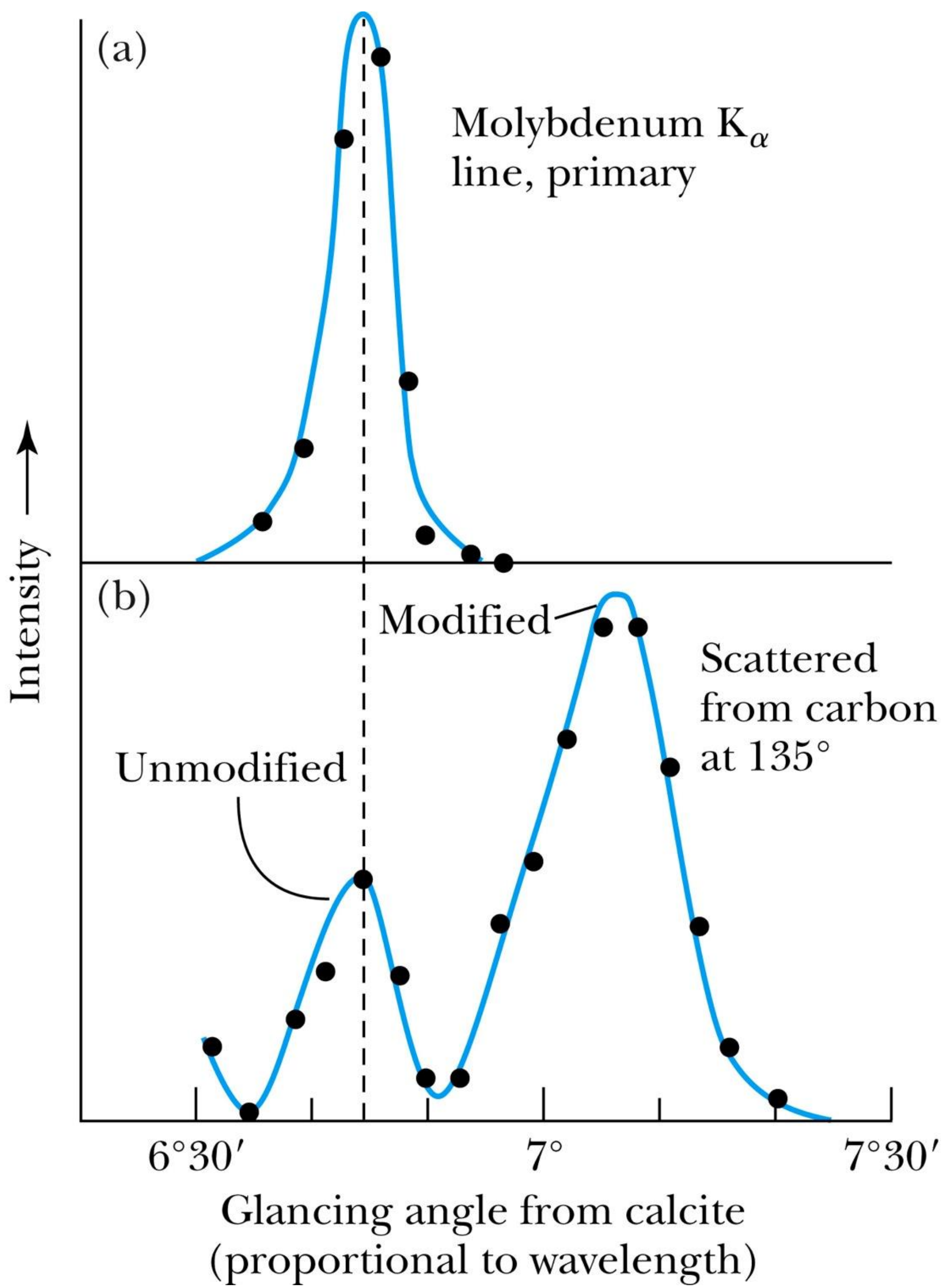


Table 3.4 Results of Compton Scattering

Energy or Momentum	Initial System	Final System
Photon energy	hf	hf'
Photon momentum in x direction (p_x)	$\frac{h}{\lambda}$	$\frac{h}{\lambda'}\cos\theta$
Photon momentum in y direction (p_y)	0	$\frac{h}{\lambda'}\sin\theta$
Electron energy	mc^2	$E_e = mc^2 + \text{K.E.}$
Electron momentum in x direction (p_x)	0	$p_e \cos\phi$
Electron momentum in y direction (p_y)	0	$-p_e \sin\phi$

Figure 3-20 p114



Thomson scattering=photon scattering from an tightly bound electron(use atom mass)

Compton scattering=photon scattering from a loosely bound electron(use electron mass)

Figure 3-21 p115

52. A 650-keV gamma ray Compton-scatters from an electron. Find the energy of the photon scattered at 110°, the kinetic energy of the scattered electron, and the recoil angle of the electron.

$$\lambda' = \lambda + \lambda_c\left(1-\cos\theta\right) = \frac{hc}{E} + \lambda_c\left(1-\cos\theta\right);$$

$$\lambda' = \frac{1240\ \text{eV}\cdot\text{nm}}{650\times10^{3}\ \text{eV}} + \left(2.43\times10^{-3}\ \text{nm}\right)\left(1-\cos 110^{\circ}\right) = 5.17\ \text{pm}$$

$$E' = \frac{hc}{\lambda'} = \frac{1240\ \text{eV}\cdot\text{nm}}{5.17\times10^{-3}\ \text{nm}} = 2.40\times10^{5}\,\text{eV} = 240\,\text{keV}$$

By conservation of energy we find: $K_e = E - E' = 650\,\text{keV} - 240\,\text{keV} = 410\,\text{keV}$ which agrees with the K formula in the previous problem. Also from the previous problem we have:

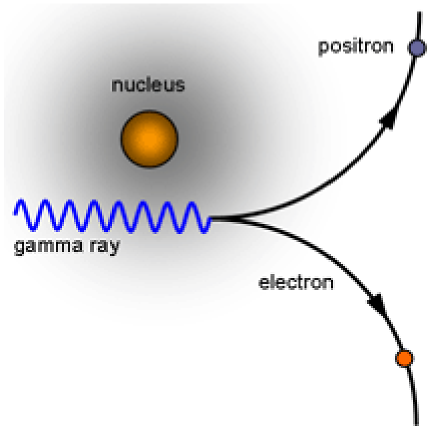
$$\cot\phi = \left[1+\frac{hf}{mc^2}\right]\tan\left(\frac{\theta}{2}\right) = \left[1+\frac{650\ \text{keV}}{511\ \text{keV}}\right]\tan\left(\frac{110^{\circ}}{2}\right) = 3.245 \text{ so}$$

$\phi = 17.1^{\circ}$.

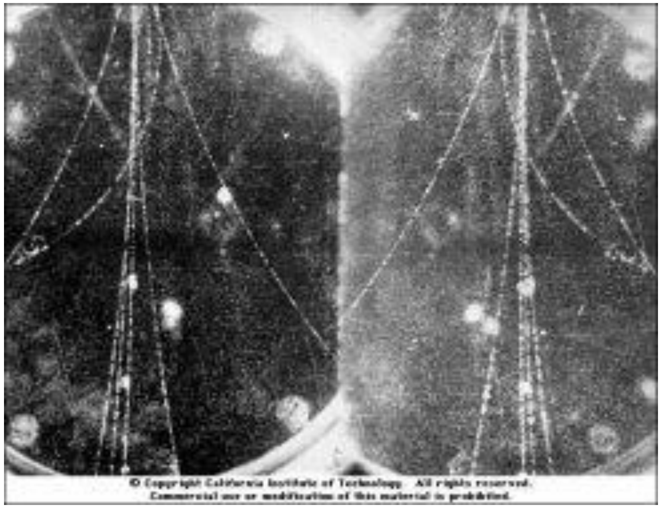
3.9: Pair Production and Annihilation

- If a photon can create an electron, it must also create a positive charge to balance charge conservation.
- In 1932, C. D. Anderson observed a positively charged electron (e^+) in cosmic radiation. This particle, called a positron, had been predicted to exist several years earlier by P. A. M. Dirac.
- A photon's energy can be converted entirely into an electron and a positron in a process called pair production.

$$\gamma \rightarrow e^{+} + e^{-}$$



Pair production from gamma ray



Cloud chamber with tracks left behind by positron and electron

Pair production

A dramatic proof of relativistic change of mass

$$E = h\nu = pc$$
$$E^2 = p^2c^2 + m^2c^4$$

Rest mass of electron

$$E = m_oc^2 = 9.1x10^{-31}(3x10^8)^2\, kg\frac{m^2}{s^2}\frac{1\ eV}{1.6x10^{-19}g}$$
$$E = 0.51\ MeV$$

For pair production (birth of positron and electron) as shown in the cloud chamber picture; an energy of at least 2x0.51MeV=1.02MeV is needed. The photon which transforms into particle must have an energy

$$E = h\nu = pc\ > 1.02\ MeV$$

Pair annihilation

Inverse process; where 2 or 3 photons are produced when

$$e^+ + e^- = nh\nu$$

Conservation of mass energy

Proton – antiproton
Electron – positron
Hydrogen – antihydrogen
Neutron – antineutron
Matter – antimatter

Pair Production in Empty Space

- Conservation of energy for pair production in empty space is

$$hf = E_+ + E_-$$

- Considering momentum conservation yields

$$hf = p_- c \cos\theta_- + p_+ c \cos\theta_+$$

- This energy exchange has the maximum value $hf_{\max} = p_- c + p_+ c$

- Recall that the total energy for a particle can be written as

$$E_\pm^2 = p_\pm^2 c^2 + m^2 c^4$$

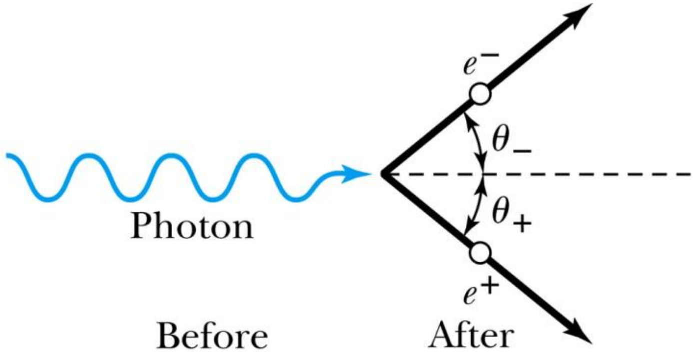
However this yields a contradiction: $hf > p_- c + p_+ c$
and hence the conversion of energy in empty space is an impossible situation.

Pair Production in Matter

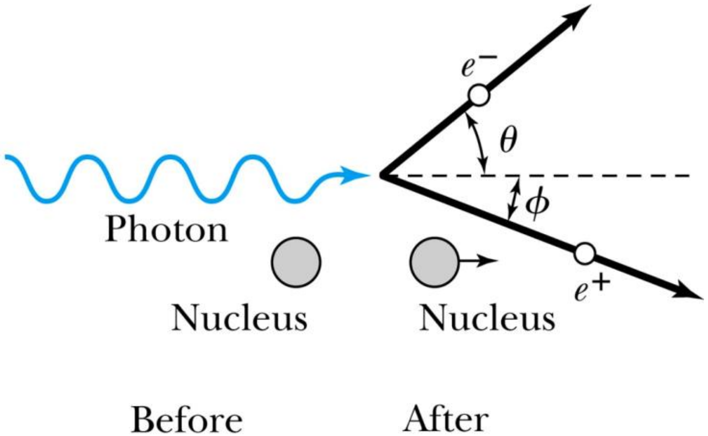
- Since the relations $hf_{max} = p_- c + p_+ c$ and $hf > p_- c + p_+ c$ contradict each other, a photon can not produce an electron and a positron in empty space.
- In the presence of matter, the nucleus absorbs some energy and momentum.

$$hf = E_+ + E_- + \text{K.E. (nucleus)}$$

- The photon energy required for pair production in the presence of matter is $hf > 2m_e c^2 = 1.022\ \text{MeV}$



(a) Free space (**cannot occur**)



(b) Beside nucleus

Pair Annihilation

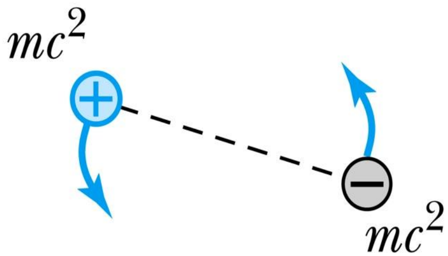
Para-positronium T=0.12 ns Ortho-positronium T=138 ns

- A positron passing through matter will likely **annihilate** with an electron. A positron is drawn to an electron by their mutual electric attraction, and the electron and positron then form an atomlike configuration called **positronium**.
- Pair annihilation in empty space will produce two photons to conserve momentum. Annihilation near a nucleus can result in a single photon.
- Conservation of energy: $2m_e c^2 \approx hf_1 + hf_2$
- Conservation of momentum: $0 = \frac{hf_1}{c} - \frac{hf_2}{c}$
- The two photons will be almost identical, so that

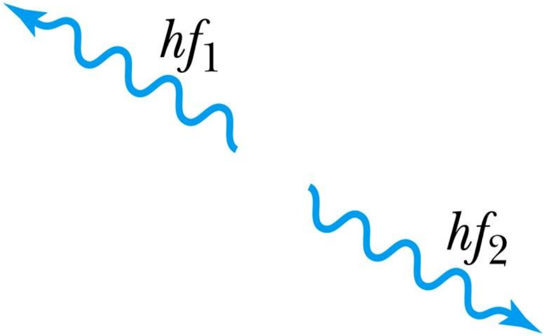
$$f_1 = f_2 = f$$

- The two photons from positronium annihilation will move in opposite directions with an energy:

$$hf = m_e c^2 = 0.511 \text{ MeV}$$



(a) Positronium, before decay (schematic only)



(b) After annihilation

Application of pair production in PET (partical-wave duality)

PET: positron emission tomography

use positron emitting pharmaceuticals (
The accumulate for instance in cancer cells, also
are transported by blood. When positron annihilates with an electron, the two γ-rays indicate
the locating

$$\boxed{e^+ + e^- = 2h\nu}$$ ⟹ opposite direction $2h\nu \simeq 1.02\,MeV$

$$E_0 = m_0 c^2 = 9.1\times10^{-31}\,kg\,(3\times10^8)^2\,\frac{m^2}{s^2}\left(\frac{1\,eV}{1.6\times10^{-19}\,J}\right) = 0.51\,MeV$$

factor of unity for conversion

E_0: rest energy; m_0: rest mass

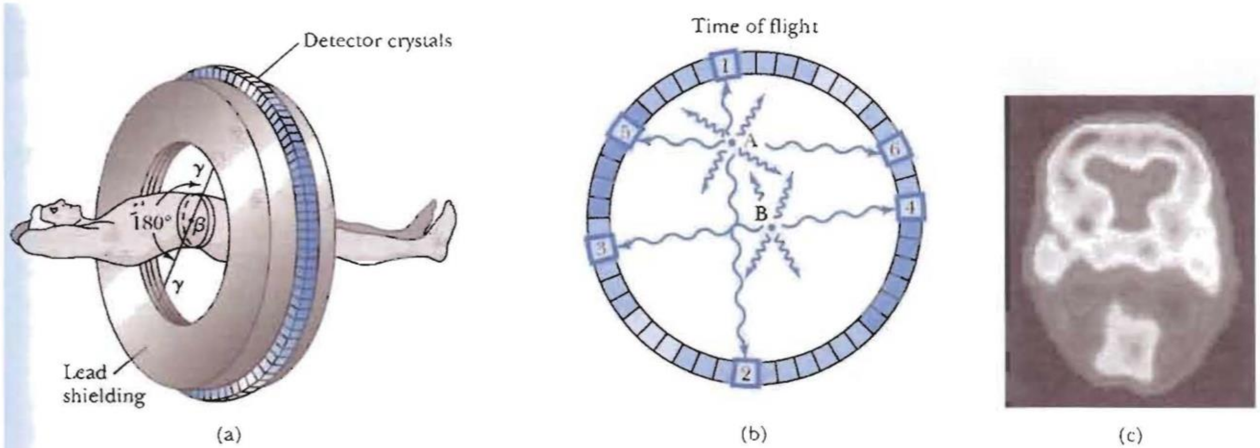


Figure 3.24 Positron emission tomography is a useful medical diagnostic tool to study the path and location of a positron-emitting radiopharmaceutical in the human body. (a) Appropriate radiopharmaceuticals are chosen to concentrate by physiological processes in the region to be examined. (b) The positron travels only a few millimeters before annihilation, which produces two photons that can be detected to give the positron position. (c) PET scan of a normal brain. *(a) and (b) are after G. L. Brownell et al., Science* **215**, *619 (1982); (c) National Institutes of Health/SPL/Photo Researchers, Inc.*