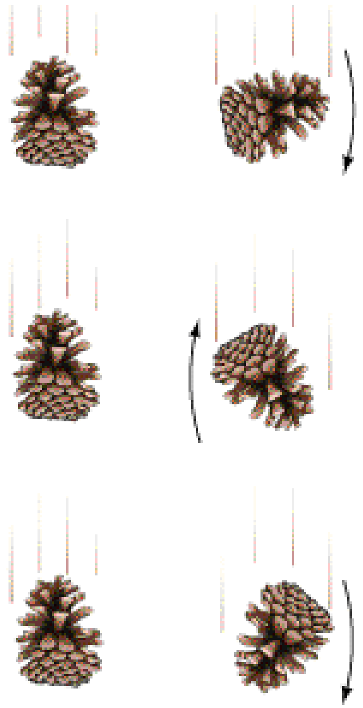
Chapter 2:Motion along a straight line

Translational Motion and Rotational Motion

Today



Later

Pearson

Goals for Chapter 2

- Become comfortable with displacement, velocity, and acceleration in one dimension.
- Explore motions at constant acceleration.
- Be able to graph and interpret graphs as they describe motion.
- Be able to reason proportionally.
- Examine the special case of freely falling bodies.
- Consider relative motion.

Describing Motion …

Coordinates

→ Position (displacement)

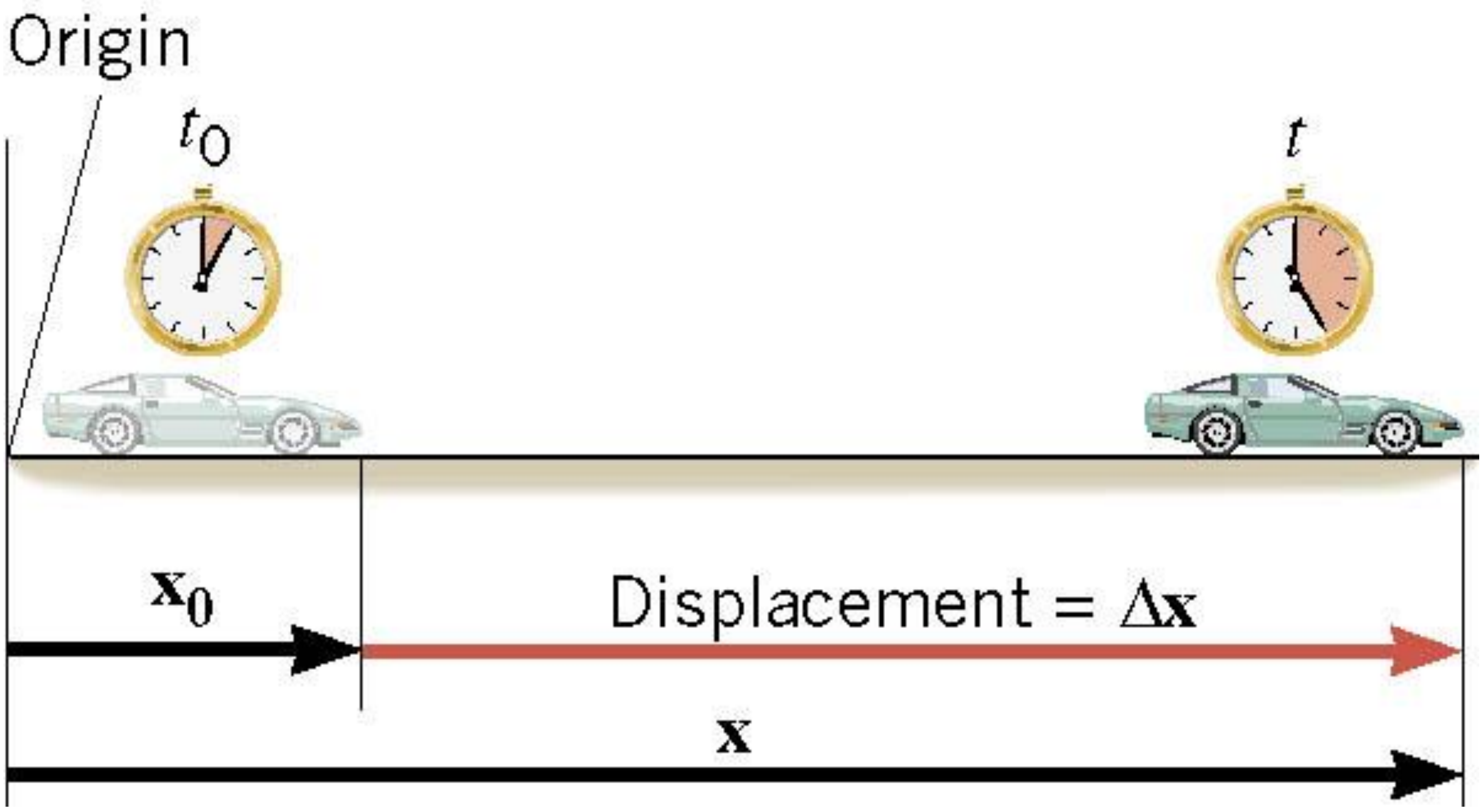
→ Velocity

→ Acceleration

a) Motion with *zero acceleration*

b) Motion with *non-zero acceleration*

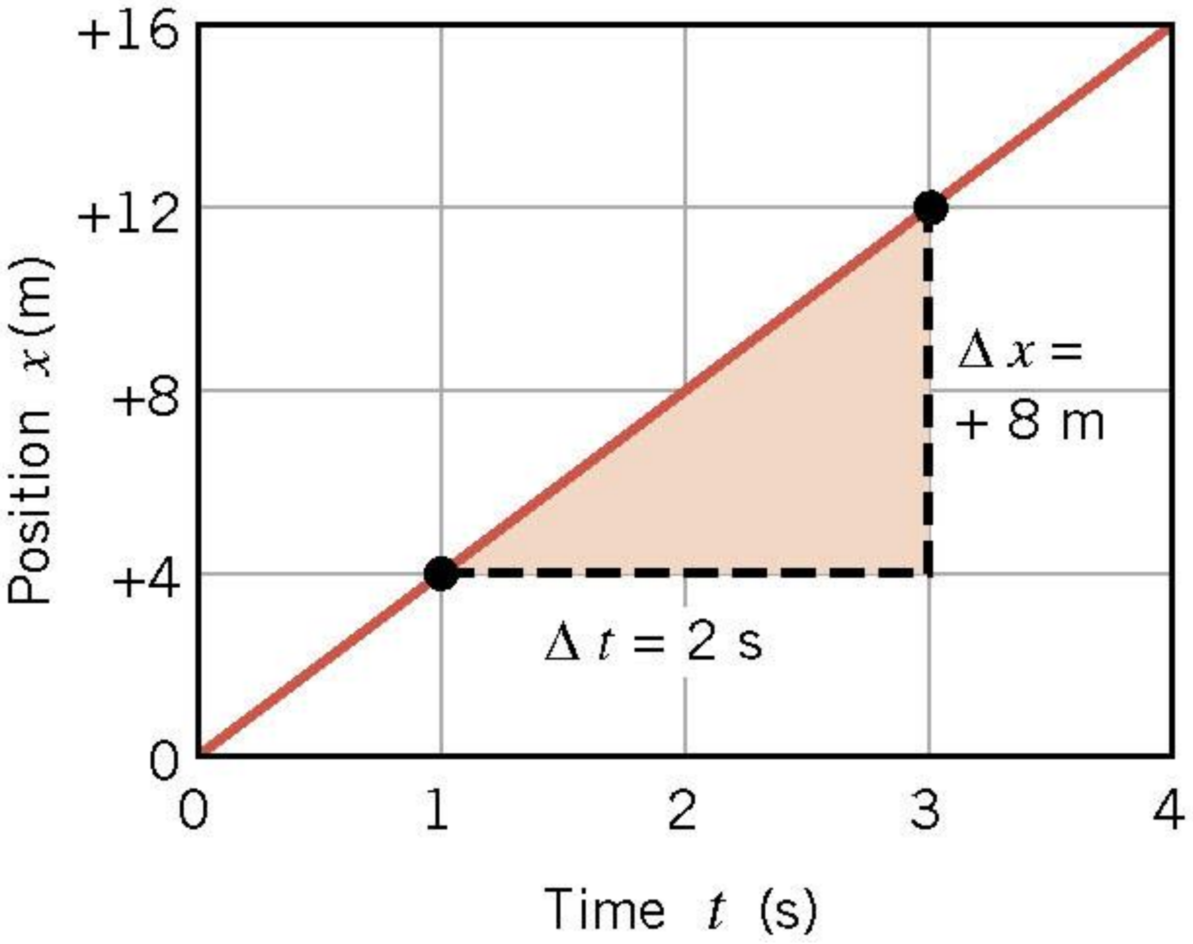
Kinematics in One Dimension: Displacement



$$\Delta x = x - x_0$$

Average velocity

Average velocity = total displacement covered divided by total elapsed time



$$\bar{v} = \frac{\Delta x}{\Delta t}$$

Unit: m/s
meter per second

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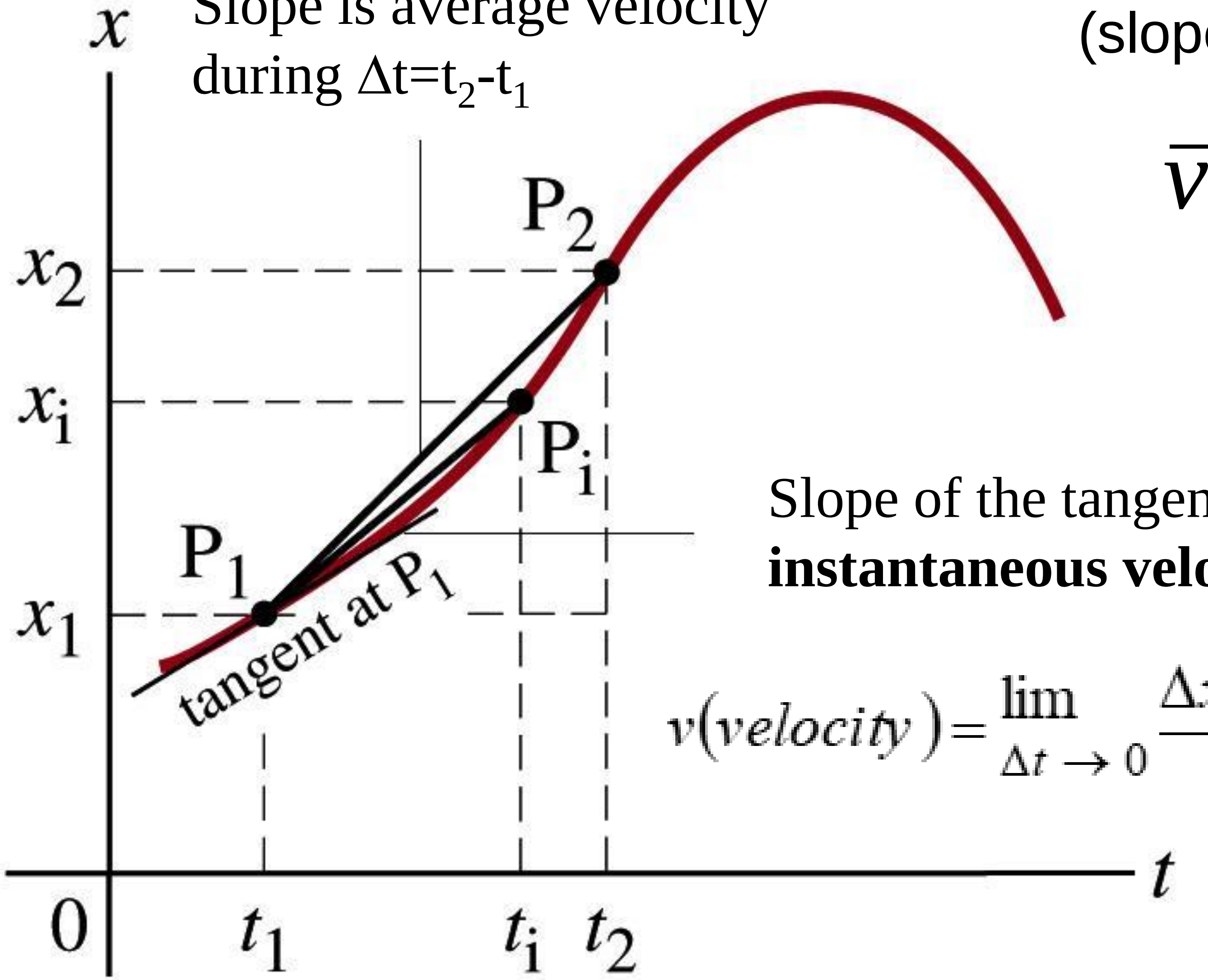
Graphic representation: average and instantaneous velocities

Slope is average velocity during $\Delta t = t_2 - t_1$

Average velocity (slope of line P_1P_2)

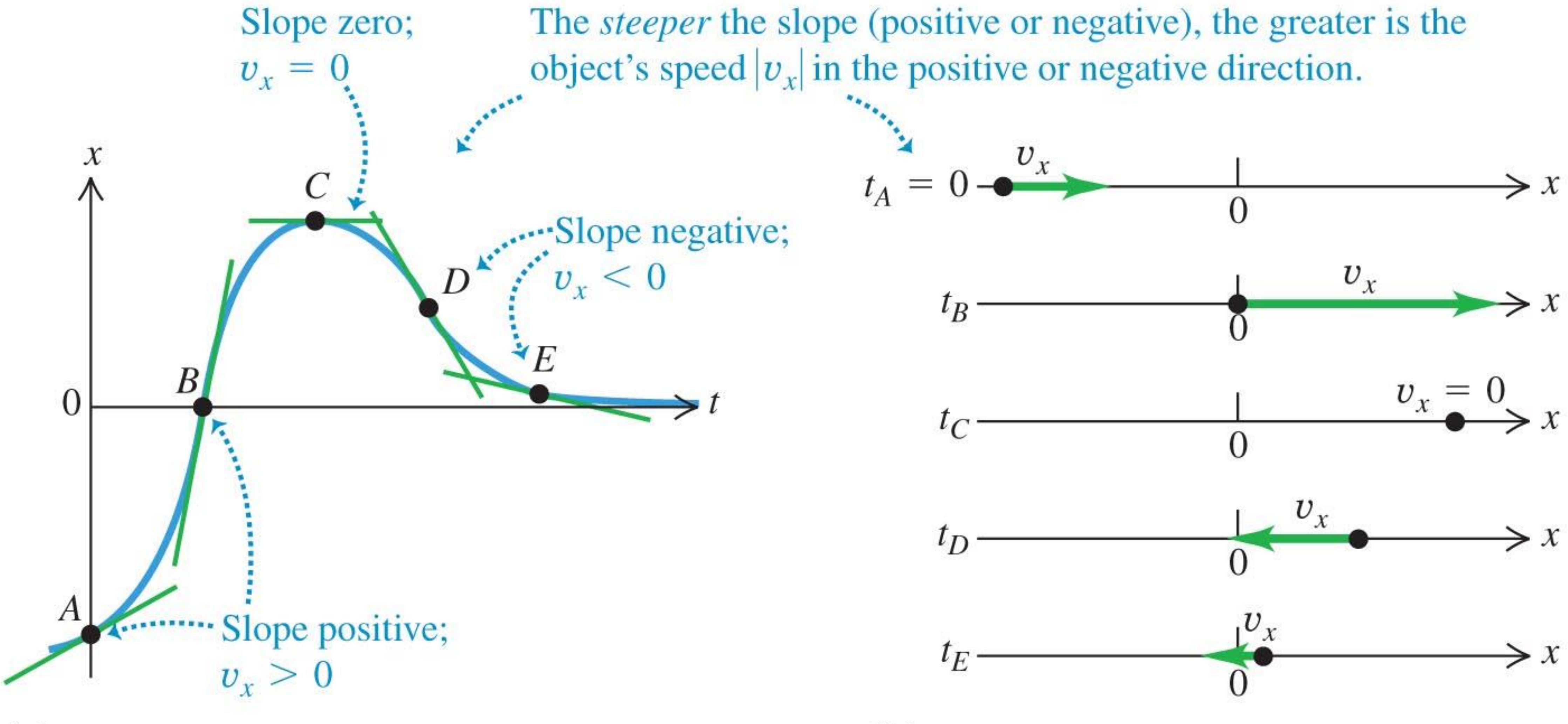
$$\bar{v} = \frac{\Delta x}{\Delta t}$$



Slope of the tangent line is **instantaneous velocity** at t_1

$$v(velocity) = \lim_{\Delta t \to 0} \frac{\Delta x(displacement)}{\Delta t(time)} = \frac{dx}{dt}$$

Understanding the direction and the magnitude of the instantaneous velocity graphically



(a) (b)

Average speed and velocity

- Average velocity = total displacement covered divided by total elapsed time,

$$\bar{v}(average\_velocity) = \frac{\Delta x(total\_displacement)}{\Delta t(total\_time)}$$

- **<u>Speed</u> is just the magnitude of <u>velocity</u>!**
  - The “how fast” without accounting for the direction.

- Average speed = total distance covered per total elapsed time,

- **Instantaneous velocity**, velocity at a given instant

$$v(velocity) = \lim_{\Delta t \to 0} \frac{\Delta x(displacement)}{\Delta t(time)} = \frac{dx}{dt}$$

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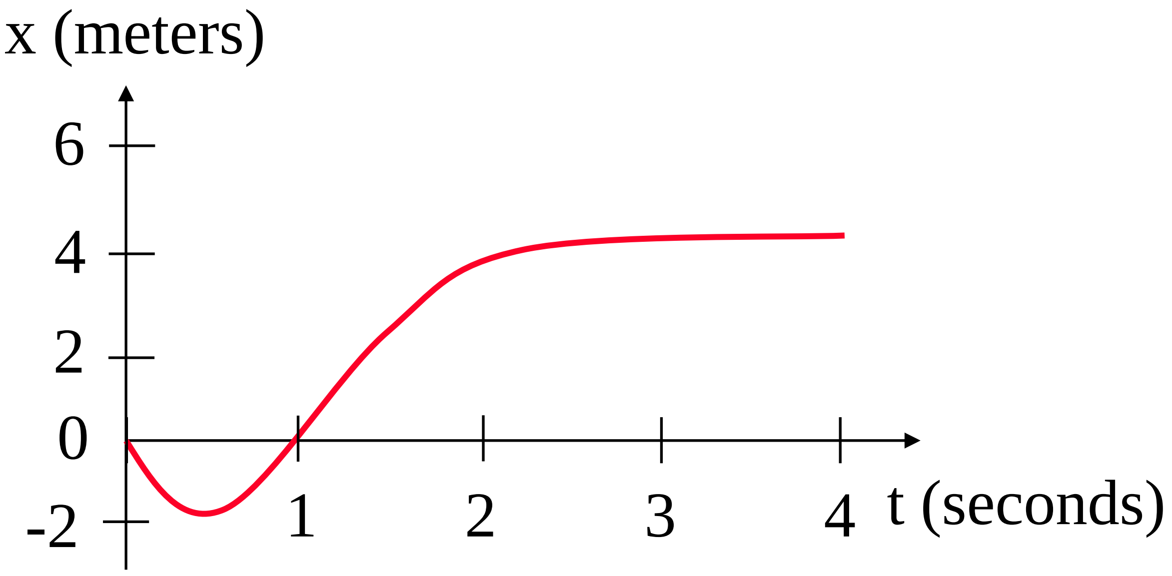
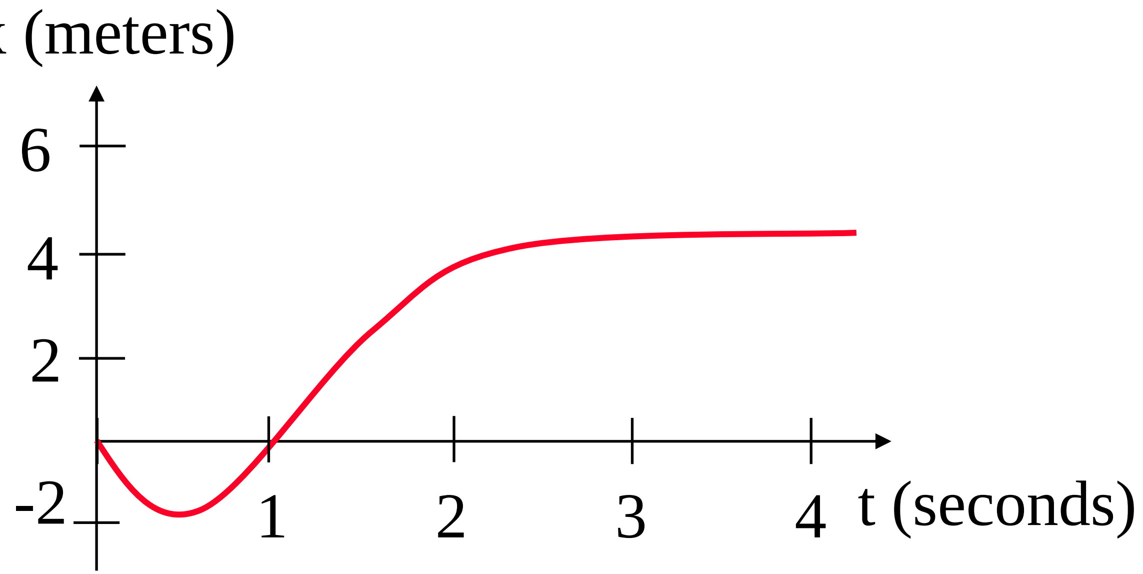
Average Velocity (example)



What is the average velocity over the first 4 seconds ?

A) -2 m/s B) 4 m/s C) 1 m/s D) not enough information to decide.

Instantaneous Velocity



What is the instantaneous velocity at the fourth second ?

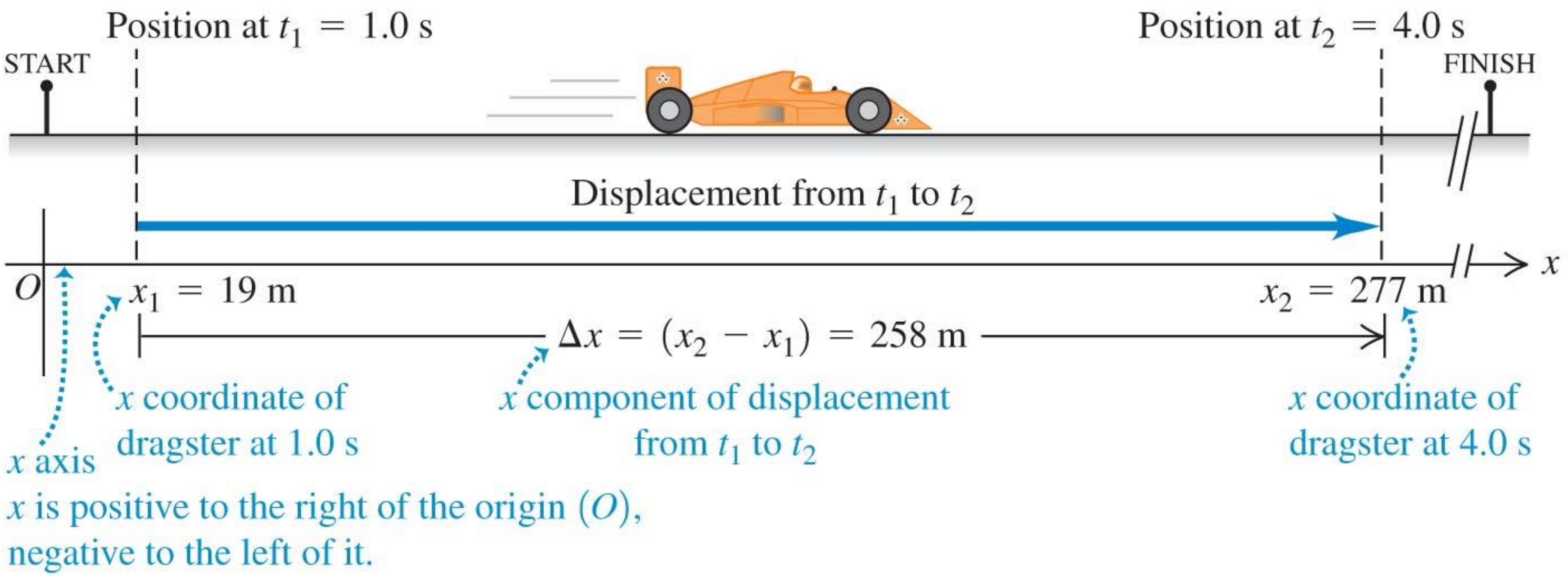
A) 4 m/s

B) 0 m/s

C) 1 m/s

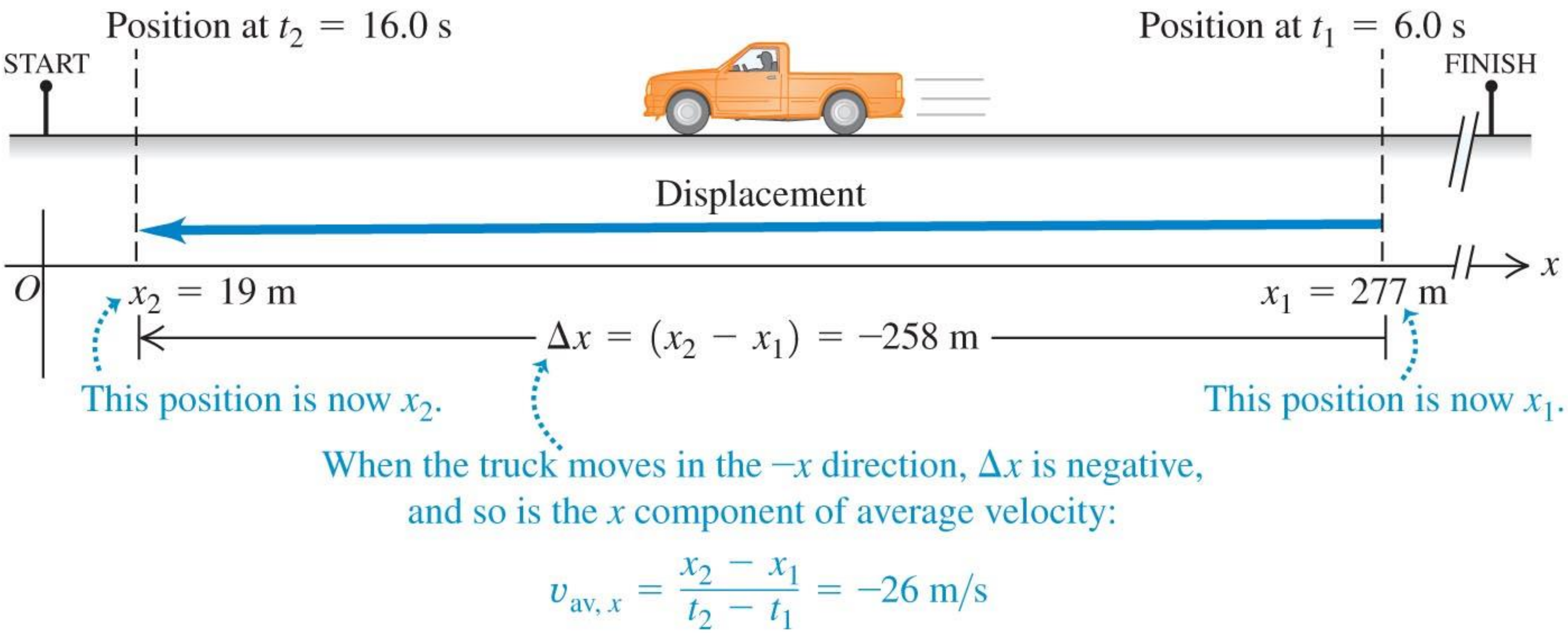
D) not enough information to decide.

Average Velocity - Figure 2.1



$$v_{av,x} = \frac{277\ \text{m} - 19\ \text{m}}{4.0\ \text{s} - 1.0\ \text{s}} = 86\ \text{m/s}$$

Displacement and Average Velocity



Courtesy of Wenhao Wu

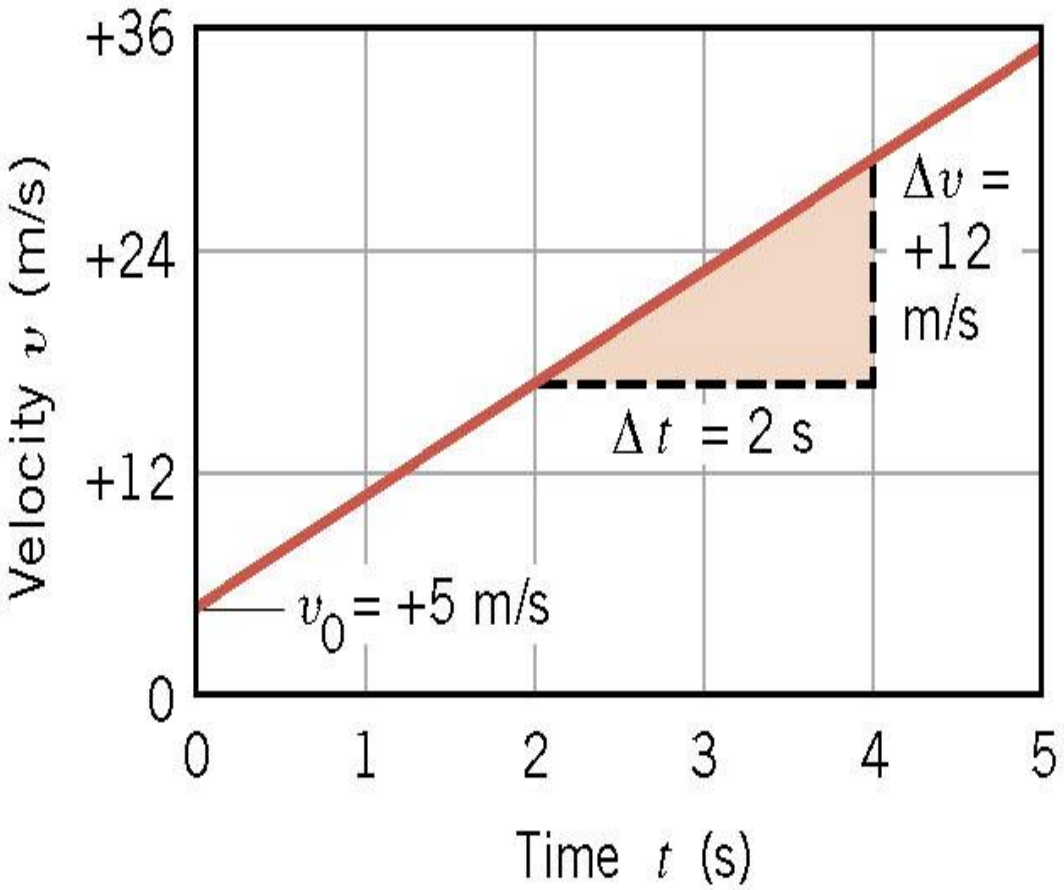
Acceleration

- We say that things which have changing velocity are *“accelerating”*
- Acceleration is the “Rate of change of velocity”
- You hit the *accelerator* in your car to speed up

  – (Ok…It’s true you also hit it to stay at constant velocity, but that’s because **friction** is slowing you down…we’ll get to that later…)

Average acceleration

Average acceleration = total change of the velocity divided by total elapsed time



$$\bar{a} = \frac{\Delta v}{\Delta t}$$

Unit of acceleration:

$(m/s)/s = m/s^2$

Meters per second squared

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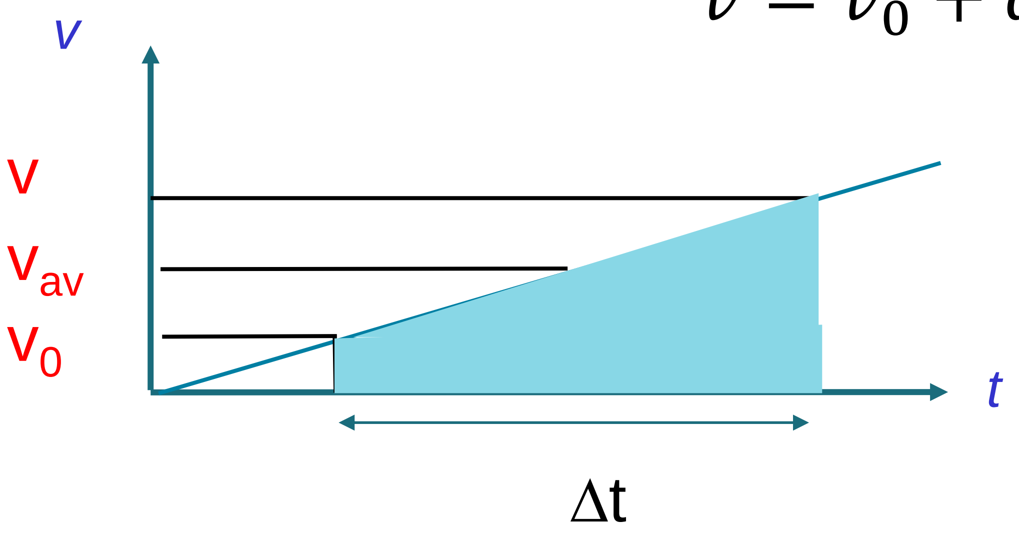
Average Velocity

- For constant acceleration

$$v = v_0 + at$$



The area under the graph v(t) is the total distance travelled

$$\Delta x = v_{av}\Delta t = \Delta t(v_0 + v)/2$$

$$v_{av} = \frac{1}{2}(v_0 + v)$$

Kinematics in one dimension

Motion with constant acceleration.

From the formula for average acceleration

$$a = \frac{v - v_0}{t}$$

We find

$$v = v_0 + at$$

$$Average\ velocity\ \bar{v} = \frac{x - x_0}{t}$$

On the other hand

$$\bar{v} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(v_0 + (v_0 + at)) = v_0 + \frac{1}{2}at$$

Then we can find

$$x - x_0 = \bar{v}t = (v_0 + \frac{1}{2}at)t = v_0 t + \frac{1}{2}at^2$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

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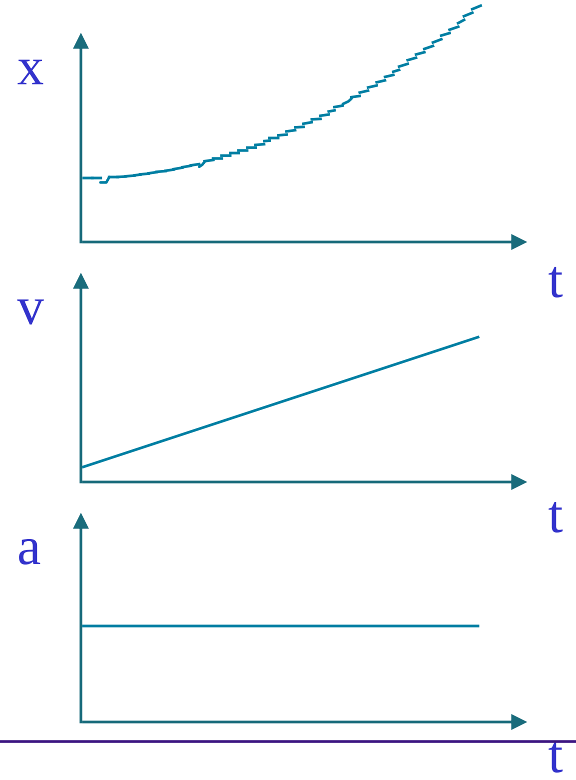
Recap

- So for constant acceleration we find:

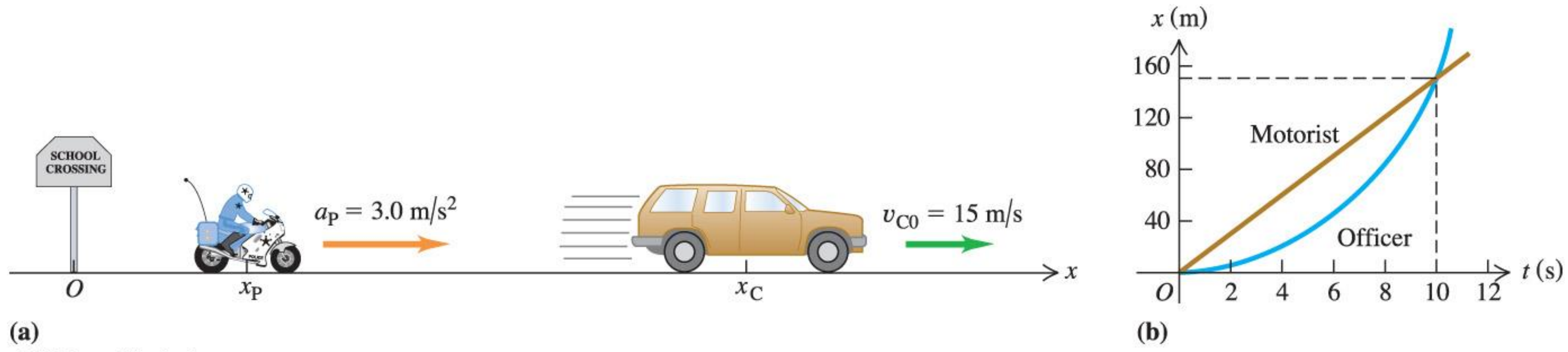
$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + at$$

$$a = const$$



Example of graphic solution: Catching a speeder



$$x_p = x_0 + v_{p0}t + \frac{1}{2}a_p t^2 = 0 + 0 + \frac{1}{2}(3)t^2$$

$$x_c = x_0 + v_{0c}t + \frac{1}{2}a_c t^2 = 0 + (15\,m/s)t + 0$$

(a) How much time elapses before the police catches up with the car? $x_p(t) = x_c(t)$

$\frac{1}{2}(3)t^2 = 15t$ $\boxed{t = 10\,s}$

(b) What is the officers speed when he catches up?

$v_p = v_{p0} + a_p t = 0 + 3t$ $t = 10\,s$ $\rightarrow$ $\boxed{v_p = 30 \text{ m/s}}$

(c) What is the total distance the police has travelled at that time?

When $t = 10\,s$ the car has traveled $x_c = 15\frac{m}{s}\,10\,s = 150\,m$

and the officer $x_p = \frac{1}{2}\,3\,\frac{m}{s^2}(10\,s)^2 = 150\,m$

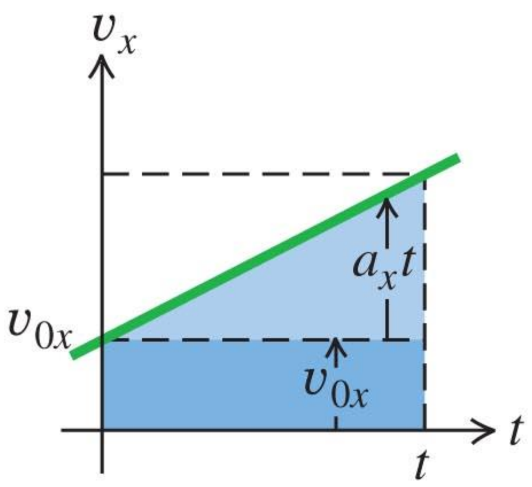
Car and police have gone the same distance

Question: How can we predict the position of the particle at any time t?

- Since the average velocity is defined by $$v_{av,x} = \frac{x(t) - x(0)}{t - 0} = \frac{x(t) - x_0}{t},$$

  we have $$x(t) = x_0 + v_{av,x}t.$$
- But, what is $v_{av,x}$?

  Since $v_x(t)$ is linear in time t, $$v_x(t) = v_{0x} + a_x t,$$



v_x versus t

we have $$v_{av,x} = \frac{1}{2}[v_x(t) + v_{0x}].$$

Therefore,
$$x(t) = x_0 + \frac{1}{2}[v_x(t) + v_{0x}]t$$
$$= x_0 + \frac{1}{2}[(v_{0x} + a_x t) + v_{0x}]t,$$
or, $$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \quad \text{......(2.10)}$$

- This is the second of the three kinematic equations.

Courtesy of Wenhao Wu

Motion with const acceleration (main equations!)

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v - v_0}{t - 0}$$

$$\boxed{v = v_0 + at}$$

$$t = \frac{v - v_0}{a}$$

$$v_{av} = \frac{v_0 + v}{2} = \frac{v_0 + v_0 + at}{2} = v_0 + \frac{1}{2}at$$

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{x - x_0}{t - 0} \Rightarrow \boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$

$$x = x_0 + v_0\left(\frac{v - v_0}{a}\right) + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2 \quad \Big| \cdot 2a$$

$$2a(x - x_0) = 2v_0(v - v_0) + (v - v_0)^2$$

$$= 2v_0 v - 2v_0^2 + v^2 + v_0^2 - 2vv_0$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

$$\left.\begin{array}{l} v_{av} = \frac{v_0 + v}{2} \\ v_{av} = \frac{x - x_0}{t} \end{array}\right\}$$

$$\boxed{x - x_0 = \left(\frac{v_0 + v}{2}\right)t}$$

Kinematics equations for constant acceleration

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)t$$

note: the mass m is not in these equations

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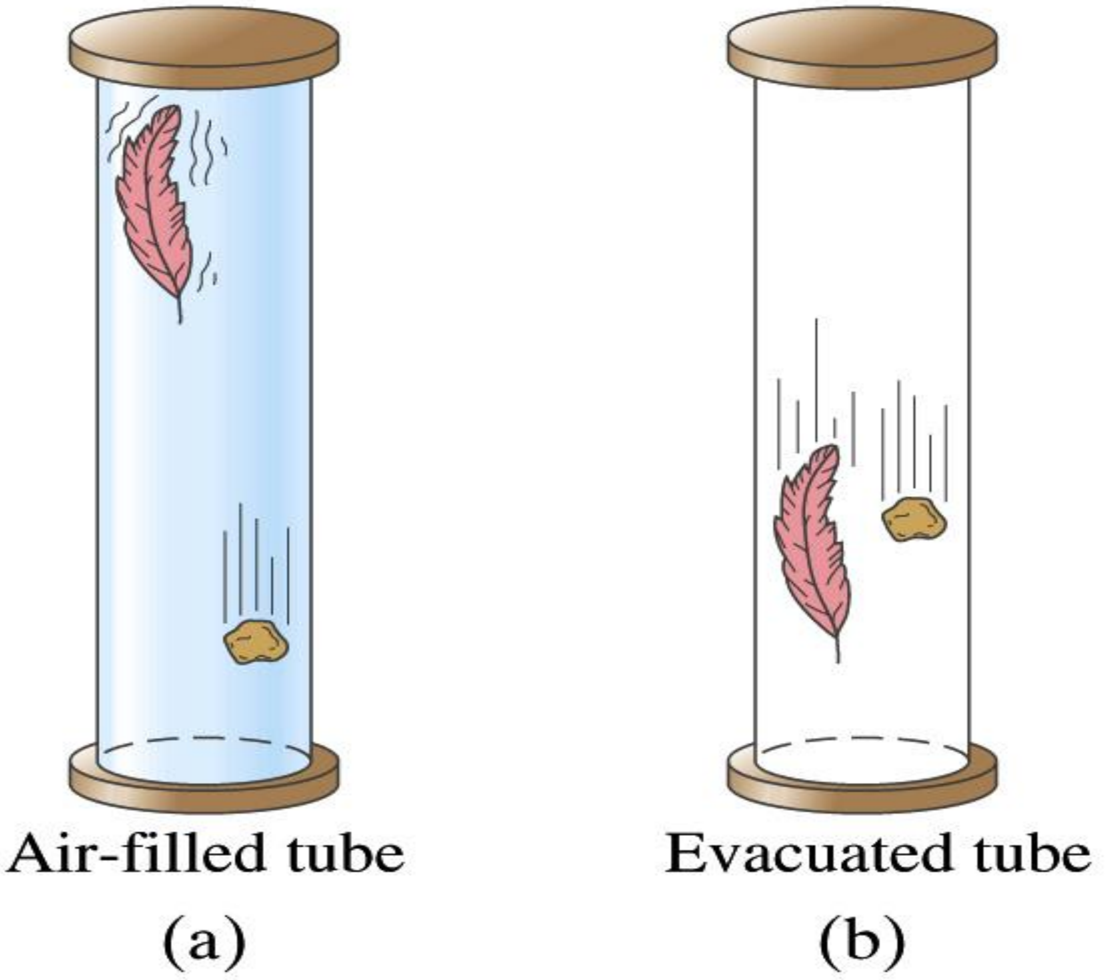
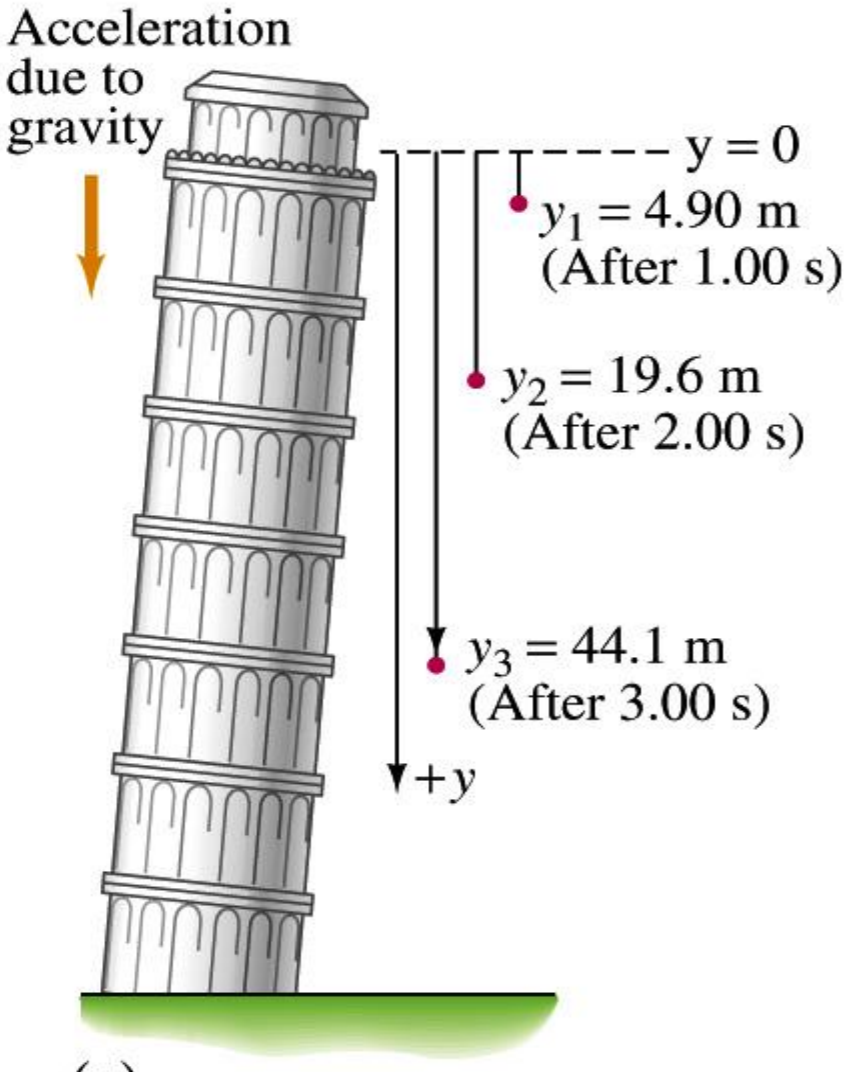
Particular case of motion with constant acceleration: Falling objects. All objects fall with the same constant acceleration!!

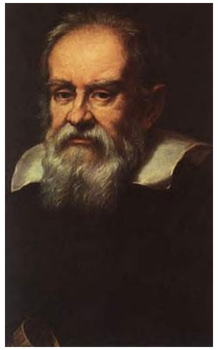


Kinematics for a=9.80m/s² (acceleration of gravity)

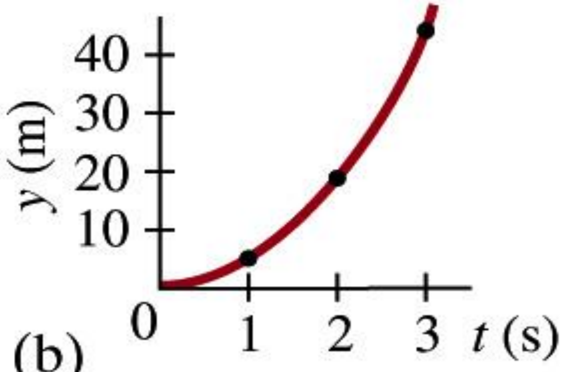


(a)

Between 1589 and 1592.



Experiments on the motion of objects falling from leaning tower of Pisa under the action of the force of gravity



(b)

$a_y = g = 9.8\ m/s^2$ if the y-axis points down

$v_y(t) = v_{0y} + gt$ ……………...(2.6)

$y(t) = y_0 + v_{0y}t + \frac{1}{2}gt^2$ ……(2.10)

$v_y^2 = v_{0y}^2 + 2g(y - y_0)$ ……..(2.11)

$v_{av,y} = \frac{1}{2}[v_y(t) + v_{0y}]$……….(2.7)

Acceleration of gravity on different planets, $a_{Earth} = 9.81 m/s^2$

$$a_{earth} = 9.81 \frac{m}{s^2}$$

$$a_{moon} = 1{,}67 \frac{m}{s^2}$$

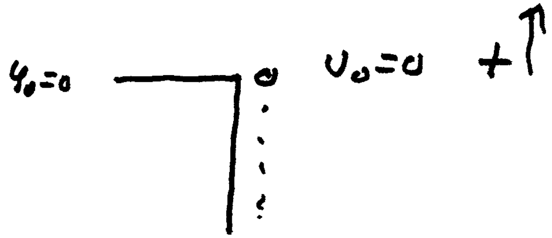
$$a_{sun} = 274 \frac{m}{s^2}$$

A brick is dropped from the roof of a building. The brick strikes the ground after 5s. (sign convention does not matter!)

a. how tall

b. velocity of brick just before it strikes the ground

choose + up:

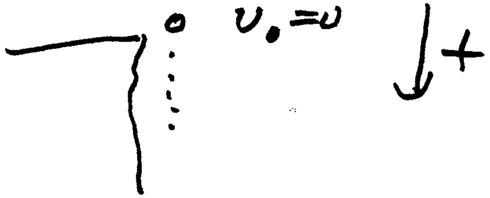


$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$y = 0 + 0 - \frac{1}{2} 9.8 \cdot 5^2 = -122\,m$$

$$v = v_0 + at = 0 - 9.8 \cdot 5 = -49\,m/s$$

choose + down:



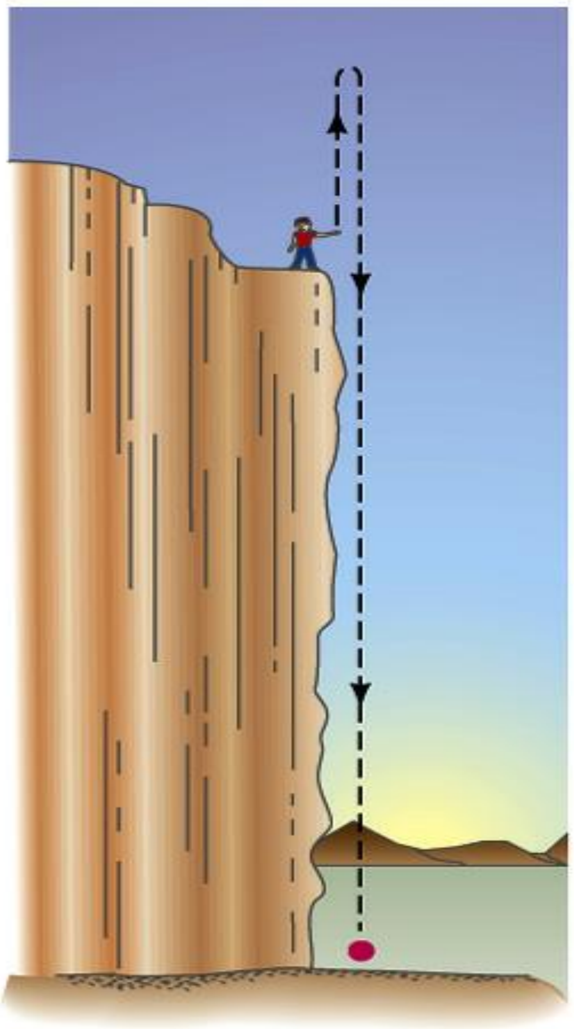
$$y - y_0 = v_0 t + \frac{1}{2} a t^2$$

$$= \frac{1}{2} 9.8 \cdot 5^2 = 122\,m$$

$$v = v_0 + at = 0 + 9.8 \cdot 5 = 49\,\frac{m}{s}$$

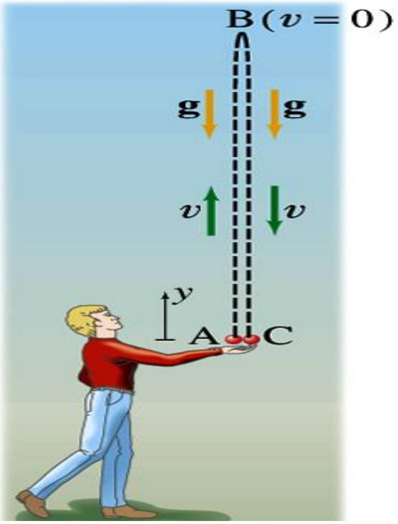
Important to have the
correct interpretation
of the results!
The optimal selection
of the reference frame helps.

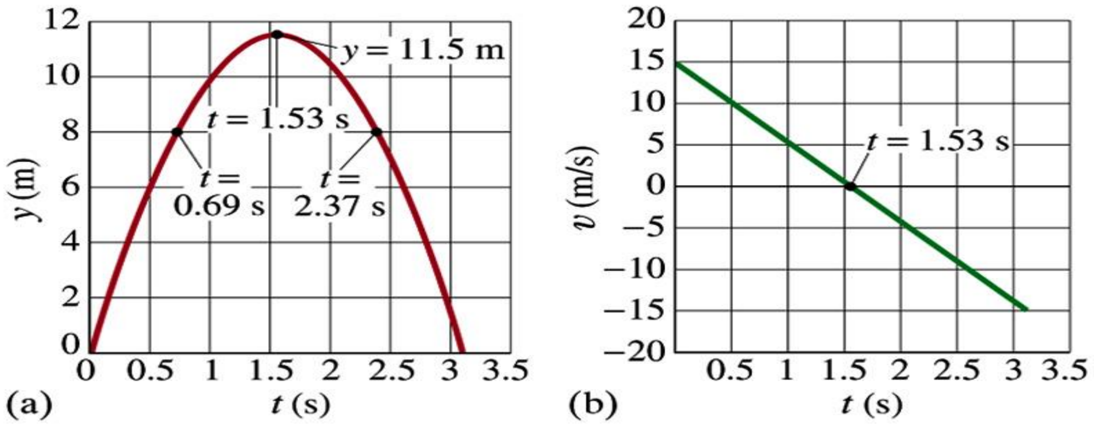
Problem: Vertical motion

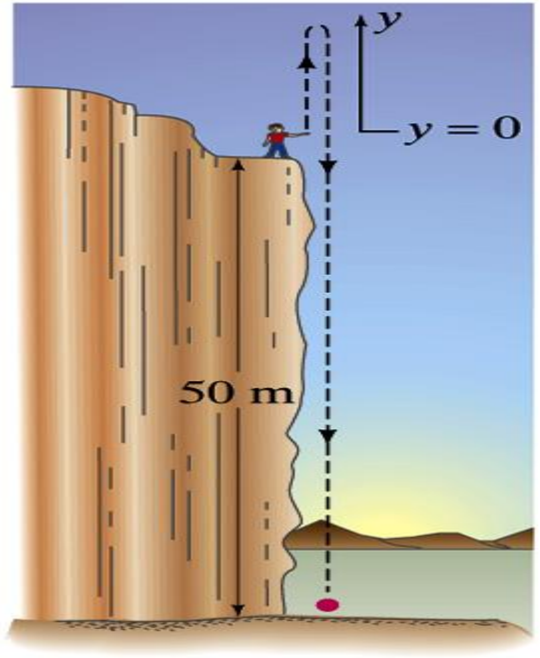


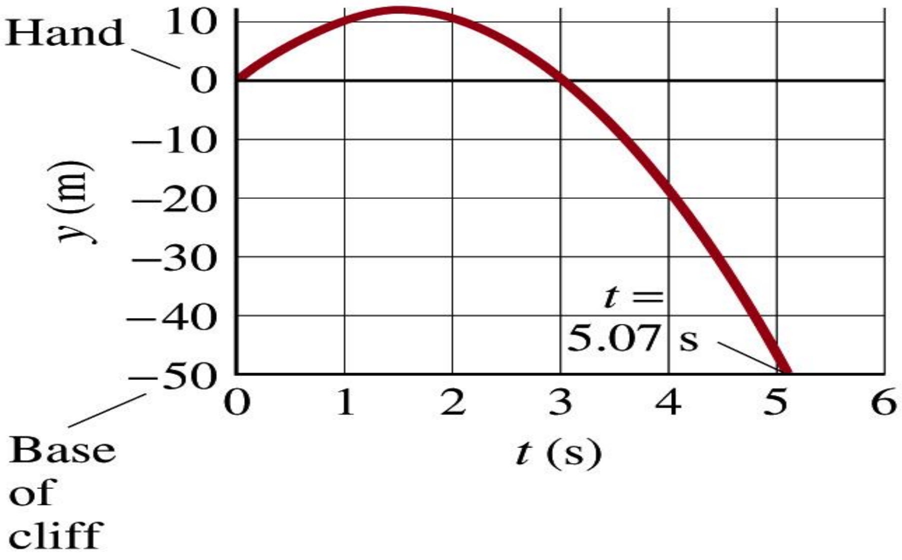
Stone is thrown vertically upward

1-D motions in the gravitational field









Ball thrown upward

height of the cliff h=50m

$v_0 = 15\,m/s$ initial velocity, sign convention: upward +

a. how high?

at point A: $t = 0$, $y_0 = 0$, $v_0 = 15\,m/s$, $a = 9.8\,m/s^2$

t, $v = 0$, $y = ?$

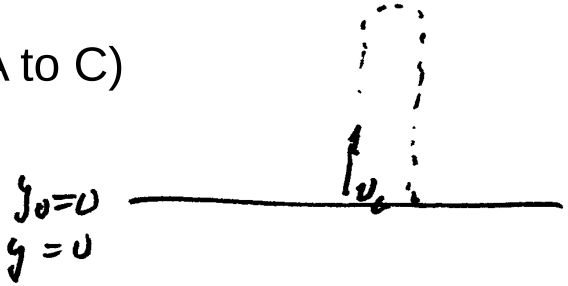
$$v^2 = v_0^2 + 2a(y - y_0)$$

~~$v = v_0$~~

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - 15^2}{2\cdot(-9.8)} = \boxed{11.5\,m}$$

b. how long in the air? (from point A to C)

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$



$$0 = 0 + 15t - \frac{1}{2} 9.8\, t^2$$
$$= (15 - 4.9t)t$$

$t_1 = 0 \qquad t_2 = \frac{15}{4.9} = \boxed{3.06\,s}$

c. how long for highest point? $\left(\text{from symmetry } t = \frac{3.06}{2} = 1.53\,s\right)$ (from point A to B)

$v = v_0 + at \qquad v = 0$ (at highest point)

$$t = -\frac{v_0}{a} = -\frac{15}{-9.8} = \boxed{1.53\,s}$$

d. velocity when ball reaches the thrower's hand (point C)

$$v = v_0 + at = 15 - 9.8 \cdot 3.06 = -15.0 \ \frac{m}{s}$$

e. at what time passes the ball a point 8m above the throwers hand

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$8 = 0 + 15t - \frac{1}{2} 9.8 t^2$$

$$4.9t^2 - 15t + 8 = 0$$

$$at^2 + bt + c = 0 \qquad t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{15 \pm \sqrt{15^2 - 4 \cdot 4.9 \cdot 8}}{2 \cdot 4.9} = \begin{cases} t_1 = 0.69\,s & \text{ball goes up} \\ t_2 = 2.37\,s & \text{ball goes down} \end{cases}$$

f. How long does it take to reach the base of the cliff and what is the velocity there? ($h = 50$ m)

t1 = (15 + (15^2 + 4*4.9*50)^0.5)/(2*4.95) = 5.0727 = 5.07 s

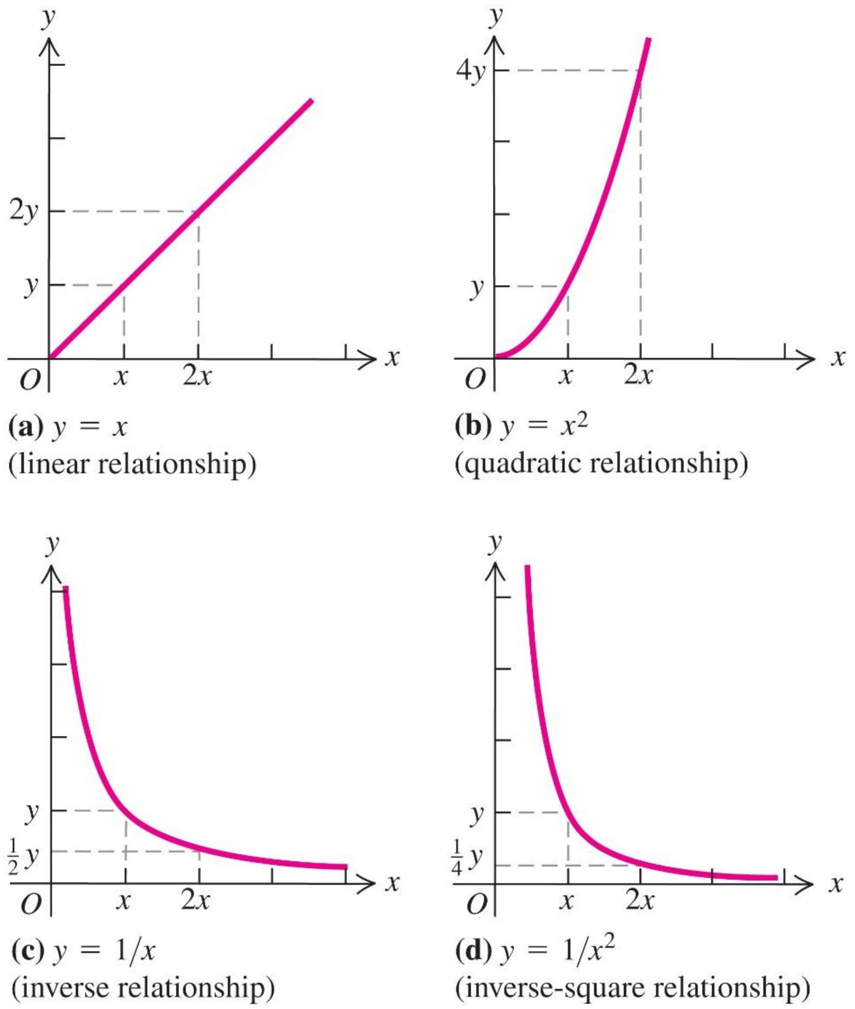
$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$-50 = 0 + 15t - \frac{1}{2} 9.8 t^2 \qquad \boxed{t_1 = 5.07\,s} \qquad \sout{t_2 = -2.1\,s} \text{ not sensible}$$

$$v = v_0 + at = 15 - 9.8 \cdot 5.07 = \boxed{-35 \ \frac{m}{s}}$$

2.5: Proportional Reasoning

- Thinking about your numbers
- For example: there are linear, quadratic, inverse, inverse-square proportions between two variables *x* and *y*.



(a) $y = x$ (linear relationship)

(b) $y = x^2$ (quadratic relationship)

(c) $y = 1/x$ (inverse relationship)

(d) $y = 1/x^2$ (inverse-square relationship)

Clicker problem

When you brake on dry pavement, your maximum acceleration is about three times greater than when you brake on wet pavement. For a given initial speed, how does your stopping distance x_{dry} on dry pavement compare with your stopping distance x_{wet} on wet pavement?

a) $x_{dry} = 1/3x_{wet}$

b) $x_{dry} = 3x_{wet}$

c) $x_{dry} = 1/9x_{wet}$

d) $x_{dry} = 9x_{wet}$