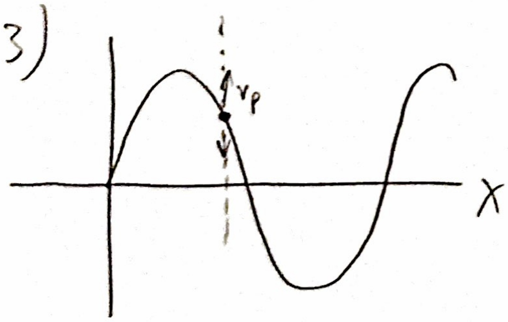
1) $P = \int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$ We want total probability of a particle being anywhere in space to be 100%.

2) $\Delta p \Delta x \geq \frac{\hbar}{2}$, The product of the momentum uncertanty & position uncertanty must be greater than or equal to $\frac{\hbar}{2}$.

3)



Phase Velocity is the speed of a point on a wave at a given phase.

4) For a harmonic osillator, $E_n = (n+\frac{1}{2})\hbar\omega$.

$\Delta E = E_{n+1} - E_n \Rightarrow (\cancel{n}+1+\cancel{\tfrac{1}{2}})\hbar\omega - \cancel{(n+\tfrac{1}{2})\hbar\omega} = \hbar\omega \Rightarrow$ Energy levels have seperation of $\hbar\omega$

5) Degeneracy $= g_n = n^2 \rightarrow \underline{n=4 \rightarrow g_n = 16}$

6) $E_n = \frac{(n\hbar\pi)^2}{2mL^2}$ For 2nd excited state, $n=3$ * Remember, the ground state starts at $n=1$.

energy levels of a particle in a box.

$$\boxed{E_3 = \frac{(3\hbar\pi)^2}{2mL^2}}$$

7) Angular momentum codes go alphabetically after first 4.

$\ell = 1\ 2\ 3\ 4\ 5\ 6 \ldots$

$\quad\ \ s\ p\ d\ f\ g\ h \ldots$

---

8) $\lambda = \frac{h}{p}$, $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Since $m_p < m_n$ & $\lambda_p = \lambda_n$

$\rightarrow \frac{h}{P_p} = \frac{h}{P_n} \Rightarrow P_p = P_n$

$\rightarrow K_p = \frac{P_p^2}{2m_p}$, $K_n = \frac{P_n^2}{2m_n}$

$\rightarrow K_p > K_n$ since $\frac{1}{m_p} > \frac{1}{m_n}$

---

9) $\vec{\mu}_n = \frac{-g_n \mu_B \vec{N}}{\hbar}$ ← angular momentum term

$n = \ell \rightarrow g_\ell = 1,\ \vec{N} = \vec{L}$

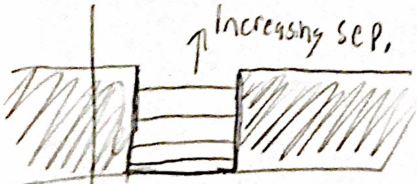
$n = s \rightarrow g_s = 2,\ \vec{N} = \vec{S}$

The gyromagnetic ratio connects magnetic moments to angular momentum.

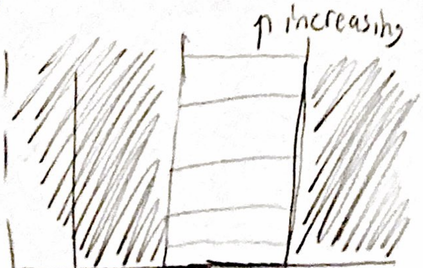
---

10)

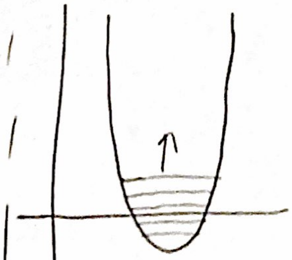
Finite square well	Particle in a Box [Infinite square well]	Harmonic Oscill.	Hydrogen Atom



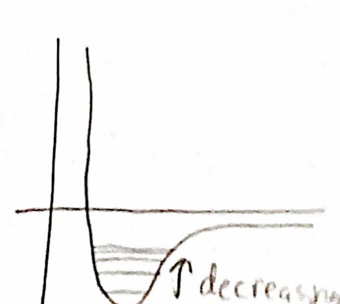
$\Delta E = \frac{(n\hbar)^2}{2m}$



$\Delta E = \frac{(n\pi\hbar)^2}{2mL^2}$



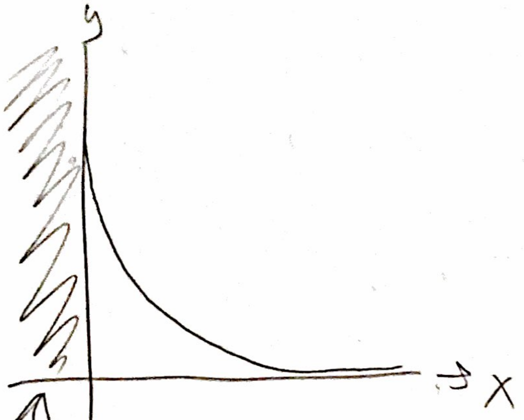
$\Delta E = \hbar\omega$



$\Delta E = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

Smaller ΔE means lower energy photons (longer wavelengths).

11) $e^{-i(kx-\omega t)} + e^{i(\omega t-kx)} \Rightarrow e^{-i(kx-\omega t)} + e^{-i(kx-\omega t)} = 2\underbrace{e^{-i(kx-\omega t)}}_{\text{is normalizable}}$



we only consider values of $+x$
as $-x$ values not possible in real system.*

*usually

Conditions

✓ 1. Must be finite over allowed values of x. [no asymptotes]

✓ 2. Must be single-valued [valid function]

✓ 3. Must be continuous. [Derivative test]

✓ 4. ψ must go to zero as x approaches $+\infty$.

$$\lim_{x\to\infty} 2e^{-i(kx-\omega t)} = 2e^{-i(\infty)} = 2\cdot 0 = 0 \checkmark$$

---

12)

$n_x^2+n_y^2+n_z^2$	3	6	6	6	9	9	9	12	11	11	11	19	...
n_x	1	1	1	2	2	2	1	2	1	1	3	3	
n_y	1	1	2	1	2	1	2	2	1	3	1	3	
n_z	1	2	1	1	1	2	2	2	3	1	1	1	
	1	3			3			1	3				

$^3E_n = {}^1E_i(n_x^2+n_y^2+n_z^2)$

By def, $E_n > E_{n-1}$

So

^3E_n	$^3E_1$	$^3E_2$	$^3E_3$	$^3E_4$	$^3E_5$
$^1E_i[^3E_n]$	3	6	9	11	12
g_n	1	3	3	3	1

---

13) $5.5E18\left(\frac{\gamma}{s}\right)$, $\Delta E = |E_f - E_i| = 2.3\,(eV)$ per γ $\rightarrow$ $\frac{2.3\,\cancel{eV}}{\gamma}\left|\frac{1.6E\text{-}19\,J}{1\,\cancel{eV}}\right| = 3.68E\text{-}19\left(\frac{J}{\gamma}\right)$

γ per second; E_f: 0eV, E_i: 2.3(eV)

$$\boxed{\text{Power} = E/t = 3.68E\text{-}19\left(\tfrac{J}{\cancel{\gamma}}\right)\cdot 5.5E18\left(\tfrac{\cancel{\gamma}}{s}\right) = 2\left(\tfrac{J}{s}\right) = \boxed{2W}}$$

14) $\lambda = \frac{h}{p}$, $p = mv \rightarrow \lambda = \frac{h}{mv} \rightarrow \lambda \propto \frac{1}{m} \rightarrow$ $m_t > m_H > m_{p^+} > m_{e^-}$

$\downarrow$

$\lambda_t < \lambda_H < \lambda_{p^+} < \lambda_{e^-}$

---

15) $\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$, $n = 3$, $L = 1\text{Å}$, $x_1 = 0.5\text{Å}$, $x_2 = 0.6\text{Å}$

wave func. of a square well.

second excited state

$$P = \int_{x_1}^{x_2} \psi^* \psi \, dx = \int_{x_1}^{x_2} \frac{2}{L} \sin^2\left(\frac{3\pi}{L}x\right) dx = \frac{2}{\cancel{L}}\left[\frac{x}{2} - \frac{\cancel{L}}{12\pi} \sin\left(\frac{6\pi x}{\cancel{L}}\right)\right]_{0.5}^{0.6} = 0.6505 - 0.5 = \boxed{0.1505}$$

use integral table

---

16) $\psi = N \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right)$, $P = 1 = \int_{-\infty}^{\infty} \psi^* \psi \, d\tau$ for in multiple dimensions

for bounded by L, new bounds become $0 \rightarrow L$

Because x & y components are unrelated, can break them into simple multiple.

$$1 = N^2 \int_0^L \sin^2\left(\frac{\pi x}{L}\right) dx \cdot \int_0^L \sin^2\left(\frac{\pi y}{L}\right) dy \Rightarrow N^2 \left[\frac{x}{2} - \frac{L}{2\pi} \sin\left(\frac{2\pi x}{L}\right)\right]_0^L \left[\frac{y}{2} - \frac{L}{2\pi} \sin\left(\frac{2\pi y}{L}\right)\right]_0^L = N^2 \left(\frac{L}{2}\right)\left(\frac{L}{2}\right)$$

use integral table

$$\Rightarrow N^2 \left(\frac{L}{2}\right)^2 = 1 \Rightarrow \boxed{N = \frac{2}{L}}$$

---

17) $(4, 2, -1, \frac{1}{2}) \rightarrow (5, 1, -1, \frac{1}{2}) \rightarrow$ Check if trans is allowed

n_i n_f

$\Delta n = 1$ ✓

$\Delta l = 1$ ✓

$\Delta m_l = 0$ ✓

$\Delta m_s = 0$ ✓

$\Delta E = \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) E_1 \leftarrow E_1 = 13.6 \text{(eV)}$

$= \left(\frac{1}{5^2} - \frac{1}{4^2}\right) E_1 = (-0.0225) \cdot (-13.6)$

$= +0.306 \text{(eV)}$

Indicates energy gain is need for transition.

(5)

18) $\varphi = Ne^{-ix}, \quad 0 \le x \le L$

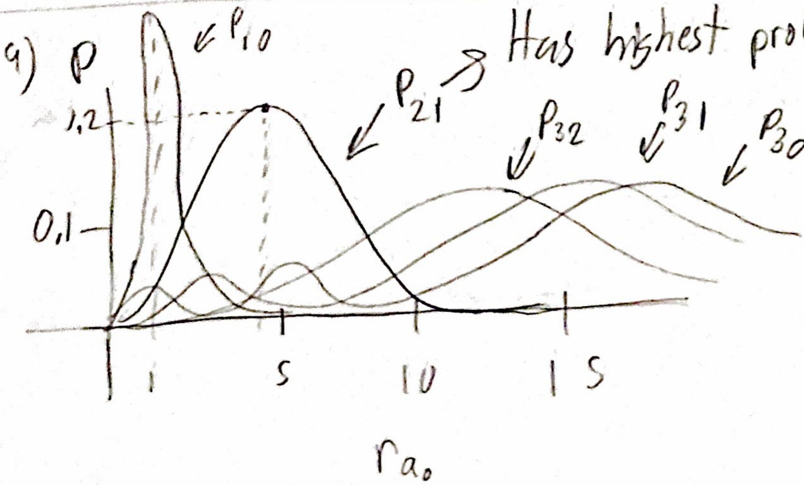
$$1 = N^2 \int_0^L e^{+ix} \cdot e^{-ix}\, dx = N^2 \int_0^L \cancel{e^0}^{\,1} dx = N^2 \, x\Big|_0^L = N^2 \cdot L$$

$$\Rightarrow N^2 = \frac{1}{L} \Rightarrow \underline{N = \sqrt{\frac{1}{L}}}$$

$$\langle x \rangle = \frac{1}{L}\int_0^L e^{+ix}\, x\, e^{-ix}\, dx = \frac{1}{L}\int_0^L x \cdot 1\, dx = \frac{1}{L}\left(\frac{x^2}{2}\right)\Big|_0^L = \frac{1}{\cancel{L}}\frac{L^{\cancel{2}}}{2} \Rightarrow \boxed{\langle x \rangle = \frac{L}{2}}$$

---

19) Has highest probability near $r = 5a_0$



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20) $n \; l \; m_l \; m_s$

$5, 2, 3, -\frac{1}{2}$

$m_l > l$

not allowed

$n =$ anything

$l = 0 \to n-1$

$m_l = -l \to +l$

$m_s = \pm\frac{1}{2}$