Find the most probable radius for the hydrogen atom in the 1s state. (Use $P_{1,0}=\frac{4r^2}{a_0^3} * e^{-\frac{2r}{a_0}}$)

Most probable is when probability is maximized. use first derivative test.

$$\frac{d(P_{1,0})}{dr} = 0 \Rightarrow \frac{d}{dr}\left[\frac{4r^2}{a_0^3} e^{-\frac{2r}{a_0}}\right] = 0$$

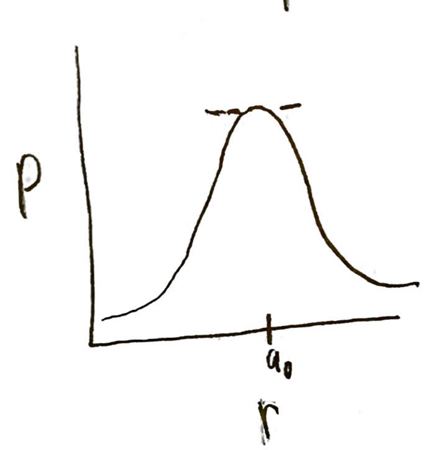
$$0 = \frac{8r}{a_0^3}\left(e^{-\frac{2r}{a_0}}\right) + \left(\frac{4r^2}{a_0^3}\right) e^{\left(-\frac{2r}{a_0}\right)} \cdot \left(-\frac{2}{a_0}\right)$$

$$0 = \left(\frac{8r}{a_0^3} - \frac{8r^2}{a_0^4}\right) e^{-\frac{2r}{a_0}}$$

Can never equal zero

$$0 = \frac{8r}{a_0^3} - \frac{8r^2}{a_0^4}$$

Graphical depection



$$\frac{\cancel{8}r^{\cancel{2}}}{a_0^{\cancel{4}}} = \frac{\cancel{8}\cancel{r}}{\cancel{a_0^3}}$$

$$\frac{r}{a_0} = 1$$

$$\boxed{r = a_0}$$

An electron is trapped in a square well of length $L = 10^{-10}$ m. What is the probability of finding the electron in the region from x = 0.5E-10 m to 0.6E-10 m.

a. If the electron is in the ground state?
b. If the electron is in the first excited state?

$\Psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$, n is state , $x_1 = 0.5E{-10}m$, $x_2 = 0.6E{-10}m$

a) n=1

$$P = \int_{x_1}^{x_2} \psi^* \cdot \psi \, dx = \int_{x_1}^{x_2} \sqrt{\frac{2}{L}}^{2} \sin^2\left(\frac{\pi}{L}x\right) dx = \frac{2}{L}\int_{x_1}^{x_2} \sin^2\left(\frac{\pi}{L}x\right) dx$$

$$\Rightarrow \frac{2}{L}\left[\frac{x}{2} - \frac{L}{4\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_{x_1}^{x_2} = \left(\frac{x_2}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi x_2}{L}\right)\right) - \left(\frac{x_1}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi x_1}{L}\right)\right)$$

$$\Rightarrow 0.6935 - 0.5 = \boxed{0.1935}$$

b) n=2

$$P = \int_{x_1}^{x_2} \psi^* \psi \, dx = \frac{2}{L}\int_{x_1}^{x_2} \sin^2\left(\frac{2\pi}{L}x\right) dx = \frac{2}{L}\left[\frac{x}{2} - \frac{L}{8\pi}\sin\left(\frac{4\pi x}{L}\right)\right]_{x_1}^{x_2}$$

$$\Rightarrow \left(\frac{x_2}{L} - \frac{1}{4\pi}\sin\left(\frac{4\pi x_2}{L}\right)\right) - \left(\frac{x_1}{L} - \frac{1}{4\pi}\sin\left(\frac{4\pi x_1}{L}\right)\right) = 0.524 - 0.5 = \boxed{0.2432}$$

Find the normalization constant A for the following wave function. $\Psi^* \Psi = A^2 r^2 e^{-\frac{2r}{\alpha}}$

$$1 = \int_0^\infty \psi^* \psi \, d_r = A^2 \int_0^\infty r^2 e^{\frac{-2r}{\alpha}} dr \rightarrow \text{use double integration by parts}$$

$$\Rightarrow A^2 \left[\frac{\alpha^3}{4} \right] = 1 \Rightarrow A^2 = \frac{4}{\alpha^3} \Rightarrow \boxed{A = \sqrt{\frac{4}{\alpha^3}} = 2\alpha^{-3/2}}$$