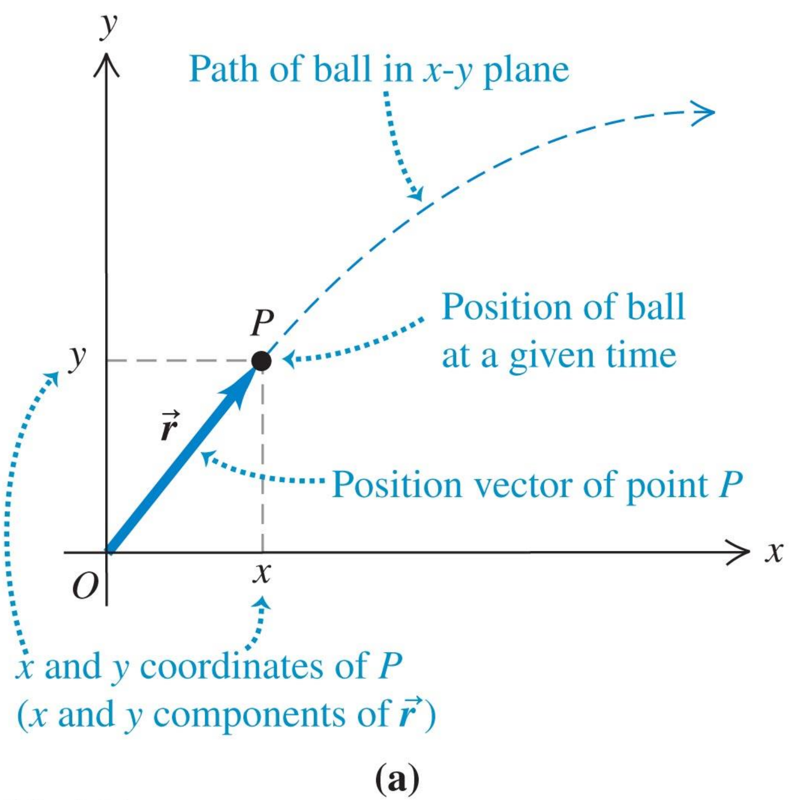
Chapter 3 Motion in a Plane (Motion in Two-Dimensions)

- calculate position, velocity, and acceleration vectors in 2D
- apply the kinematic equations to understand 2D projectile motion
- apply the kinematic equations to solve for unknown quantities for an object moving with constant acceleration in 2D
- study the relative velocity of an object for observers in different frames of reference in 2D

Courtesy of Wenhao Wu

Velocity in a Plane (1 of 2)

- Vectors in terms of Cartesian *x*- and *y*- coordinates may now also be expressed in terms of magnitude and angle.

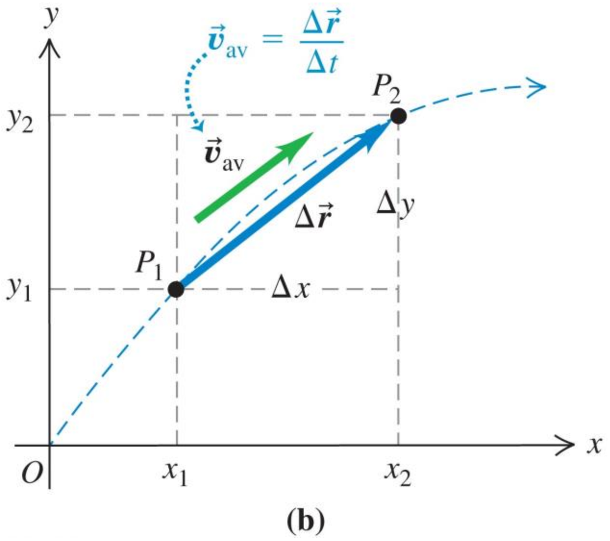


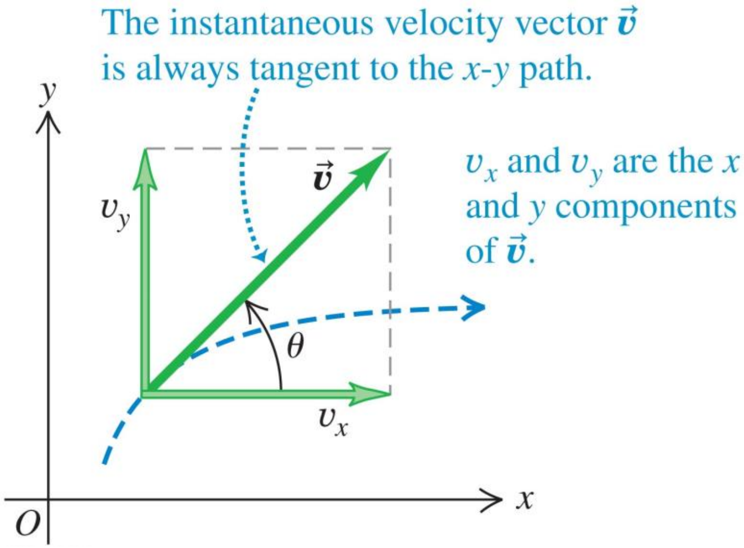
distance of point *P* from the origin is the magnitude of vector $\vec{r}$

$$r = \left|\vec{r}\right| = \sqrt{x^2 + y^2}$$

Velocity in a Plane (2 of 2)

- From the graphs, we see both average and instantaneous velocity vectors.





Average velocity of a particle over displacement $\Delta\vec{r}$

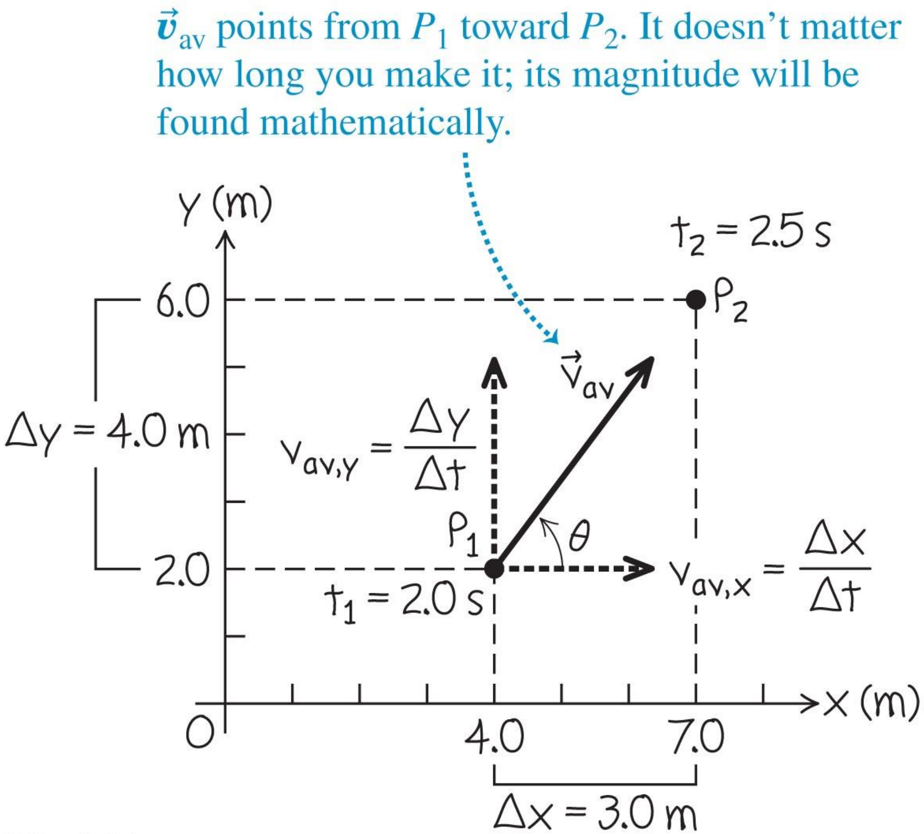
$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$$

Instantaneous velocity of particle at point $\vec{r}$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$

The Motion of a Model Car – Example 3.1

- See the worked example on page 67.



Velocity in a Plane

average velocity	$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t}$	
components	$v_{av,x} = \frac{\Delta x}{\Delta t}$	$v_{av,y} = \frac{\Delta y}{\Delta t}$

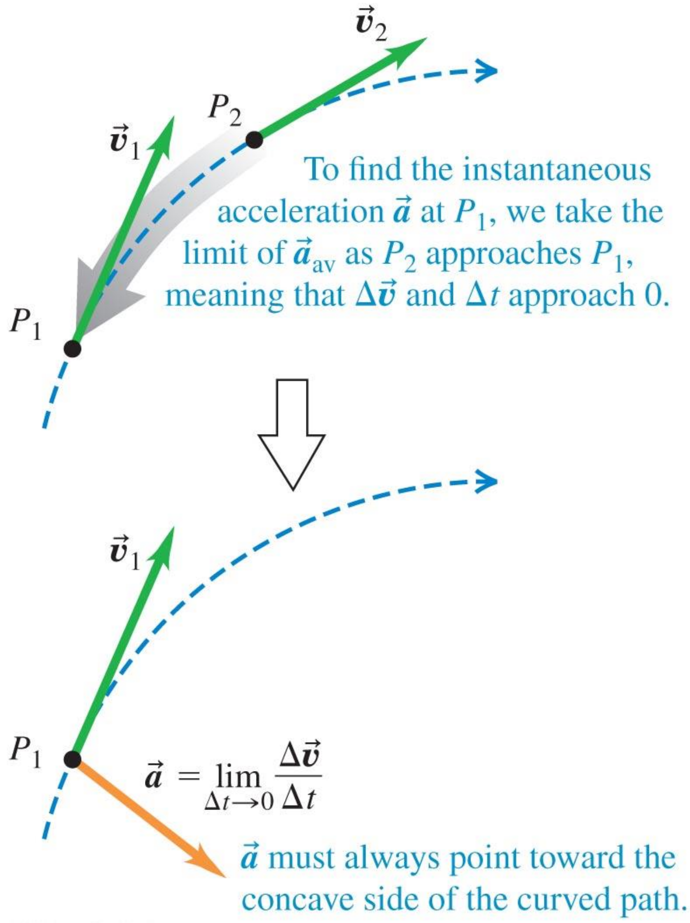
instantaneous velocity	$\vec{v} = \lim_{\Delta t \to 0} \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$	
components	$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	$v_y = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$

magnitude	$\lvert\vec{v}\rvert = \sqrt{v_x^2 + v_y^2}$	and direction	$\theta = tan^{-1} \frac{v_y}{v_x}$

Courtesy of Wenhao Wu

Accelerations in a Plane

- Acceleration must now be considered during change in magnitude AND/OR change in direction.



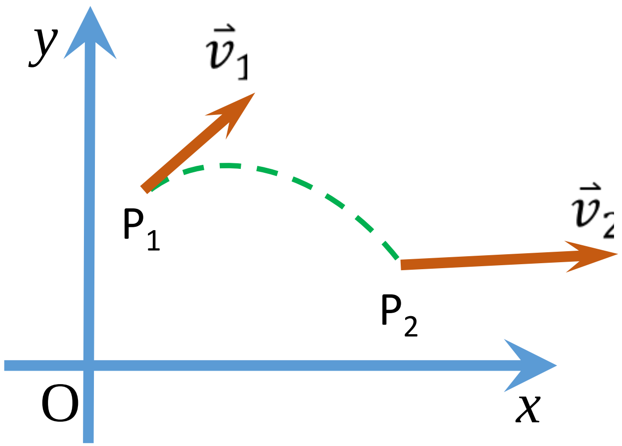
Average acceleration of a particle over displacement $\Delta\vec{r}$

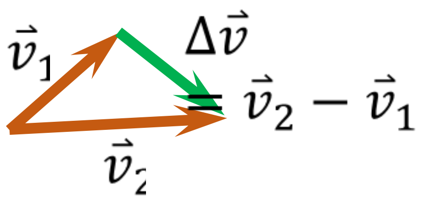
$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous acceleration of particle at point $\vec{r}$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

3.2 Acceleration in a Plane





average acceleration

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

components

$$a_{av,x} = \frac{\Delta v_x}{\Delta t} \qquad v_{av,y} = \frac{\Delta v_y}{\Delta t}$$

instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

components

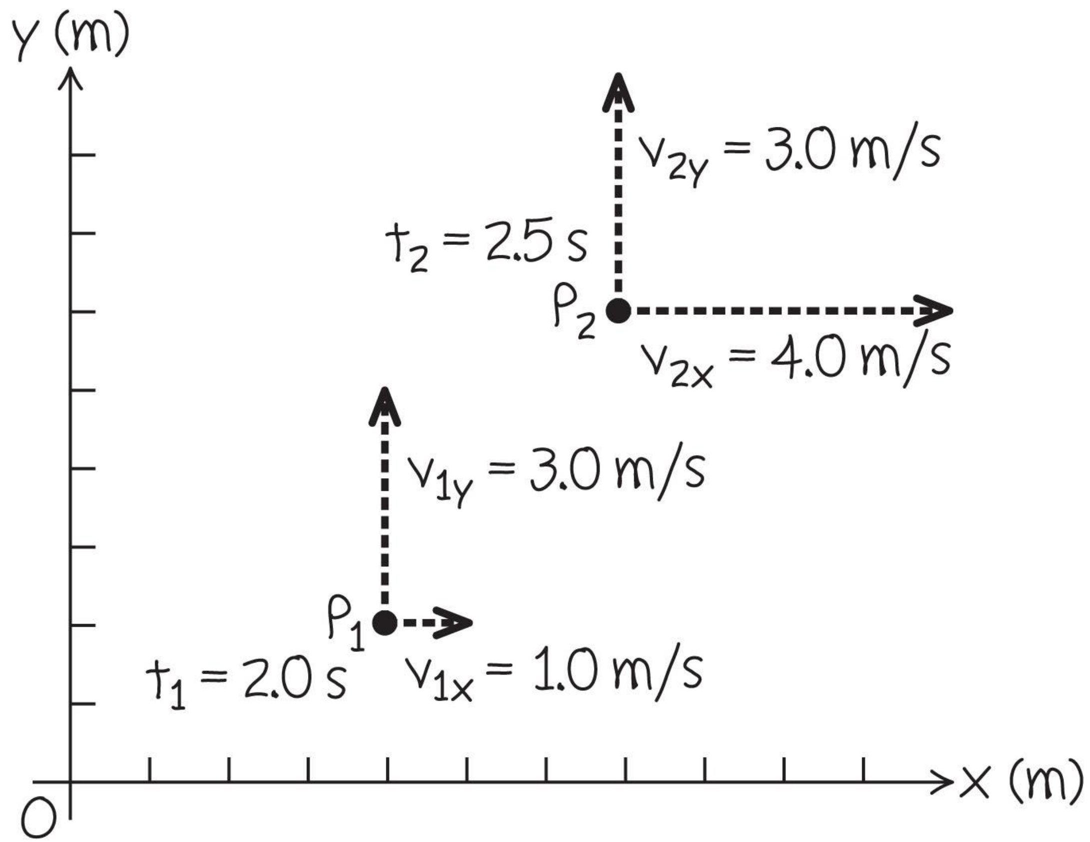
$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \qquad a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}$$

magnitude	direction
$\lvert\vec{a}\rvert = \sqrt{a_x^2 + a_y^2}$	$\theta = tan^{-1}\frac{a_y}{a_x}$

Courtesy of Wenhao Wu

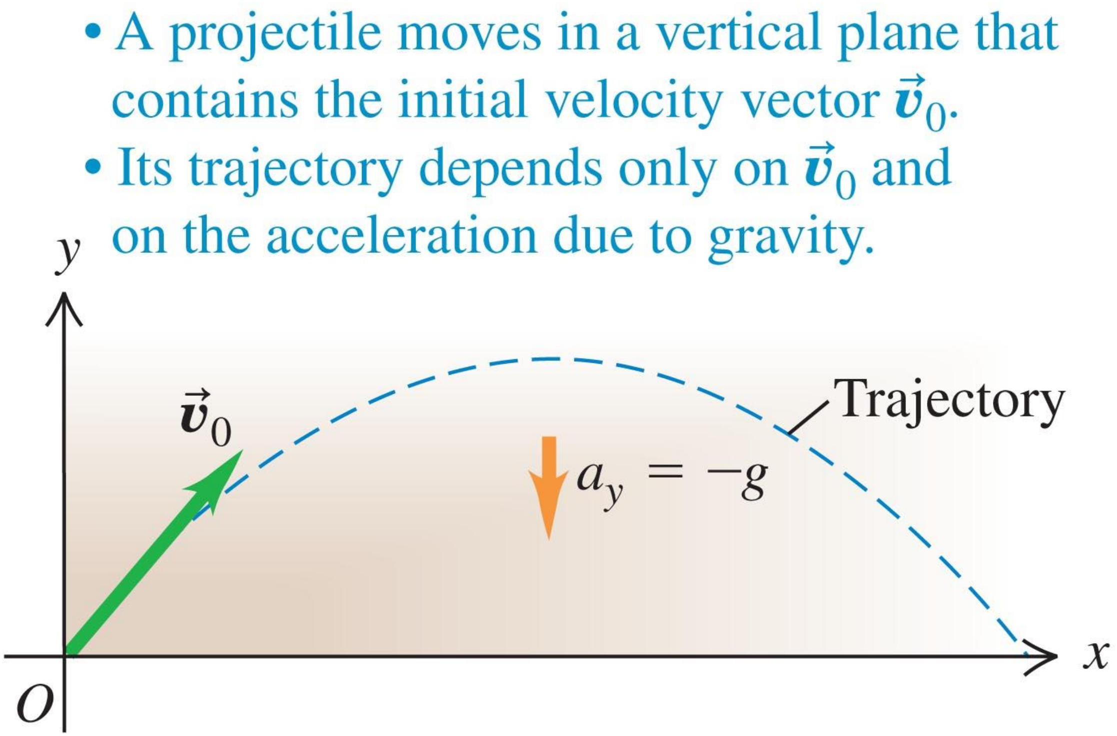
The Model Car Revisited – Example 3.2

- See the worked example on page 69.



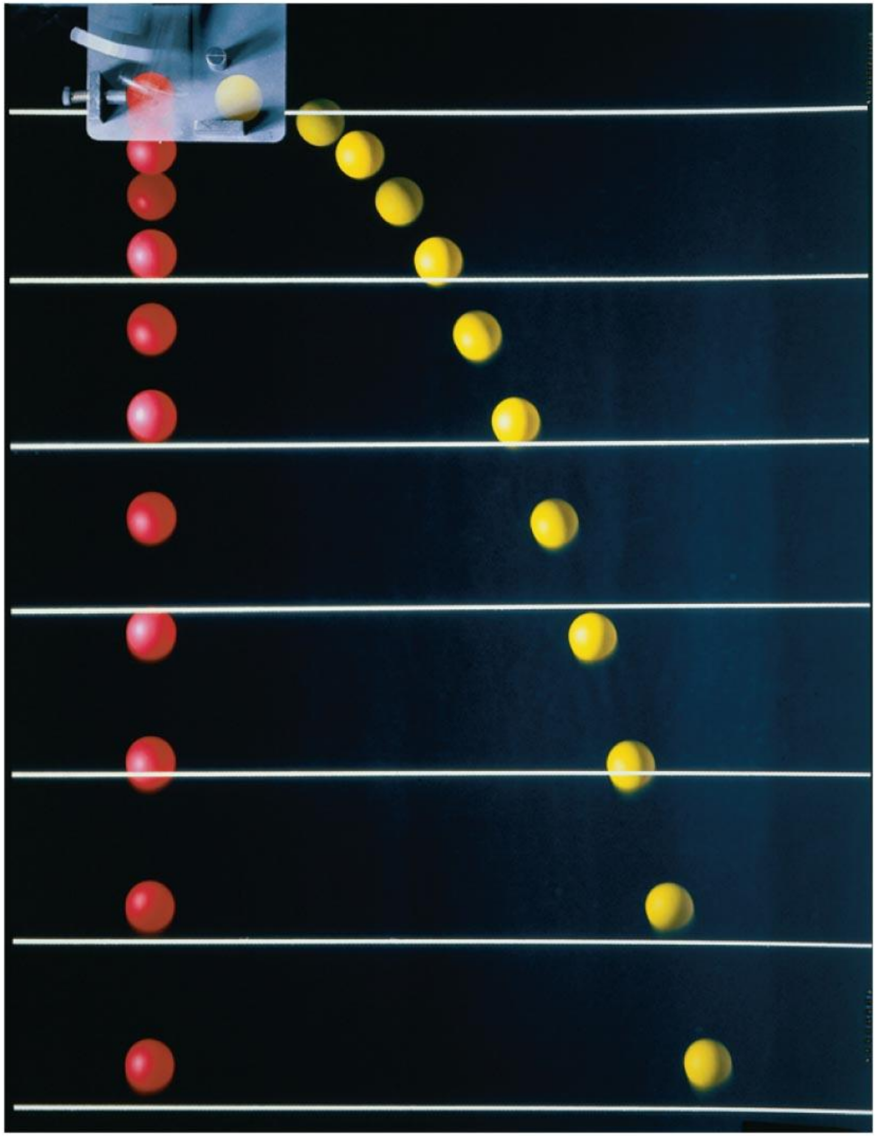
Projectile Motion

- Determined by the initial velocity, gravity, and air resistance.
- Footballs, baseballs … any projectile will follow this *parabolic* **trajectory** in the *x*-*y* plane.



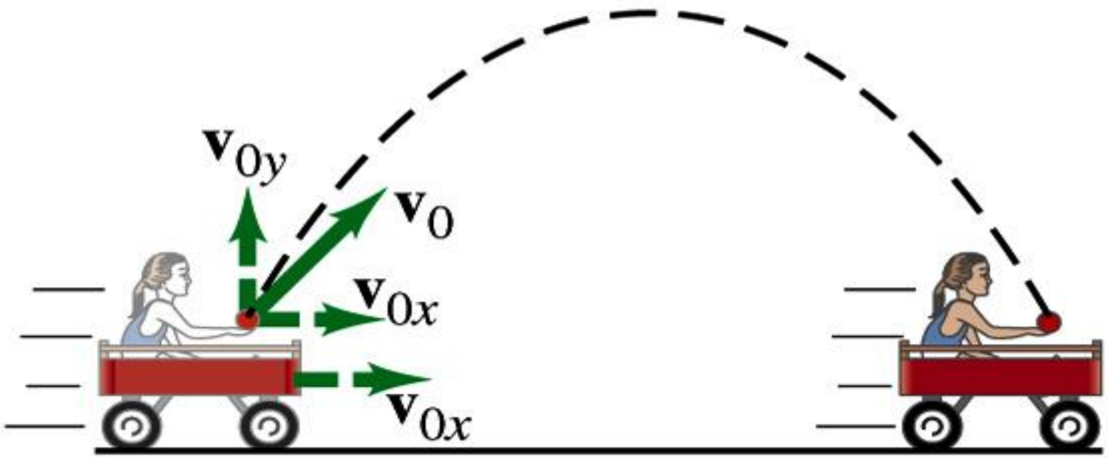
The Independence of *x*- and *y*-Motion – Figure 3.9

- Notice how the vertical motion under free fall spaces out exactly as the vertical motion under projectile motion.
- We can treat the *x*- and *y*- coordinates separately!



Where does the apple land?

(a) Wagon reference frame



(b) Ground reference frame

Motion in a Plane (2D) with Constant Acceleration

A General Rule: Two sets of quantities are treated separately/independently.

All the x-components of the quantities are related to each other in one set of kinematic equations:

$$v_x(t) = v_{0x} + a_x t \quad \ldots\ldots\ldots\ldots\ldots(2.6)$$

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \ldots\ldots\ldots(2.10)$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \quad \ldots\ \ldots\ldots(2.11)$$

$$v_{av,x} = \frac{1}{2}[v_x(t) + v_{0x}] \ldots\ldots\ldots\ldots(2.7)$$

All the y-components of the quantities are related to each other in another set of kinematic equations:

$$v_y(t) = v_{0y} + a_y t \quad \ldots\ldots\ldots\ldots\ldots(2.6)$$

$$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \ldots\ldots\ldots(2.10)$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \quad \ldots\ \ldots\ldots(2.11)$$

$$v_{av,y} = \frac{1}{2}[v_y(t) + v_{0y}] \ldots\ldots\ldots\ldots(2.7)$$

Important initial steps for solving 2D motion with constant acceleration:

- Set up a convenient *x*-*y* coordinate system.
- Identify the *x* and *y* components of initial position, initial velocity, and acceleration.
- Apply the rule set above and NEVER mix *x*-component and *y*-component quantities in the same kinematic equation.

Courtesy of Wenhao Wu

3.3 Projectile Motion

Projectile Motion: The motion in a vertical plane of a point particle, given an initial velocity, under the influence of a constant gravitation acceleration, with other factors such as air friction and wind, etc., all neglected.

A Summary of the Parameters
(with the given coordinates)

Acceleration:

$a_x = 0$ $\quad\quad$ $a_y = -g = -9.8\ \frac{m}{s^2}$

Initial Conditions:

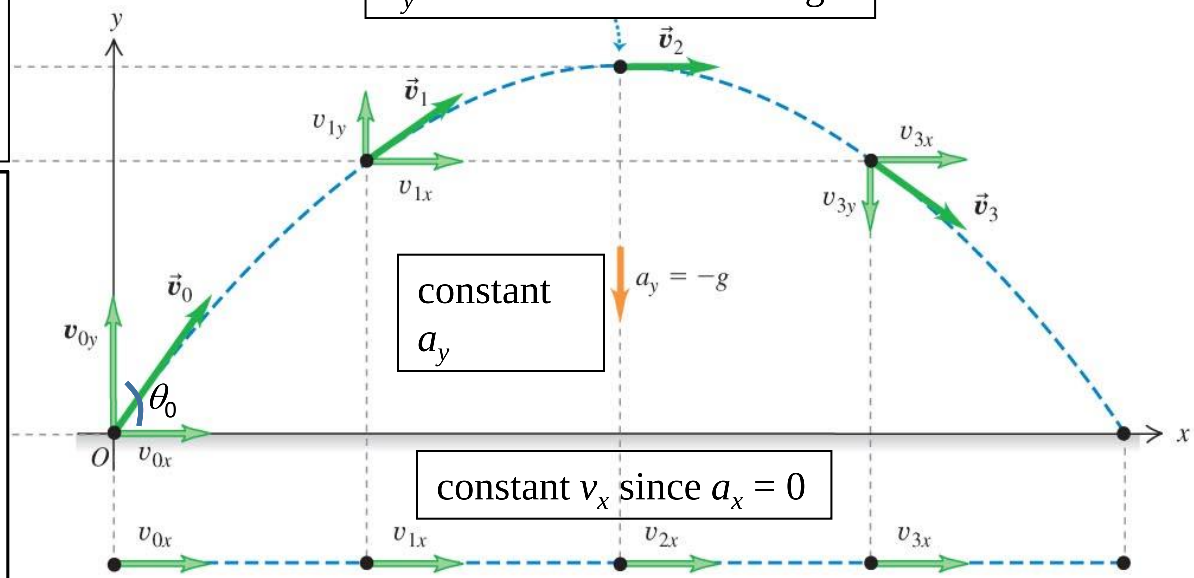
$x_0 = 0$ $\quad\quad$ $y_0 = 0$

$v_{0x} = v_0\cos(\theta_0)$ $\quad\quad$ $v_{0y} = v_0\sin(\theta_0)$

Other Examples
- airplane dropping a package
- motorcycle running off a cliff
- rock sliding off the edge of a roof

etc.

$v_y = 0$ at the maximum height



constant a_y

constant v_x since $a_x = 0$

x-components

$$v_x(t) = v_0\cos(\theta_0)$$
$$x(t) = v_0\cos(\theta_0)t$$

y-components

$$v_y(t) = v_0\sin(\theta_0) - gt$$
$$y(t) = v_0\sin(\theta_0)t - \frac{1}{2}gt^2$$
$$v_y^2 = (v_0\sin(\theta_0))^2 - 2gy$$

Courtesy of Wenhao Wu

Problem 3.3 12

. **I** A tennis ball rolls off the edge of a tabletop 0.750 m above the floor and strikes the floor at a point 1.40 m horizontally from the edge of the table. (a) Find the time of flight of the ball. (b) Find the magnitude of the initial velocity of the ball. (c) Find the magnitude and direction of the velocity of the ball just before it strikes the floor.

Take $+y$ downward, so $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$ and $\upsilon_{0y} = 0$. When the ball reaches the floor, $y - y_0 = 0.750$ m.

Solve: (a) $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(0.750\ \text{m})}{9.80\ \text{m/s}^2}} = 0.391\ \text{s}$.

(b) $x - x_0 = \upsilon_{0x}t + \frac{1}{2}a_x t^2$ gives $\upsilon_{0x} = \frac{x - x_0}{t} = \frac{1.40\ \text{m}}{0.391\ \text{s}} = 3.58\ \text{m/s}$. Since $\upsilon_{0y} = 0$, $\upsilon_0 = \upsilon_{0x} = 3.58\ \text{m/s}$.

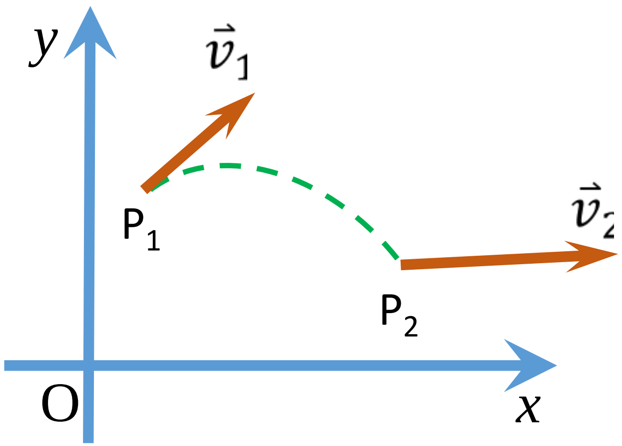
(c) $\upsilon_x = \upsilon_{0x} = 3.58\ \text{m/s}$. $\upsilon_y = \upsilon_{0y} + a_y t = (9.80\ \text{m/s}^2)(0.391\ \text{s}) = 3.83\ \text{m/s}$. $\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = 5.24\ \text{m/s}$.

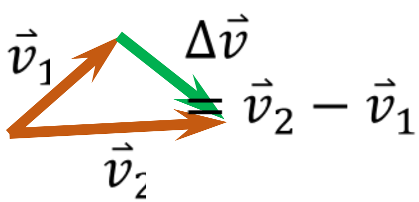
$$\tan\theta = \frac{|\upsilon_y|}{|\upsilon_x|} = \frac{3.83\ \text{m/s}}{3.58\ \text{m/s}}$$

and $\theta = 46.9°$. The final velocity of the ball has magnitude 5.24 m/s and is directed at 46.9° below the horizontal.

Reflect: The time for the ball to reach the floor is the same as if it had been dropped from a height of 0.750 m; the horizontal component of velocity has no effect on the vertical motion.

3.2 Acceleration in a Plane





average acceleration

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

components

$$a_{av,x} = \frac{\Delta v_x}{\Delta t} \qquad v_{av,y} = \frac{\Delta v_y}{\Delta t}$$

instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

components

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \qquad a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}$$

magnitude	direction
$\lvert\vec{a}\rvert = \sqrt{a_x^2 + a_y^2}$	$\theta = tan^{-1}\frac{a_y}{a_x}$

Courtesy of Wenhao Wu

Problem 3.2 7

. **II** A particle starts from rest at the origin with an acceleration vector that has magnitude $4\ \mathrm{m/s^2}$ and direction 30° above the positive x axis. (a) What are the components of its velocity vector 20 s later? (b) What is the particle's position at that time?

: **(a) At *t* = 20 s, the magnitude of the velocity is** $\upsilon = \upsilon_0 + at = at,$ **where we have used the fact that the initial velocity** υ_0 **is zero. Inserting the given time and acceleration gives** $\upsilon = at = \left(4\ \mathrm{m/s^2}\right)\left(20\ \mathrm{s}\right) = 80\ \mathrm{m/s}.$ This velocity is oriented at 30° above the positive *x* axis, so its components are

$$\upsilon_x = \upsilon\cos\theta = \left(80\ \mathrm{m/s}\right)\cos\left(30^\circ\right) = 7\times10^1\ \mathrm{m/s}$$

$$\upsilon_y = \upsilon\sin\theta = \left(80\ \mathrm{m/s}\right)\sin\left(30^\circ\right) = 4\times10^1\ \mathrm{m/s}$$

(b) The displacement *d* of the particle is $d = \upsilon_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2.$ Inserting the given acceleration and time gives

$$d = \frac{1}{2}at^2 = \frac{1}{2}\left(4\ \mathrm{m/s^2}\right)\left(20\ \mathrm{s}\right)^2 = 800\ \mathrm{m}$$

Again, this displacement is at 30° above the $+x$ axis, so the coordinates of the particle's position at this point in time are

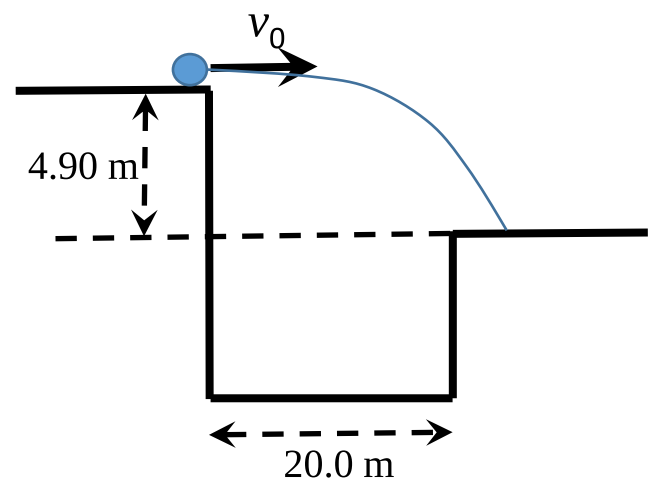
$$d_x = d\cos\theta = \left(800\ \mathrm{m}\right)\cos\left(30^\circ\right) = 7\times10^2\ \mathrm{m}$$

$$d_y = d\sin\theta = \left(800\ \mathrm{m}\right)\sin\left(30^\circ\right) = 4\times10^2\ \mathrm{m}$$

Thus, the projectile's position is $\left(7\times10^2\ \mathrm{m}, 4\times10^2\ \mathrm{m}\right)$.

More Examples in 2D Motion:
Minimum speed required for a stunt performance

A stunt performer on a motorcycle attempts to jump across a trench as shown in the diagram. What is the minimum speed that is required to perform this safely?



Consider this:

(a) After the motorcycle leaves the higher platform, it under goes a free fall under the influence of gravity. It takes a certain amount of time for the motorcycle to fall to the height level of the lower platform. This time is independent of the horizontal velocity. It is given by

$4.90 = \frac{1}{2}gt^2,$ from which this time can be calculated.

(b) To jump across the trench safely, during the above time, the motorcycle needs to cove a horizontal distance

larger than 20.0m so that it lands safely before it falls below the lower platform. Or, the requirement is

$v_0 t > 20.0$ m.

Therefore, the minimum horizontal velocity is $v_0 = (20.0\text{ m})/t$.

Courtesy of Wenhao Wu

Determination of Key Items

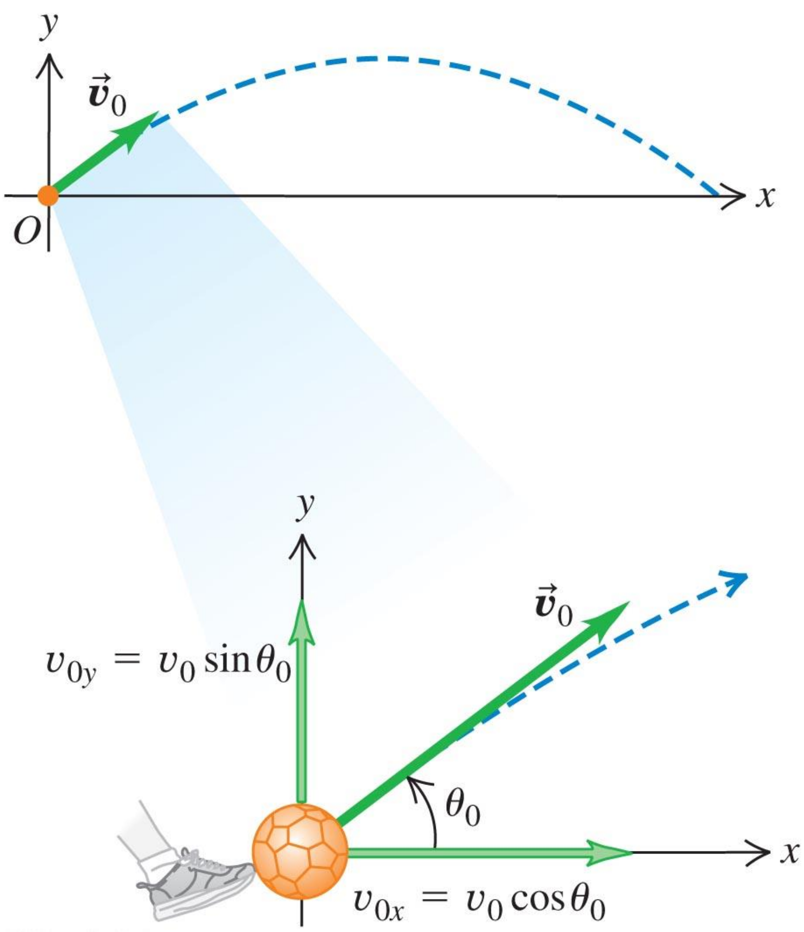
Don't forget: Initial velocity is a 2D vector.

Vector. $\rightarrow$ $\vec{v}_0 = \vec{v}_0 + \vec{v}_{0y}$

Components. $\rightarrow \begin{cases} v_{oy} = v_0 \sin\theta_0 \\ v_{ox} = v_0 \cos\theta_0 \end{cases}$

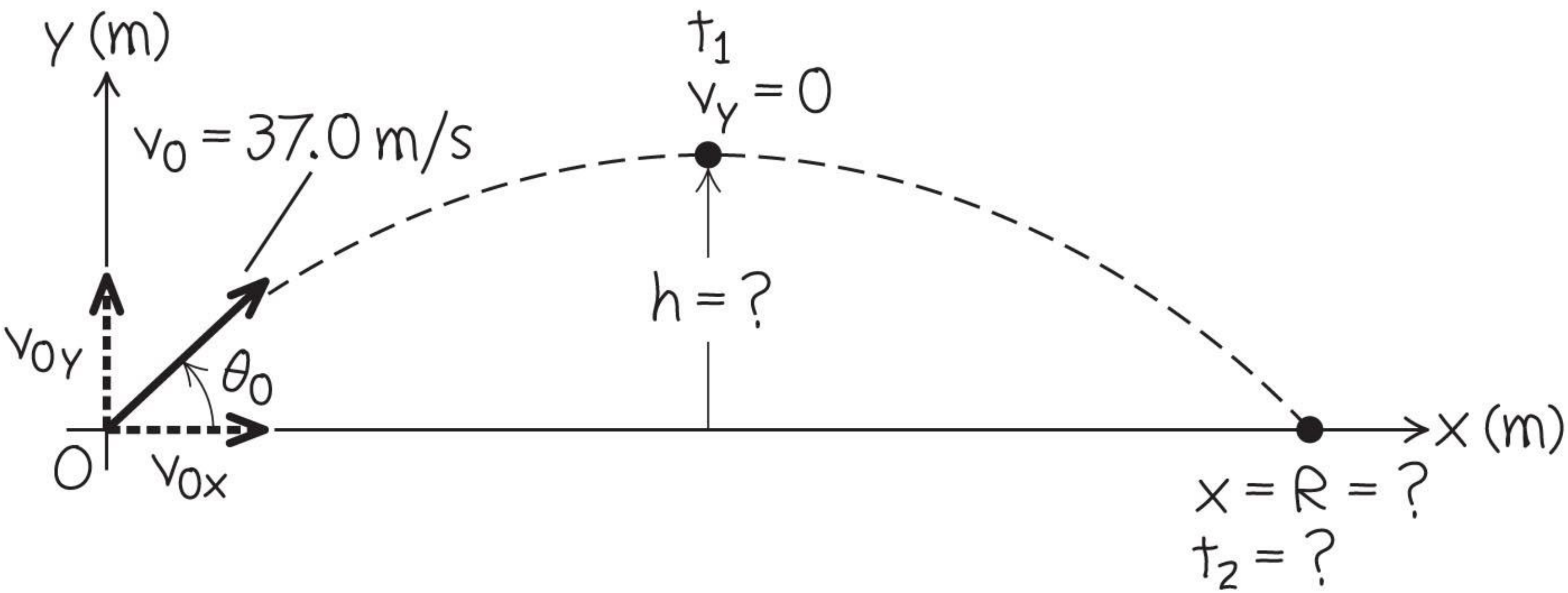
Initial speed. $\rightarrow$ $v_0 = \sqrt{v_{0x}^2 + v_{0y}^2}$

Launch angle. $\rightarrow$ $\tan\theta_0 = v_{0y} / v_{0x}$



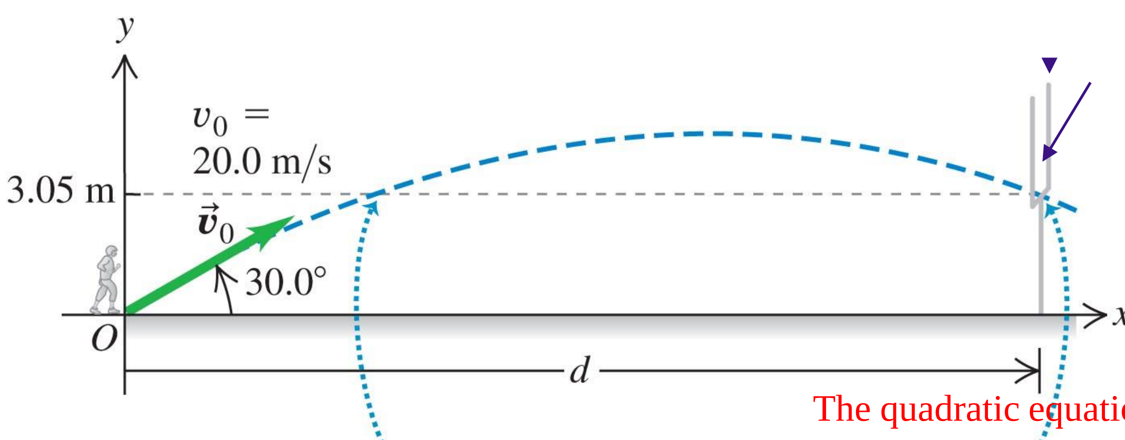
A Home-Run Hit – Example 3.4

- See the fully worked example on pages 75–76.
- Details examined for the flight of a baseball hit from home plate toward a fence hundreds of feet away.



Vertical/Horizontal Displacement – Example 3.6

- After a given horizontal displacement, a projectile will have a vertical position.
- Sports provide a large number of excellent examples.
- For a field goal, see worked example on page 76.

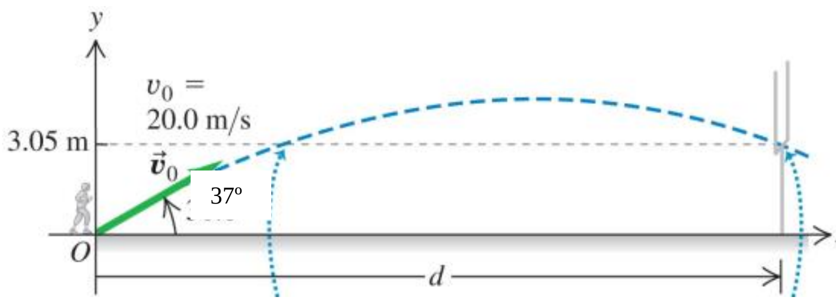


v=?

The quadratic equation has 2 solutions

The ball will have sufficient height to clear the goalpost at two different values of x. However, only the height at $x = d$ describes the ball on its way back down.

Kicked football



What is the speed at the highest point $|v_{x0}|$?

What is the max height?

$$v_{xo} = v_0 \cos 37° = 20 \cdot \cos 37° = 16\,\frac{m}{s}$$

$$v_{yo} = v_0 \sin 37° = 20 \cdot \sin 37° = 12\,\frac{m}{s}$$

Time it takes to reach the ground?

$$t = \frac{v_{yo}}{g} = 1.22\,s \qquad v_y = v_{y0} - gt$$

Time to reach highest point

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

$$0 = 0 + 12t - \frac{1}{2}9.8 \cdot t^2 \qquad t = 0$$

$$\frac{1}{2}9.8t - 12 = 0 \qquad t = \frac{2.12}{9.8} = 2.45s$$

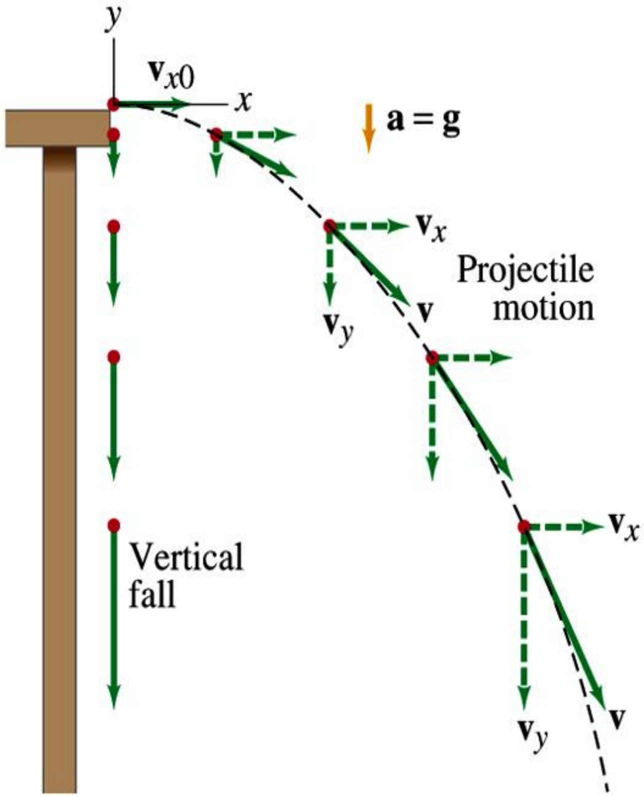
$$y = v_{y0} \cdot t - \frac{1}{2}gt^2$$

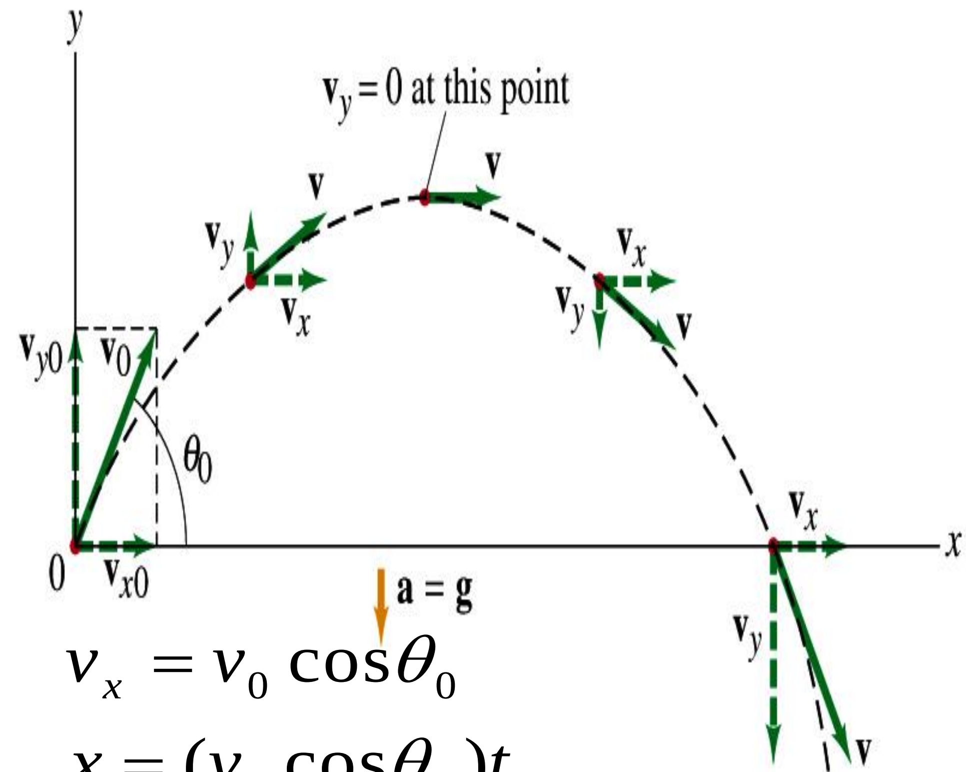
$$= 12 \cdot 1.22 - \frac{1}{2}9.8(1.22)^2 = 7.35m$$

What is the range?

$$x = v_{x0} \cdot t = 16 \cdot 2.45s = 39.2m \cdot \frac{3\text{ feet}}{1\text{m}} = 120\text{ feet}$$

Projectile Motion





$$v_x = v_{0x}$$

$$x = x_0 + v_{0x} t$$

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

$$v_x = v_0 \cos\theta_0$$

$$x = (v_0 \cos\theta_0) t$$

$$v_y = v_0 \sin\theta_0 - gt$$

$$y = (v_0 \sin\theta_0) t - \frac{1}{2} g t^2$$

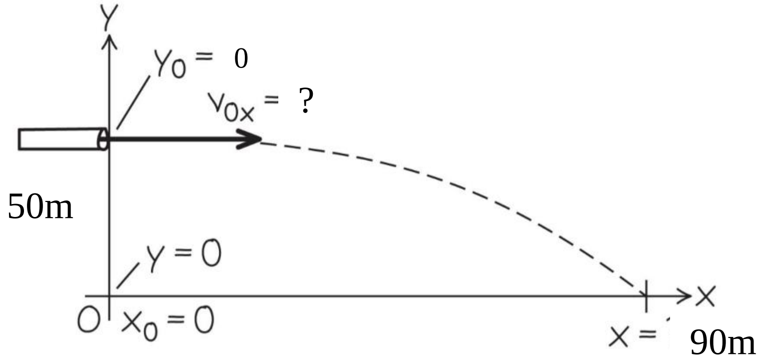
Clicker question

You throw a ball horizontally off a roof. Assuming the ball behaves as an ideal projectile, the time until it lands is determined only by

a) its initial speed and the horizontal distance to the point where it lands.
b) the height of the roof and its initial speed.
c) the height of the roof.

Driving off a cliff

$$\text{Vertical } y_0 = 0 \quad y = -\frac{1}{2}gt^2$$



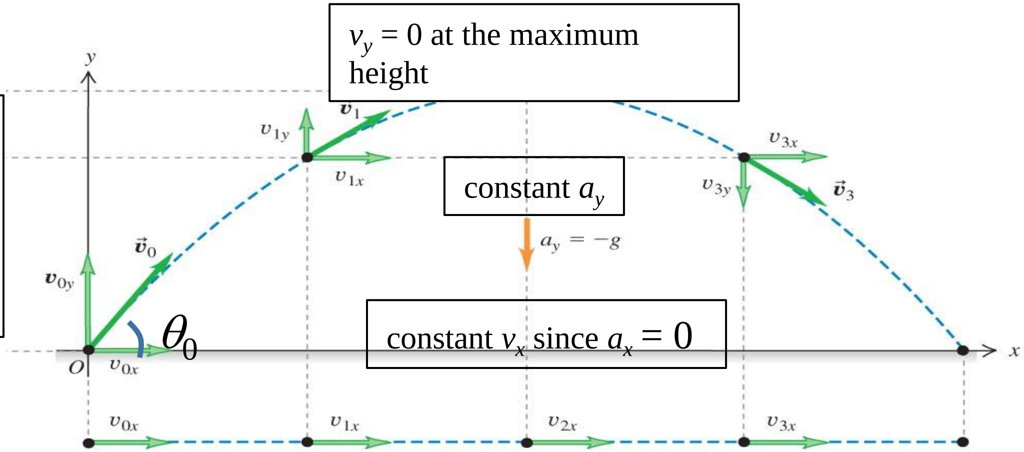
$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(-50)}{-9.8}} = 3.19s$$

Time for vertical motion

$$x = v_{x0} \cdot t$$

$$v_{x0} = \frac{x}{t} = \frac{90}{3.19} = 28.2\,\frac{m}{s}$$

3.3 Projectile Motion



<u>x-components</u>

$v_x(t) = v_0\cos(\theta_0)$

$x(t) = v_0\cos(\theta_0)t$

<u>y-components</u>

$v_y(t) = v_0\sin(\theta_0) - gt$

$y(t) = v_0\sin(\theta_0)t - \frac{1}{2}gt^2$

$v_y^2 = (v_0\sin(\theta_0))^2 - 2gy$

<u>Examples 3.4&3.5</u>: A home run hit

<u>Given</u>: v_0 and θ_0

<u>Find</u>: (a) x and y, and, v and θ, at t = 2.00 s

(b) t_{max} and h_{max}

(c) Horizontal range R

<u>Solutions</u>:

(a) $x(t) = v_0\cos(\theta_0)t$

$y(t) = v_0\sin(\theta_0)t - \frac{1}{2}gt^2$

$v_x(t) = v_0\cos(\theta_0)$

$v_y(t) = v_0\sin(\theta_0) - gt$

$v = (v_x^2 + v_y^2)^{\frac{1}{2}}$

$\theta = tan^{-1}(v_y/v_x)$

(b) Set $v_y(t) = v_0\sin(\theta_0) - gt = 0$

to get $t_{max} = v_0\sin(\theta_0)/g$.

Then, set $v_y^2 = (v_0\sin(\theta_0))^2 - 2gy = 0$

to get $h_{max} = (v_0\sin(\theta_0))^2/2g$.

(c) Set $y(t) = v_0\sin(\theta_0)t - \frac{1}{2}gt^2 = t(v_0\sin(\theta_0) - \frac{1}{2}gt) = 0$

to get the time of flight $t_f = 2v_0\sin(\theta_0)/g = 2t_{max}$.

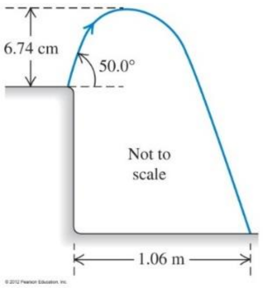
Then, using $x(t) = v_0\cos(\theta_0)t$

to get the range $R = 2v_0^2\sin(\theta_0)\cos(\theta_0)/g = v_0^2\sin(2\theta_0)/g$

Courtesy of Wenhao Wu

Grasshopper problem

What is the initial speed??
What is the height of the cliff??



Note: $a_x = 0, a_y = -9.8\, m/s$ $\quad v_y = 0$ when $y - y_0 = 0.0674\, m$

a) $$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.8\frac{m}{s^2})(0.0674\, m)} = \boxed{1.15\frac{m}{s}}$$

y-component of initial speed of grasshopper

$$v_{0y} = v_0 \sin\theta_0 \rightarrow v_0 = \frac{v_{0y}}{\sin\theta_0} = \frac{1.15\, m/s}{\sin 50°} = \boxed{1.5\frac{m}{s}}$$

initial speed

b) Use horizontal motion to find the time in air. The grasshopper travels in constant motion horizontally

$$x - x_0 = 1.06\, m$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \qquad t = \frac{x - x_0}{v_{0x}} = \frac{x - x_0}{v_0 \cos 50°} = \boxed{1.10\, s}$$

Time in air

Find the vertical displacement at t=1.1s.This is now accelerated motion

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 1.15\frac{m}{s} \times 1.1\, s + \frac{1}{2}\left(-9.8\frac{m}{s^2}\right) \times \left(1.15\frac{m}{s}\right)^2 = \boxed{-4.66\, m}$$

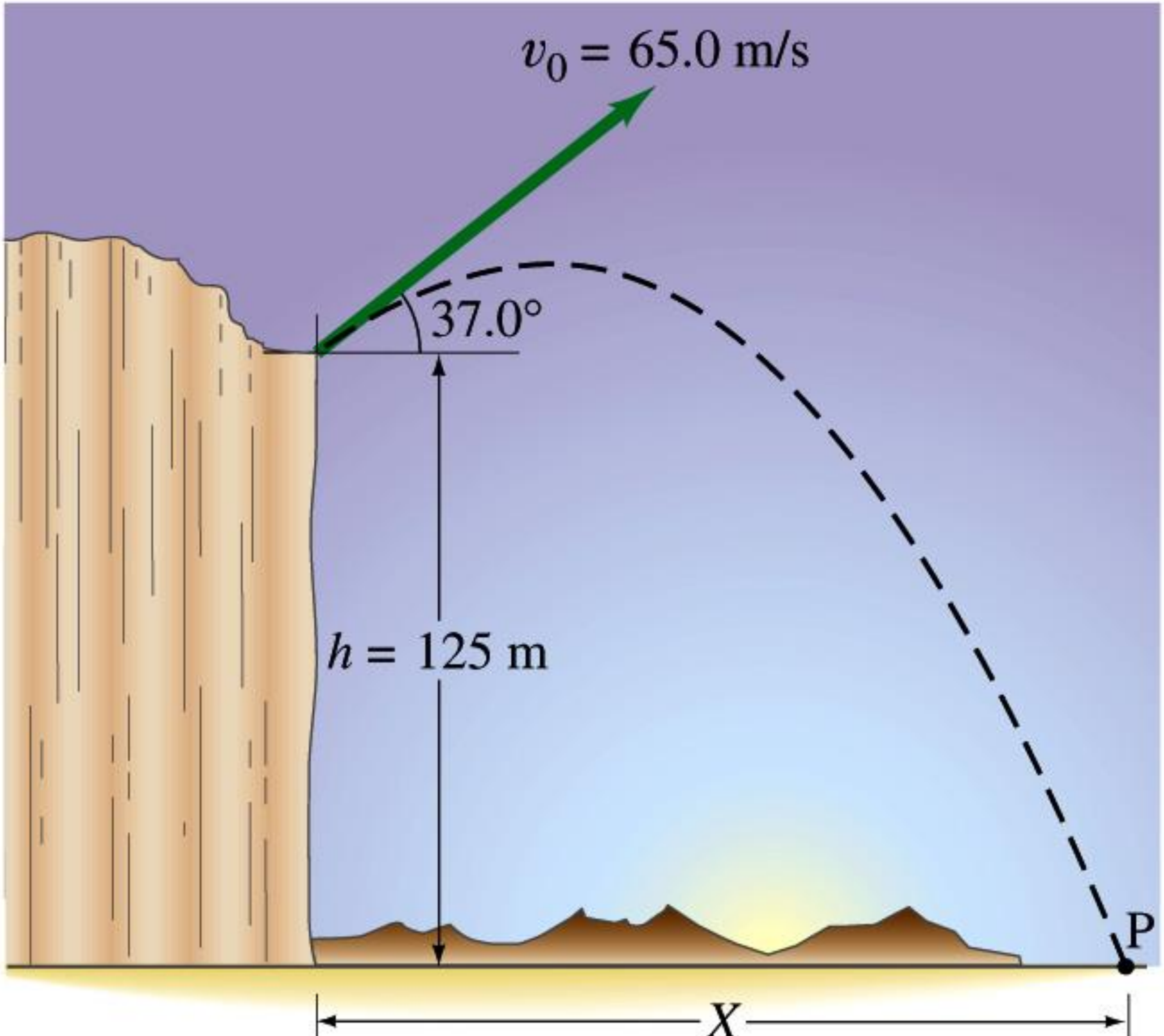
Height of cliff

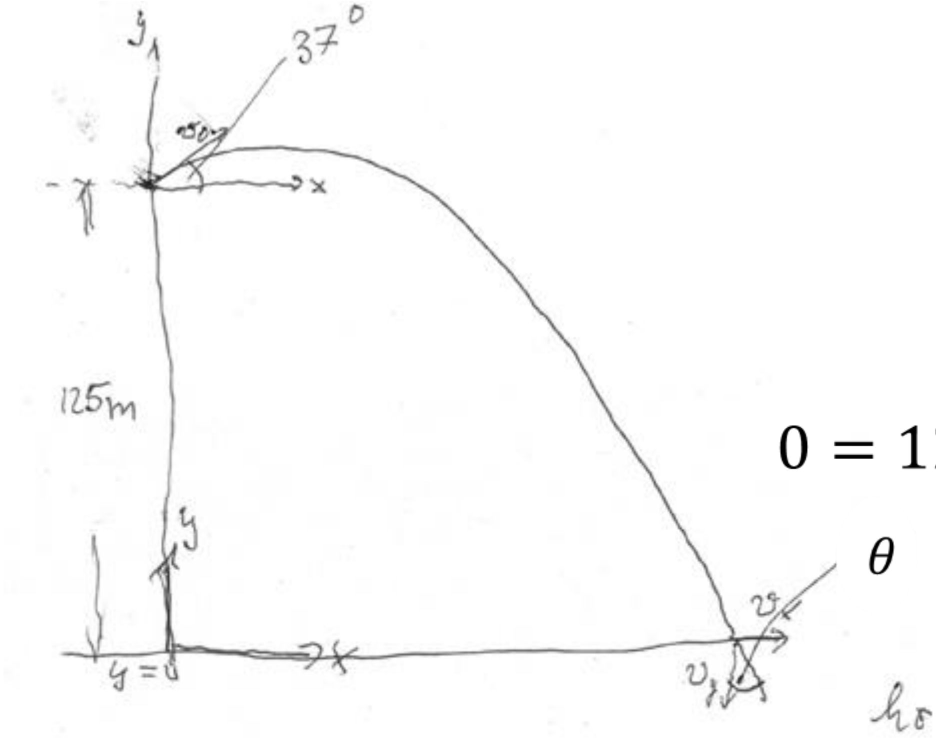
Clicker question

You and a friend throw two rocks off a bridge. Your friend throws hers with an initial direction 30º below the horizontal. You throw yours with the same initial speed but in a direction 30º above the horizontal. When the two rocks hit the water

a) your friend's is moving faster.
b) yours is moving faster.
c) they are moving at the same speed.

Projectile launched from a cliff





Workout problem:

Projectile $v_0 = 65\,\frac{m}{s}\,,\alpha = 37°$

Vertical motion

$$y = y_0 + v_{oy}t + \frac{1}{2}a_y t^2$$

$$0 = 125 + 65(\sin 37)t + \frac{1}{2}(-9.8)t^2$$

Solve quadratic equation:

$\cancel{t_1 = -2.45s}$ Not a sensible root

$$\boxed{t_2 = 10.4s}$$

Horizontal motion

$$x = v_{0x}\cdot t = 65\cos 37\cdot 10.4$$
$$= 540\;m$$

$v_x = v_{x0} = 65\cos 37 = 51.9\,\frac{m}{s}$ (does not change)

$$v_y = 65\sin 37 - \underbrace{9.8\cdot 10^4}_{g\,\cdot\,t} = -62.8\,\frac{m}{s}$$

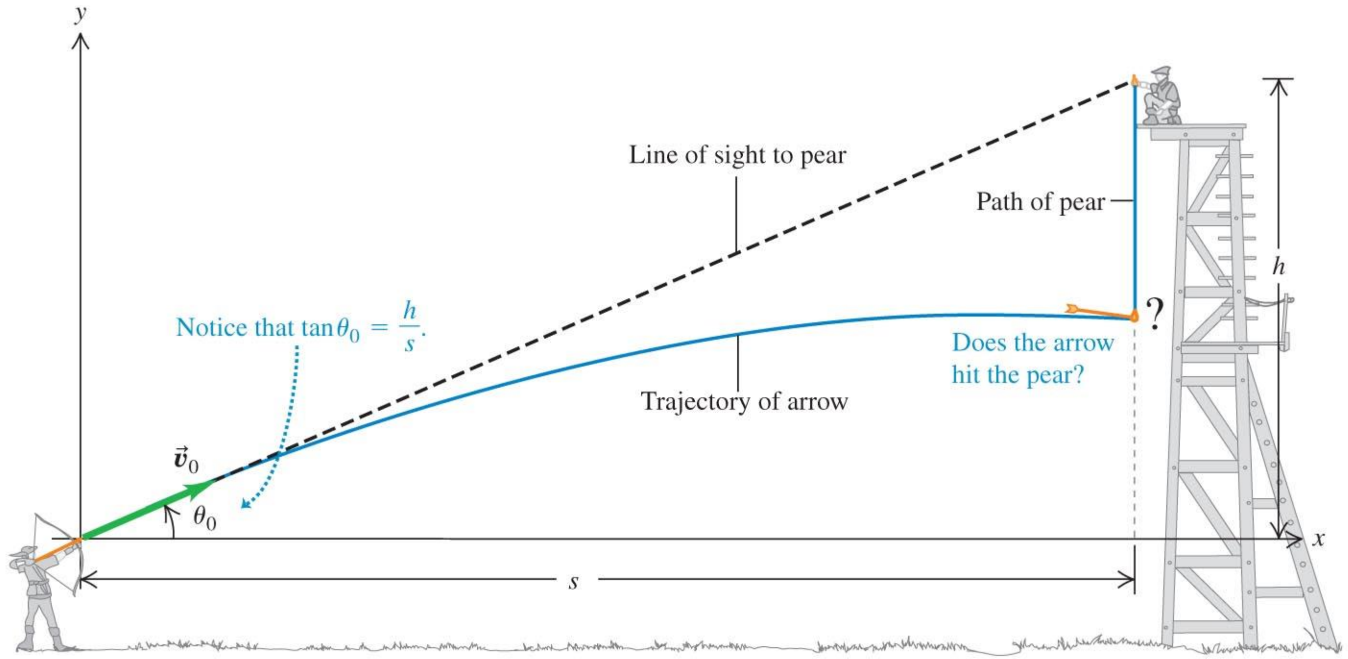
What is |velocity| at the bottom of the cliff?

$$|v| = \sqrt{v_x^2 + v_y^2} = 81.5\,\frac{m}{s}$$

$$\tan\theta = \frac{v_y}{v_x} = 1.21$$

$$\boxed{\theta = 50.4°}$$

Example 3.7 (and demo) Shooting a falling pear, on page 77



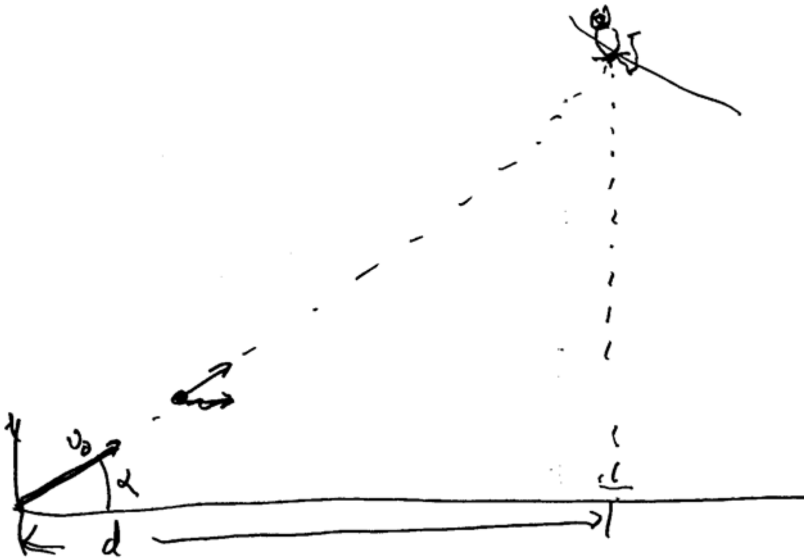
Note: Without the pull of gravity, the pear would not fall and the arrow would hit the pear along the dashed straight line.

With the pull of gravity, both the *y*-positions of the pear and the arrow as a function of time are lowered by an equal amount $-\frac{1}{2}gt^2$.

Conclusion: The arrow will hit the pear.

The monkey jumps from the tree at the moment when the tranquilizer arrow leaves the barrel:

Is there a hit?



x,y motions are independent

$$x_m = d$$

$$x_B = (v_0 \cos\alpha)t$$

$$d = v_0 \cos\alpha$$

$$\boxed{t = \frac{d}{v_0 \cos\alpha}}$$

$$y_m = d\tan\alpha - \frac{1}{2}gt^2$$

$$y_B = t(v_0 \sin\alpha) - \frac{1}{2}gt^2$$

Condition:

For a hit, the x and y coordinates must be the same for a particular time

$$t v_0 \sin\alpha = d\tan\alpha$$

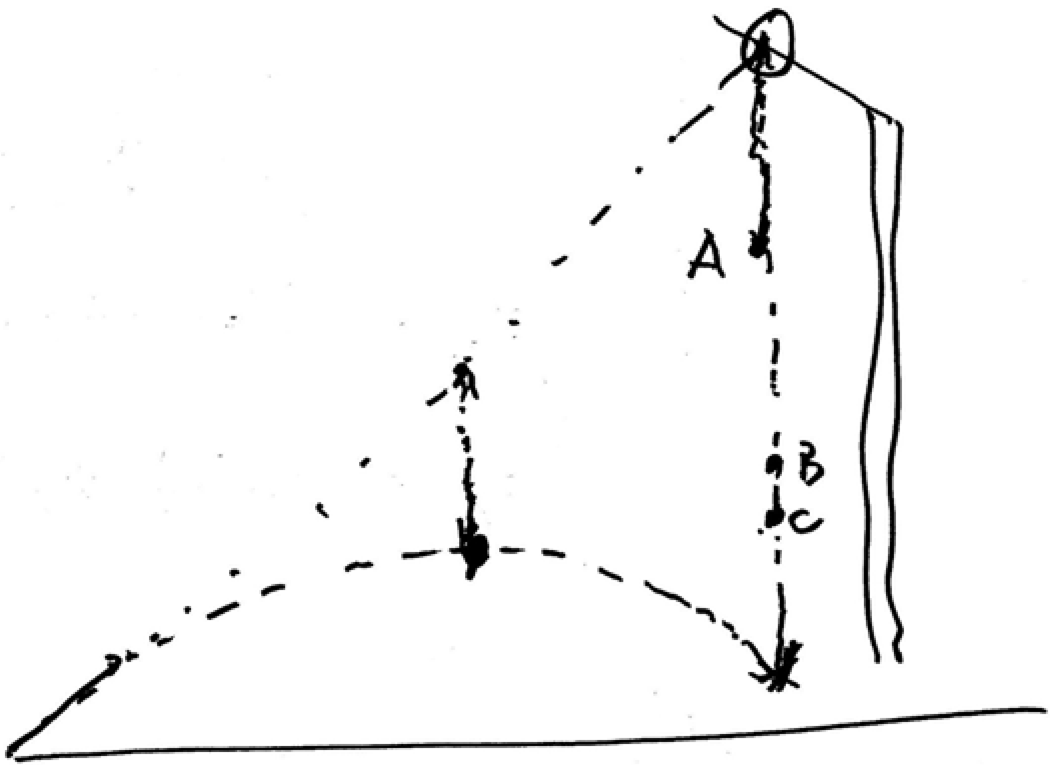
?

$$\frac{\cancel{d}}{\cancel{v_0} \cos\alpha} \cdot \cancel{v_0} \sin\alpha = \cancel{d} \tan\alpha$$

Yes it is equal

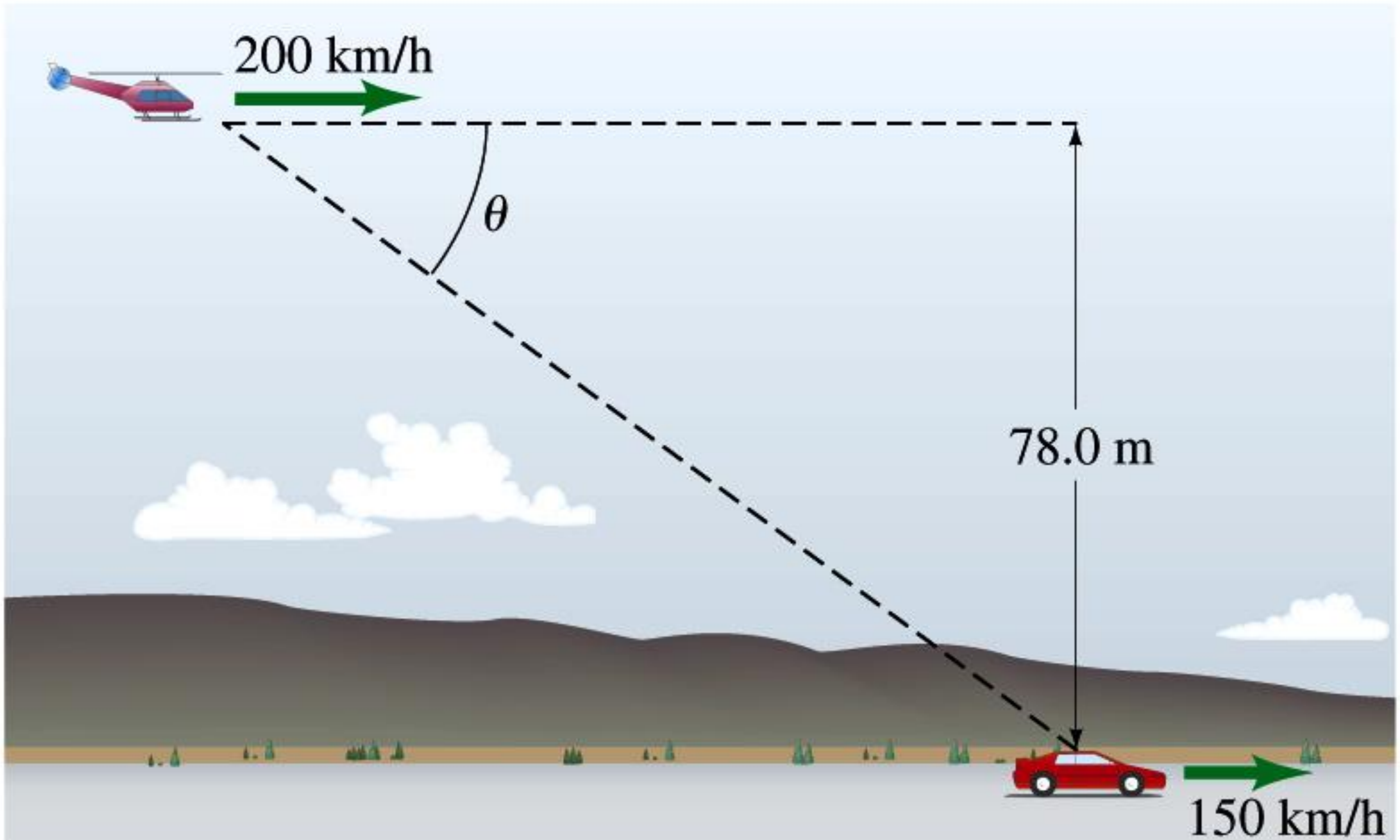
There is a hit

Where is the monkey??



A

Helicopter drops parcel



Parcel delivered by helicopter

v_{heli}, v_{car} go in the same direction: difference

$$\boxed{v_{0x} = v_{HG} - v_{AG}} = 200 - 150 = 50\frac{km}{h} = 13.9\frac{m}{s}$$

ground

Vertical motion:

$$y = y_1 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$78 = 0 + 0 + \frac{1}{2}9.8t^2$$

Time to hit the ground

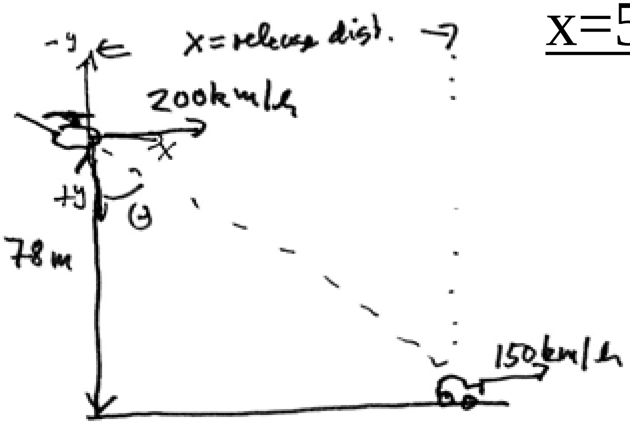
t=3.99s

Horizontal motion:

$$v_{0x} \cdot t = x = 13.9 \times 3.99s$$

Release distance is 55.4m behind the car

x=55.4m



Reference from: helicopter
Positive is down

$$\tan\theta = \frac{y}{x} = \frac{78}{55.4}$$

$$\boxed{\theta = 54.6°}$$

The parcel is always below the helicopter

You throw a ball horizontally off a roof. Assuming the ball behaves as an ideal projectile, the time until it lands is determined only by

a. its initial speed and the horizontal distance to the point where it lands.

b. the height of the roof and its initial speed.

c. the height of the roof.

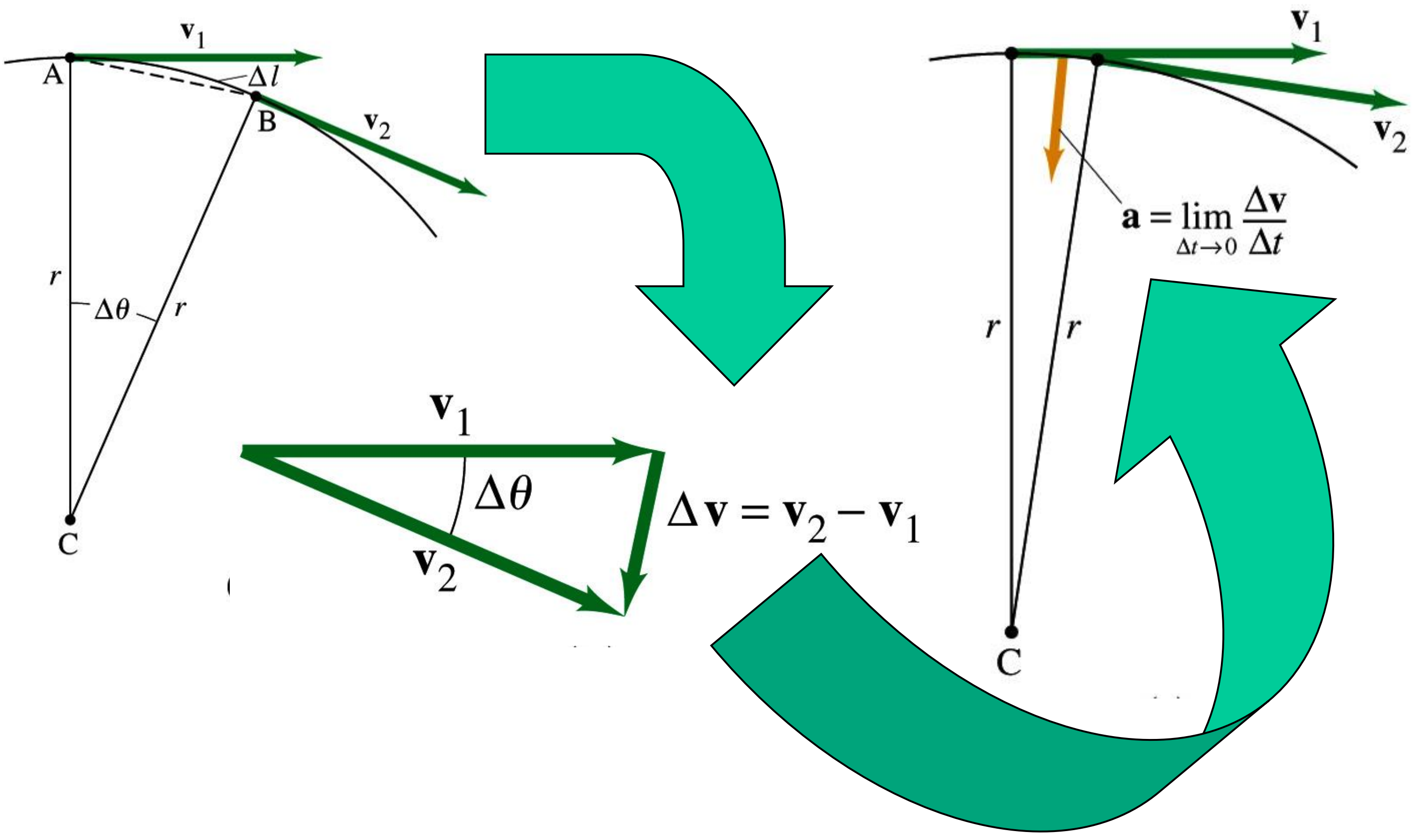
You and a friend throw two rocks off a bridge. Your friend throws hers with an initial direction 30° below the horizontal. You throw yours with the same initial speed but in a direction 30° above the horizontal. When the two rocks hit the water

a. your friend's is moving faster.

b. yours is moving faster.

c. they are moving at the same speed.

Center-seeking Acceleration



3.4 Uniform Circular Motion

An object moving along a circular path with a constant speed *v* (magnitude of velocity)

Velocity and Acceleration Vectors in Uniform Circular Motion

Velocity: tangent to the circle with constant magnitude $v_1 = v_2 = v$

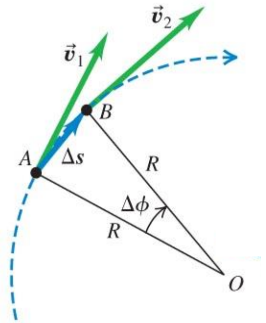
Accelerarion: $a_{\text{rad}} = \dfrac{v^2}{R}$, always pointing toward the center of the circle.

$$\frac{\Delta\vec{v}}{v} = \frac{\Delta\vec{s}}{R} \quad \rightarrow \quad \Delta\vec{v} = \frac{v}{R}\Delta\vec{s}$$

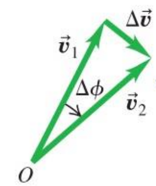
$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{v}{R}\frac{\Delta\vec{s}}{\Delta t}$$

$$\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{v}{R}\lim_{\Delta t \to 0}\frac{\Delta\vec{s}}{\Delta t} = \frac{v}{R}\vec{v}$$

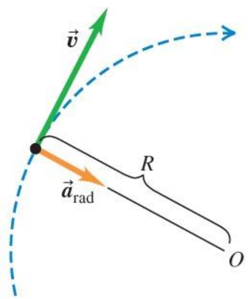
$$a_{\text{rad}} = |\vec{a}| = \frac{v}{R}|\vec{v}| = \frac{v^2}{R}$$



(a) A point moves a distance Δ*s* at constant speed along a circular path.



(b) The corresponding change in velocity



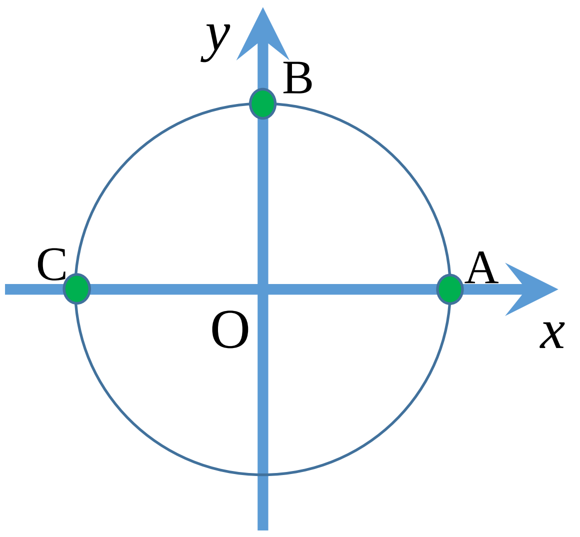
(c) The acceleration in uniform circular motion always points toward the center of the circle.

Courtesy of Wenhao Wu

Example

Answer the following questions using the given coordinate system.

Part (1): A circular track is 50.0 m in diameter. A runner completes a full lap on the track in $t_1 = 32.0$ s in a *ccw* rotation.



(a) Her average speed for the full lap?

Let $R = 50.0/2 = 25.0$ m

average speed: $s_1 = 2\pi R/t_1$

(b) Her average velocity for the full lap?

average velocity: $v_{av} = 0$

Part (b): If she runs the first half of the lap from A through B to C in $t_2 = 14.0$ s at a constant speed.

(c) Her average speed for this half lap? average speed: $s_2 = \pi R/t_2$

(d) Her average velocity for this half lap? average: $v_{av,x} = [(-R) - (R)]/t_2 = -2R/t_2$ (pointing in $-x$)

(e) The magnitude of her acceleration? magnitude of the acceleration: $a_{rad} = s_2^2/R$

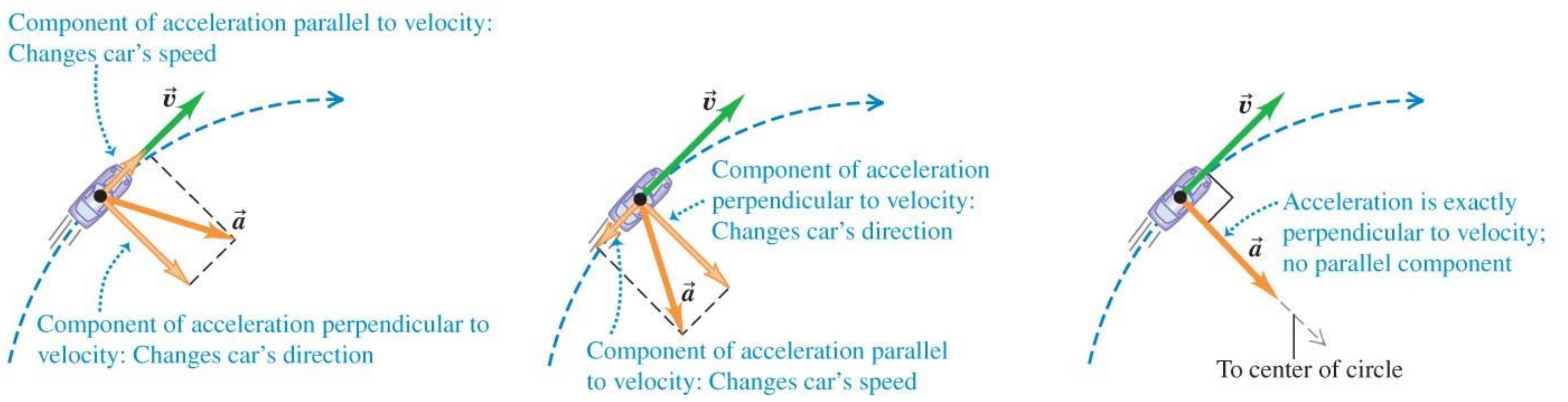
(f) The direction of acceleration at point B? pointing downward toward the center of the circle

(e) Her average acceleration? average: $a_{av,y} = [(-s_2) - (s_2)]/t_2 = -2s_2/t_2$ pointing in $-y$ axis

Courtesy of Wenhao Wu

Circular Motion as a Special Application – Figure 3.20

- Two dimensional motion in a plane takes on unique features when it is confined to a circle.
- The acceleration is centripetal ("center seeking").
- The velocity vector retains the same magnitude, it changes **direction**.

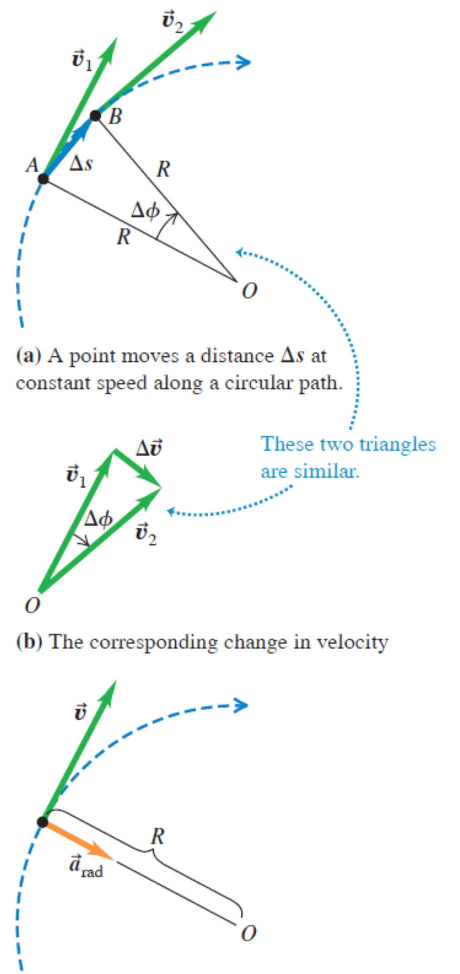


(a) Car speeding up along a circular path **(b) Car slowing down along a circular path** **(c) Uniform circular motion:** Constant speed along a circular path

It Is Possible to Solve for the Velocity – Figure 3.21

- The problem may be treated by extracting a small portion of the motion such that the $\Delta\vec{v}$ arc may be approximated as a straight line.
- Acceleration in a uniform circular motion has magnitude:

$$a_{rad} = \frac{v^2}{r}$$



(a) A point moves a distance Δs at constant speed along a circular path.

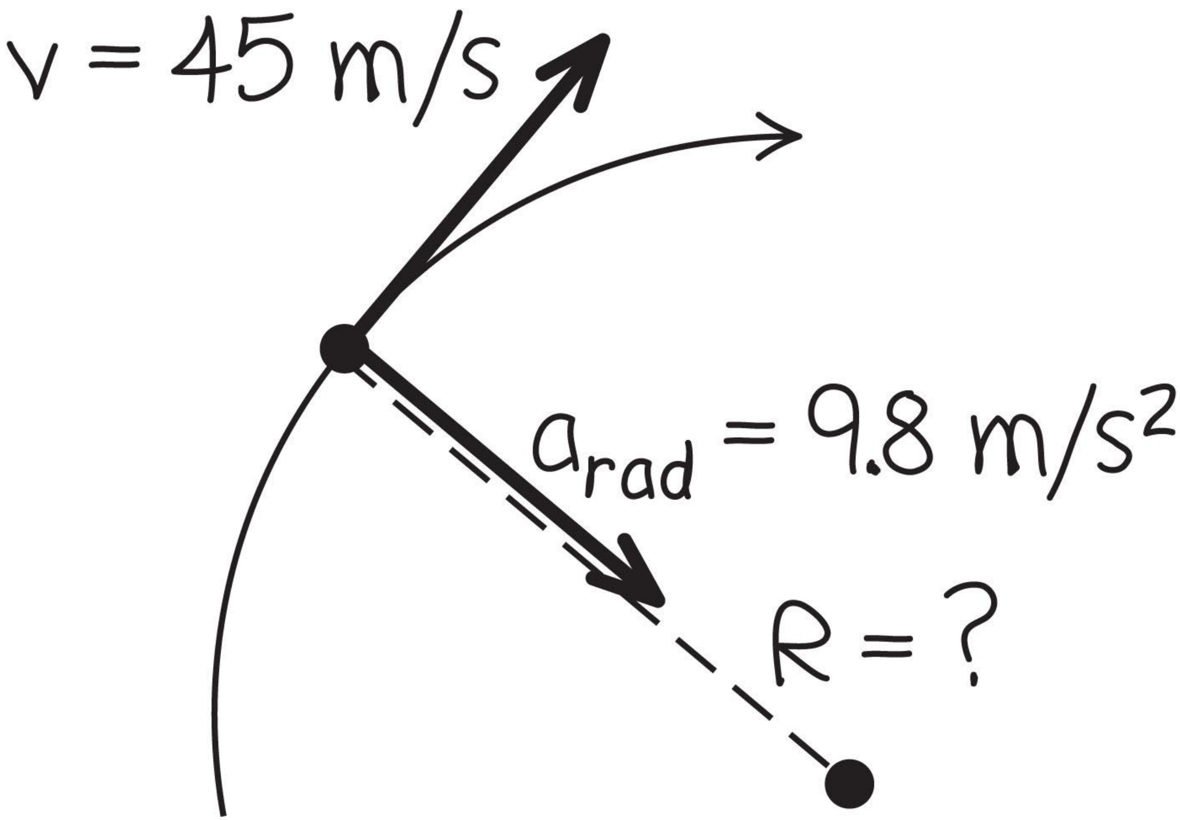
(b) The corresponding change in velocity

(c) The acceleration in uniform circular motion always points toward the center of the circle.

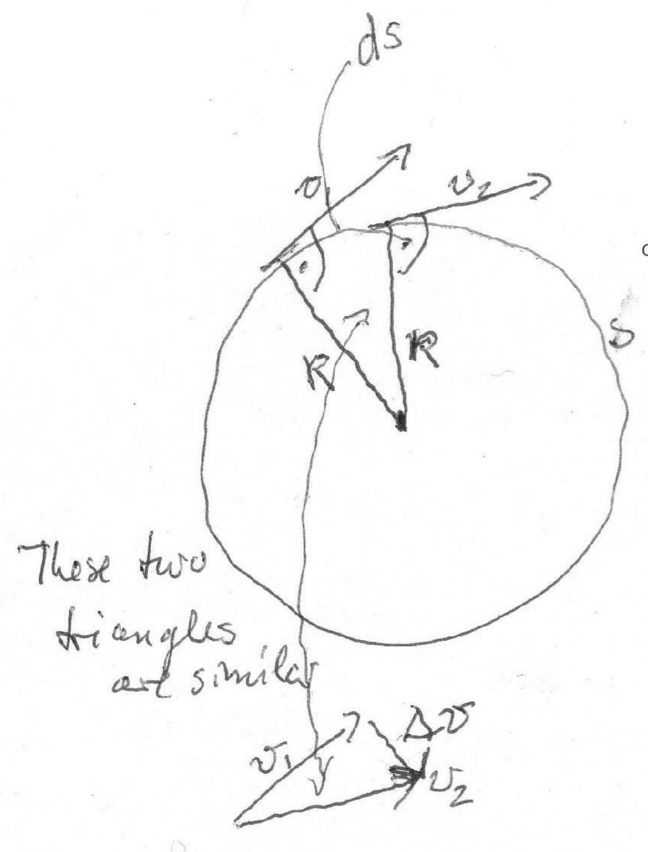
Pearson

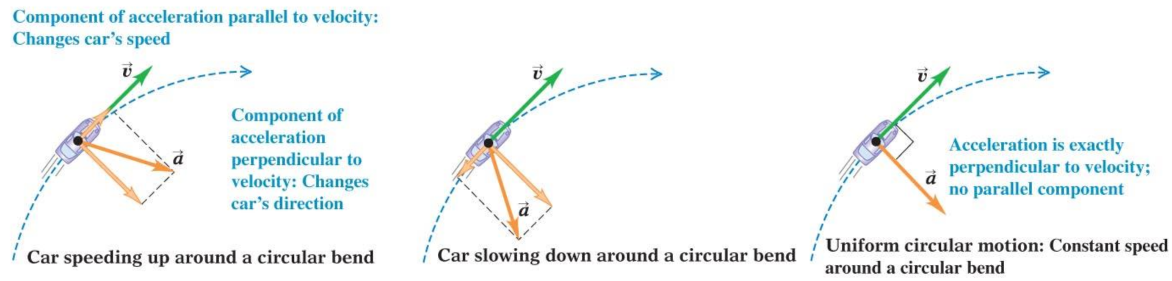
An Example You Can Go Try Right Now – Example 3.8

- Refer to the worked example on page 80



Circular motion





$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R} \qquad \text{or} \qquad |\Delta v| = \frac{v_1}{R}\Delta s$$

The magnitude of the average acceleration a_v for Δt is

$$a_v = \frac{|\Delta v|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}$$

Instantaneous acceleration:

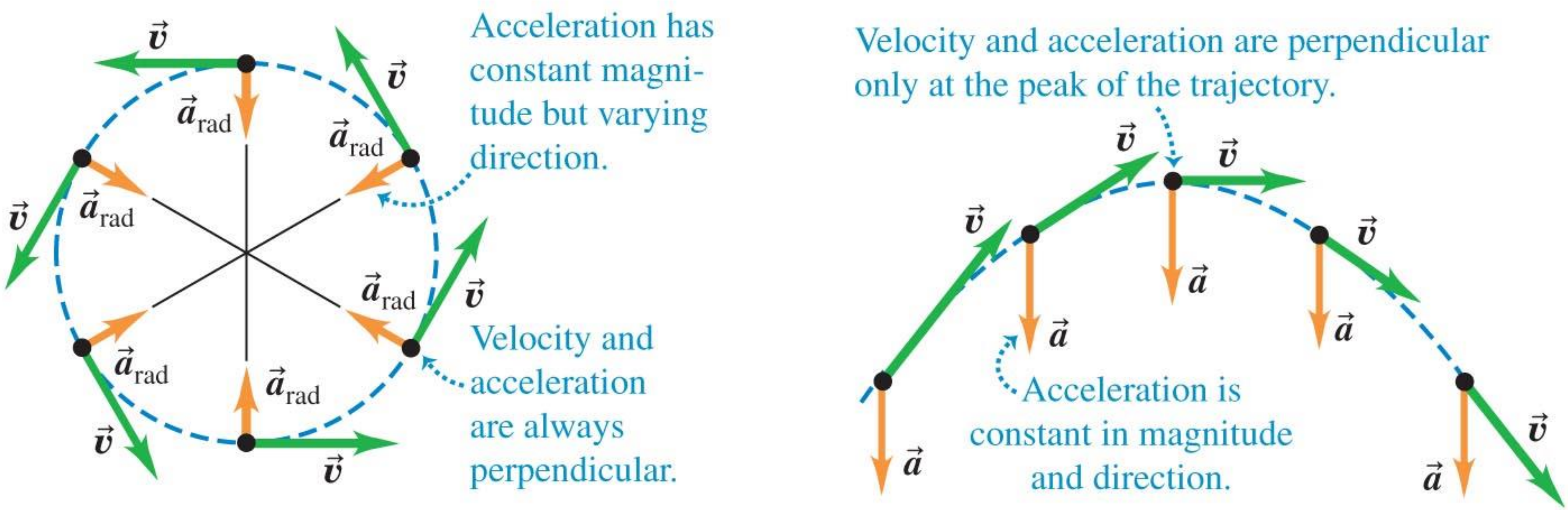
$$a = \lim_{\Delta t=0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v}{R}\frac{ds}{dt} = \frac{v^2}{R}$$

Radial acceleration: $\boxed{a_{rad} = \frac{v^2}{R}}$

James Webb Space telescope

After traveling nearly one million miles, the James Webb Space Telescope arrived at its new home on Monday January 24 2022. The spacecraft's arrival checks off another tricky step as scientists on Earth prepare to spend at least a decade using the observatory to study distant light from the beginning of time.

Launch date: December 24, 2021

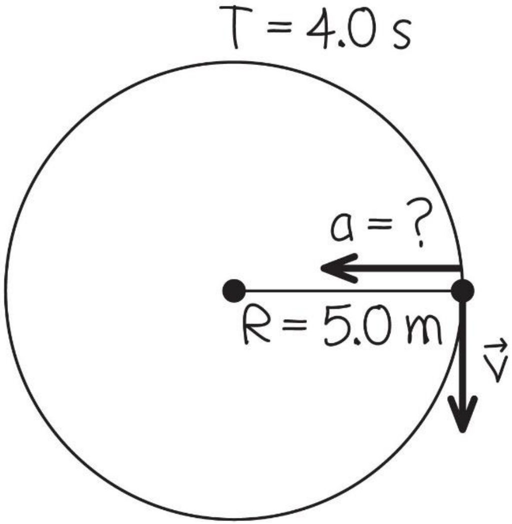


(a) Uniform circular motion

(b) Projectile motion

A Problem to Try on Your Next Vacation – Example 3.9

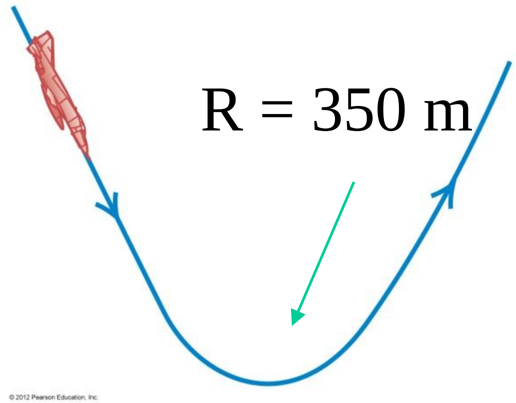
- Uniform circular motion applied to a daring carnival ride. [1 period = time for one revolution]



$$v = \frac{2\pi R}{T} \qquad T = \text{period}$$

$$a_{rad} = \frac{v^2}{r}$$

Pilot Blacks-Out



Stuka=
German war plane
(History Channel)

$$a_{rad} = \frac{v^2}{R} \quad v = \sqrt{a_{rad} \cdot R} = \sqrt{350m \times 53.9\frac{m}{s^2}} = 140\frac{m}{s} = 310\,mph$$

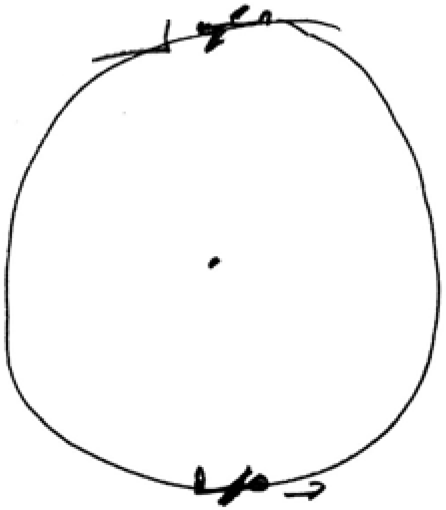
R 6g

a(rad)= 5.5g =53.9 m/s^2 1mph = 0.4470 m/s

Loss of consciousness for an acceleration of 5.5 g

The pilot takes a vertical loop
6g! do not exceed

Has a velocity of 700 km/hr



use

$$a_R = \frac{v^2}{R}$$

$$g = 9.8\frac{m}{s^2}$$

$$6g = 60m/s^2 \cdot 1km/10^3m = 6\cdot 10^{-2}km/s^2$$

What radius should the plane fly and not exceed 6g?

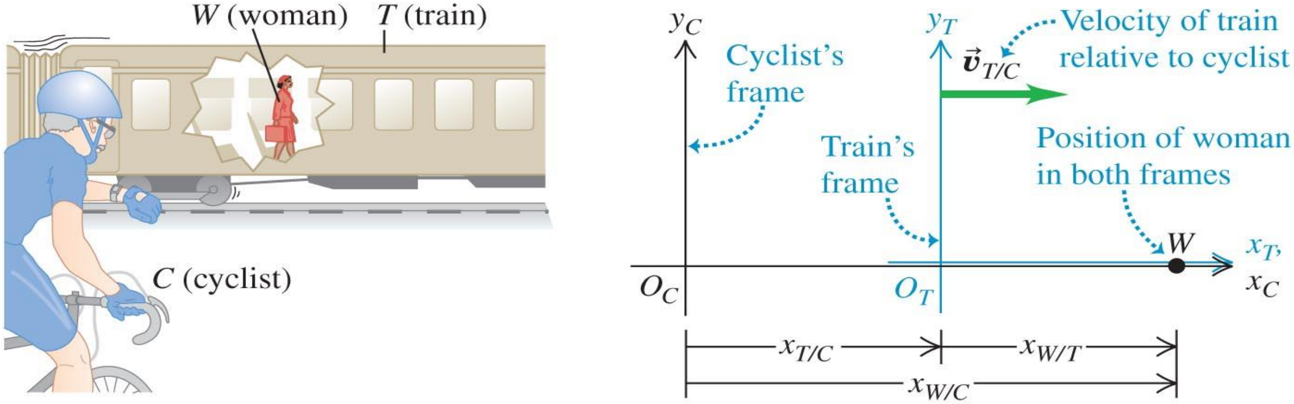
$$|v| = 700\frac{km}{h} \text{ at the bottom}$$

$$\text{v}= 700\frac{km}{h}\frac{10^3m}{1km}\frac{1h}{60\cdot 60}$$

$$\text{R}= \frac{v^2}{a_r} = \frac{v^2}{6\cdot 10^{-2}} = 0.64km = 640m$$

$$= 6\cdot 10^{-2}km/s^2$$

3.5 Relative Velocity in a Plane (in Two-Dimension)



Positions are relative:

$$x_{W/C} = x_{W/T} + x_{T/C}$$
$$y_{W/C} = y_{W/T} + y_{T/C}$$

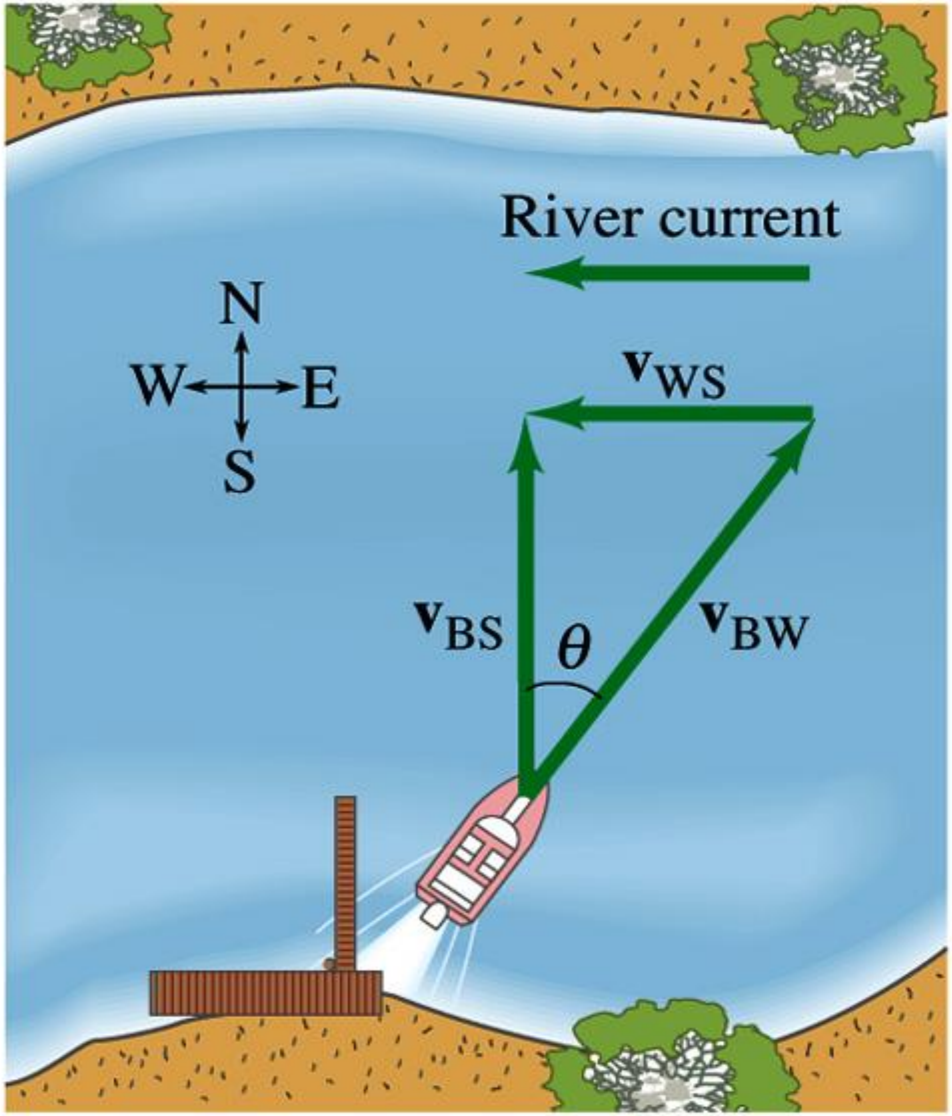
or, in vector form $\vec{r}_{W/C} = \vec{r}_{W/T} + \vec{r}_{T/C}$

Velocities are also relative: $\vec{v}_{W/C} = \vec{v}_{W/T} + \vec{v}_{T/C}$

Pay attention to the directions (signs): positions and velocities are vectors.

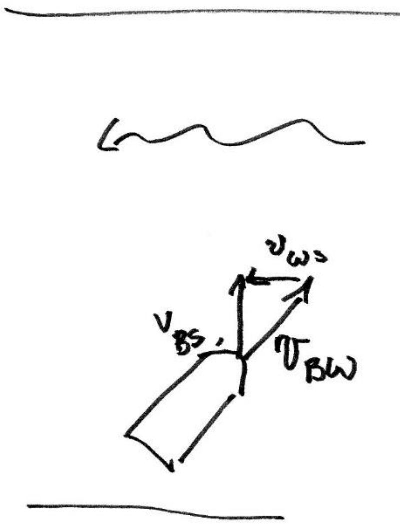
Courtesy of Wenhao Wu

crossing a stream



Boat crossing a river

S = shore



$$\boxed{\overrightarrow{v_{Bs}} = \overrightarrow{v_{Bw}} + \overrightarrow{v_{ws}}}$$

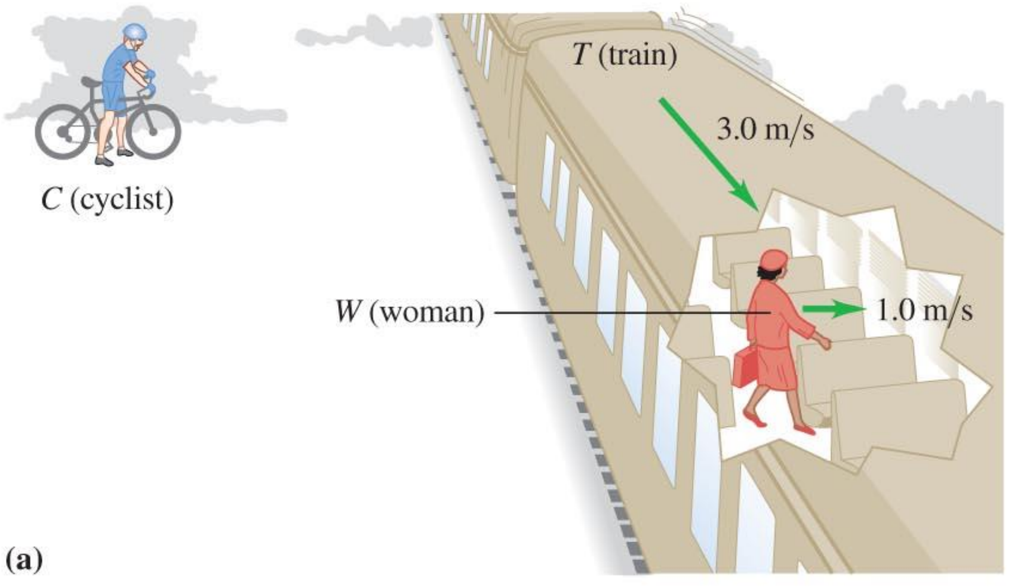
$\overrightarrow{v_{Bs}}$ = velocity of boat to shore
$\overrightarrow{v_{Bw}}$ = velocity of boat in water
$\overrightarrow{v_{ws}}$ = velocity of water with respect to shore

Remember for addition of two vectors, you find the x and y components of each vector, add them quadratically, and take the square root to get the magnitude of the resultant vector

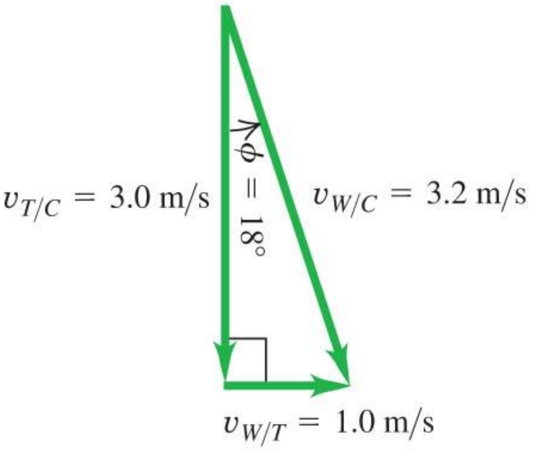
Relative Velocity: A Matter of Perspective – Figure 3.26

- Velocities can carry multiple values depending on the position and motion of the object and the observer.

$$\vec{v}_{W/C} = \vec{v}_{W/T} + \vec{v}_{T/C}$$



(a)



(b) Relative velocities and their corresponding magnitudes as seen from above

Example: The current in a river flows due south with a speed of 3.00 m/s. The river is 24.0 m wide. A boat travels due east from point A on one bank to point B on the other bank that is exactly opposite of point A. The speed of the boat relative to the water is 6.00 m/s. How long does it take the boat to travel from A to B?

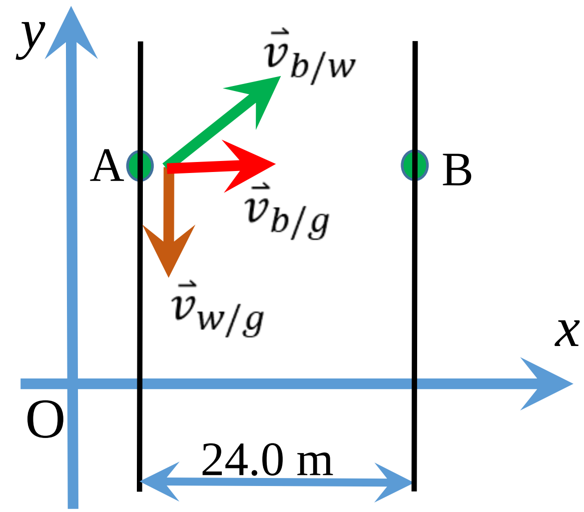
Question to ask: How can the boat travel east with the current flowing south at a speed of 3.00 m/s?

Intuitive answer: The boat must have a velocity component of 3.00 m/s toward north relative to the water, or the velocity must point in an angle north of east. Therefore, related to the water, the boat should have a east-ward velocity component of

$$\sqrt{6.00^2 - 3.00^2} = 5.20\ m/s.$$

Time takes to cross the river is

$$24.0/5.20 = 4.62\text{ s}$$



$$\vec{v}_{b/g} = \vec{v}_{b/w} + \vec{v}_{w/g}$$

Since $v_{b/g,y} = v_{b/w,y} + v_{w/g,y} = 0$,

$v_{b/w,y} = -\, v_{w/g,y} = +\,3.00$ m/s.

Therefore,

$$v_{b/g,x} = v_{b/w,x} = \sqrt{{v_{b/w}}^2 - {v_{b/w,y}}^2}$$

$$= \sqrt{6.00^2 - 3.00^2} = 5.20\text{ m/s}.$$

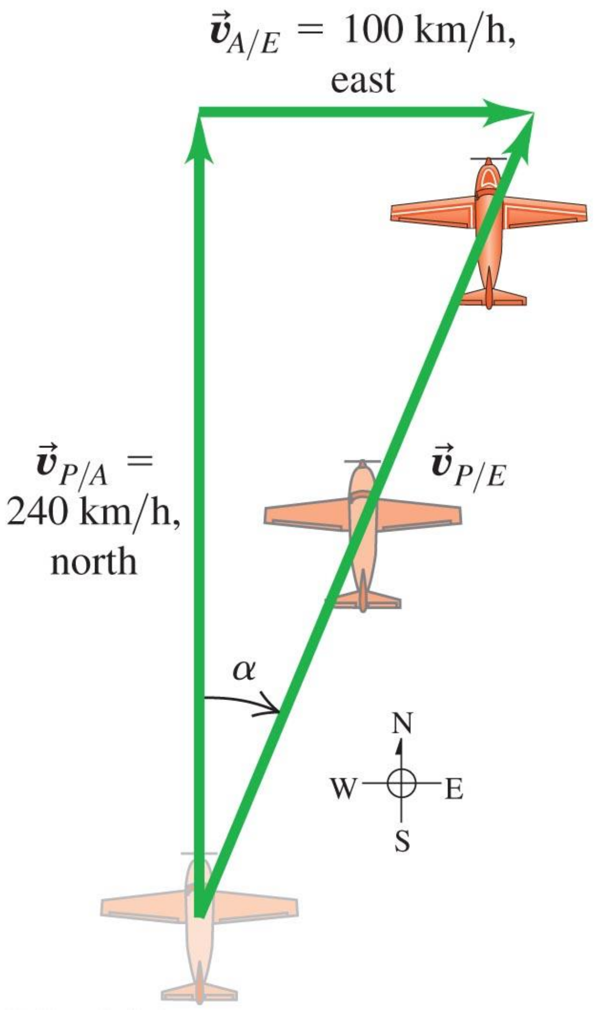
Courtesy of Wenhao Wu

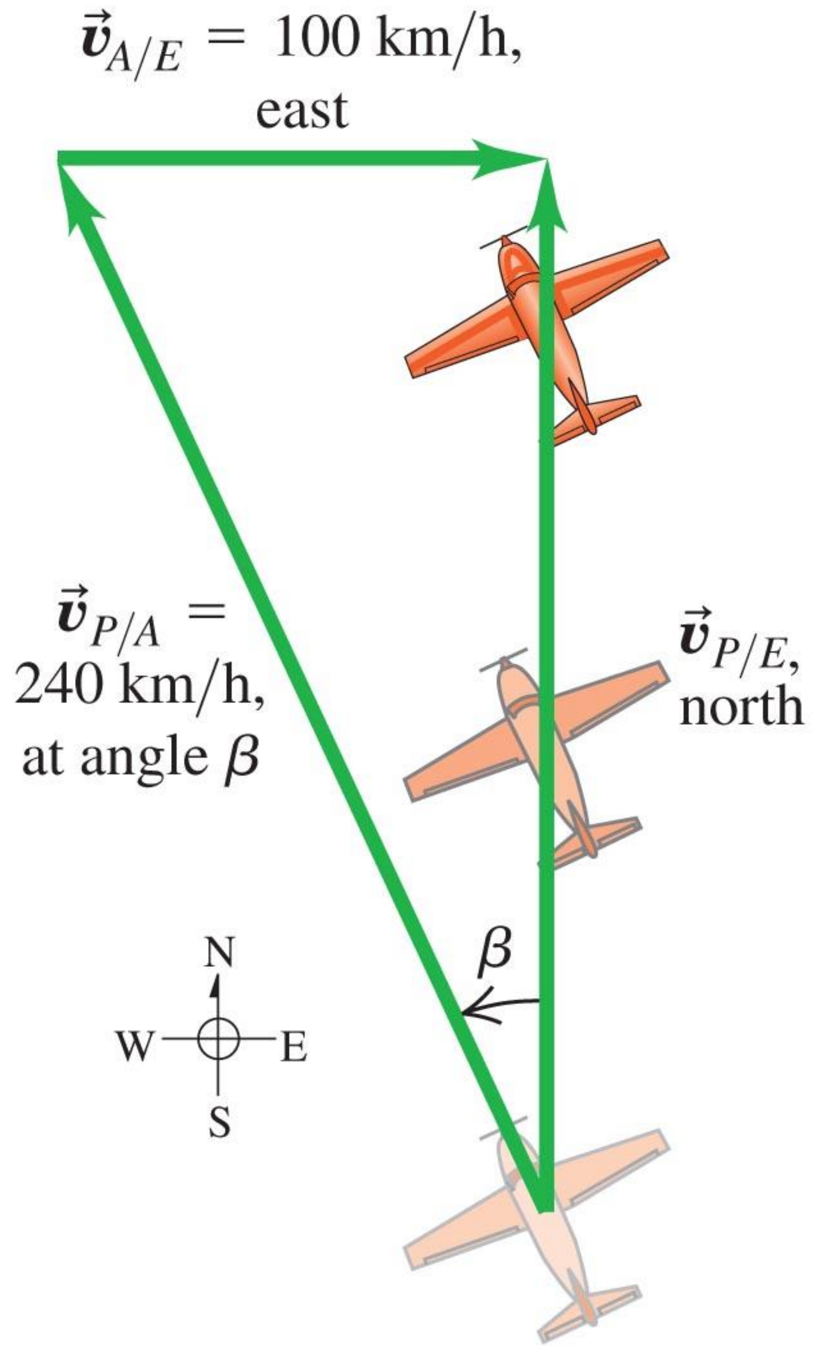
An Airplane in a Crosswind – Example 3.10

- A solved application of relative motion.

$$\vec{V}_{P/E} = \vec{V}_{P/A} + \vec{V}_{A/G}$$

- This is just velocity vector addition.




$\vec{v}_{A/E} = 100$ km/h,
east
$\vec{v}_{P/A} =$
240 km/h,
at angle β
$\vec{v}_{P/E}$,
north
β
N
W
E
S

Questions 3.3 Projectile Motion

A projectile is launched at some angle to the horizontal with some initial speed vi; air resistance is negligible.

Is the projectile a freely falling body?

What is its acceleration in the vertical direction?

What is its acceleration in the horizontal direction?

A student throws a heavy red ball horizontally from a balcony of a tall building with an initial speed v0. At the same time, a second student drops a lighter blue ball from the same balcony. Neglecting air resistance, which statement is true?

The blue ball reaches the ground first.

The red ball reaches the ground first.

Both balls hit the ground with the same speed.

As a projectile moves in its path, is there any point along the path where the velocity and acceleration vectors are

perpendicular to each other?

Parallel to each other?

Questions 3.3 Projectile Motion

A passenger sitting in the rear of a bus claims that she was injured as the driver slammed on the brakes, causing a suitcase to come flying toward her from the front of the bus. If you were the judge in this case, what disposition would you make? Explain.

As an apple tree is transported by a truck moving to the right with a constant velocity, one of its apples shakes loose and falls toward the bed of the truck. Of the curves shown

- which best describes the path followed by the apple as seen by a stationary observer on the ground, who observes the truck moving from his left to his right?
- Which best describes the path as seen by an observer sitting in the truck?