Chapter 4 Newton’s Laws of Motion

- To understand the concept of force – individual forces, the net force, and the components of a force.
- To study and apply Newton's first law.
- To study and apply Newton's second law.
- To study and apply Newton's third law & identify action-reaction force pairs.
- To draw a free-body diagram representing the forces acting on an object.
- To differentiate between mass and weight.

Courtesy of Wenhao Wu

Dynamics, a New Frontier

- Stated previously, the onset of physics separates into two distinct parts:
  - statics
  - dynamics
- So, if something is going to be dynamic, what causes it to be so?
  - A force is the cause. It could be either:
    - pushing
    - pulling

4.4 Mass and Weight

- Mass is a measure of "how much material do I have?"
- Weight is a force: "how hard do I pull on a scale?"
- Weight of an object with mass m must have a magnitude w equal to the mass times the magnitude of acceleration due to gravity:

$$w = mg \qquad \text{magnitude}$$

$$w_y = -mg \qquad \text{vector component}$$

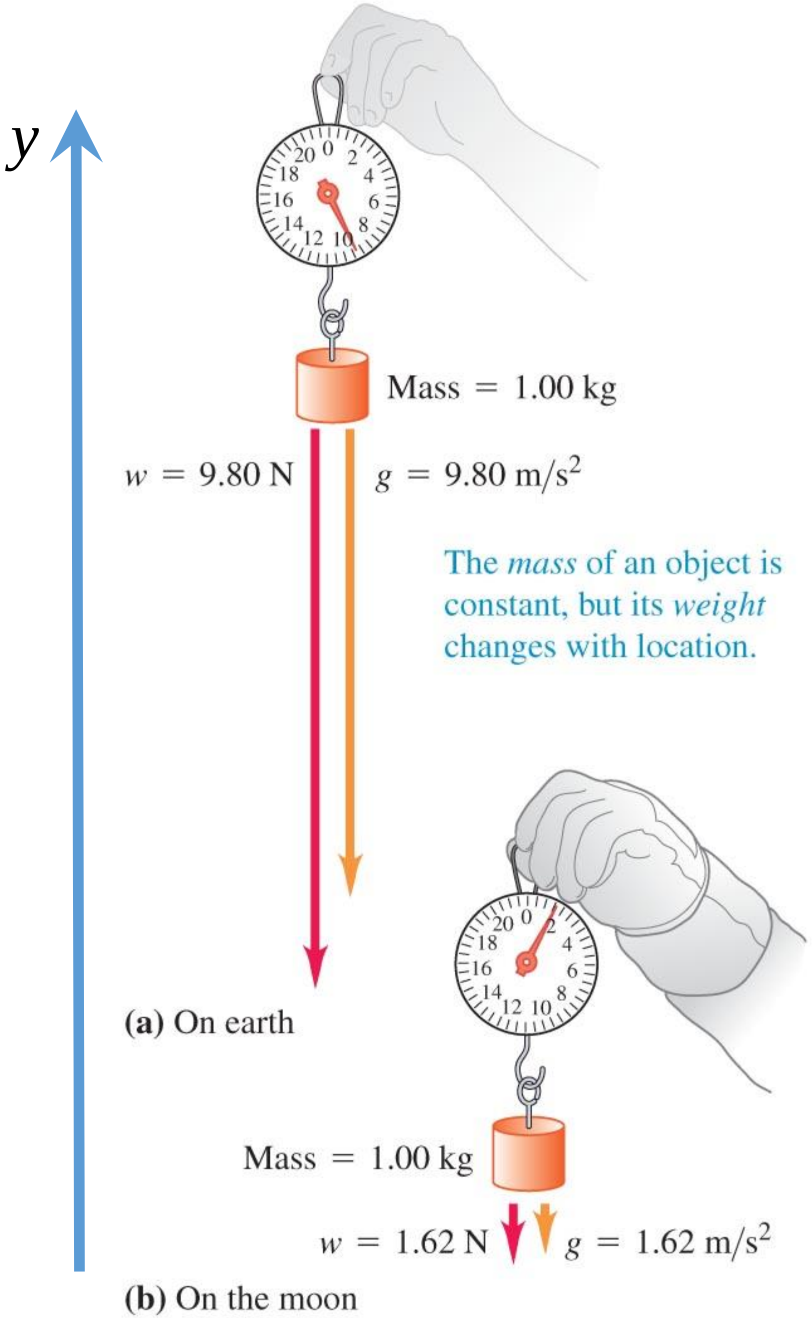
- Why???

  Consider an object of mass m falling under the influence of gravity only. In terms of the magnitude:

$$F = ma \qquad \text{and} \qquad a = g.$$

  Therefore

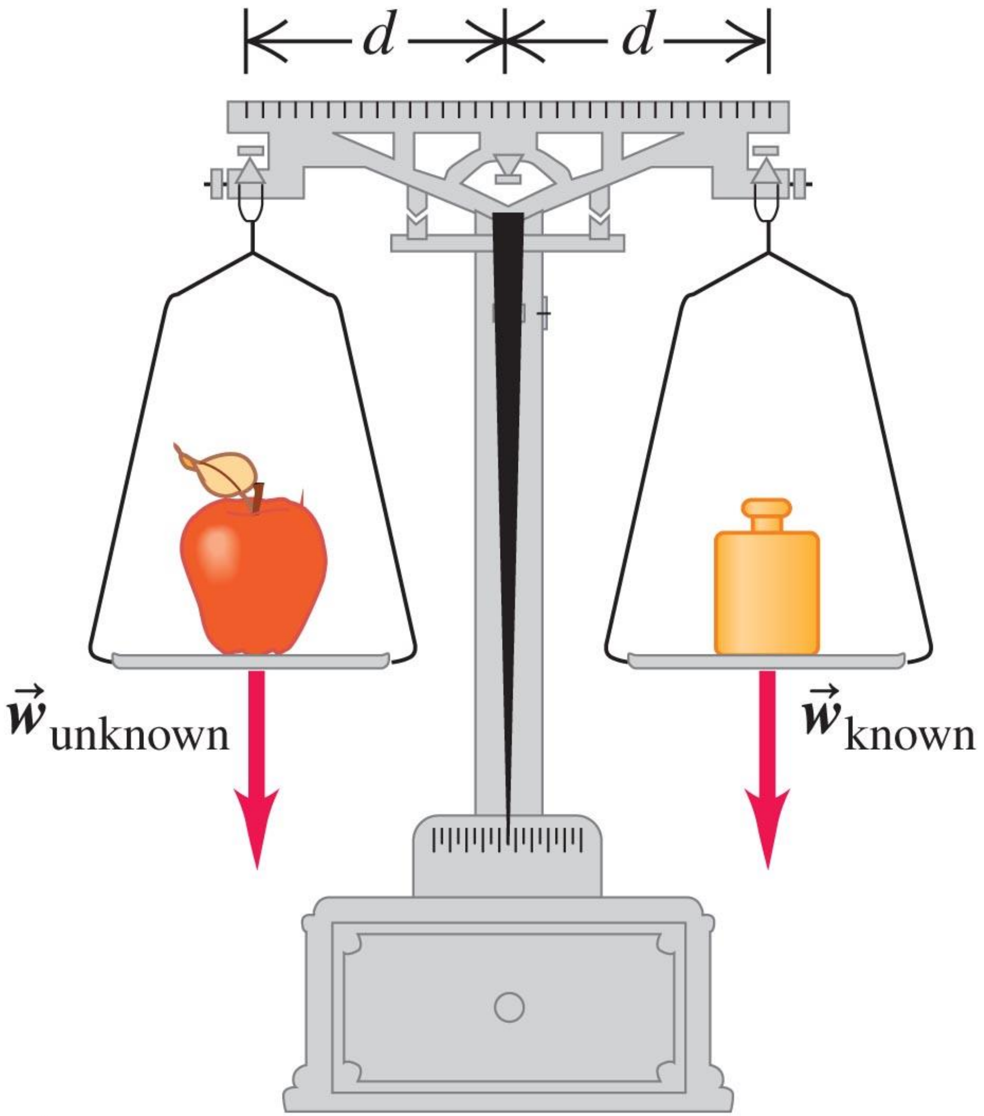
$$w = F = mg$$



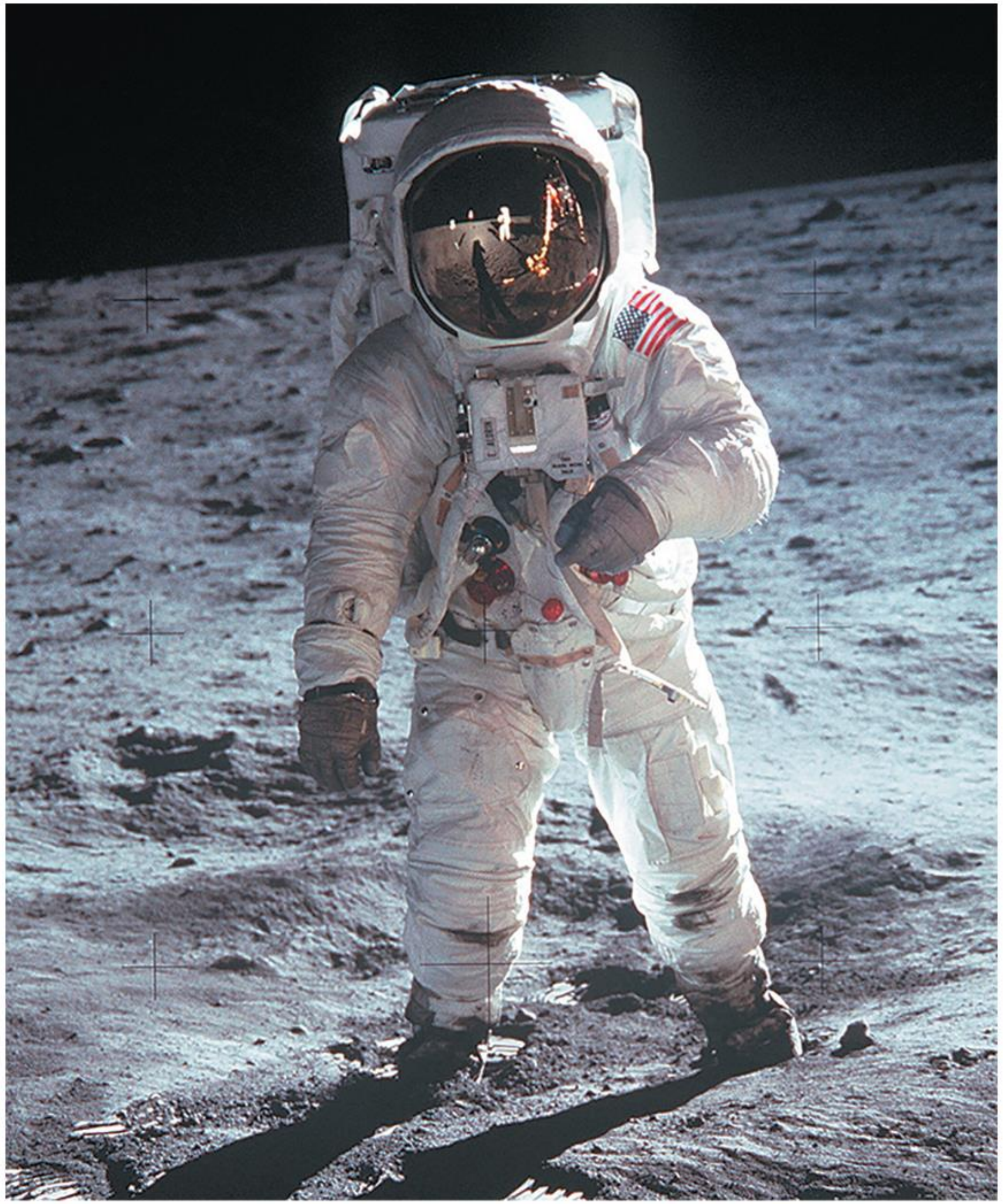
Courtesy of Wenhao Wu

Measurement of Mass

- Since gravity is constant, we can compare forces to measure unknown masses.

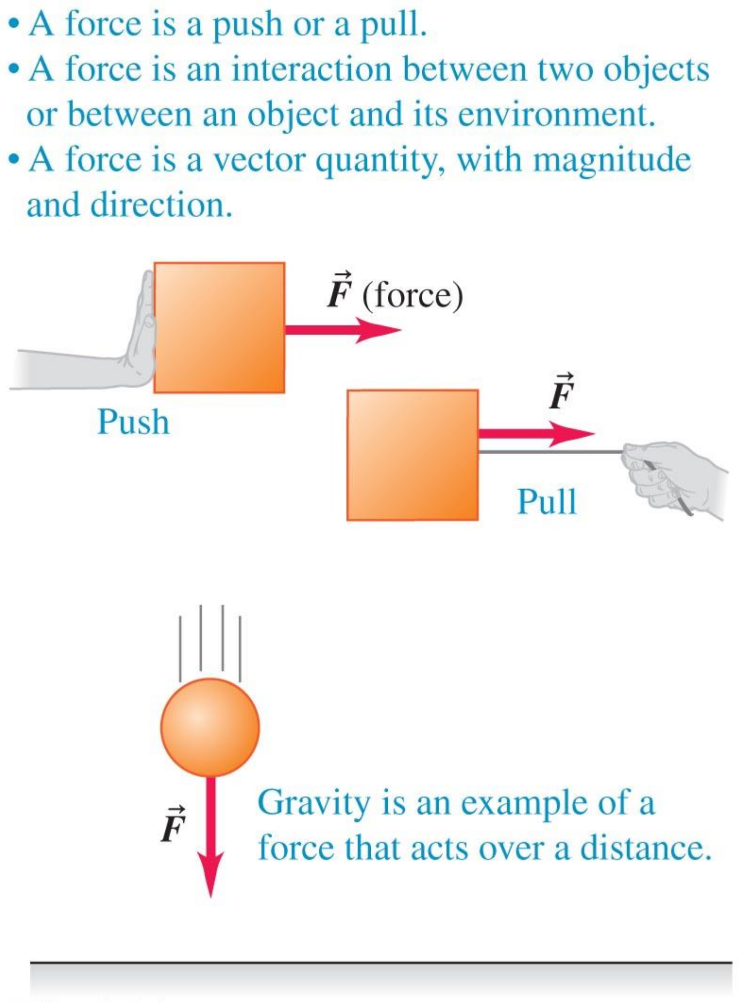


Astronauts on the moon



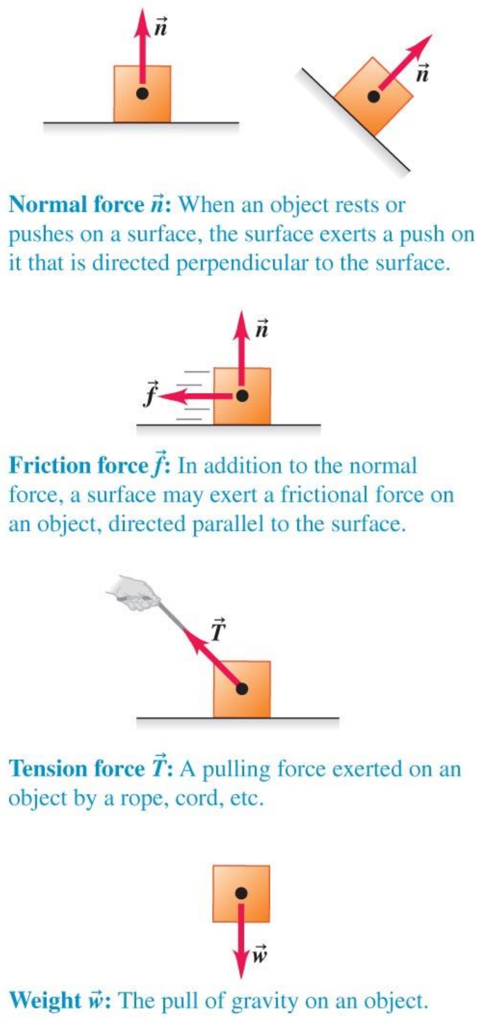
Pearson

Types of Force Illustrated – Figure 4.1



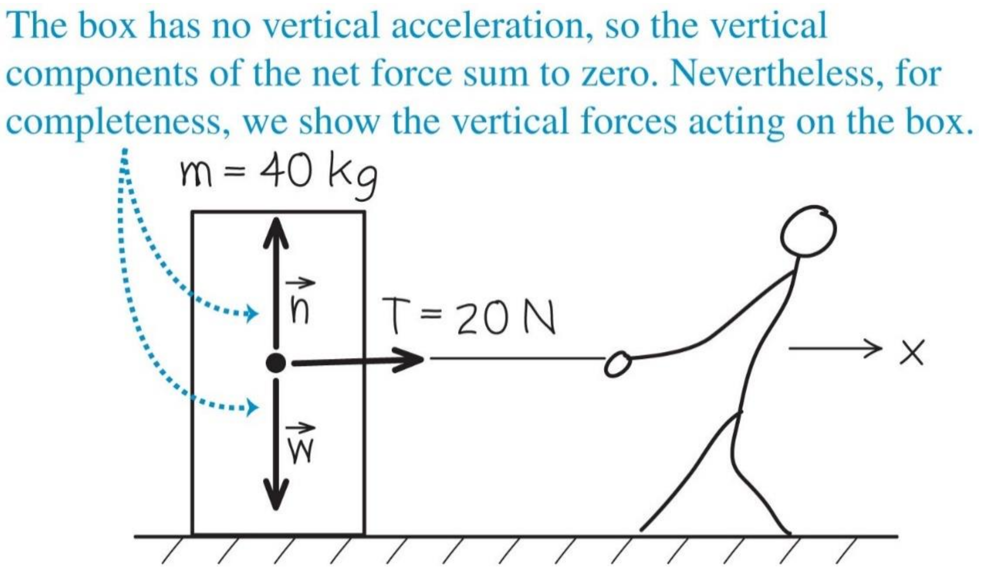
Types of Force – Figure 4.2

- Single or net
  - Contact force
  - Normal force
  - Frictional force
  - Tension
  - Weight



Forces and Free Body Diagrams – Example 4.1

- Observe the worked example on page 103.
- The vertical forces are in **equilibrium** so there is no vertical motion.
- But there is a net force along the horizontal direction, and thus acceleration.



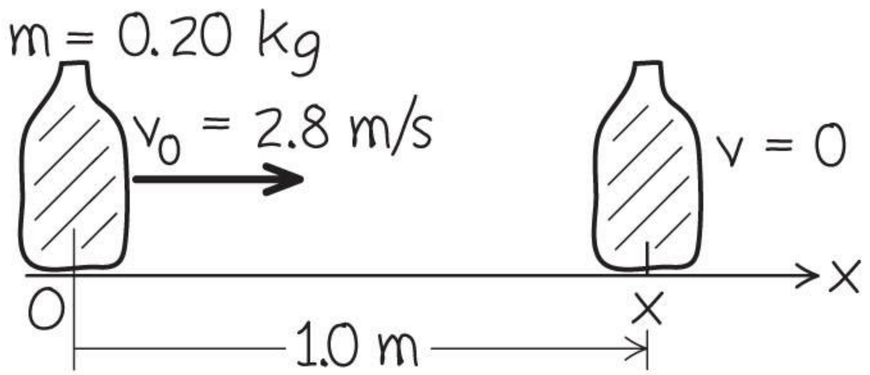
$$\sum F_y = n_y + W_y = 0 \Rightarrow a_y = 0$$

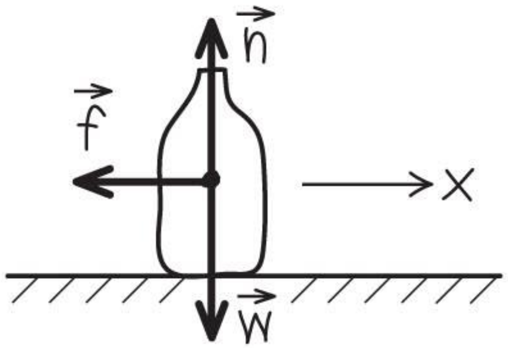
$$\sum F_x = T_x \Rightarrow a_x = \frac{T_x}{m} \neq 0$$

Forces and Free Body Diagrams – Example 4.2 ketchup-slide

- Like the previous example, we account for the forces and draw a free body diagram.
- Again, in this case, the net horizontal force is unbalanced.
- In this case, the net horizontal force opposes the motion and the bottle slows down (decelerates) until it stops.

We draw one diagram for the bottle's motion and one showing the forces on the bottle.





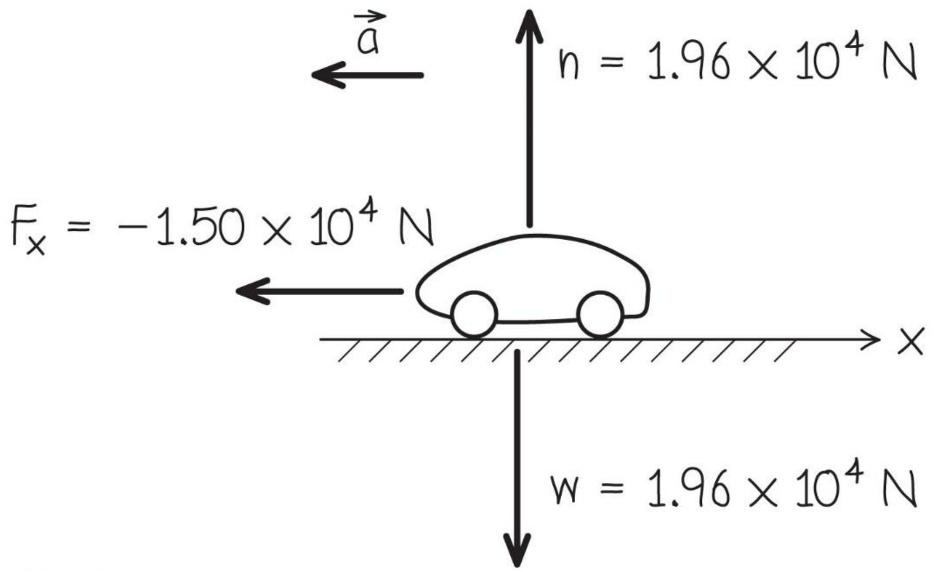
$$v_x^2 - v_{0x}^2 = 2a_x(x - x_0)$$
$$0 - \left(2.8\frac{m}{s}\right)^2 = 2a_x(1.0\,m\ - 0)$$
$$a_x = -3.9\frac{m}{s^2}$$

$$\Sigma F_x = -f = ma_x$$
$$-\vec{f} = (0.20\,kg)\left(-3.9\frac{m}{s^2}\right)$$
$$= -0.78\frac{m}{s^2} = -0.78\,N$$

Newton
1 N = (1 kg) (1 m/s^2)

We Can Solve for Dynamic Information – Example 4.4

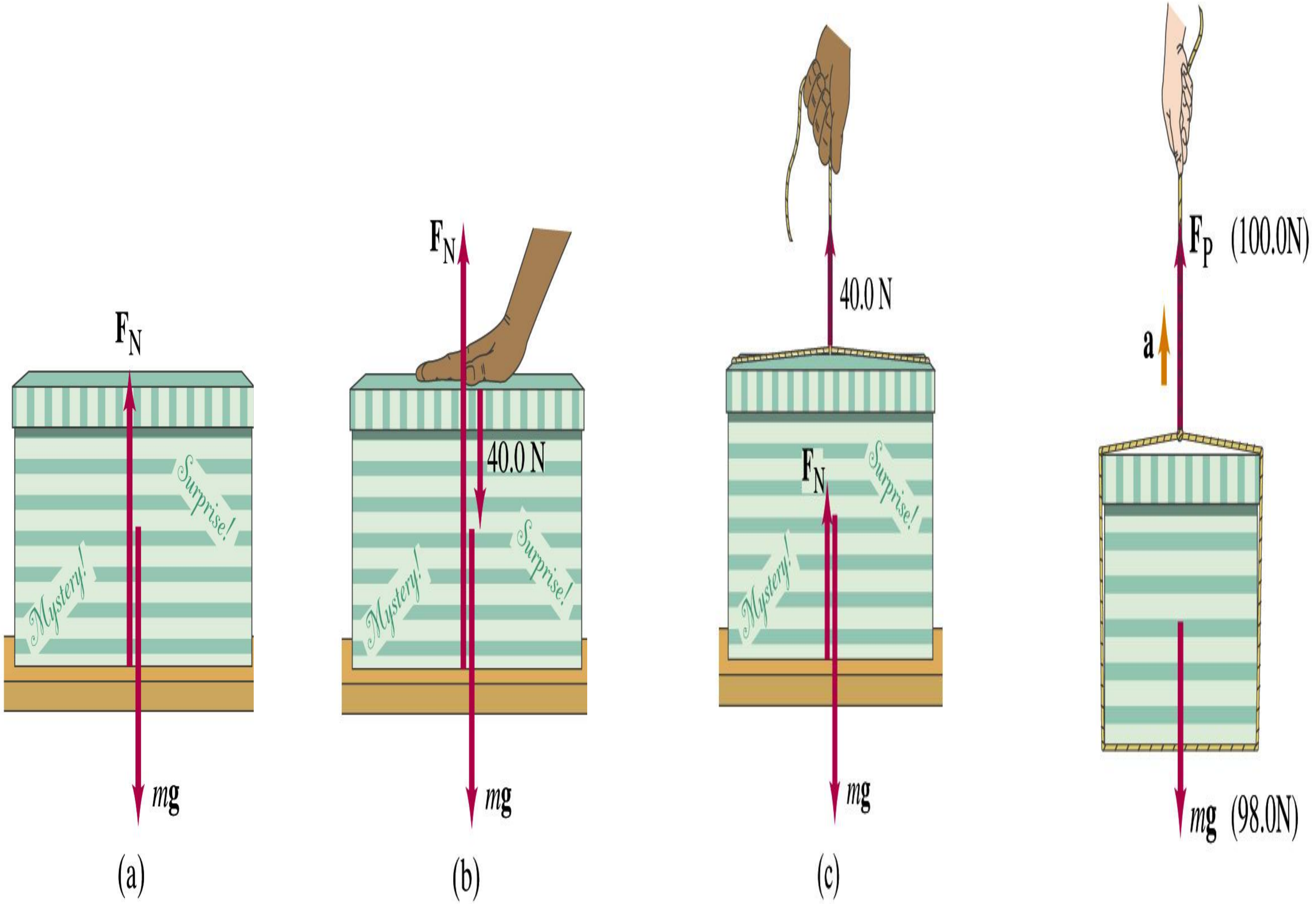
- Knowing force and mass, we can sketch a free body diagram and label it with our information.
- We can solve for acceleration.



$$\text{Along vertical } \hat{y}: \; n_y + W_y = 0 \quad \Rightarrow \quad \text{m} = \frac{w}{g}$$

$$\text{Along vertical } \hat{x}: \; F_x = ma_x \quad \Rightarrow \quad a_x = \frac{F_x}{m} = \frac{F_x}{w} g$$

Weight, normal force and a box



Normal force = F_N and mystery box

$\sum F = mg$ and (mass) $m = 10\ kg$

a) $\sum F = F_N - mg = ma$ and $a = 0$

$F_N - mg = 0$ So, $F_N = mg$ (normal force)

b) $F_N - mg - 40N = 0;\ F_N = 98N + 40N = 138N$

c) $F_N - mg + 40N = 0;\ F_N = 98N - 40N = 58N$

d) $F_y = F_P - mg = 100N - 98N = 2N = ma_y$

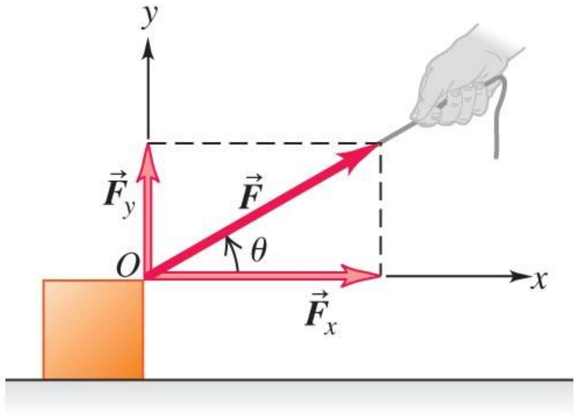
$$a_y = \frac{F_y}{m} = \frac{2N}{10kg} = 0.2\,\frac{m}{s^2}$$

A Force May Be Resolved Into Components

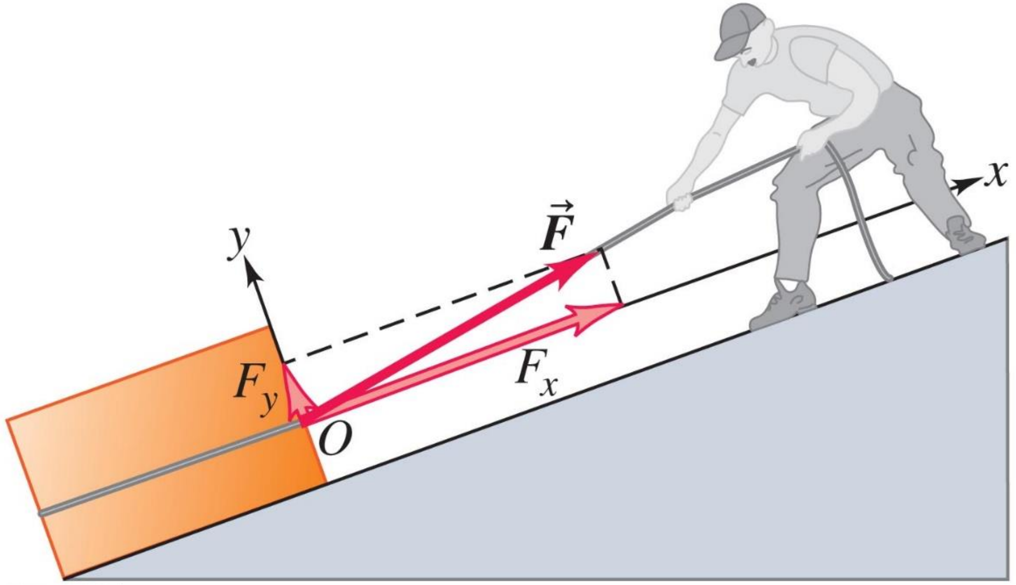
$$F_x = F\cos\theta$$

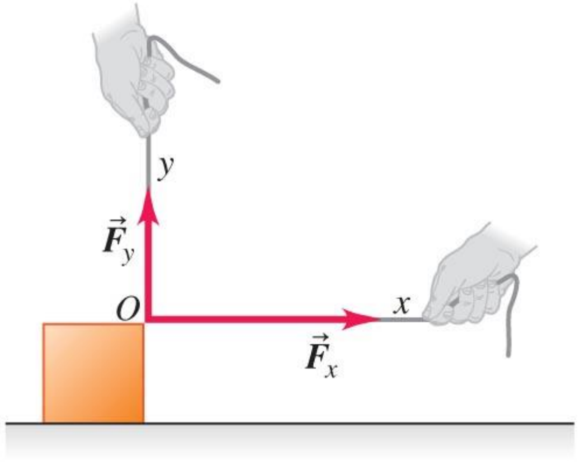
$$F_y = F\sin\theta$$

- The *x*- and *y*-coordinate axes don't have to be vertical and horizontal.



(a) Component vectors: $\vec{F}_x$ and $\vec{F}_y$
Components: $F_x = F\cos\theta$ and $F_y = F\sin\theta$



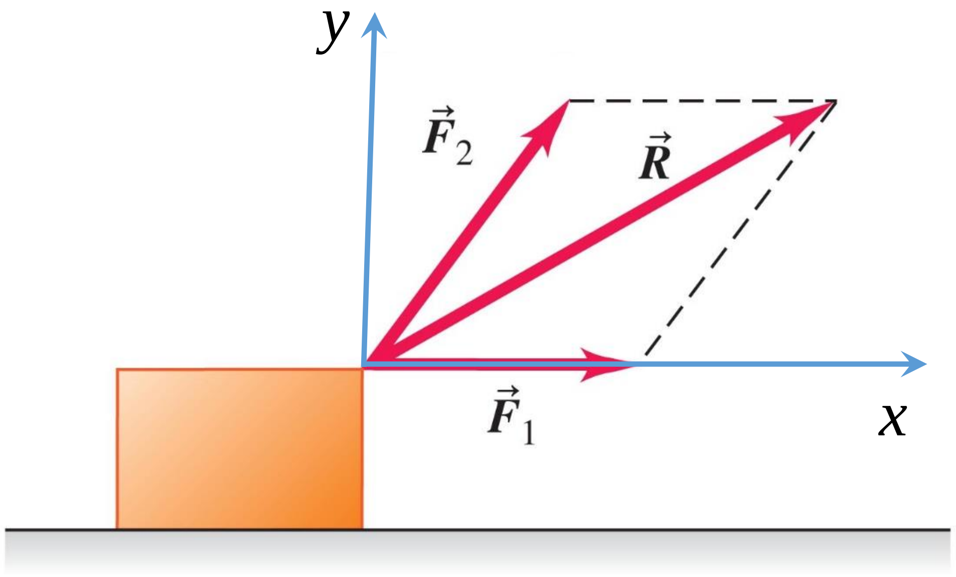


(b) Component vectors $\vec{F}_x$ and $\vec{F}_y$ together have the same effect as original force $\vec{F}$.

Superposition of Forces: Resultant and Components of Force Vectors

- An example of superposition of forces:

$$\vec{R} = \vec{F}_1 + \vec{F}_2$$

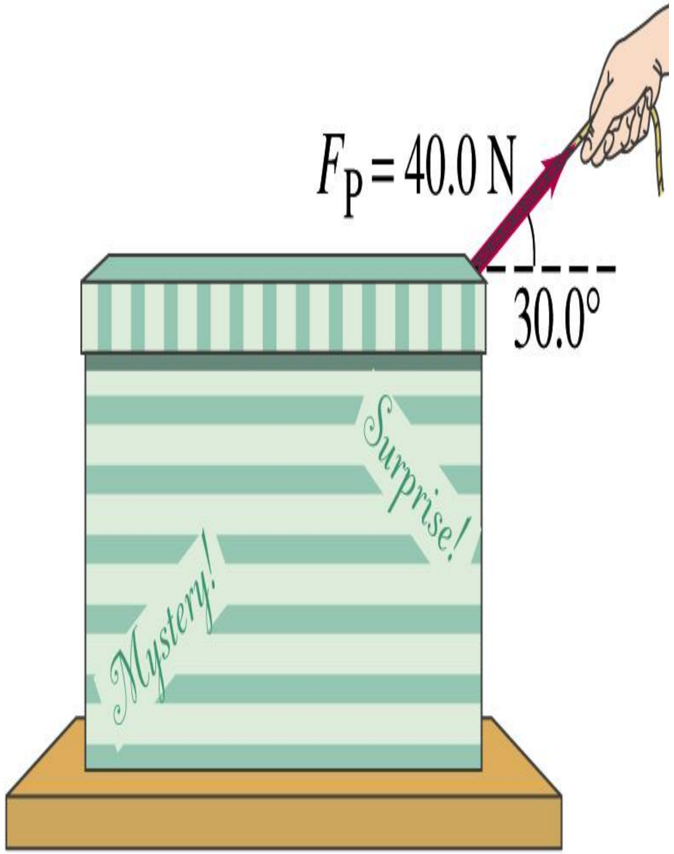


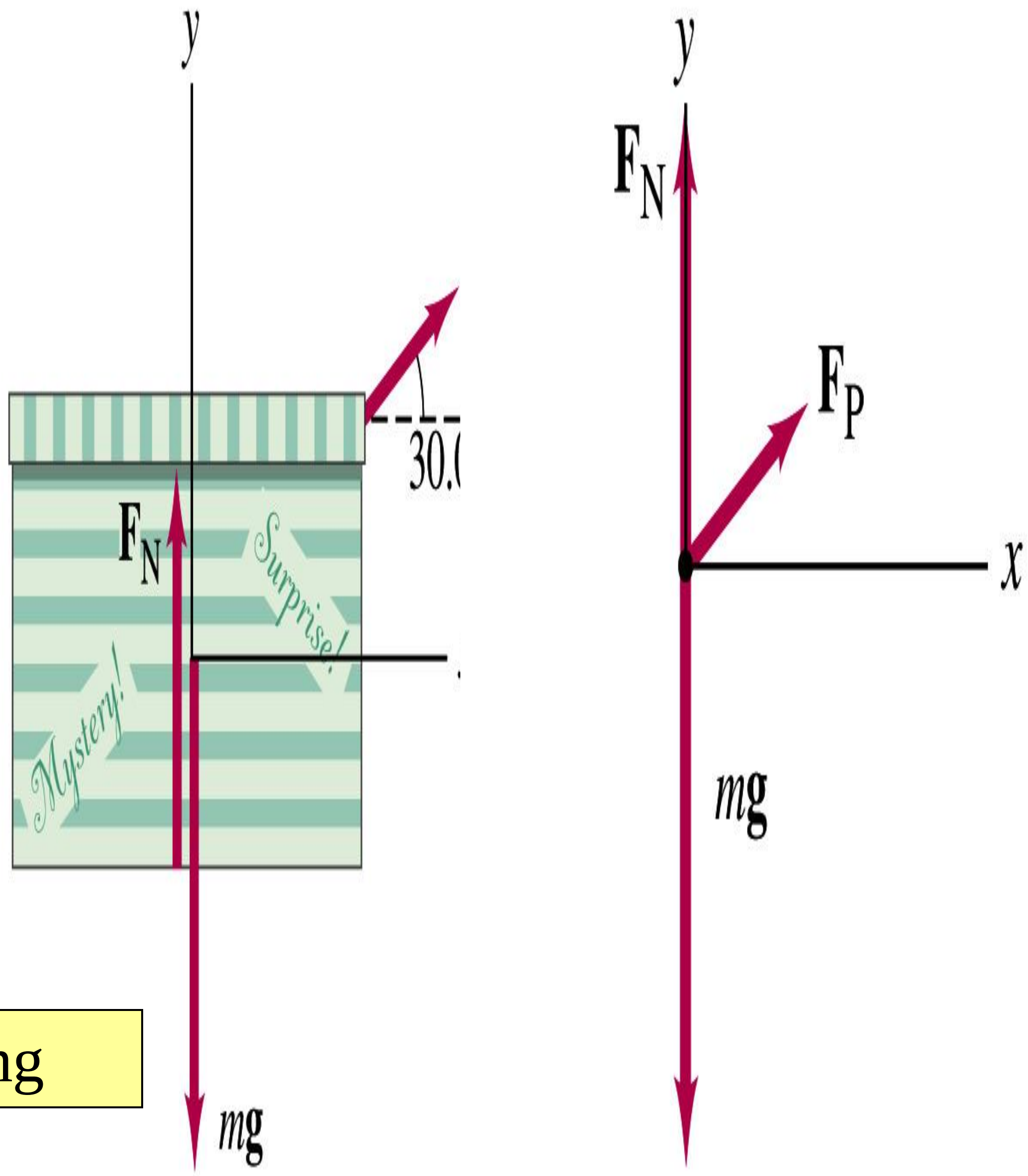
- In general, the resultant, or vector sum of forces, is:

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \cdots = \sum \vec{F}$$

$$\text{where} \begin{cases} R_x = \sum F_x \\ R_y = \sum F_y \end{cases} \Rightarrow \begin{array}{ll} \vec{R} = \vec{R}_x + \vec{R}_y & \text{vector} \\ R = \sqrt{R_x^2 + R_y^2} & \text{magnitude} \\ \theta = tan^{-1}\frac{R_y}{R_x} & \text{direction} \end{array}$$

Pulling a box





Note: F_N is less than mg

Pulling the box

$\sum F = ma$ and $m = 10\ kg$

$$F_{px} = 40 * cos30 = 34.6N$$
$$F_{py} = 40 * sin30 = 20.0N$$

Horizontal

$$F_{px} = ma_x \rightarrow a_x = \frac{F_{px}}{m} = \frac{34.6}{10} = 3.46\frac{m}{s^2}$$

box accelerates horizontally

Vertical

$$\sum F_y = ma_y$$

$\rightarrow a_y = 0$ box does not accelerate vertically

and $F_N - mg + F_{py} = ma_y$

$$F_N - 98N + 20N = 0 = 78N$$

4.1 Force: a vector quantity describing the interaction between two objects

Type of Forces (that we will deal with in this course)

Contact forces

- Normal Force $\vec{n}$

  normal to the contacting surface

- Friction Force $\vec{f}_k$

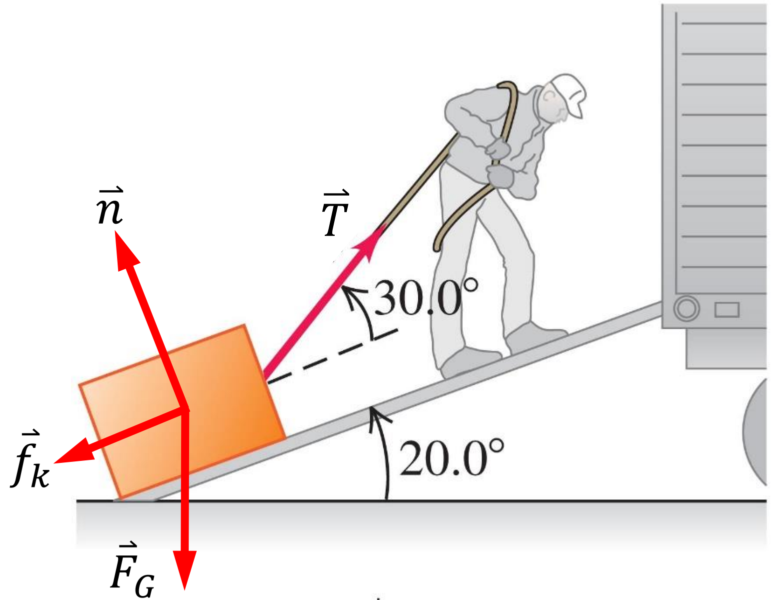
  tangent to the contacting surface

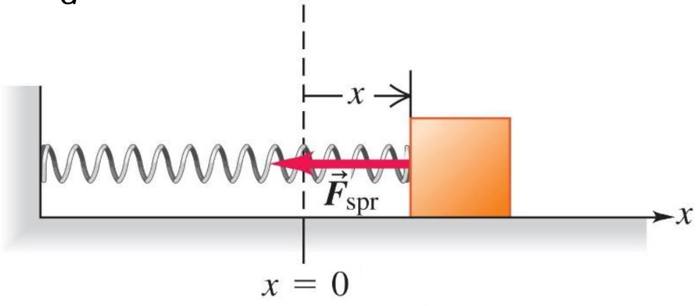
- Tension Force $\vec{T}$

  (in a rope) along the rope

- Spring Force $\vec{F}_{spr}$

  opposite to the deformation





Action-at-a-Distance Forces (non-contacting)

- Gravitational Force $\vec{F}_G$

  (along the line connecting the two centers of mass)

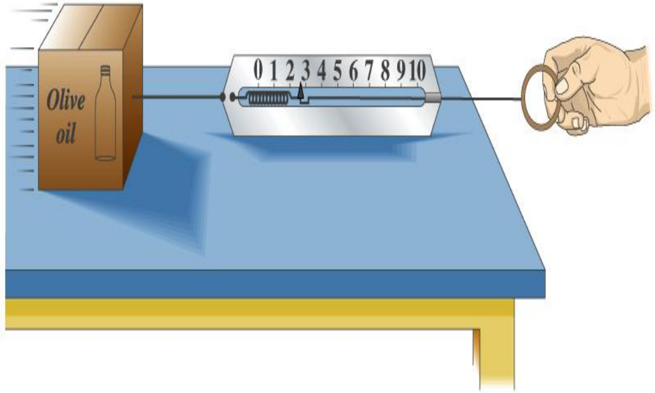
Courtesy of Wenhao Wu

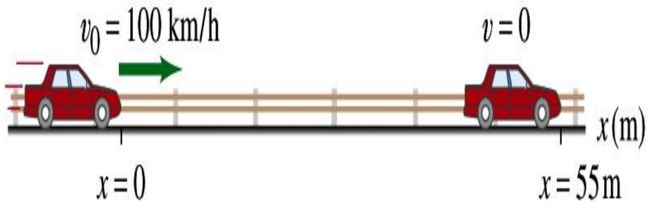
Dynamics, Newton’s Laws

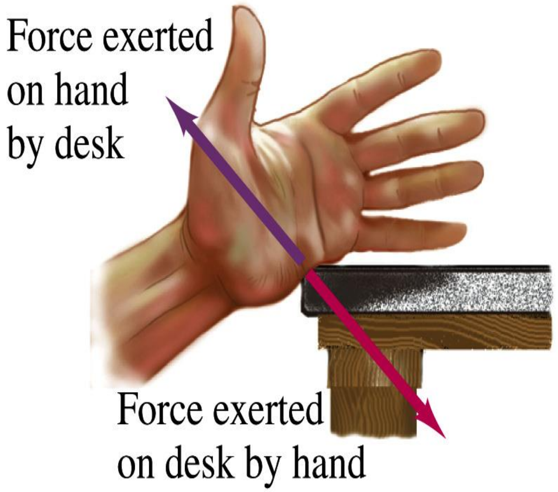
First Law: Every body stays in its state of motion(rest or uniform speed) unless acted on by a non zero net force.

Second Law: The acceleration is directly proportional to the net force and inversely proportional to the mass: $a = \dfrac{\sum F}{m}$

Third Law: Action is equal to reaction.







Newton’s laws and concept of mass

Famous equation:

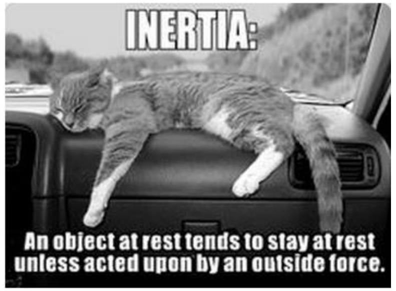
$E = mc^2$ (Einstein) $\qquad F = m\,a$ (Newton)

$$[N] = [kg]\left[\frac{m}{s^2}\right]$$

1N=1kg*1m/s^2 (mkgs-systems)

Force=1dyn (cmgs-systems)

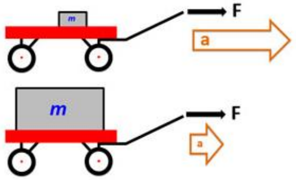
$$1N = \frac{1kg * 10^3 g * 1m * 10^2 cm}{1kg * s^2 * 1m} = 10^5 g\frac{cm}{s^2} = 10^5 dyn$$



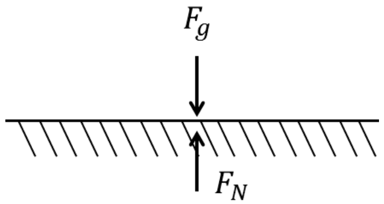
<u>Newton’s Laws</u>

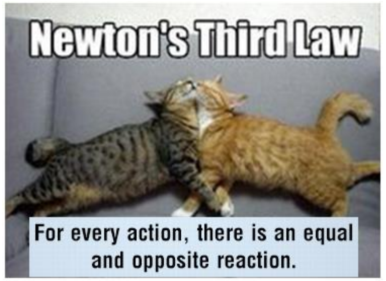
1st law: Without a force the state of motion is the same

2nd law: $F \propto a$, and $a = \frac{\sum F}{m}$



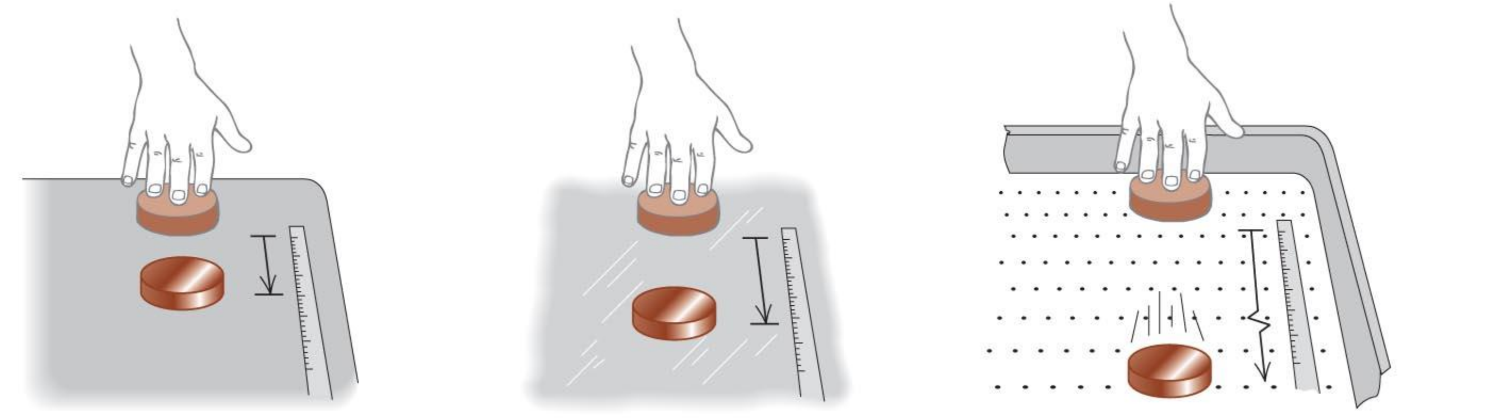
3rd law: Action = Reaction





Newton's First Law – Figure 4.7

- "Every object continues either at rest or at a constant speed in a straight line.…"
- What this common statement of the first law often leaves out is the final phrase "…unless it is forced to change its motion by forces acting on it."
- In one word, we say "inertia."



(a) Table: puck stops short (b) Ice: puck slides farther (c) Air-hockey table: puck slides even farther

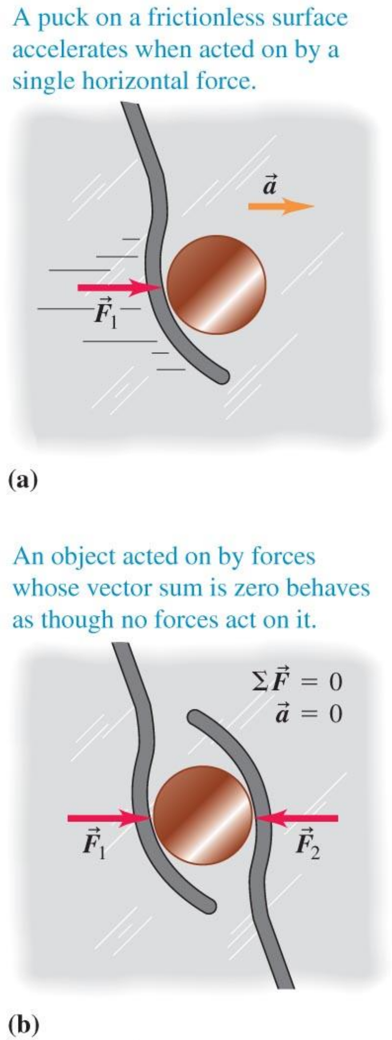
We Determine Effect with the Net Force – Figure 4.8

- The top puck responds to a non-zero net force (resultant force) and accelerates.
- The bottom puck responds to two forces whose vector sum is zero:

$$R = F1 + F2 = \sum F = 0$$

$$\text{Where} \quad \begin{cases} Rx = \sum F_x = 0 \\ Ry = \sum F_y = 0 \end{cases}$$

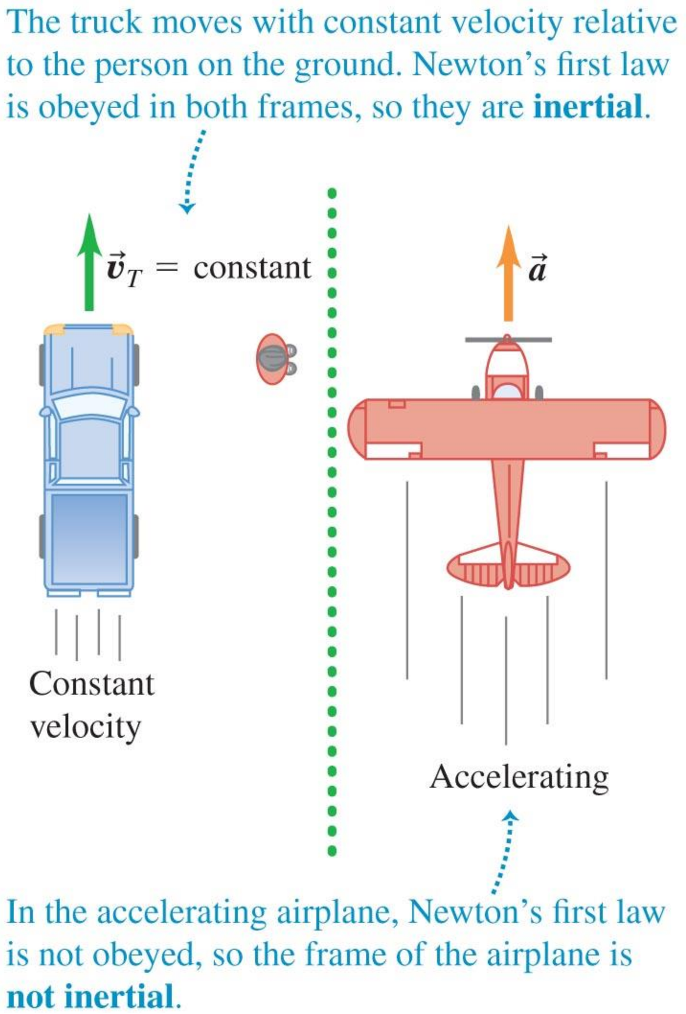
- The bottom puck is in **equilibrium,** and does NOT accelerate.



Pearson

Forces are Inertial and Non-inertial – Figure 4.9

- The label depends on the position of the object and its observer.
- A frame of reference in which Newton's first law is valid is called an **inertial frame of reference (no acceleration**).



4.2 Newton's First Law (Law of Inertia)

An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.

If $\sum \vec{F} = 0$ or $\sum F_x = 0$, $\sum F_y = 0$

then $\vec{a} = 0$ or $a_x = 0$, $a_y = 0$; $v_x = constant$, $v_y = constant.$

Inertia: a tendency to maintain the state of motion.

Mass: a quantitative measure of inertia.

Equilibrium

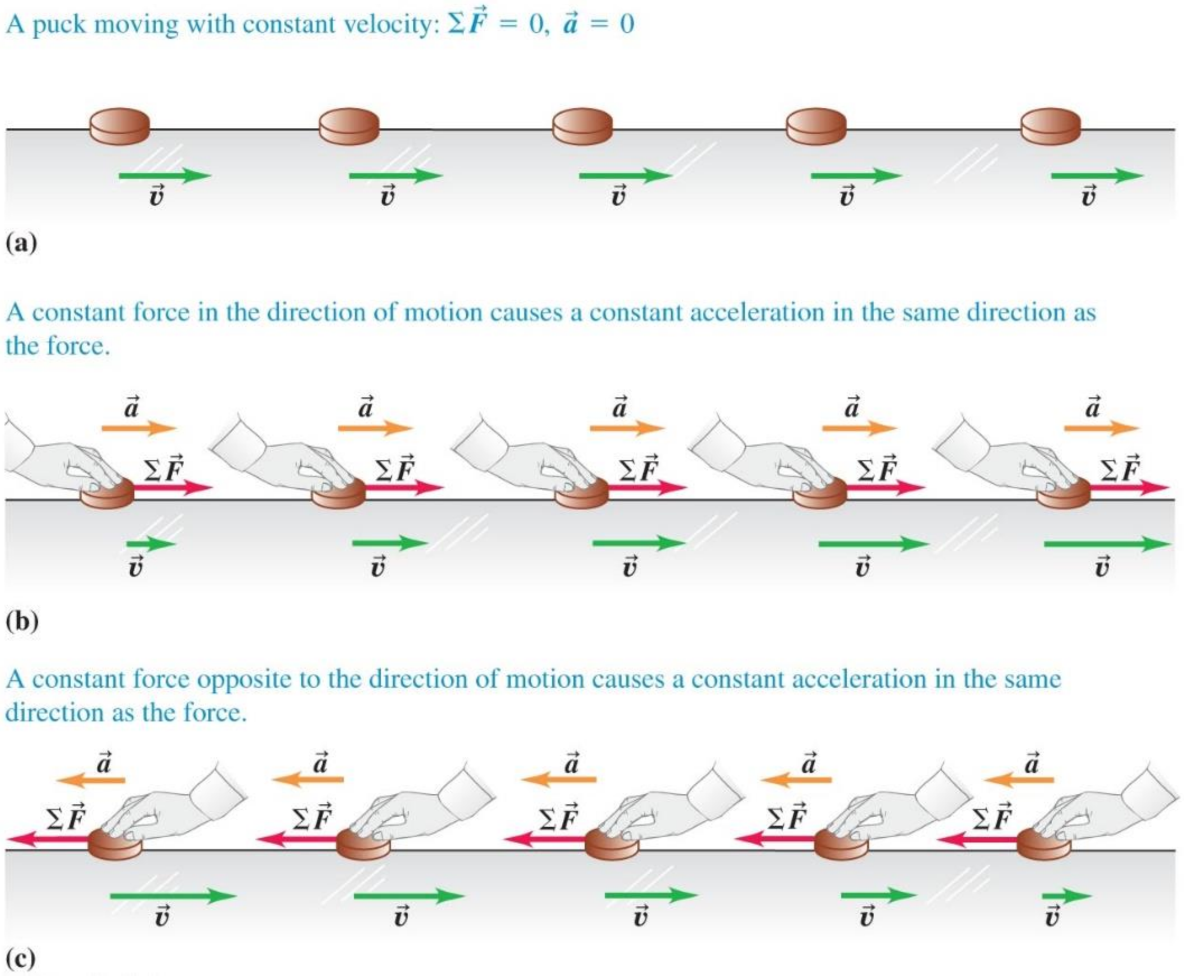
In mechanics, equilibrium means

$\vec{a} = 0$ or $a_x = 0$, $a_y = 0$; $v_x = constant$, $v_y = constant.$

Courtesy of Wenhao Wu

Mass and Newton's Second Law I –Figure 4.11

- $\vec{F} = m\vec{a}$
- Object's acceleration is in same direction as the net force acting on it.
- We can examine the effects of changes to each component.



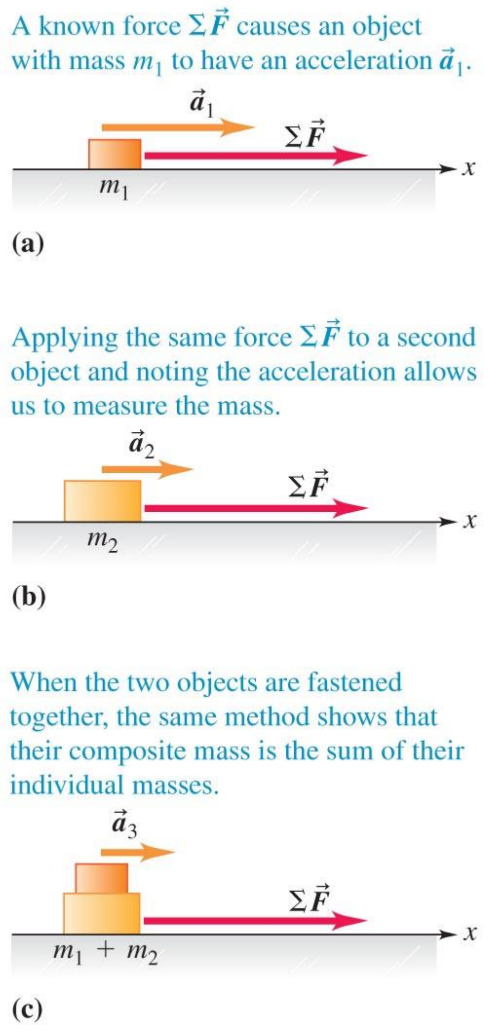
Mass and Newton's Second Law II – Figure 4.12

- Let's examine some situations with more than one mass.
- In each case we have Newton's second law of motion:

$$\sum \vec{F} = m\vec{a}$$

- Force in N, mass in kg, and acceleration in m/s^2.

$$1\,N = (1\,kg)(1\,m/s^2)$$



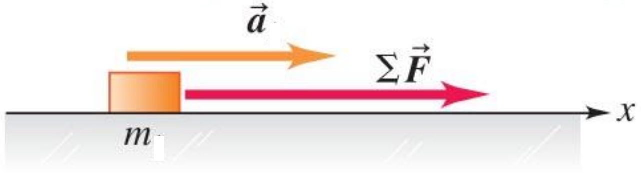
Pearson

4.3 Newton's Second Law

The vector sum of all the forces acting on an object equals the object's mass times its acceleration:

$$\sum \vec{F} = m\vec{a}$$

A known force $\Sigma \vec{F}$ causes an object with mass m to have an acceleration $\vec{a}$.



- The net force $\sum \vec{F}$ and the acceleration $\vec{a}$ have the same direction.
- The magnitude of the acceleration is proportional to the magnitude of the net force.
- One may also express Newton's Second Law in this format:

$$\vec{a} = \frac{1}{m}\sum \vec{F}.$$

Therefore, in terms of magnitude:

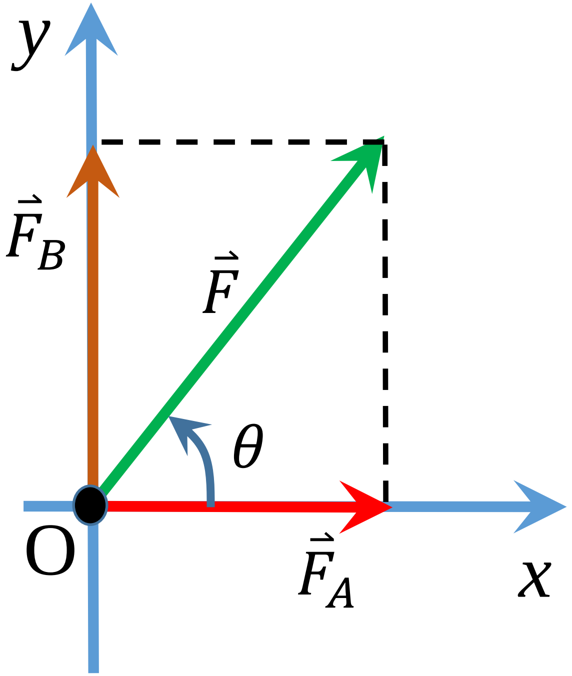
- The magnitude of the acceleration is proportional to the magnitude of the net force.
- The magnitude of the acceleration is inversely proportional to the mass.
- Units: Force is measured in Newton or N: 1 N = (1 kg)(1 m/s^2)

Courtesy of Wenhao Wu

The Component Form: $\sum \vec{F} = m\vec{a} \quad \rightarrow \quad \sum F_x = ma_x$ and $\sum F_y = ma_y$

Example: An object of mass 5.0 kg is acted upon by two forces, F_A and F_B. F_A is directed toward east and has a magnitude 3.0 N. F_B is directed toward north and has a magnitude 4.0 N.

(a) Draw diagram that describes this situation.
(b) Set up a coordinate system.
(c) Calculate the components, the magnitude, and the direction of the net force.
(d) Calculate the components and the magnitude of the acceleration.



(a) Sketch a diagram that describes this situation.
(b) Set up a coordinate system.
(c) $F_x = F_{Ax} = 3.0$ N $\qquad F_y = F_{By} = 4.0$ N

$F = (3.0^2 + 4.0^2)^{1/2} = 5.0$ N $\qquad \theta = tan^{-1}\left(\frac{4.0}{3.0}\right) = 53°$

(d) $a_x = \frac{F_x}{m} = \frac{3.0}{5.0} = 0.60$ m/s^2 $\qquad a_y = \frac{F_y}{m} = \frac{4.0}{5.0} = 0.80$ m/s^2

$a = (0.60^2 + 0.80^2)^{1/2} = 1.0$ m/s^2 or $a = F/m = 1.0$ m/s^2

Newton's Third Law   Action – Reaction Pair

- "For every action, there is an equal and opposite reaction."
- Rifle recoil is a wonderful example.

When shooting, press it hard against your shoulder !

An action–reaction pair: $\vec{F}_{\text{ball on foot}} = -\vec{F}_{\text{foot on ball}}$.
The two forces represent a *mutual interaction* of two objects, and each acts on a *different* object.

Clicker question

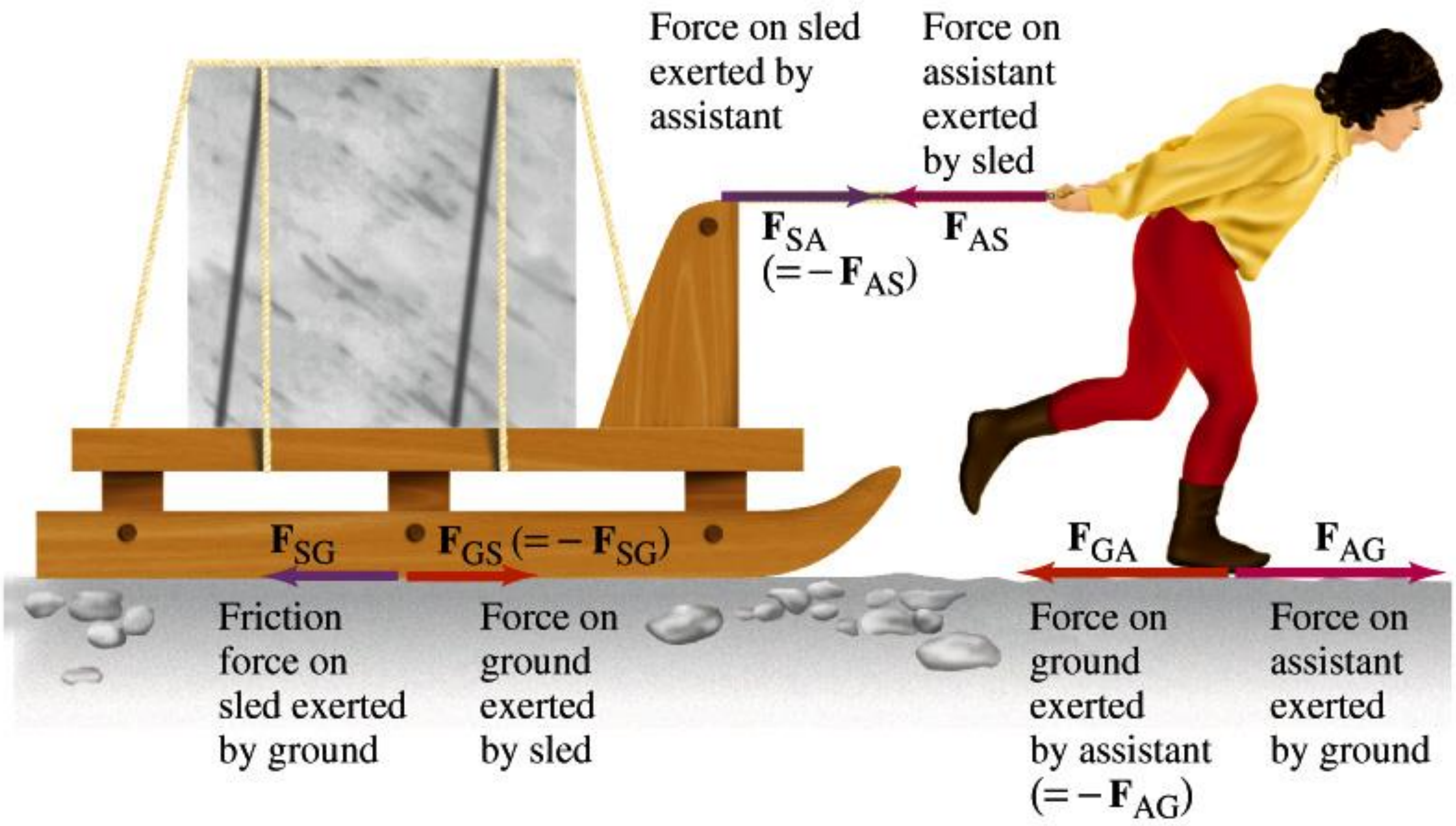
You use a cord and pulley to raise boxes up to a loft, moving each box at a constant speed. You raise the first box slowly. If you raise the second box more quickly, what is true about the force exerted by the cord on the box while the box is moving upward?

a) The cord exerts more force on the faster-moving box.
b) The cord exerts the same force on both boxes.
c) The cord exerts less force on the faster-moving box.
d) The cord exerts less force on the slower-moving box.

b) The cord exerts the same force on both boxes

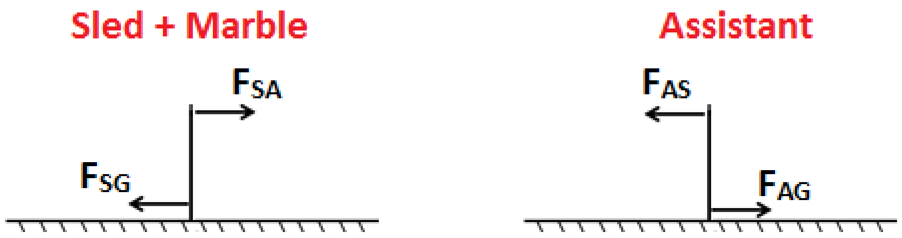
pulling a sled, Michelangelo's assistant

The two forces in an action-reaction pair always act on different objects



For forward motion: $F_{AG} > F_{AS}$ $F_{SA} > F_{SG}$

$\boxed{\text{weight} = mg}$



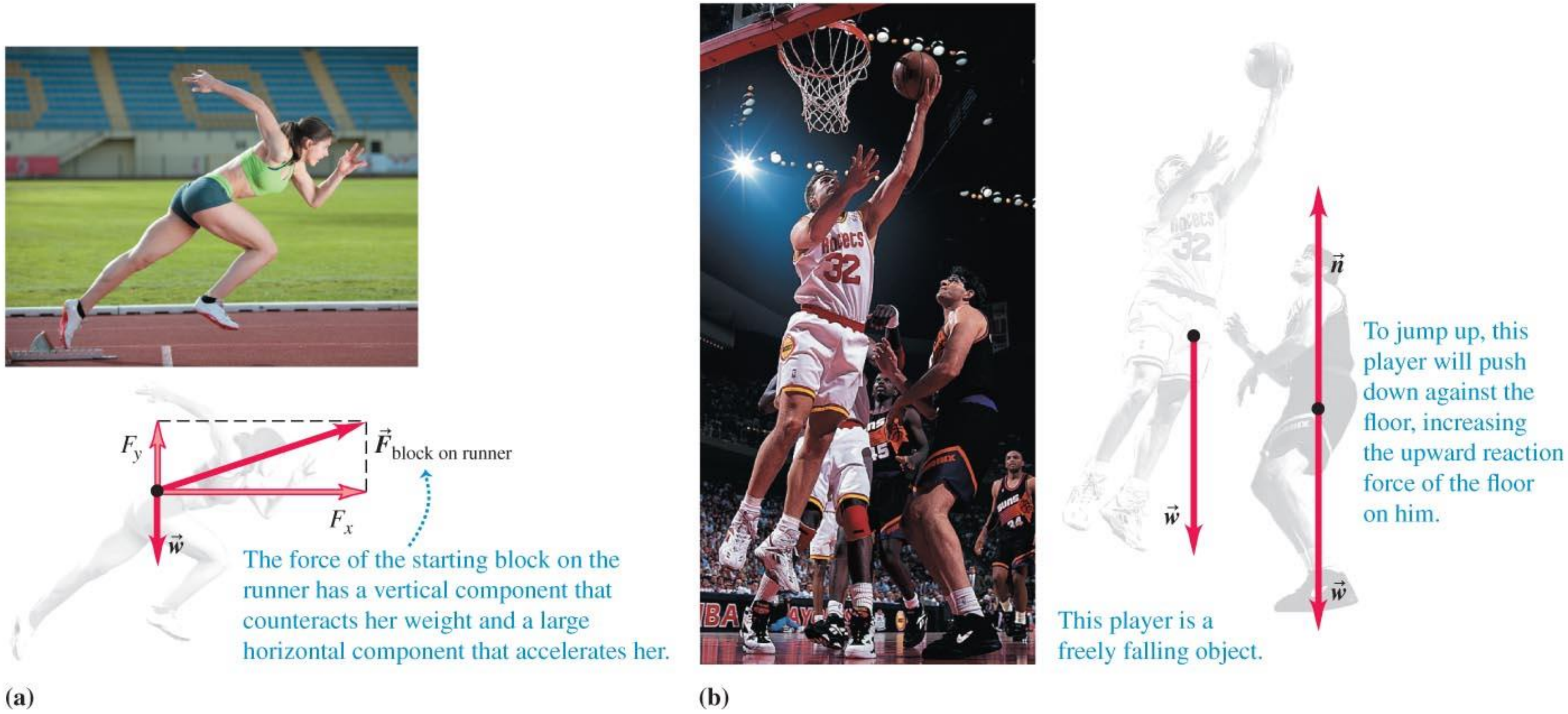
When drawing force diagrams consider only forces acting on the same object

For forward motion:

$$F_{SA} > F_{SG}$$
$$F_{AG} > F_{AS}$$

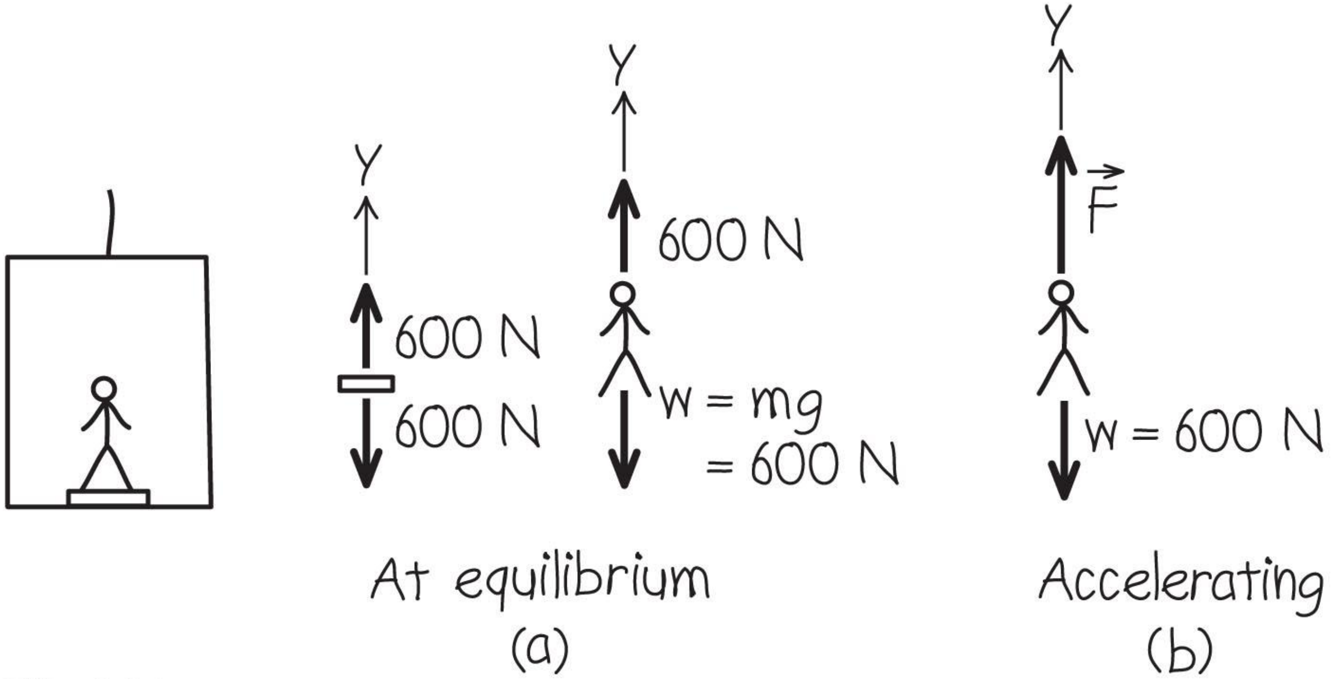
Use Free Body Diagrams In Any Situation – Figure 4.24

- Find the object of the focus of your study, and collect all forces acting upon it.

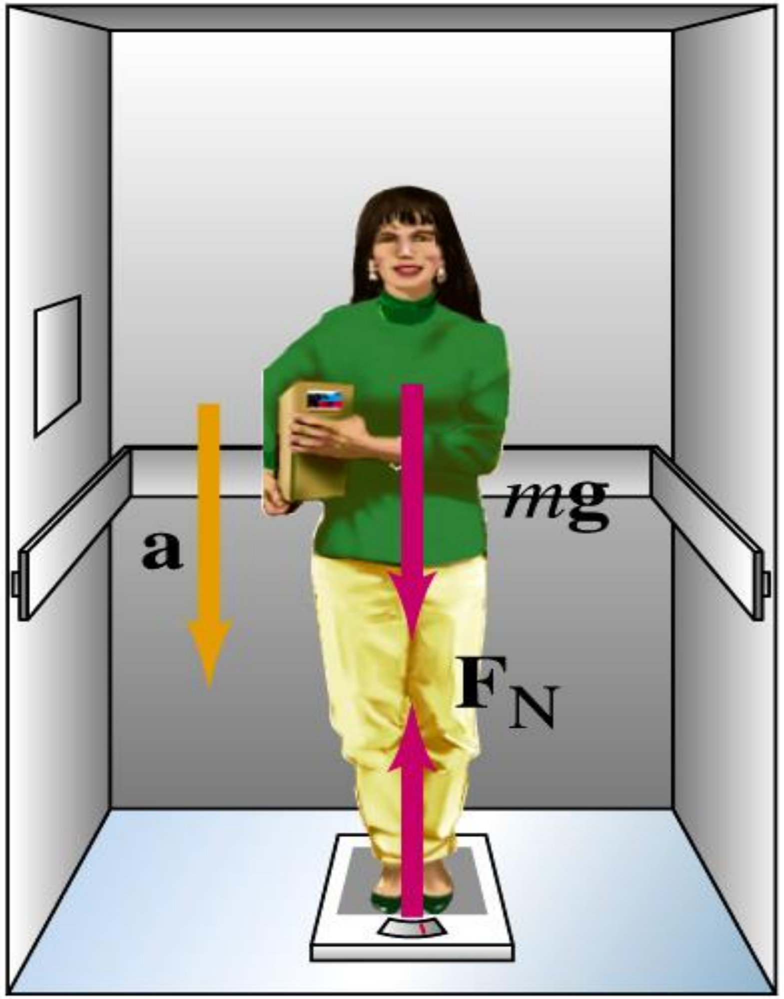


Forces Transmit Themselves as Tension – Example 4.9

- We can solve for several outcomes using the elevator as our example.
- Follow the worked problem on page 114.



Unwanted weight loss(descending)



$$\sum F = ma$$

Weight loss in a descending elevator

$\sum F = ma$ and $m = 65\ kg$ with $a = 0.2 * g$ (elevator)

F_N=normal force indicated by scale

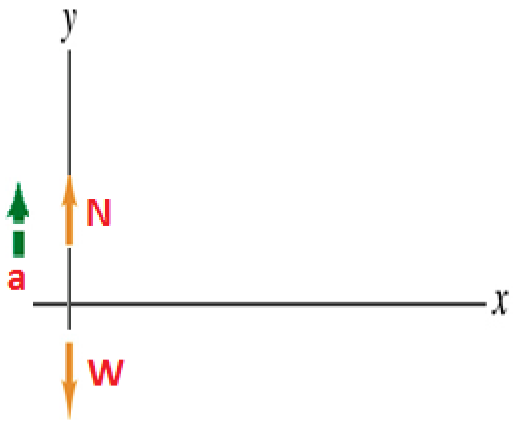
$F_N - mg = m(-a) \rightarrow F_N = mg - 0.2 * g * m = m0.8g = 509.6N$ (apparent weight)

$F_N = mg = 65\ kg * 9.8 \frac{m}{s^2} = 637\ N$ (real weight)

$m = \frac{0.8*637}{9.8} = 52\ kg$ (apparent mass)

Stepping on a scale in an elevator and push "up". Your normal weight is 625N.

Your weight going up



a) Work out your mass

$$m = \frac{W}{g} = 63.8\ kg$$

Elevator goes up

$$a = +2.5\frac{m}{s^2}$$

What does the scale read?

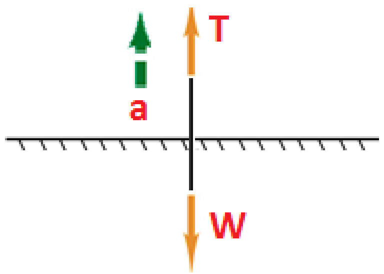
$$\sum F_y = ma_y = N - W = ma$$

Scale reading;

$$\mathrm{N} = ma + W = 63.8 * 2.5 + 63.8 * 9.8 = 784N$$

Apparent mass $= \frac{784}{9.8} = 80\ kg$

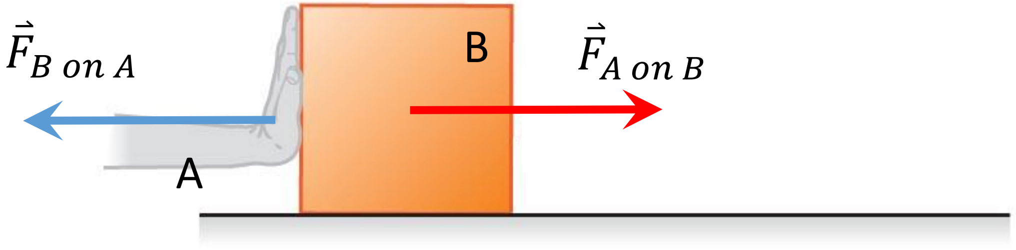
Tension in the string



b) What is the tension in a string holding a 3.85kg package?

$$\sum F_y = ma_y = T - W$$

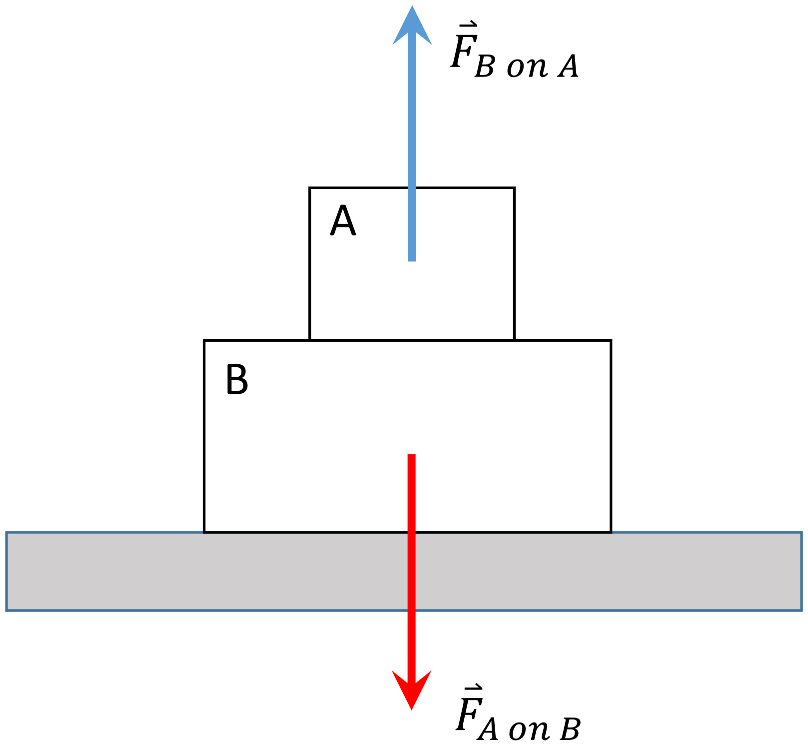
$$\mathrm{T} = ma + W = 3.85 * 2.5 + 3.85 * 9.8 = 47.4N$$



4.5 Newton's Third Law

"For every action, there is an equal and opposite reaction."

$$\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$$



Newton's Second Law:

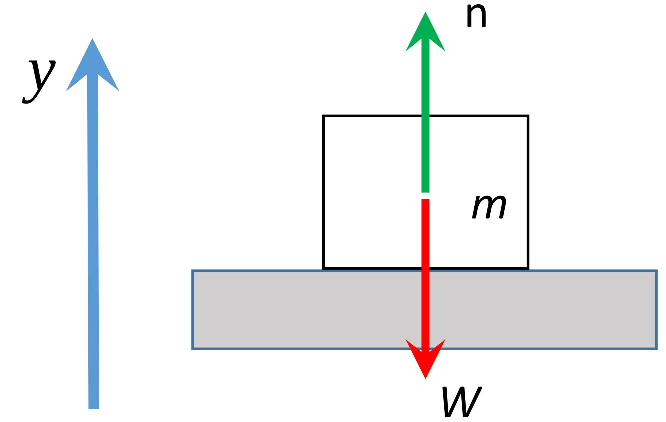
in vector form $\sum \vec{F} = m\vec{a}$

in component form $\sum F_x = ma_x$ $\sum F_y = ma_y$

Strategy for Solving Newton's Law Problems

- Isolate the bodies in a system.
- Analyze all the forces acting on each body and draw one free-body diagram for each body.
- Based on the free-body diagram, set up a most convenient *x*-*y* coordinate system.
- Break each force into components with these coordinates.
- For each body, sum up all the *x*-components of the forces to an equation: $\sum F_x = ma_x$.
- For each body, sum up all the *y*-components of the forces to an equation: $\sum F_y = ma_y$.
- Use these equations to solve for unknown quantities.

Example 1: A box of known mass m is resting on a level table surface. Find all the forces acting on this box and their action-reaction counterparts.



Two forces act on the box:

weight (known) $W = -mg$

normal force n

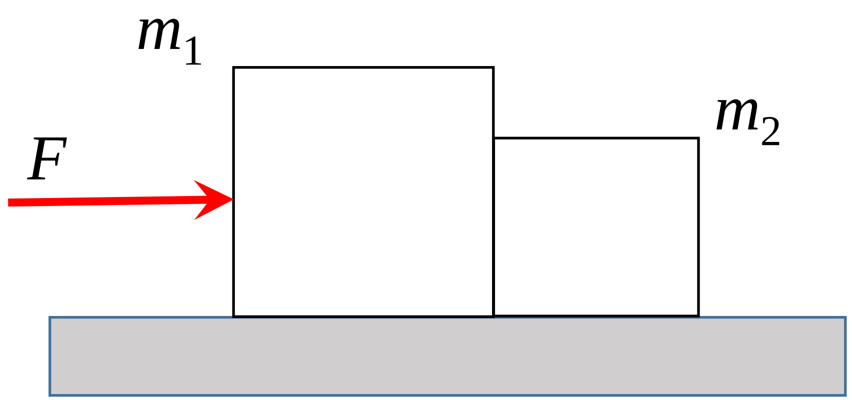
Sum up the $y-$component forces:

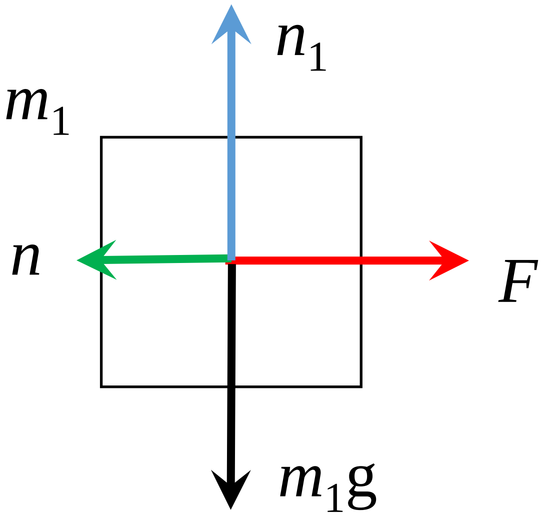
$$n + (-mg) = 0.$$

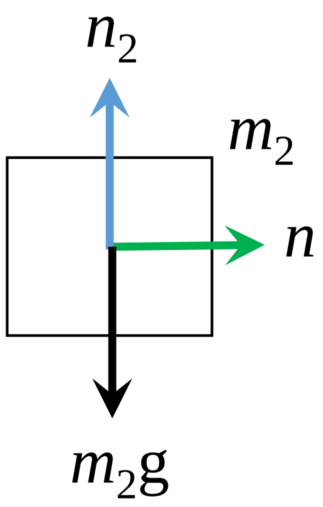
Therefore, $n = mg$.

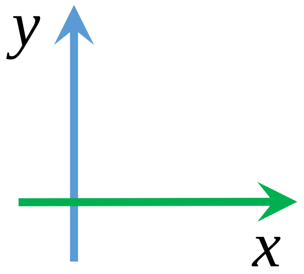
Courtesy of Wenhao Wu

Example: Two boxes of masses m_1 and m_2 are pushed together by a horizontal force F to accelerate to the right on a level frictionless table surface as shown. Calculate the acceleration and the normal force between the boxes.









x-axis: $F - n = m_1a$

y-axis: $n_1 - m_1g = 0$

$n_1 = m_1g$

x-axis: $n = m_2a$

y-axis: $n_2 - m_2g = 0$

$n_2 = m_2g$

$F - m_2a = m_1a$

$F = m_1a + m_2a = (m_1 + m_2)a$

$a = F/(m_1 + m_2)$

$n = m_2a = [m_2/(m_1 + m_2)]F$

Since $m_2/(m_1 + m_2) < 1 \longrightarrow n < F$

Courtesy of Wenhao Wu

Example: Box of mass m on a frictionless incline

Given: m and θ

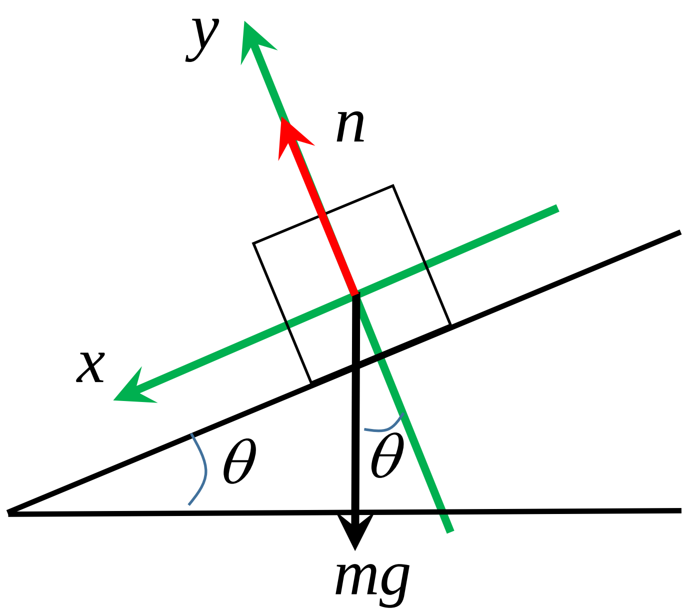
Find: n and a

Solutions:

Weight: $W = mg$ $\quad W_x = mg\sin\theta \quad W_y = -\,mg\cos\theta$

x-axis: $mg\sin\theta = ma \qquad a = g\sin\theta$

y-axis: $n - mg\cos\theta = 0 \qquad n = mg\cos\theta$



Another Question: Near the bottom of the incline, the box is given an initial velocity of a known magnitude v_0 pointing up the incline. What distance will it slide before it turns around and slides downward?

Solution: The acceleration is given above $a = g\sin\theta$

Use the kinematic equation $v_2^2 - v_1^2 = 2a(x_2 - x_1)$

$$(x_2 - x_1) = -v_1^2/2a = -v_0^2/2g\sin\theta$$

The distance that it will slide up the incline is $v_0^2/2g\sin\theta$.

Courtesy of Wenhao Wu

A gymnast climbs a rope

In which case does the rope break first? Need to see when rope tension is largest.

Consider the following cases;

a) He climbs with a constant rate

$$\sum F = ma \text{ and } a = 0 \text{ So, } T = mg$$

b) He just hangs on the rope

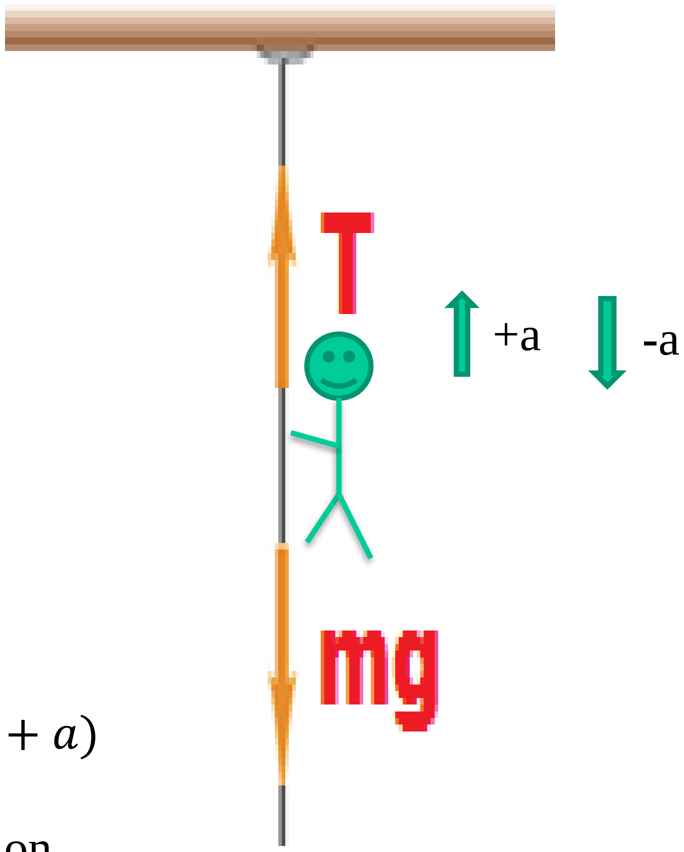
$$\sum F = T - mg = 0; \text{ So, } T = mg$$

c) He climbs up with constant acceleration

$$\sum F = T - mg = ma; \text{ So, } T = m(g + a)$$

d) He slides downward with constant acceleration

$$\sum F = T - mg = -ma; \text{ So, } T = m(g - a)$$



Clicker question

In which case does the rope break first

a) He climbs with a constant rate

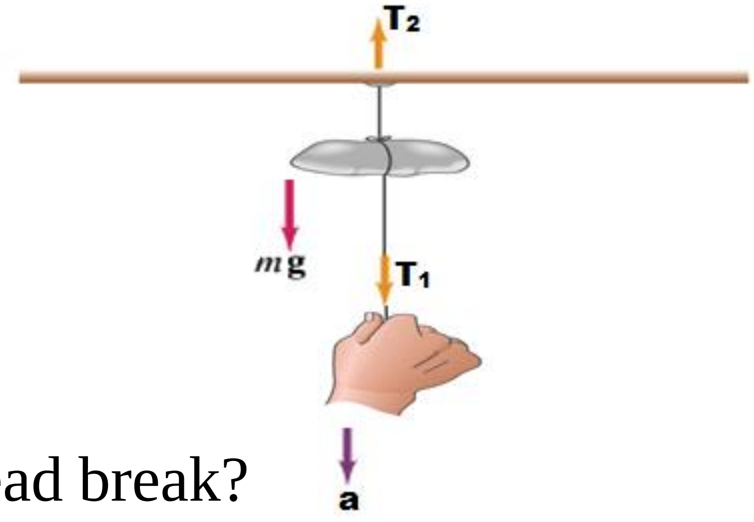
b)He just hangs in the rope

c)He accelerates upward with constant a

d) He decelerates downward with the same constant a

Where does the thread break?

Similar to Multiple Choice 8



If the lower string is pulled slowly, upper string eventually breaks, because it supports both the force and the hanging weight. However, if the lower string is suddenly jerked, it is the one breaks. The traditional
that the inertia of the mass keeps it from moving and hence from stretching the upper string.

Where does the tread break?

$$\sum F = ma$$

1) $T_2 - T_1 - mg = m(-a) \rightarrow T_1 = T_2 - mg + ma$

$a > g$ and $T_1 > T_2$

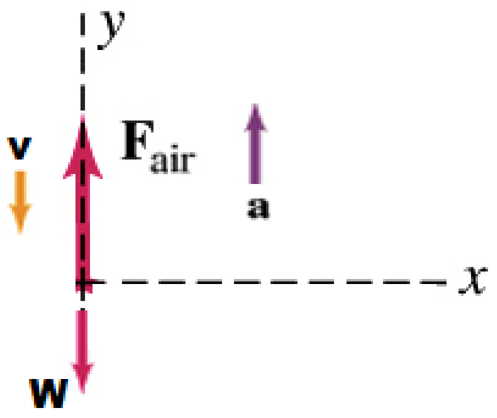
Threads break below

2) $T_2 = T_1 + mg - ma$

$a < g$ and $T_2 > T_1$

Threads break above

42 A parachutist relies on air. Let upward direction be positive and F_{air} =620N is the force of air resistance.



$$\sum F = ma$$

a) What is the weight of the parachutist?

$$W = mg = 55kg * 9.8\frac{m}{s^2} = 539N$$

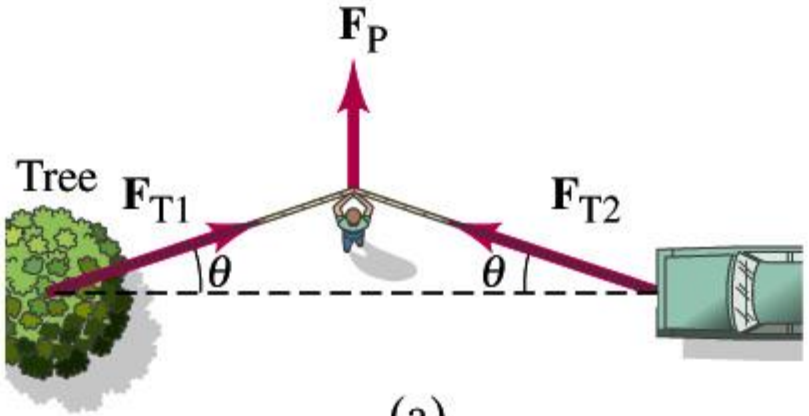
b) What is the net force on the parachutist?

$$\sum F_y = F_{air} - W = 620N - 539N = 81N$$

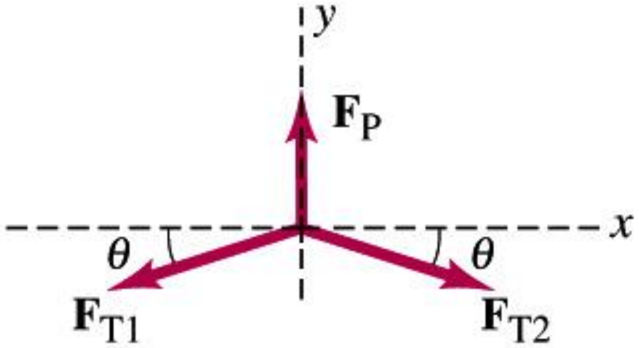
c) What is the acceleration of the parachutist?

$$a_y = \frac{\sum F_y}{m_1 + m_2} = \frac{81N}{55kg} = 1.5\frac{m}{s^2} \text{ (upward)}$$

Getting the car out of the sand



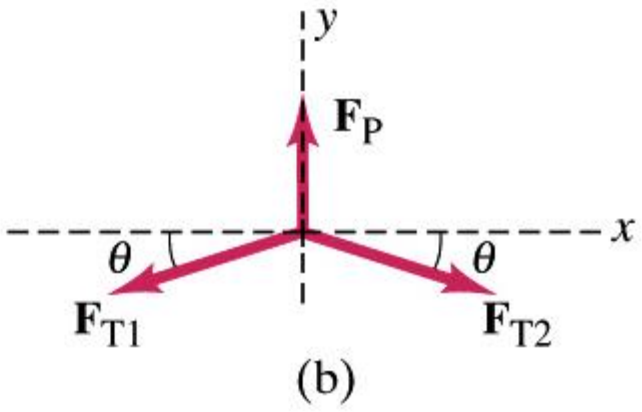
(a)

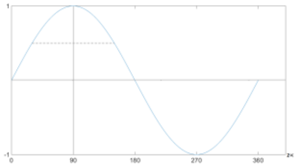


(b)

Getting the car out of the sand. All you need is a rope and Newton's second law;

$$\sum F = ma \quad \text{and} \quad \sum F_x = -F_{T1}\cos\theta + F_{T2}\cos\theta \quad \text{and} \quad F_{T1} = F_{T2}$$





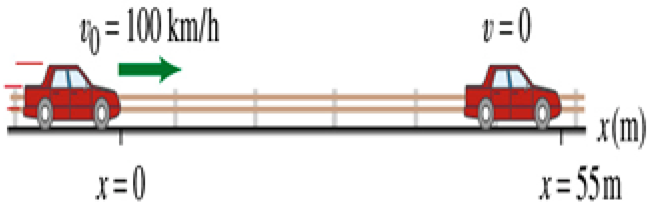
$$\sum F_y = F_P - 2F_T \sin\theta = ma = 0 \text{ (At the break or loose point)}$$

$$F_T = \frac{F_P}{2\sin\theta} \quad \text{and} \quad F_P = 300N, \theta = 5^o$$

$$F_T = \frac{300}{2sin5} = 1700N$$ (almost 6 times large)

Force to stop a car

What force is required to bring a 1500kg car to rest to rest from a speed of 100 km/h within 55m?



Conversion of units; $100\,\frac{km}{h} * \frac{10^3 m}{1km} * \frac{1h}{3.6x10^3 s} = 28\,\frac{m}{s}$

$\vec{F} = m\vec{a}$ and $v^2 = v_o^2 + 2a(x - x_o)$

So, $a = \frac{v^2 - v_o^2}{2(x - x_o)} = \frac{0 - 28^2}{2*55} = -7.1\,\frac{m}{s^2}$

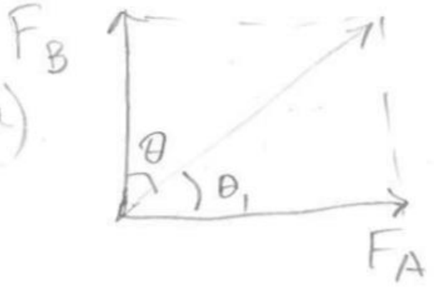
$$\vec{F} = 1500(-7.1) = -1.1x10^4 N$$

problems with solution

7. (10 pts) Forces F_A and F_B are the only forces that act on an object that has a mass of 5kg. F_A has its direction to the east and a magnitude of 3N. F_B is directed to the north and has a magnitude of 4N.

(a.) Make a sketch of the problem
(b.) What is the magnitude of the total force?
(c.) What is the magnitude of the objects acceleration?
d ~~(b.)~~ What is the direction of the acceleration due north (find the angle)?

(a)



(b) $F = \sqrt{F_A^2 + F_B^2} =$

$= \sqrt{9N^2 + 16N^2} = 5N$

(c) $a = \frac{F}{m} = \frac{5N}{5kg} = 1 \frac{m}{s^2}$

(d) $tg\,\theta = \frac{3}{4}$ $\quad \theta = tg^{-1}\left(\frac{3}{4}\right) = 36.9^\circ$

or $tg\,\theta_1 = \frac{4}{3}$ $\quad \theta_1 = tg^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$

3. (5 pts) A ball thrown horizontally from the top of a building hits the ground in 0.50 s. If it had been thrown with four times the speed in the same direction, it would have hit the ground in

(a.) 4.0 s.

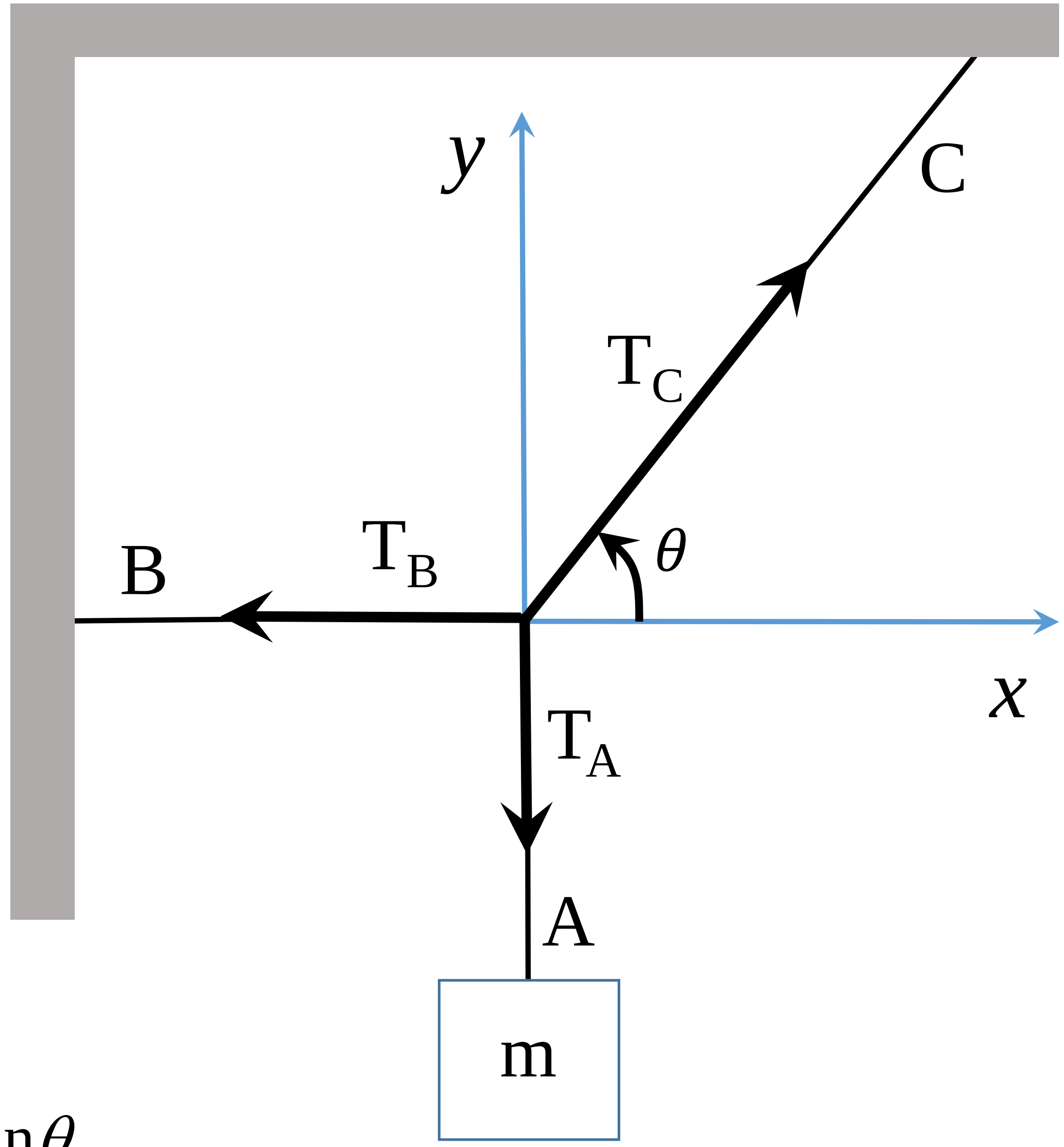
(b.) 0.5 s.

(c.) 1.5 s.

(d.) more than 3.0 s.

Example (Sections 5.1: Equilibrium of a Particle)

A box of known mass m is hung at rest by three ropes as shown. Rope A is vertical. Rope B is horizontal. Rope C forms an angle of $\theta = 60^\circ$ with the horizontal.



Find: the magnitude of tension T_C in Rope C
the magnitude of tension T_B in Rope B

Solutions:

y-axis: $T_C \sin\theta - mg = 0$ $T_C = mg/\sin\theta$

x-axis: $T_C \cos\theta - T_B = 0$ $T_B = mg\cos\theta/\sin\theta$