Chapter 5 Applications of Newton's Laws

- To draw free-body diagrams and analyzing forces on individual objects.
- To solve for unknown quantities using Newton's 2^{nd} law.
- To understand kinetic and static friction forces acting on an object.
- To use Hooke's law to relate the restoring force of a spring to the amount of stretching or compression of a spring.

Courtesy of Wenhao Wu

Applications of Newton's first law

The Conditions for a Particle to be in Equilibrium

When an object is at rest or moving with constant velocity in an inertial frame of reference (i.e. it is in equilibrium), the vector sum of all forces acting on it must be zero.

- Necessary conditions for an object to settle into equilibrium:

  $$\sum \vec{F} = 0 \Rightarrow \vec{a} = 0$$

  Or in component form:

  $$\sum F_x = 0 \quad \& \quad \sum F_y = 0$$

- Note: **a**=0 means that an object in equilibrium may be at *rest* or moving with a *constant velocity*.

Equilibrium of a "particle"

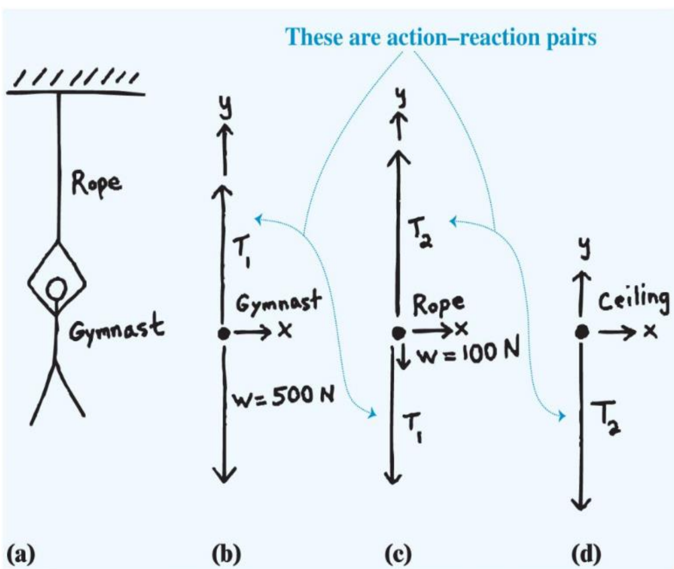
One-dimensional equilibrium

A gymnast has just begun climbing up a rope hanging from a gymnasium ceiling. She stops, suspended from the lower end of the rope by her hands. Her weight is 500 N, and the weight of the rope is 100 N. Analyze the forces on the gymnast and on the rope.

Condition for stable equilibrium: $\sum \vec{F} = 0 \leftrightarrow \sum F_x = 0 \,; \sum F_y = 0$

a) This problem is about recognizing action-reaction pairs according to Newton's 3rd law; $F_{action} = F_{reaction}$

Note: The force diagrams on the left are showing forces on the same object



b) Forces on hanging gymnast

$$\sum F_y = 0 \leftrightarrow T_1 - W = 0;$$

$$T_1 = W; T_1 = 500N$$

c) Forces on rope

2

$$\sum F_y = T_{\cancel{1}2} - 100N - 500N = 0;$$

$T_2 = 600N$; Tension on the top of the rope.

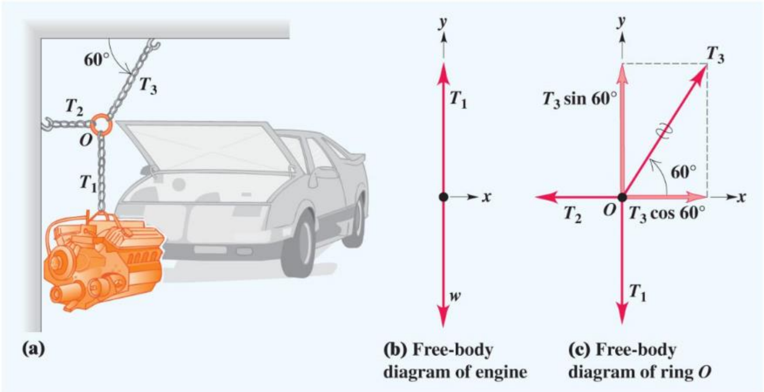
d) Forces on ceiling (partial)

Rope exerts a downward force T_2 on the ceiling.

5.1 Equilibrium of a Particle

Two Dimensional Equilibrium – Example 5.2 A cherry-picker

A car engine with weight w=2200N hangs from a chain that is linked at point O to two other chains, one fastened to the ceiling and the other to the wall. Find the tension in each of the three chains, assuming that w is given and the weights of the chains themselves are negligible.



(a) (b) Free-body diagram of engine (c) Free-body diagram of ring O

b) $T_1 - W = \sum F_y = 0\,;$

$T_1 = W;$

c)

$\sum F_x = 0;\ T_3 cos60 + (-T_2) = 0\ldots_{(1)}$

$\sum F_y = 0;\ T_3 sin60 + (-T_1) = 0\ldots_{(2)}$

1) $T_3 = \dfrac{T_1}{sin60} = \dfrac{w}{sin60} = 1.155w$

T_3 is larger than the weight, because it is at an angle and its vertical component must support ‘w’.

2) $T_2 = T_3 cos60 = 1.155w * cos60 = 0.577w$

Summarize;

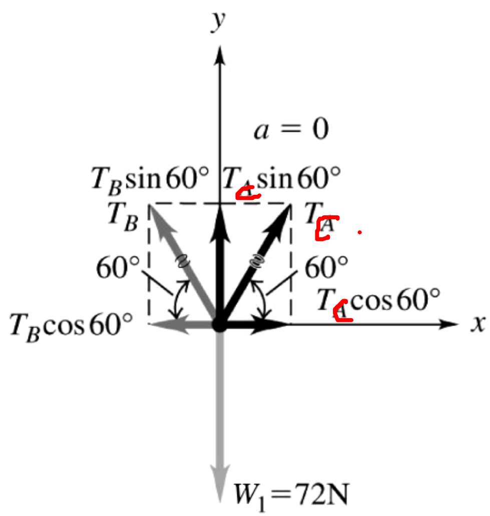
$$\left.\begin{aligned} T_1 &= w \\ T_2 &= 0.577w \\ T_3 &= 1.155w \end{aligned}\right\} \text{with w} \left\{\begin{aligned} T_1 &= 2200N \\ T_2 &= 1270N \\ T_3 &= 2540N \end{aligned}\right.$$

These tensions are proportional to the weight.

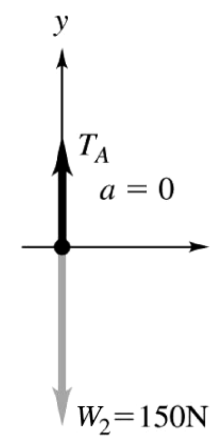
Problem 5. 1 4

Set Up: For each object use coordinates where $+y$ is upward. Each object has $a = 0$. Call the objects 1 and 2, with $W_1 = 72\text{ N}$ and $W_2 = 150\text{ N}$.

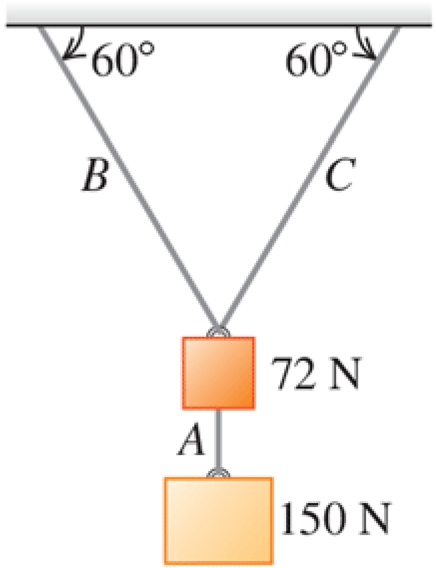
Solve: **(a)** The free-body diagrams for each object are shown in the figure below. T_B and T_C are replaced by their x and y components.



object 1



object 2



(b) For object 2, $\sum F_y = ma_y$ gives $T_A - W_2 = 0$ and $T_A = 150\text{ N}$.

(c) For object 2, $\sum F_x = ma_x$ gives $T_C \cos 60° - T_B \cos 60° = 0$ so $T_B = T_C$

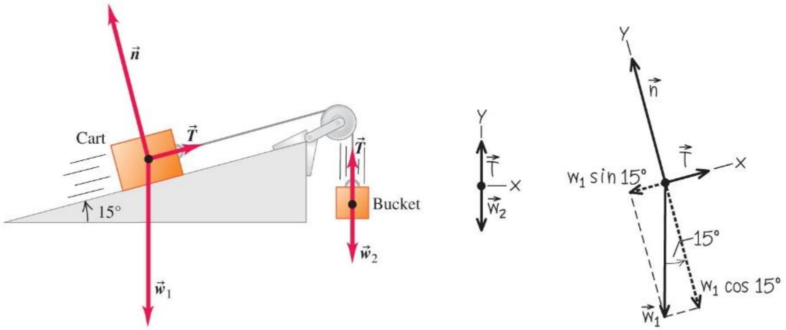
$\sum F_y = ma_y$ gives $T_B \sin 60° + T_C \sin 60° - T_A - W_1 = 0.$

$$T_B = \frac{T_A + W_1}{2\sin 60°} = \frac{150\text{ N} + 72\text{ N}}{2\sin 60°} = 128\text{ N};\quad T_C = 128\text{ N}$$

Pearson

An Example Involving Two Systems –Example 5.4

- See the worked example on page 126.
- This example brings nearly every topic we have covered so far in the course.
- This is an **equilibrium problem** because system moves with **constant speed**!!



(a) Idealized model
(b) Free-body diagram for bucket
(c) Free-body diagram for cart

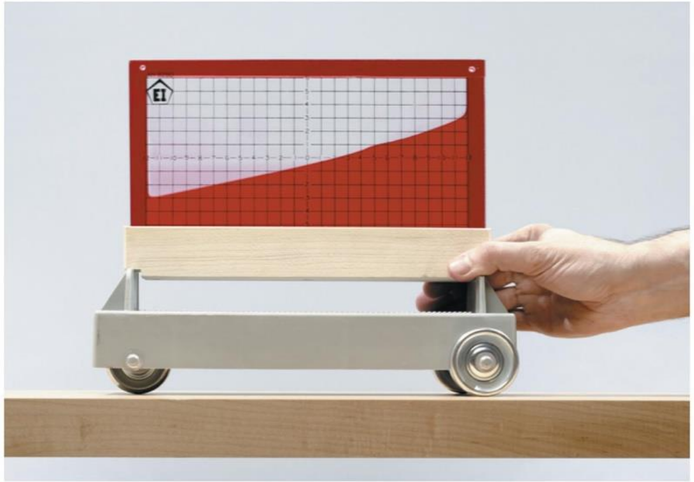
- Note: *x*-axis for the cart does not have to align with the horizontal direction and is different from the bucket.

Let's Examine Applications of Newton's Second Law.
→ Non-equilibrium or Dynamic Problems

$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x \qquad \& \qquad \sum F_y = ma_y$$

- Although this container is on a level surface, the liquid surface is on a slant because the apparatus is being accelerated to the left.



Can you tell that th container is accelerated to the left ?

5.2 Applications of Newton's Second Law

A Review of Problem-Solving Strategy

<u>Newton's Second Law:</u>

in vector form $\sum \vec{F} = m\vec{a}$

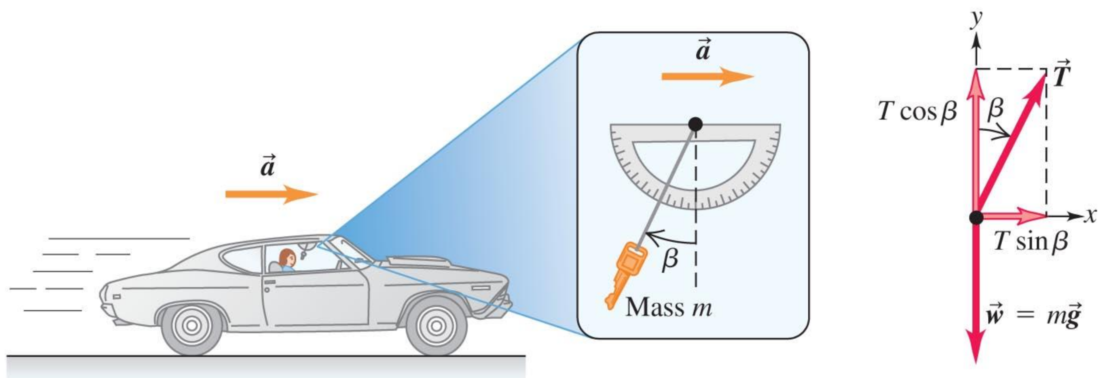
in component form $\sum F_x = ma_x$ $\sum F_y = ma_y$

Strategy for Solving Newton's Law Problems

- Isolate the bodies in a system.
- Analyze all the forces acting on each body and draw one free-body diagram for each body.
- Apply Newton's Third Law to identify action-reaction force pairs.
- Based on the free-body diagram, set up a most convenient *x*-*y* coordinate system.
- Break each force into components using this coordinate system.
- For each body, sum up all the *x*-components of the forces to an equation: $\sum F_x = ma_x$.
- For each body, sum up all the *y*-components of the forces to an equation: $\sum F_y = ma_y$.
- Use these equations to solve for unknown quantities.

Application I – Example 5.5

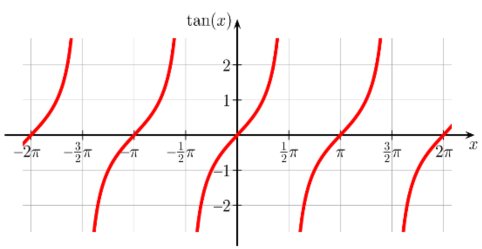
- This experiment works in your car, a bus, or even an amusement park ride!

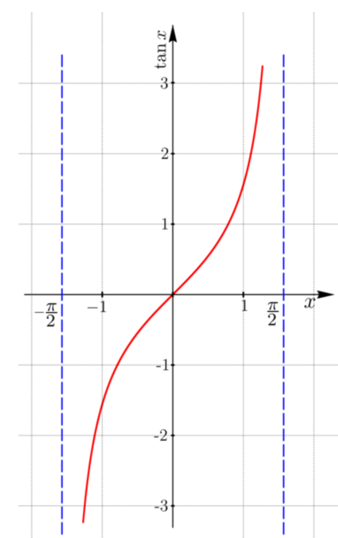


(a) Low-tech accelerometer

(b) Free-body diagram for the key

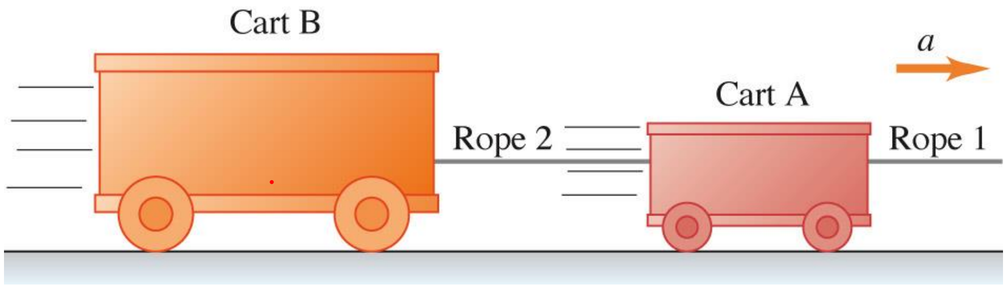
$$\left.\begin{aligned} \sum F_x &= ma_x, & T\sin\beta &= ma_x \\ \sum F_y &= 0, & T\cos\beta + (-mg) &= 0 \end{aligned}\right\} \quad a_x = g\tan\beta$$





Clicker question

The carts in the figure are held together by ropes and are accelerating to the right. The ropes do not stretch and have negligible mass. Which of the following statements is true about the magnitudes of the tension forces exerted by the ropes on the carts?



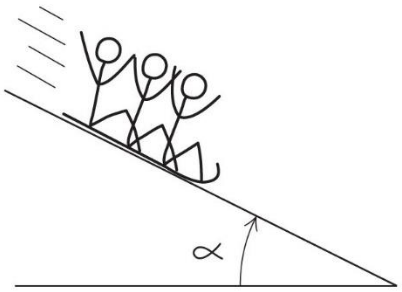
a) Rope 2 pulls harder on cart B than on cart A.

b) Rope 1 pulls harder on cart A than rope 2 pulls on cart B.

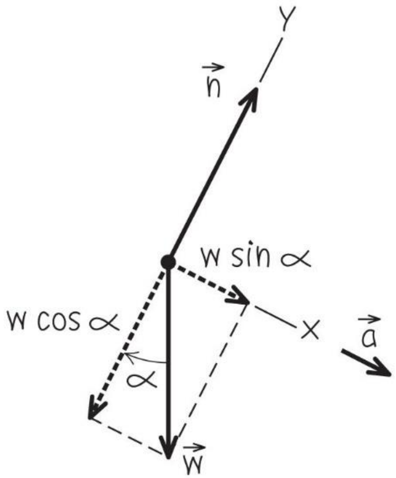
c) Rope 2 pulls harder on cart B than rope 1 pulls on cart A.

Application II – Example 5.6

- Acceleration down a hill, no friction



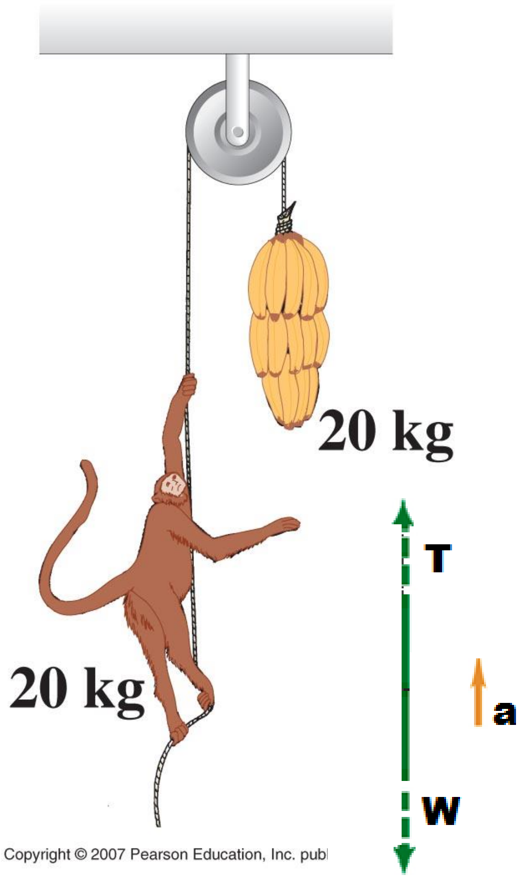
(a) The situation



(b) Free-body diagram for toboggan

$$\left.\begin{array}{ll}\sum F_x = ma_x, & (mg)\sin\alpha = ma_x \\ \sum F_y = 0, & n + (-mg\cos\alpha) = 0\end{array}\right\}\ a_x = g\sin\alpha$$

What happens to the monkey and the bananas as the monkey climbs up, or releases the rope and falls, and grabs the rope to stop the fall.



(a) For the monkey to move up, T>mg. the bananas also move up.

(b) The bananas and monkey move with the same acceleration and the distance between them remains constant.

(c) Both the monkey and bananas are in free fall. They have the same initial velocity and as they fall the distance between them doesn't change.

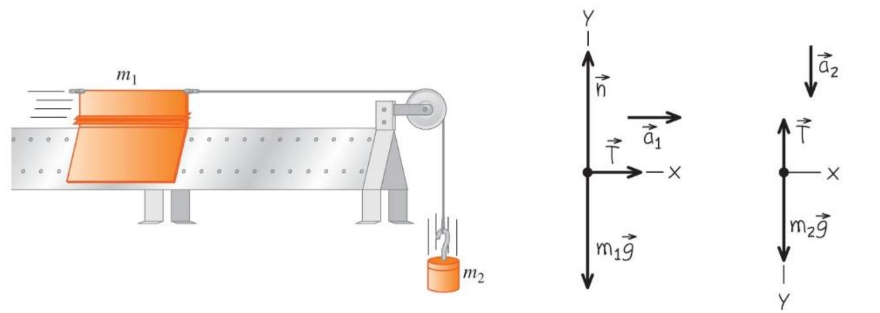
(d) The bananas will slow down at the same rate as the monkey. If the monkey comes to a stop, so will the bananas.

Just as in the gymnast climbing the rope problem; $T - mg = ma \rightarrow a = constant$

$$T = m(g + a) \rightarrow T > mg$$

Application – Example 5.7 An air track

- This problem involves two interactive systems in a common lab experiment.



(a) Apparatus **(b)** Free-body diagram for glider **(c)** Free-body diagram for weight

From Glider free-body diagram:

$$\sum F_x = ma_x, \quad T = m_1 a_x$$
$$\sum F_y = 0, \quad n + (-m_1 g) = 0$$

From Weight free-body diagram:

$$\sum F_y = m_2 a_y, \quad m_2 g + (-T) = m_2 a_y$$

Here, $a_{x\_Glider} = a_{y\_Weight} = a_{system}$

$$a = (m_2 g) / (m_1 + m_2) \qquad T = (m_1 m_2) g / (m_1 + m_2)$$

What happens for $m_1 = 0$?

Pearson

5.3 Contact Force and Friction

Friction force exits at the contact surfaces of two objects. There are two regions of friction:

- when an object is sliding with respect to another → *kinetic-friction* force
- when there is no relative motion → *static-friction* force

Kinetic Friction Force f_k

Magnitude:

It is directly related to the normal force.

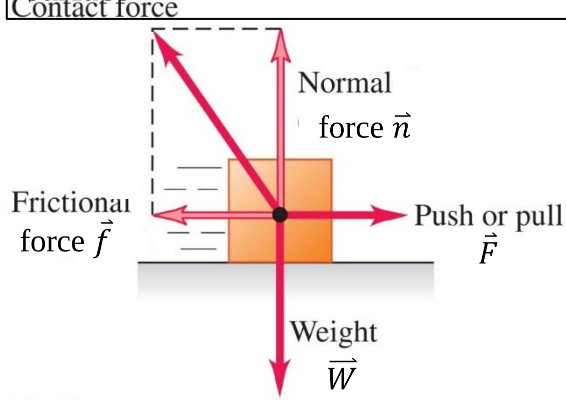
$$f_k = \mu_k n$$

μ_k: coefficient of kinetic friction

Direction:

- Parallel to the contact surface
- Opposite to the direction of relative motion

A box being pushed on a table surface



Static Friction Force f_s

Magnitude:

It depends. It is not fixed.

Maximum value

$$f_{s,max} = \mu_s n$$

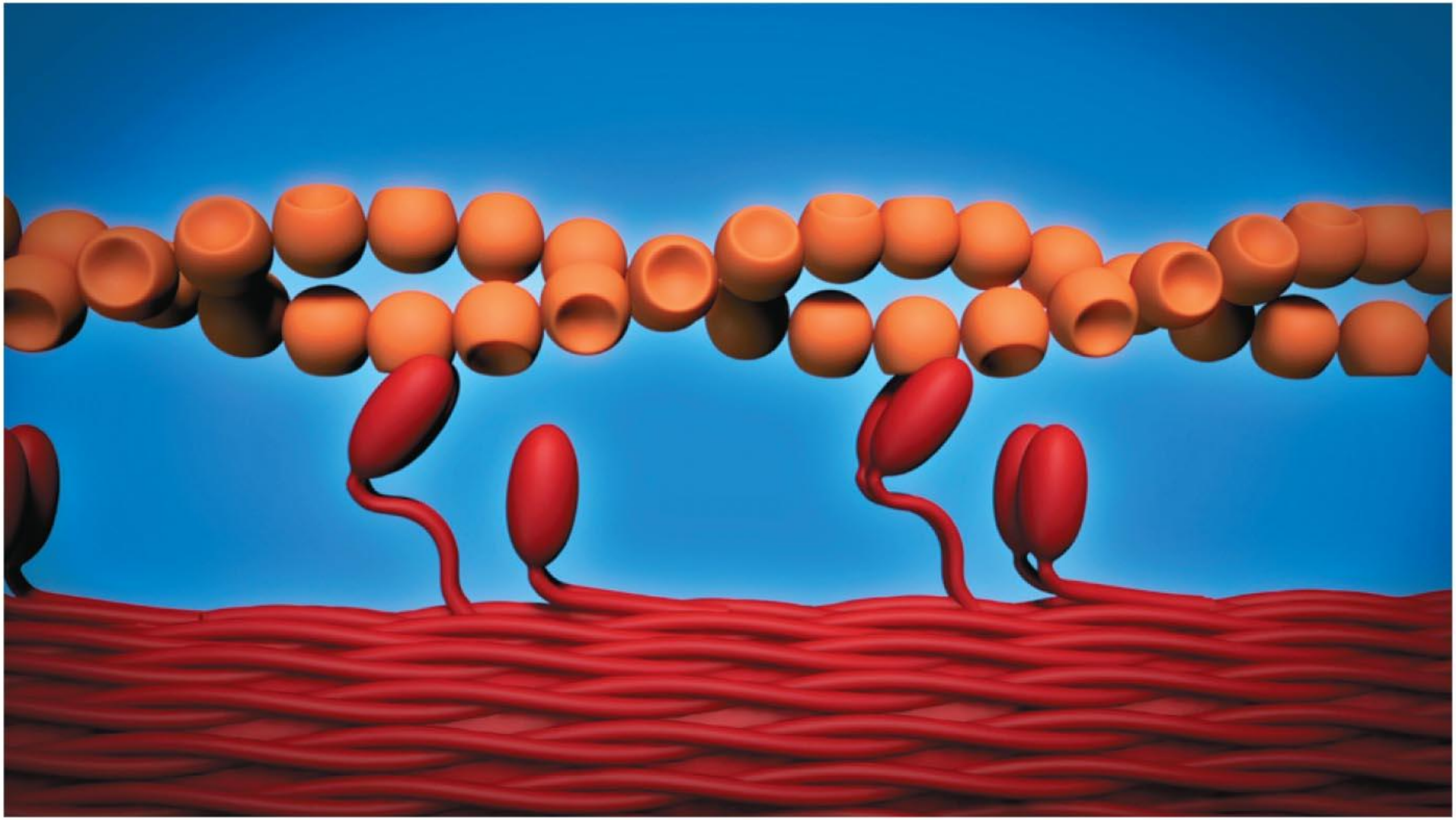
μ_s: coefficient of static friction

Direction:

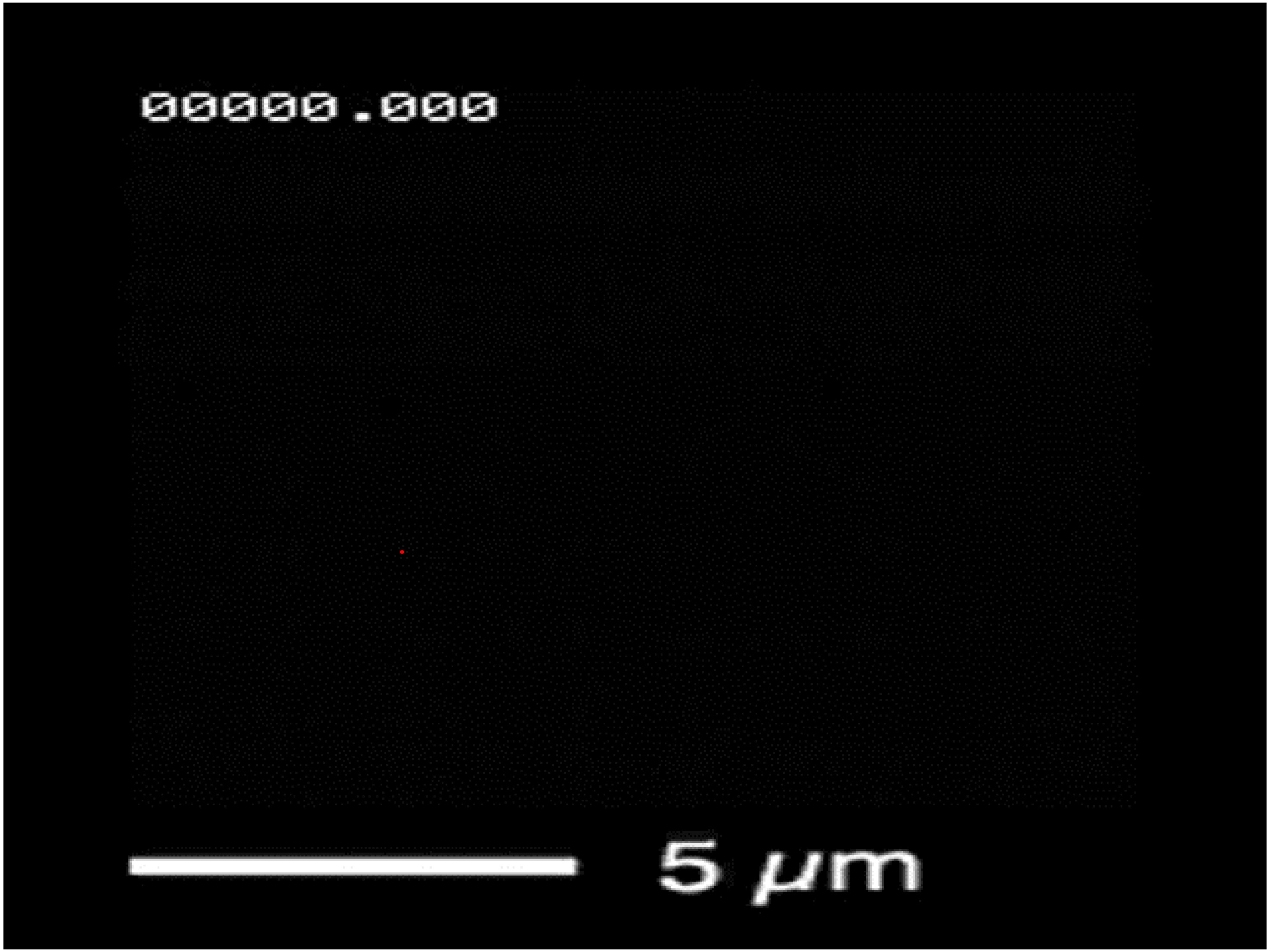
- Parallel to the contact surface
- Opposed to the tendency to change the state of motion

Courtesy of Wenhao Wu

An angle of 90 degrees between the climber and the rock gives maximum friction. Why?



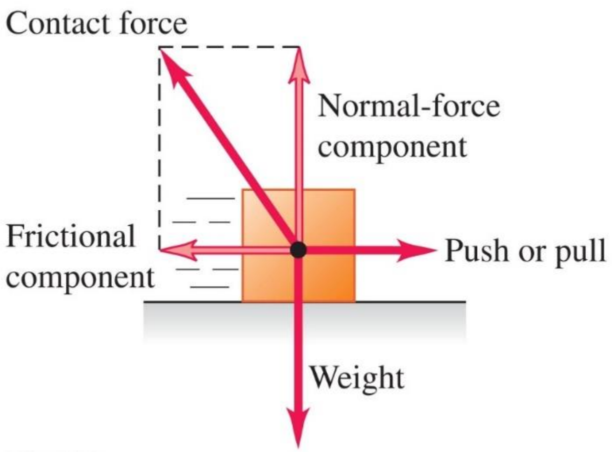
Cooperative Binding of Myosin to Tubulin



Molecular rail (tubulin) and motor (kinesin)

5.3 Contact Force and Friction

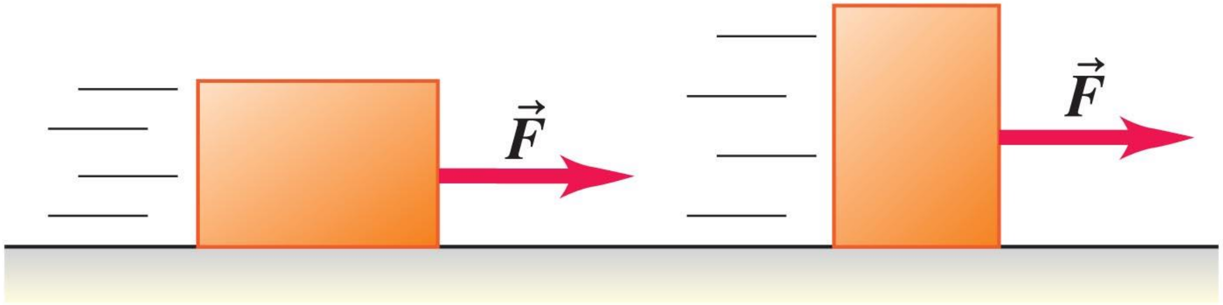
The frictional and normal forces are really components of a single contact force.



- We need to re-examine problems we formerly did as "ideal."
- We need to be able to find frictional forces given the mass of the object and the nature of the surfaces in contact with each other.
- There are two regions of friction:
  1) when an object is sliding with respect to a surface → ***kinetic-friction*** force
  2) when there is no relative motion → ***static-friction*** force

No Dependence on Surface Area

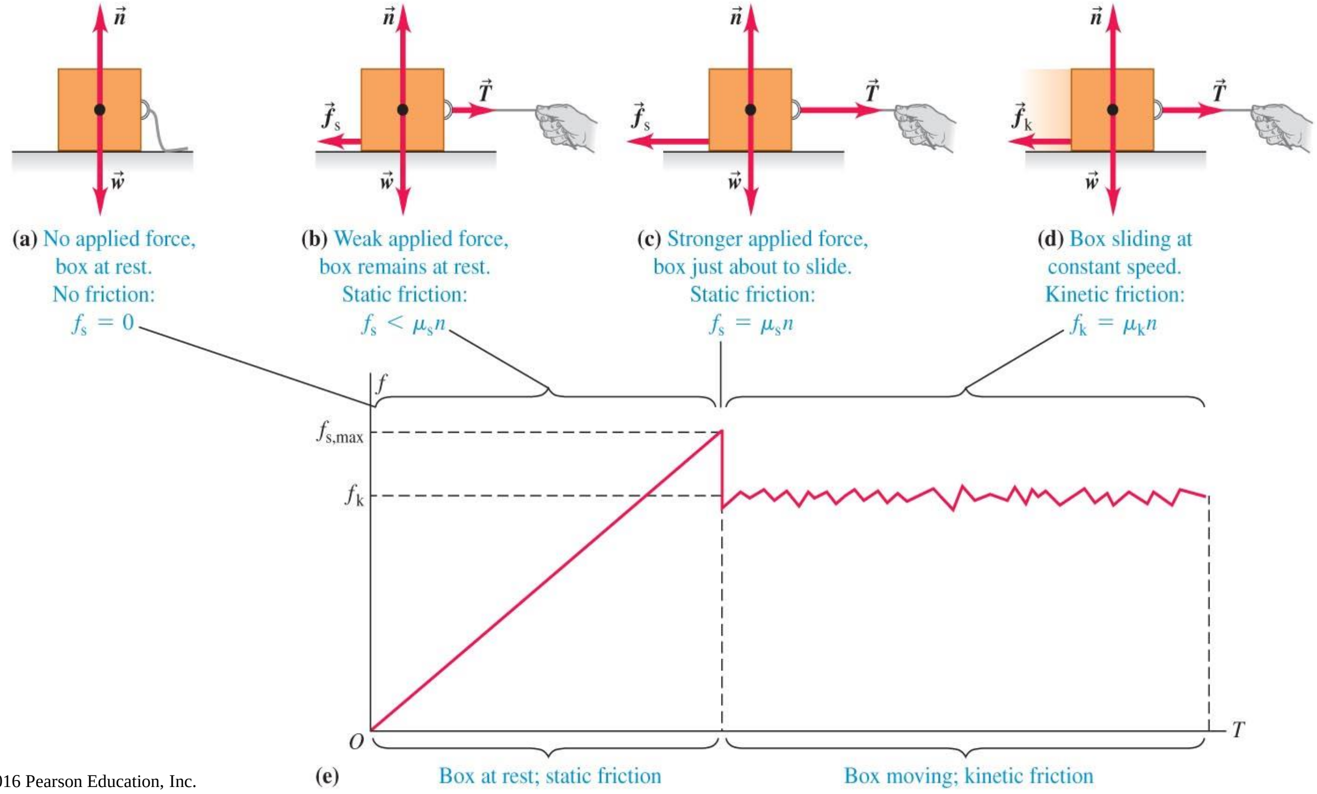
- The normal force determines friction.



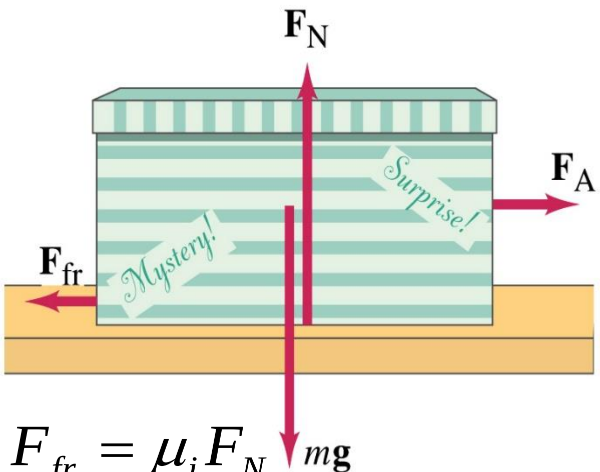
$$\left.\begin{array}{lll} f_s \leq \mu_s n & \rightarrow & \text{no relative movement} \\ f_{s,max} = \mu_s n & \rightarrow & \text{interface "breaks loose"} \\ f_k = \mu_k n & \rightarrow & \text{sliding with friction} \end{array}\right\} \mu_k \leq \mu_s$$

Friction Changes as Forces Change – Figure 5.13

- Forces from static friction increase as force increases while forces from kinetic friction are *relatively* constant.



Applications of Newton's laws

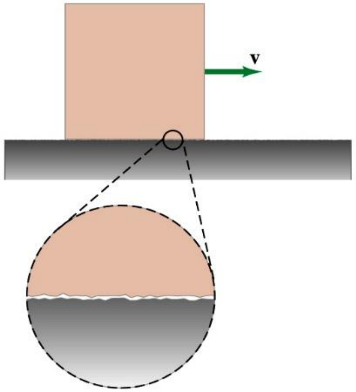


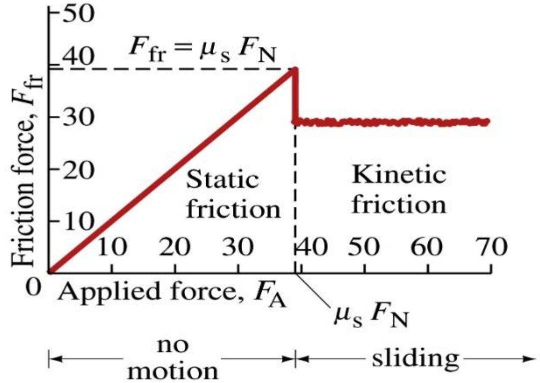
$$F_{fr} = \mu_i F_N$$

TABLE 5–1 Coefficients of Friction†

Surfaces	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	< 0.01	< 0.01
Synovial joints (in human limbs)	0.01	0.01

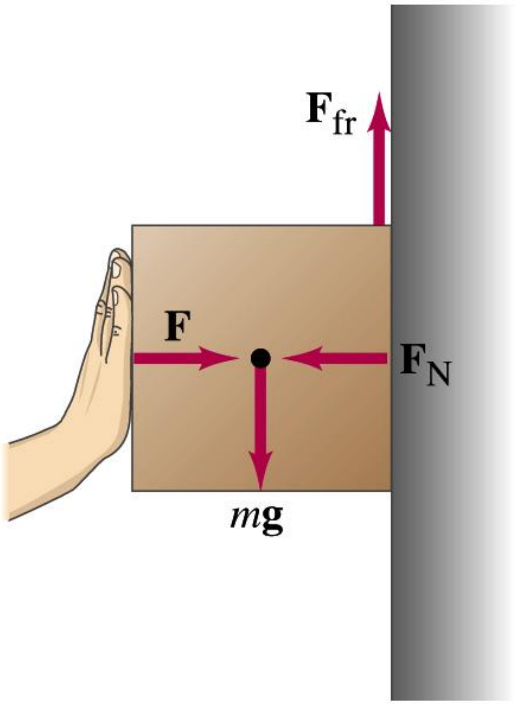
† Values are approximate and are intended only as a guide.





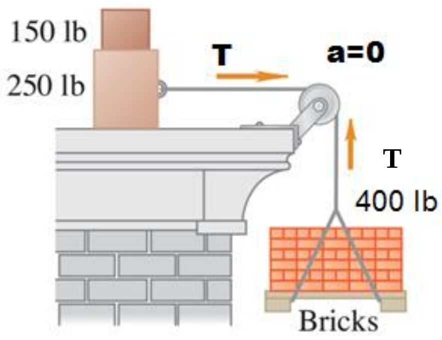
A box against a wall

With what force F one should press on the box that it will not slide down and will be at rest?



When $f_s=\mu_s F_N > mg$,

the box is at rest



5.27

Construction site has a pallet of bricks hanging on a rope, where the other side is tied to a heavy crate. $\mu_s = 0.666$

Note: The system is not moving: Static friction

$$f_s = \mu_s n;$$

a) What is the weight of the heaviest pallet of bricks, that can be supported as not moving?

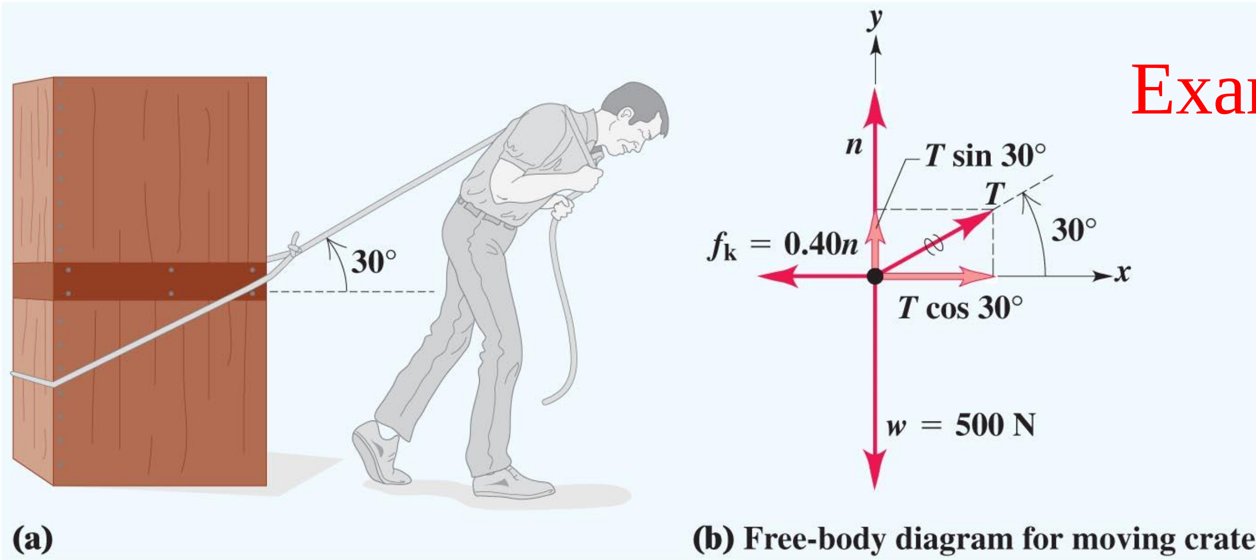
$F_y = ma_y = n - w_c = 0$ So, $n = w_c = 250 + 150 = 400$ lbs

$f_s = \mu_s n = 0.666 * 400 = 266$ lbs;

$$\sum F_x = ma_x \rightarrow T - f_s = 0 \quad \because T = f_s = 266 \text{ Ibs}$$

b) What is the friction force on the upper crate under these conditions?

$$a_x = 0; \sum F_x = 0 \rightarrow f_x = 0$$



Example 5.10

Moving the box by pulling upward with a rope at an angle of 30°. Is it easier or harder than pulling horizontally, what pulling force is needed in both cases? $\mu_k = 0.4$ and $w = 500N$

Note: The magnitude of $n < w$, due to the upward component.

$$f_k = \mu_k n;$$

For constant velocity moving;

$$\sum F_x = Tcos30 - f_k = Tcos30 - 0.4 * n = 0$$

$$\sum F_y = Tsin30 + n - 500N = 0 \because n = 500 - Tsin30$$

$$Tcos30 - 0.4(500 - Tsin30) = 0$$

$$T = 188N \text{ and } n = 406N$$

θ=0, then n=w and *T*-0.4x500=0 *T*=200N

The weight of the box is 500N. For "breaking loose", a horizontal force of 230N is needed. For "keep moving" at constant velocity a force of 200N is needed. Work out μ_s and μ_k (friction coefficients)

Moving the box by pulling horizontally.

b)

$$f_{s,max} = \mu_s n;$$

$$\sum F_x = ma \rightarrow a = 0; v = constant$$

$$\sum F_y = n - W = n - 500N = 0 \rightarrow n = 500N$$

$$\sum F_x = T - f_s = 230N - f_{s,max} = 0 \rightarrow f_{s,max} = 230N$$

$$\boxed{\mu_s = \frac{f_{s,max}}{n} = \frac{230N}{500N} = 0.46}$$

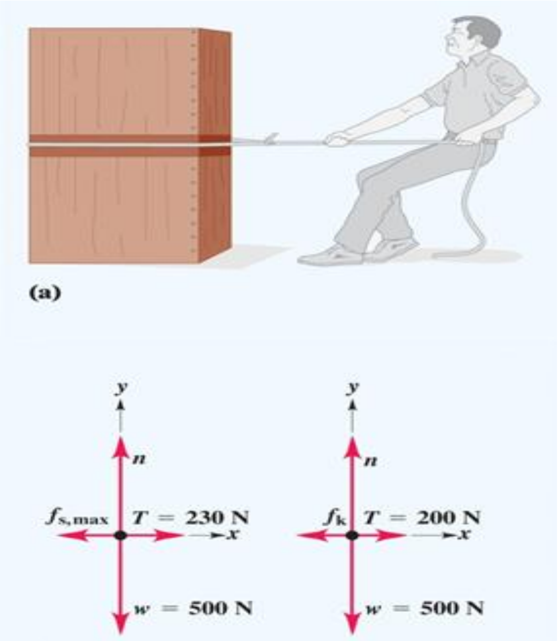
c) Crate moves, so

$$f_k = \mu_k n;$$

$$\sum F_y = n - W = n - 500N = 0 \rightarrow n = 500N$$

$$\sum F_x = T - f_k = 200N - f_k = 0 \rightarrow f_k = 200N$$

$$\boxed{\mu_k = \frac{f_k}{n} = \frac{200N}{500N} = 0.4}$$



(b) Crate just as it starts to move
(c) Crate moving at constant speed

Clicker question

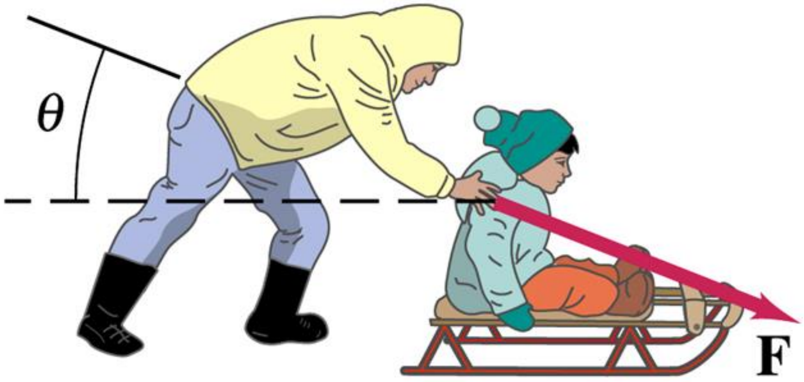
To push or to pull a sled, what is more efficient?

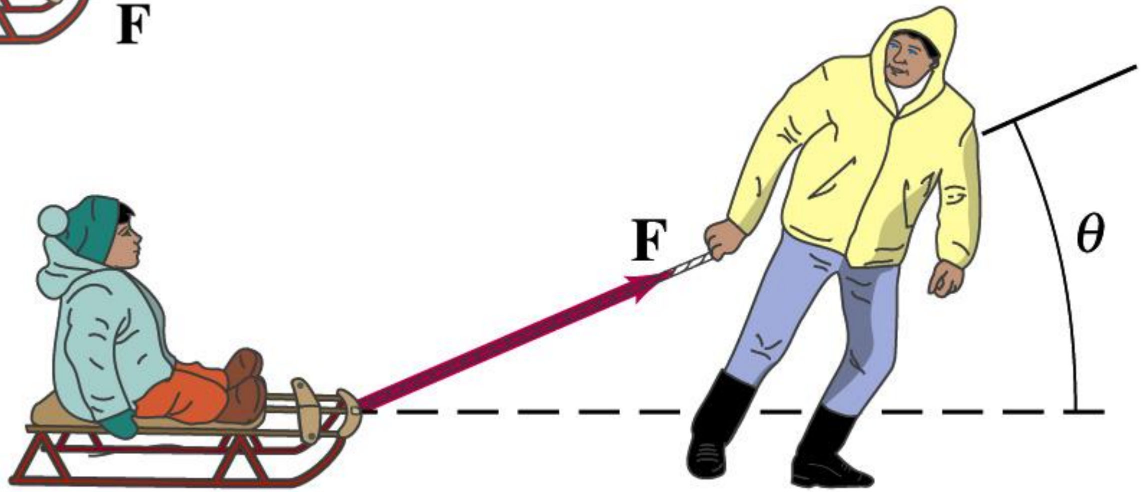
a) pushing

b) pulling

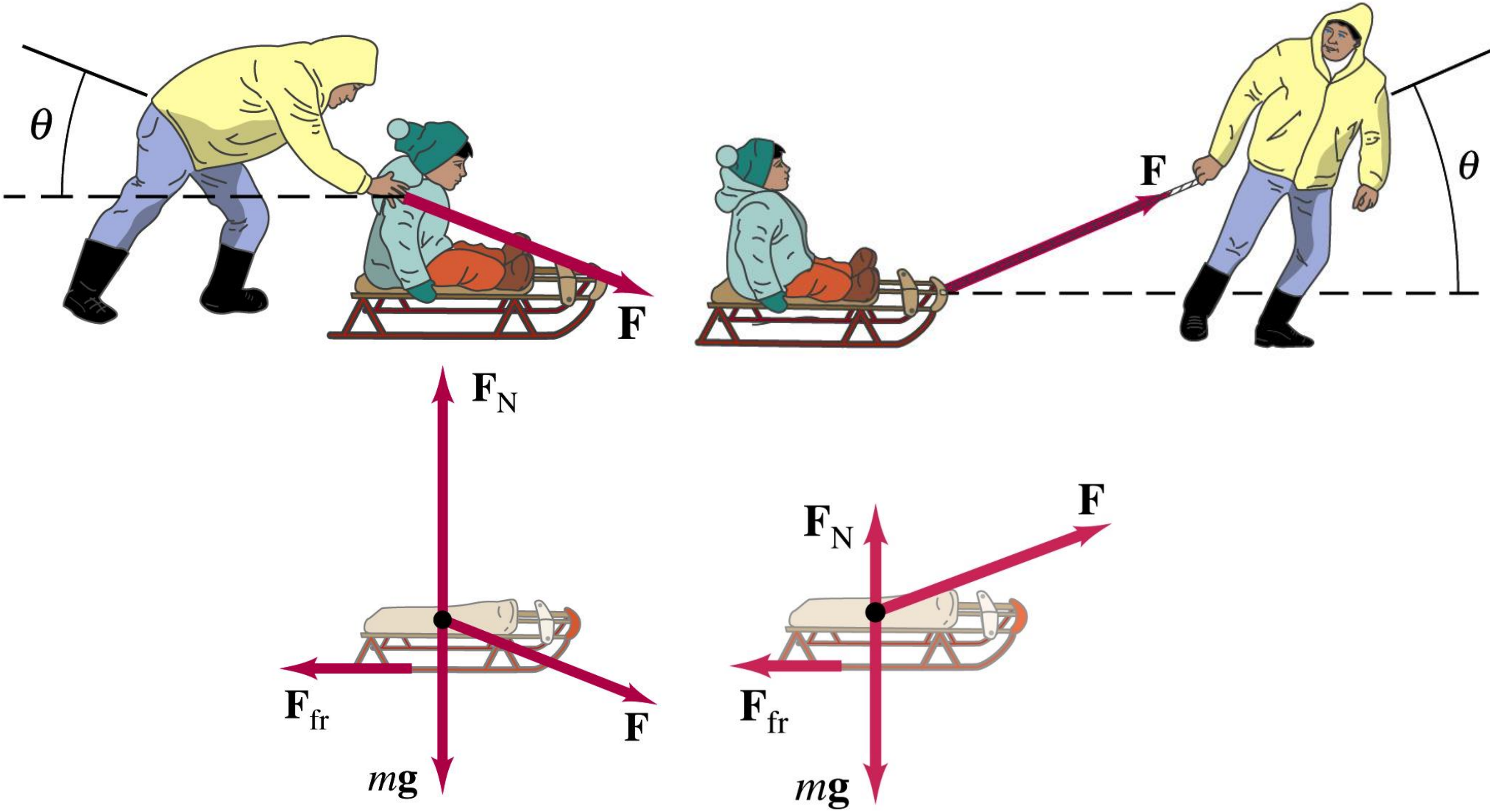
c) the same

d) depends on the magnitude of the force





To push or to pull a sled?

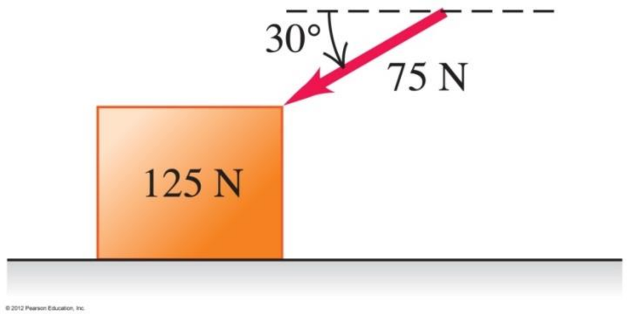


Friction is less when you pull, since F_N is less

5-40:

Note: Nothing is moving. The coefficient of Static friction is

$$f_s = \mu_s n; \mu_s = 0.8$$

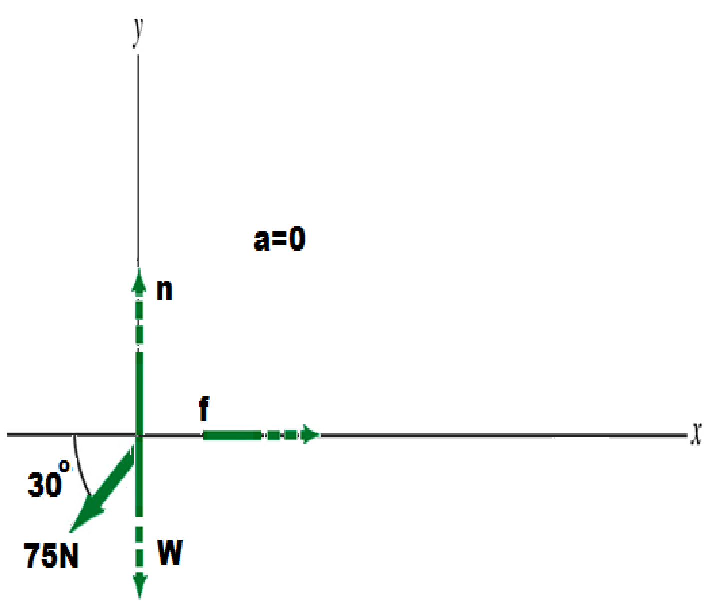


b) What is the normal force exerted by surface on the box?

$$\sum F_y = ma_y = 0 \rightarrow n - Fsin30 - W = 0$$

$$n = W + Fsin30 = 125N + 75 * sin30 = 163N$$

a) Make a free body diagram.



c) What is the friction force on the box

$$\sum F_x = ma_x \rightarrow f - Fcos30 = 0$$

$$f = Fcos30 = 65N$$

d) What is the largest possible friction force?

$$f_s = \mu_s n = 0.8 * 163 = 150\text{N}$$

e) Replace the push with a 75N pull, what is n?

$$\sum F_y = ma_y \rightarrow n + Fsin30 - W = 0$$

$$n = W - Fsin30 = 88N$$

f) What is the largest possible friction force for the75N pull?

$$f_s = \mu_s n = 0.8 * 88 = 70\text{N}$$

Box on a horizontal table

An 80 N box is pulled at an horizontal table. Kinetic friction is 1/4 ,static friction is ½. What is the friction f on the box if the tension T on the rope is a. 0 N, b. 25 N, c.39 N,d.41 N, e.150 N ?

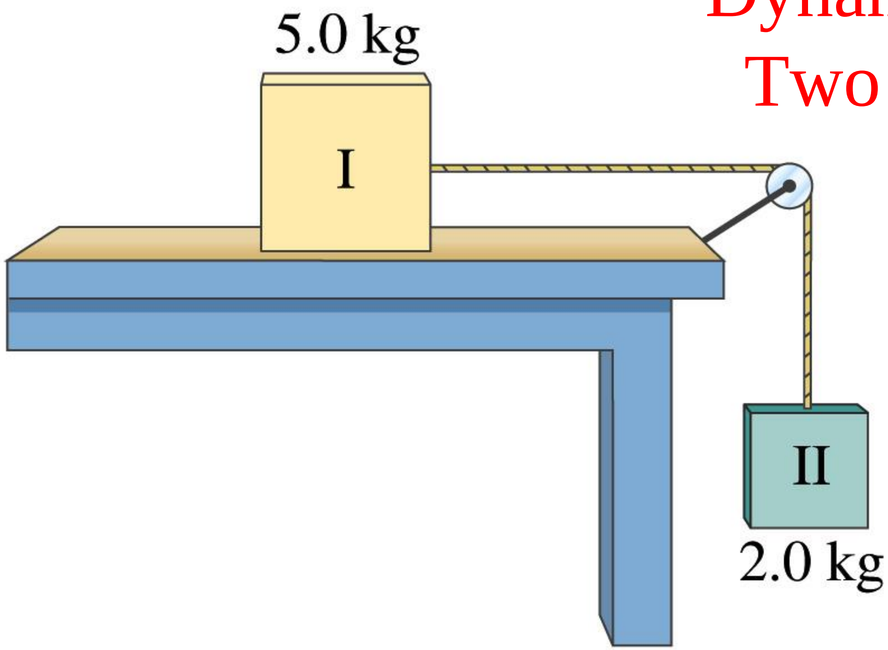
5.25. Set Up: Assume the box is initially at rest. The box remains at rest and friction is static if the pull is less than the maximum possible static friction force. If the pull is larger than this, the box moves and the friction is kinetic.

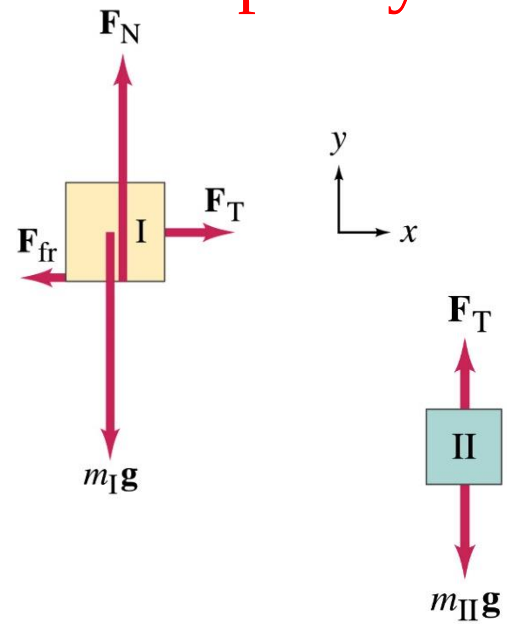
Solve: $n = W = 80\text{ N}$. The maximum possible static friction force is $\mu_s n = 40\text{ N}$. The kinetic friction force is $\mu_k n = 20\text{ N}$. The actual static friction force is only as large as necessary to prevent motion.

(a) $f_s = 0$ and the box remains at rest. **(b)** $f_s = 25\text{ N}$ and the box remains at rest.

(c) $f_s = 39\text{ N}$ and the box remains at rest. **(d)** The box slides and $f_k = 20\text{ N}$. **(e)** The box slides and $f_k = 20\text{ N}$.

Dynamic motion problem: Two boxes and a pulley





Two boxes and a pulley with friction $\mu_k = 0.2$

(a) What is the tension in the rope?

(b) What is the tension in the rope?

Y: $F_N = m_1 g = 49N$ and $F_{fr} = \mu_k F_N = 0.2 * 49 = 9.8N$

$$F_T - F_{fr} = m_1 a \rightarrow F_T = F_{fr} + m_1 a \qquad \textit{(box I)}$$

$$m_2 g - F_T = m_2 a \rightarrow F_T = m_2(g - a) \qquad \text{(box II)}$$

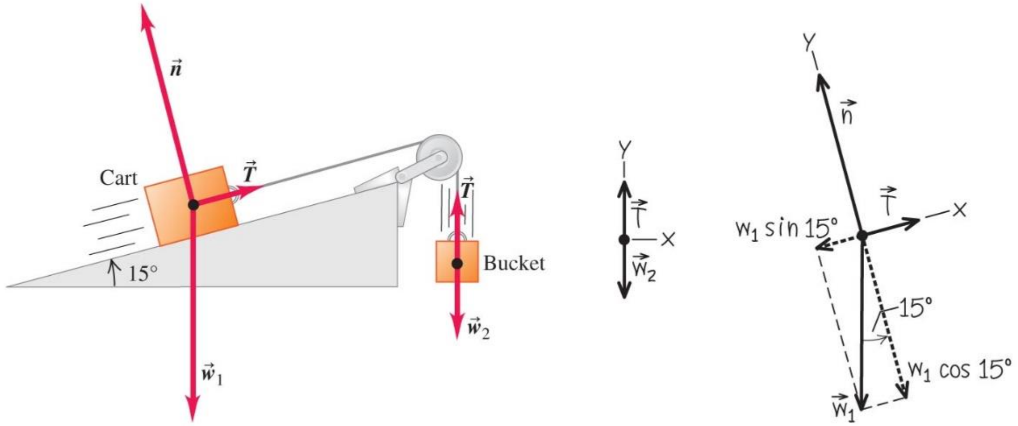
→ $m_1 a + m_2 a = m_2 g - F_{fr}$

$$(a)\ a = \frac{m_2 g - F_{fr}}{m_1 + m_2} = 1.4\,\frac{m}{s^2}$$

(b) $F_T = m_2(g - a) = 17N$

An Example Involving Two Systems – Example 5.4 Two boxes

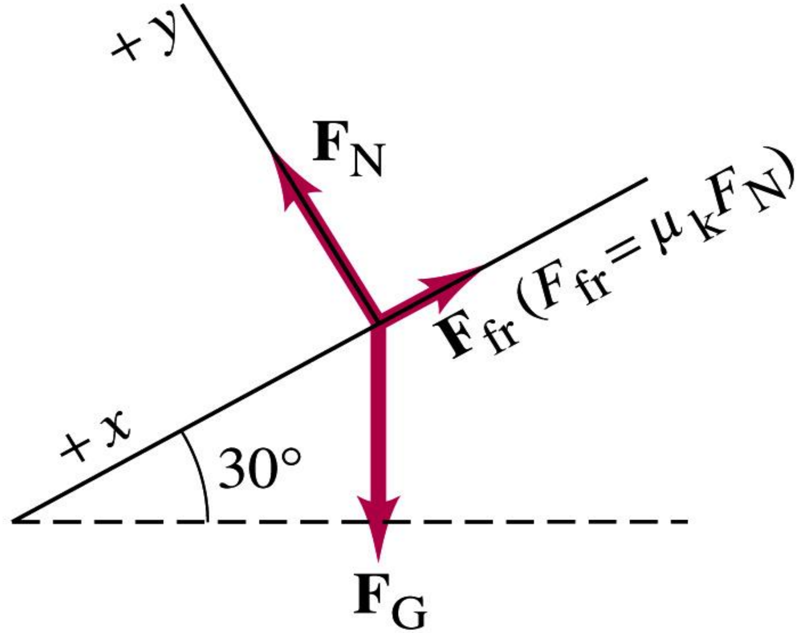
- This example brings nearly every topic we have covered so far in the course.
- This is an *equilibrium problem* because system moves with <u>constant speed</u>!!

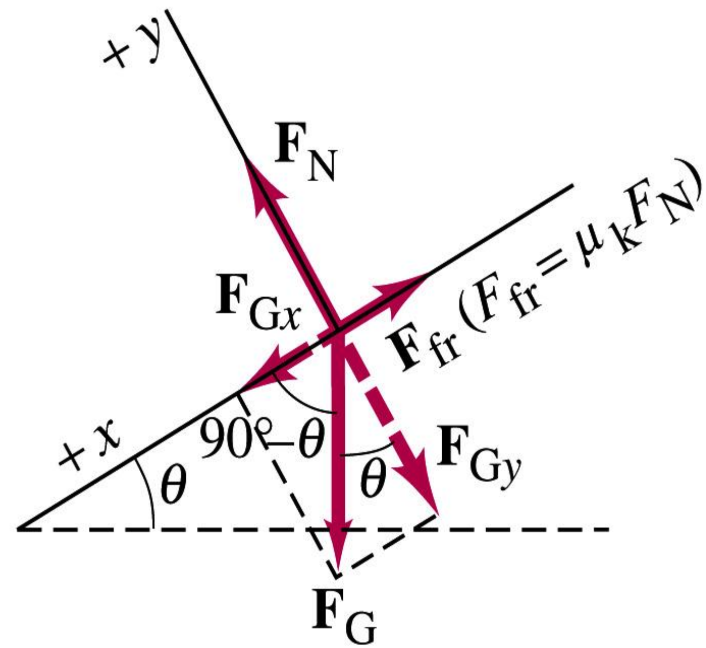


(a) Idealized model (b) Free-body diagram for bucket (c) Free-body diagram for cart

- Note: *x*-axis for the cart does not have to align with the horizontal direction and is different from the bucket.

The skier with friction





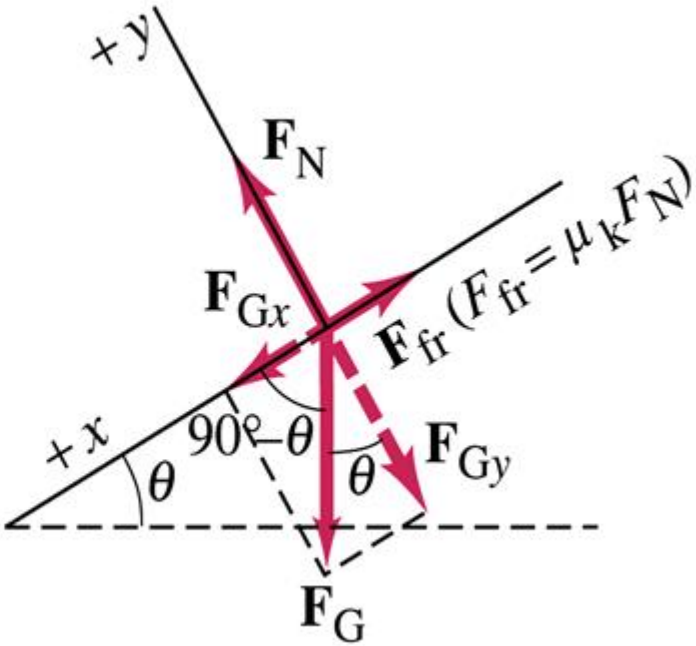
a) What is acceleration?

$\mu_K = 0.01$

Winning (acceleration) does not depend on mass!

Skier with friction

Initial step: select the right reference frame: it's up to us, and is a matter of convenience!



a) What is acceleration? ($\mu_k = 0.1$ and $\theta = 30^o$)

$$\sum F_x = ma_x \rightarrow F_{gx} = mgsin30 \;\; and \; F_{gy} - mgcos30 = 0 \qquad \because F_{gy} = mgcos30 = F_N$$

$$mgsin30 - \mu_k F_N = ma_x \rightarrow mgsin30 - \mu_k mgcos30 = ma_x$$

$$a_x = 0.41g \quad \because g = 9.8\frac{m}{s^2}$$

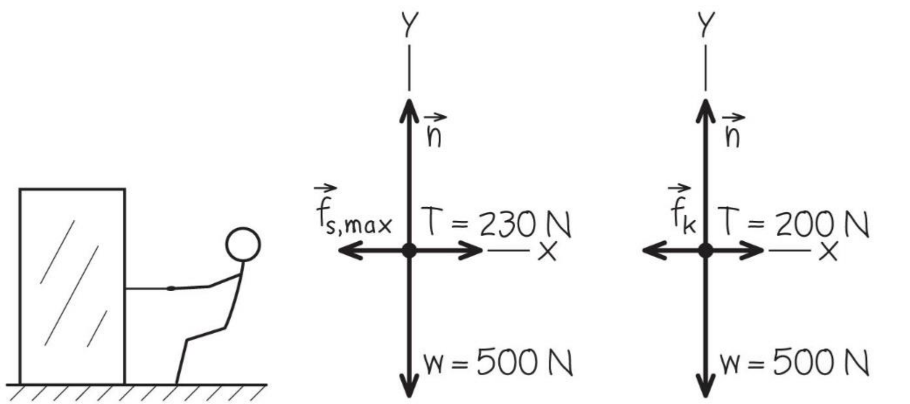
$$\rightarrow a_x = 4\frac{m}{s^2}$$

b) Starting from rest, what is the velocity of the skier after 4 second?

$$v_x = v_o - a_x t = 0 + a_x t = 4 * 4 = 16\frac{m}{s}$$

How Much Effort to Move the Crate?

- Dynamics as in the last chapter with a new force.
- See the worked solution on page 135.



(a) Pulling a crate

(b) Free-body diagram for crate just before it starts to move

(c) Free-body diagram for crate moving at constant speed

Pearson

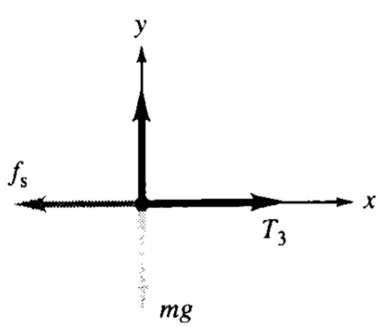
5.59 A block friction $\mu_s = 0.25$, a knot and two weights $W(A) = 60N; w = 12N$

a) Find the friction force on the block at $a = 0$, for both objects. The tension in the vertical wire is 'w'. Applying $\sum F = ma$ to the knot and to the block.

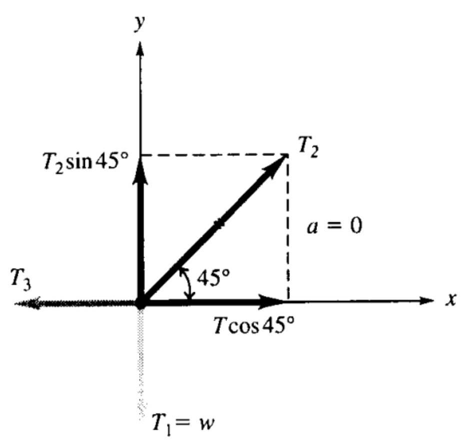
$$\left.\begin{array}{l} \sum F_y = ma_y \leftrightarrow T_2 sin45 = w \\ \sum F_x = ma_x \leftrightarrow T_2 cos45 = w \end{array}\right\} \sin 45 = \cos 45 \rightarrow T_3 = w = 12N$$

b) Find the maximum weight for which the system remains at rest.

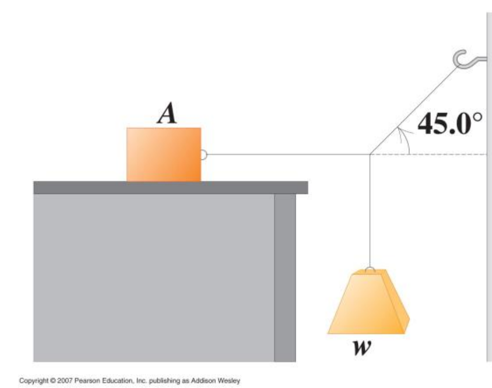
$$f_{s,max} = \mu_s mg = 0.25 * 60N = 15N \rightarrow w = f_{s,max} = 15N$$



(b)

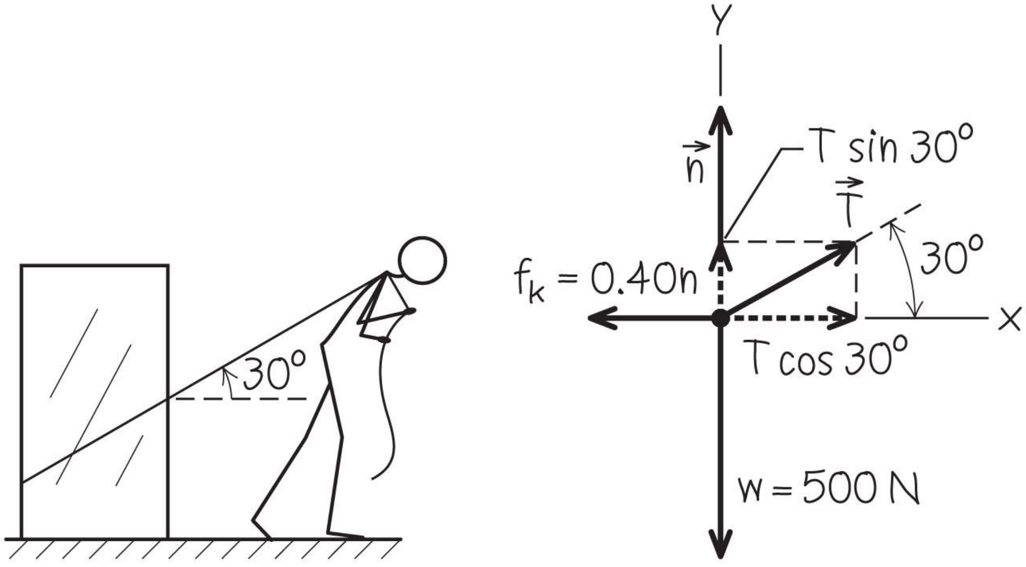


(a)



Forces Applied at an Angle

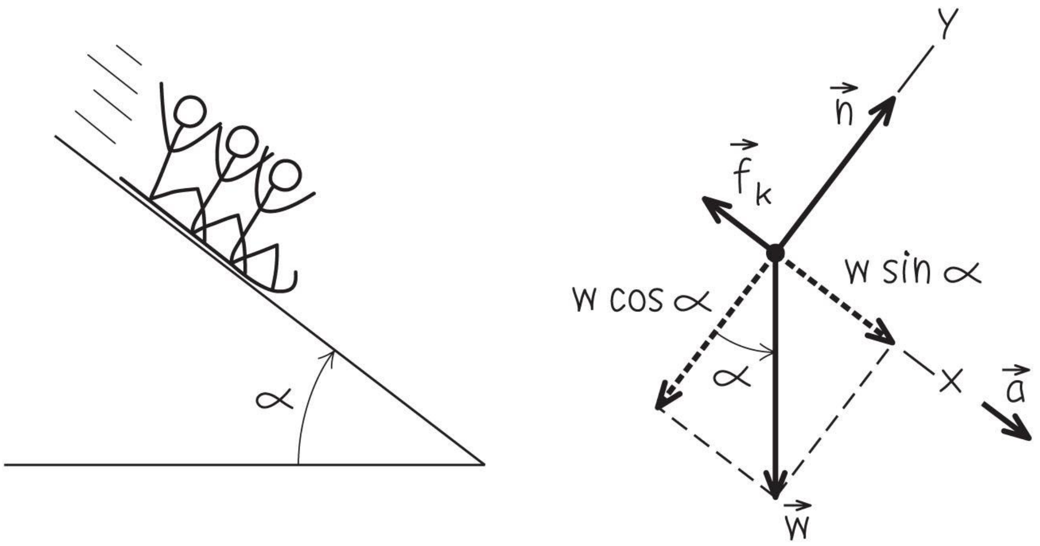
- The previous example has one new step if the force is applied at an angle.



(a) Pulling a crate at an angle **(b)** Free-body diagram for moving crate

A Toboggan on a Steep Hill with Friction –

- now at constant speed.

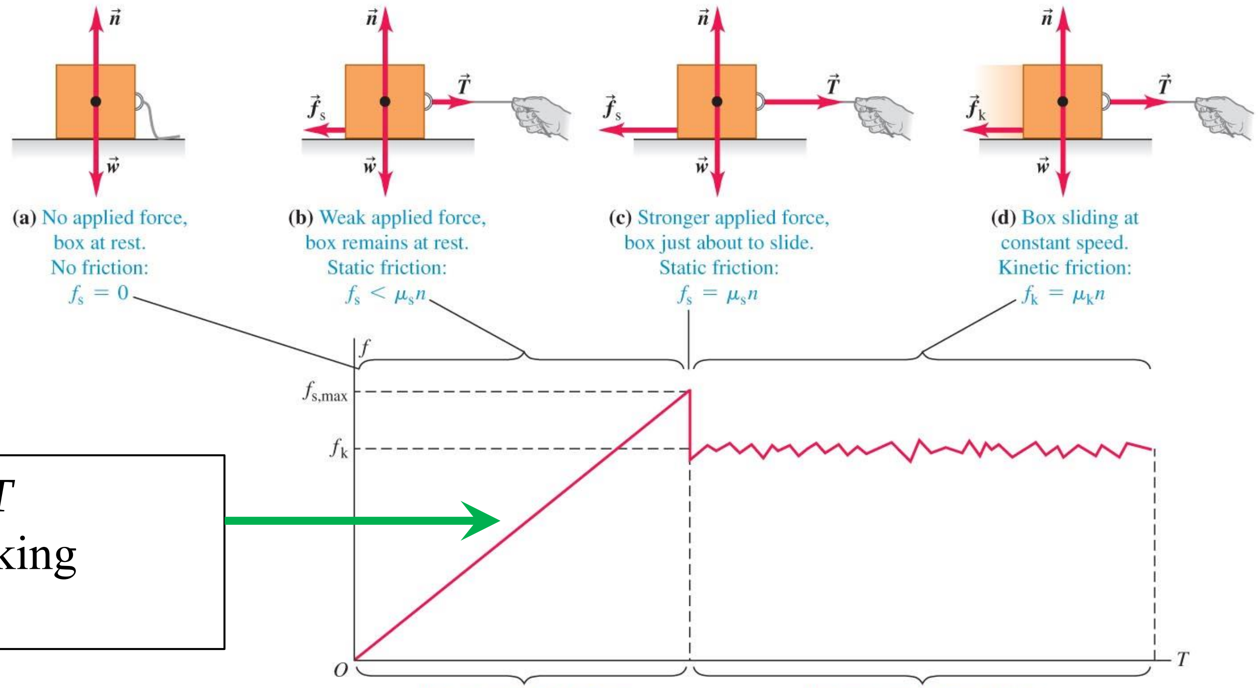


(a) The situation

(b) Free-body diagram for toboggan

$$\left.\begin{aligned}\sum F_x = 0, &\quad (mg)\sin\alpha + (-f_k) = (mg)\sin\alpha + (-\mu_k n) = 0 \\ \sum F_y = 0, &\quad n + (-mg\cos\alpha) = 0\end{aligned}\right\} \mu_k = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

Distinguishing Static and Kinetic Friction Forces



(a) No applied force, box at rest. No friction: $f_s = 0$

(b) Weak applied force, box remains at rest. Static friction: $f_s < \mu_s n$

(c) Stronger applied force, box just about to slide. Static friction: $f_s = \mu_s n$

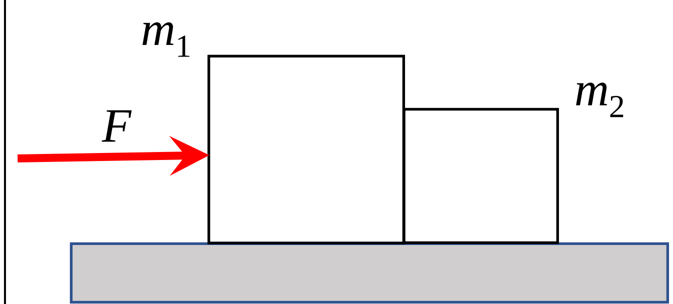
(d) Box sliding at constant speed. Kinetic friction: $f_k = \mu_k n$

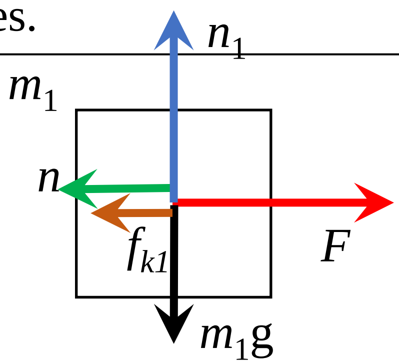
(e) Box at rest; static friction | Box moving; kinetic friction

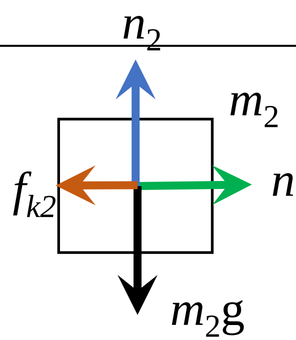
$f_s = T$
before "breaking loose"

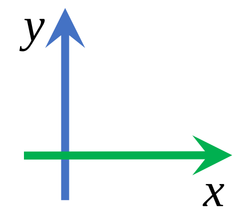
$$\left.\begin{array}{ll} f_s \leq \mu_s n & \rightarrow \text{no relative movement} \\ f_{s,\max} = \mu_s n & \rightarrow \text{interface "breaks loose"} \\ f_k = \mu_k n & \rightarrow \text{sliding with friction} \end{array}\right\} \mu_k \leq \mu_s$$

Example: Two boxes of masses m_1 and m_2 are pushed together by a horizontal force F to accelerate to the right on a leveled table surface as shown. The coefficient of kinetic friction between either of the boxes and the table surface is μ_k. Calculate the acceleration and the normal force between the boxes.









x-axis: $F - n - f_{k1} = m_1 a$

$f_{k1} = \mu_k m_1 g$

y-axis: $n_1 - m_1 g = 0$

$n_1 = m_1 g$

x-axis: $n - f_{k2} = m_2 a$

$f_{k2} = \mu_k m_2 g$

y-axis: $n_2 - m_2 g = 0$

$n_2 = m_2 g$

$F - f_{k1} - f_{k2} = m_2 a + m_1 a = (m_1 + m_2)a$

$a = (F - f_{k1} - f_{k2})/(m_1 + m_2)$
$= (F - \mu_k m_1 g - \mu_k m_2 g)/(m_1 + m_2)$

$n = f_{k2} + m_2 a = \mu_k m_2 g + m_2 a$
$= [m_2/(m_1 + m_2)]F$

Courtesy of Wenhao Wu

Additional Examples

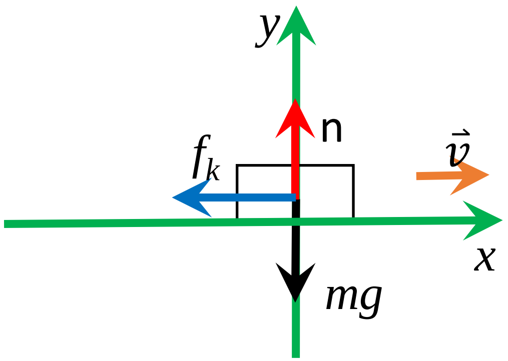
A box having an initial velocity 10 m/s enters a region of a leveled rough surface and slides a distance of 20 m before it reaches a stop. Assume that the acceleration due to friction is a constant. Calculate the coefficient of kinetic friction.

Solution:

- Initial velocity $v_1 = 10$ m/s; Final velocity $v_2 = 0.0$ m/s
- What is the acceleration?

$$v_2^2 - v_1^2 = 2ad \qquad a = (v_2^2 - v_1^2)/2d = -v_1^2/2d = -2.5 \text{ m/s}^2$$

- Free-body diagram and Newton's Second Law



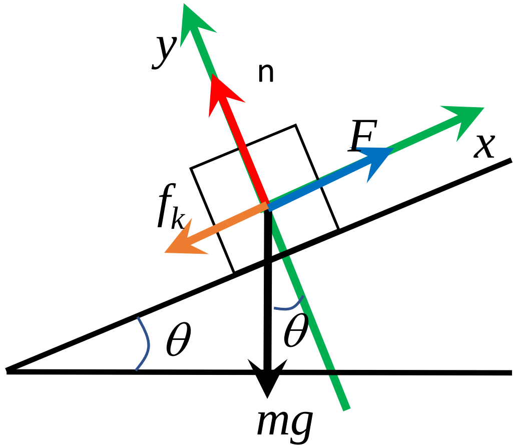
y-axis: $n - mg = 0$

x-axis: $-f_k = ma$

and $f_k = \mu_k n$

$$\mu_k = f_k/n = -ma/mg = -a/g = 2.5/9.8 = 0.26$$

Example: Box of mass m is pushed up an incline by a force F parallel to the incline. The coefficient of kinetic friction between the box and the incline is μ_k.



Given: F, m, μ_k, and θ

Find: n and a

Solutions:

Weight and its components $W = mg$ $W_x = -\, mg\sin\theta$ $W_y = -\, mg\cos\theta$

x-axis: $F - mg\sin\theta - f_k = ma$ $f_k = \mu_\mathrm{k} n = \mu_\mathrm{k} mg\cos\theta$

$a = (F - mg\sin\theta - \mu_\mathrm{k} mg)/m$

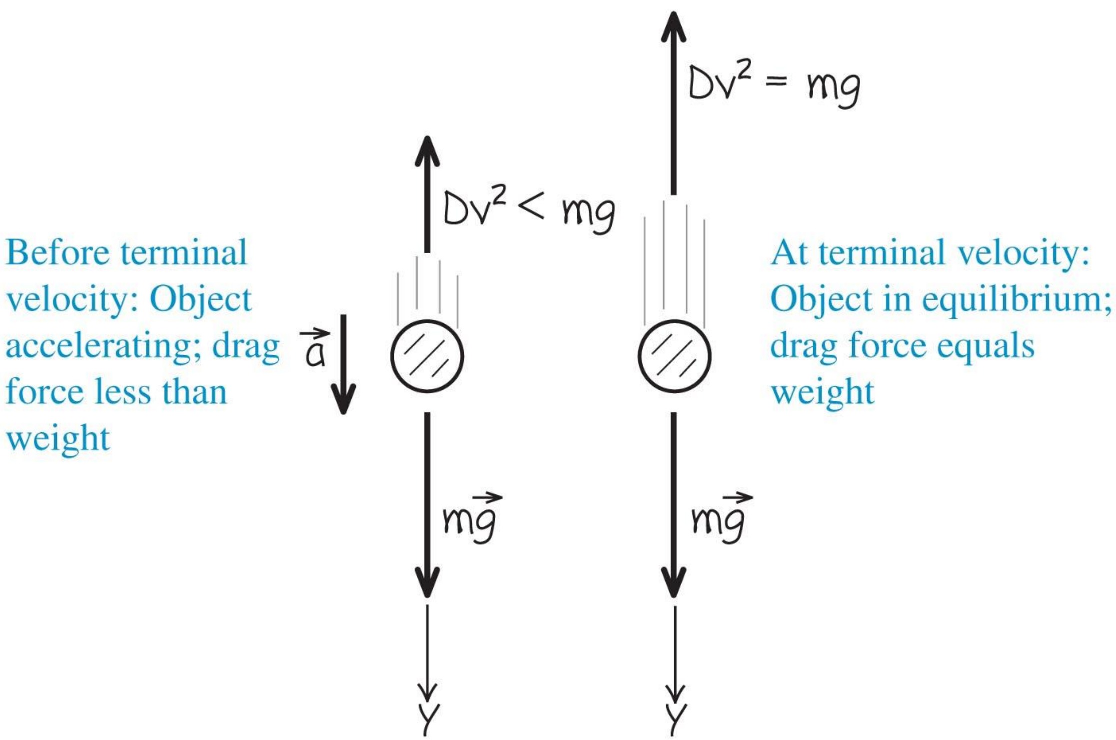
y-axis: $n - mg\cos\theta = 0$ $n = mg\cos\theta$

What if some other quantities are given and you are asked to calculate some different quantities?

Consider Problem 9 on Term Exam 1 from 2015.

Forces in Fluids – Figure 5.19

- This topic is fully developed in advanced courses.
- Conceptually, as objects fall through "thicker" liquids, they experience a drag force.



Clicker question

An adult and a teenager go skydiving. The adult weighs twice as much as the teen, but they use identical parachutes that exert identical drag forces. After the parachutes have opened and the two people have reached terminal speed, how do their speeds compare?

a) They are equal.

b) $v_{adult} = 2v_{teen}$.

c) $v_{adult} = \sqrt{2}v_{teen}$.

At terminal velocity
$mg - D\, v_T^{\ 2} = 0$

5.4 Elastic Force

Spring, Spring Restoring Force, and Hooke's Law

Hooke's Law on spring restoring force:

- In magnitude $F_{spr} = k\Delta L$
- Direction: Opposed to length change → restoring force
- A general expression taking care of the direction

$$F_{spr} = -kx$$

- x is measured with respect to the equilibrium position

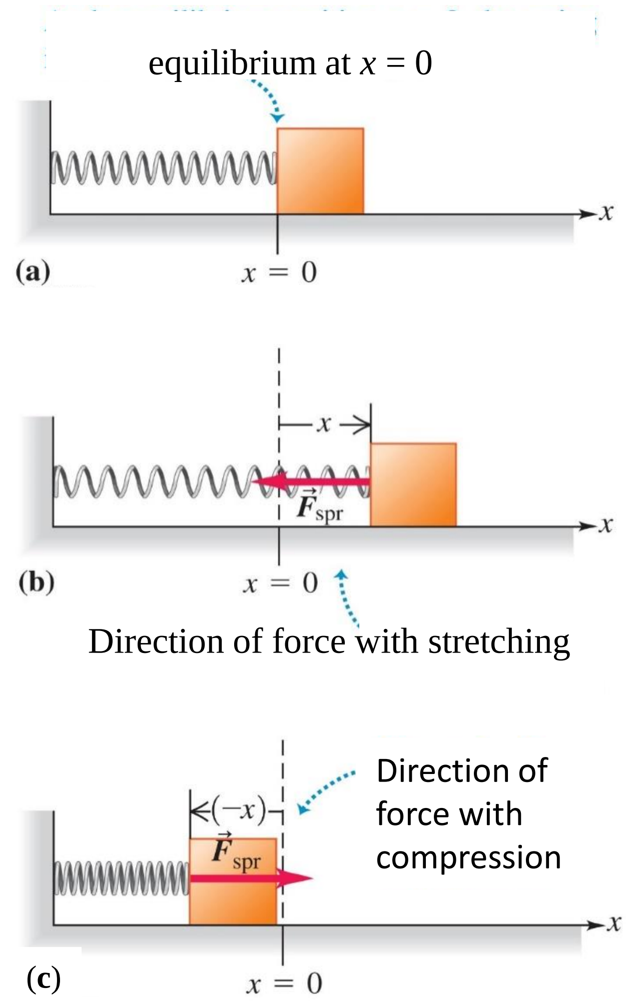
Example 5.14

A vertical spring balance scale stretches 1.00 cm when a 12.0 N weight is hung on it. If the 12.0 N weight is replaced by a 1.50 kg fish, by what amount is the spring stretched?

Answer:

Spring constant $k = F/\Delta L$ = (12.0 N)/(0.0100 m) = 1200 N/m

Stretching = $F/k = mg/k$ = 0.0123 m = 1.23 cm



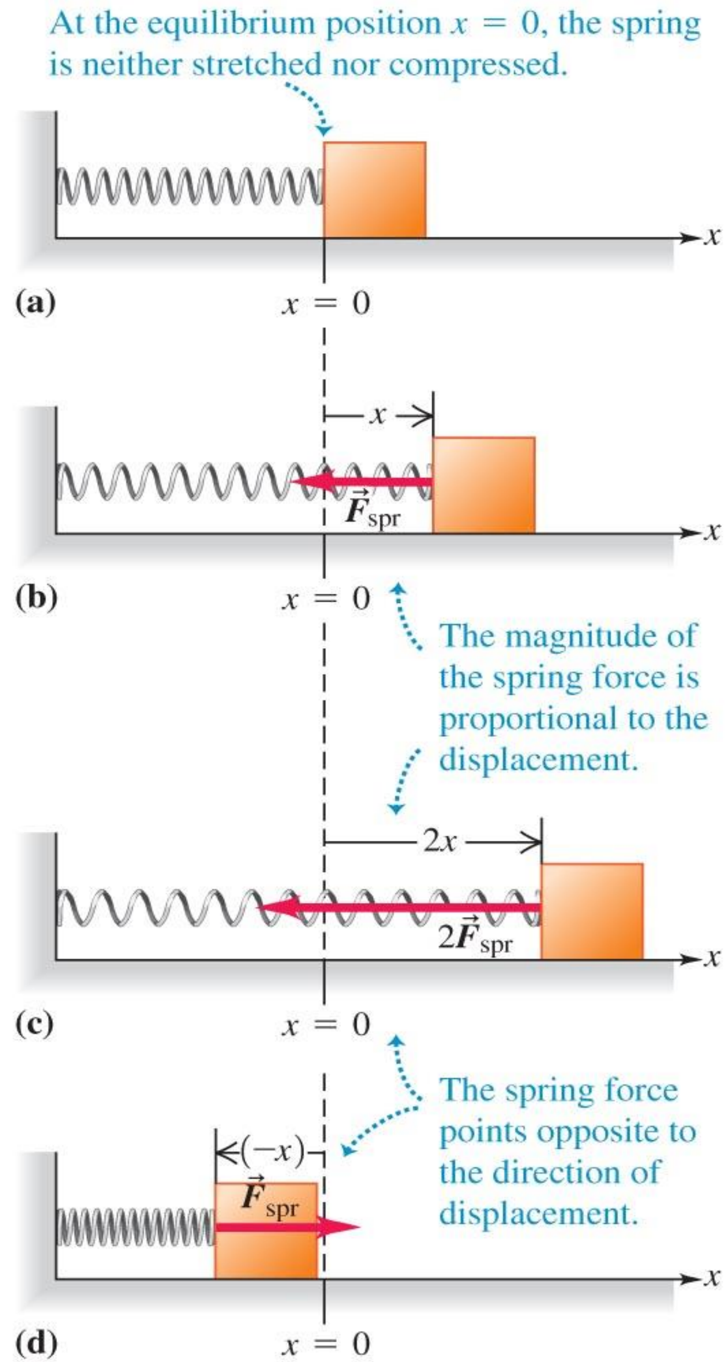
Courtesy of Wenhao Wu

Elastic Forces

- Springs or other elastic material will exert force when stretched or compressed.
- The magnitude of the spring force F_{spring} is given by Hooke's Law:

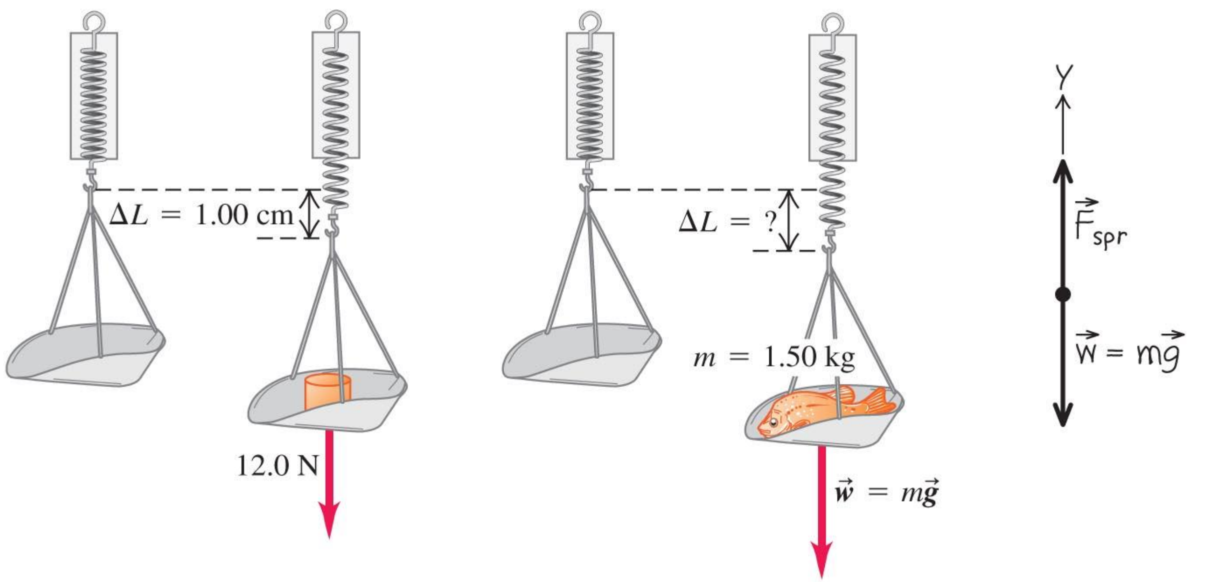
$$F_{\text{spring}} = k\Delta L$$

Where k is spring constant [N/m] and ΔL [m] is distance the spring is stretched or compressed from its equilibrium length.



Stretch a Spring to Weigh Objects – Example 5.14 Fishy business

- The force settings on the spring are calibrated with mass standards at normal earth gravity.
- The spring scales are often calibrated in force (N) and mass (kg).

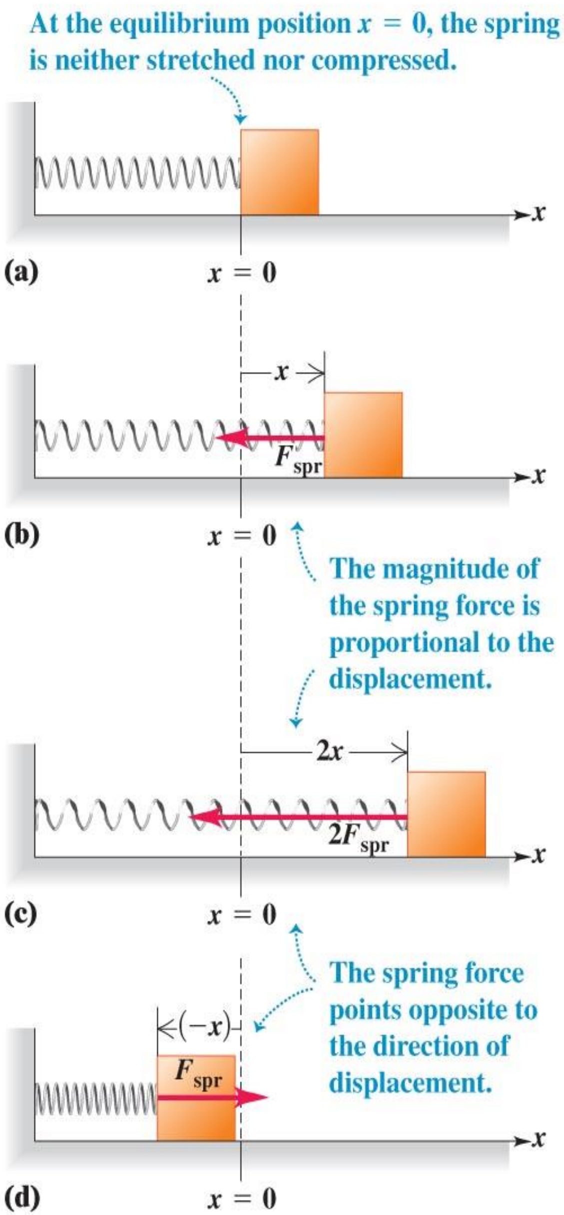


(a) The scale stretched by a known weight

(b) The scale stretched by a known mass

(c) Free-body diagram for the fish

Elastic forces



Hooke's Law: $F_{spr} = -\mathrm{kx}$,

where $k = spring\ force\ constant; dimension \left[\frac{N}{m}\right]$

<u>Example 5-14:</u>

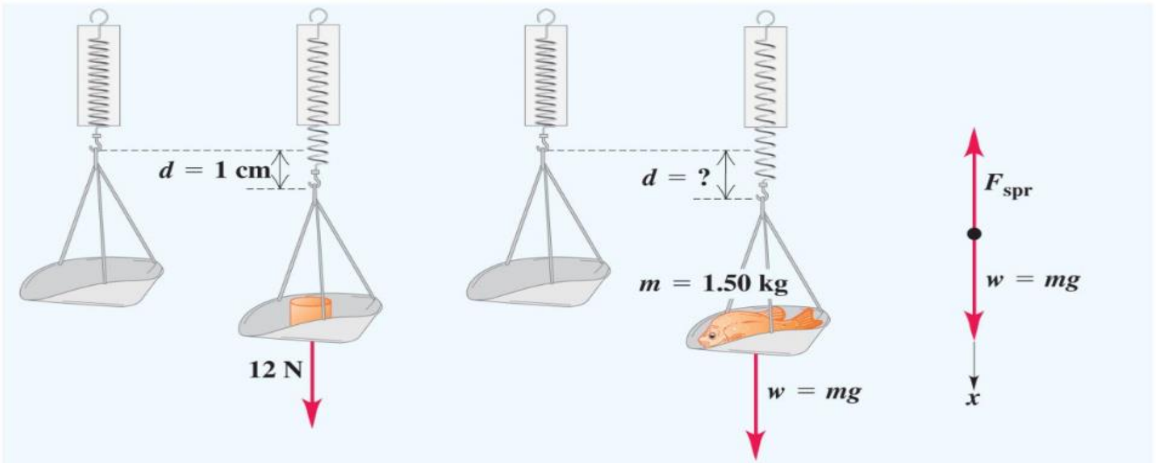
The spring stretches 1 cm for a 12N weight. When the 12N weight is replaced with 1.5kg fish; what distance does the spring stretch ?

$$k = -\frac{-12N}{1x10^{-2}m} = 1200\frac{N}{m}$$

$$w_{fish} = mg = 1.5kg * 9.8\frac{m}{s^2} = 14.7N$$

$$\sum F_x = 0 = -mg + F_{spr} \rightarrow mg - kx = 14.7N - 1200x = 0$$

$$\text{So, } x = \frac{14.7}{1200}m = 1.23\ cm$$



(a) The scale stretched by a known weight

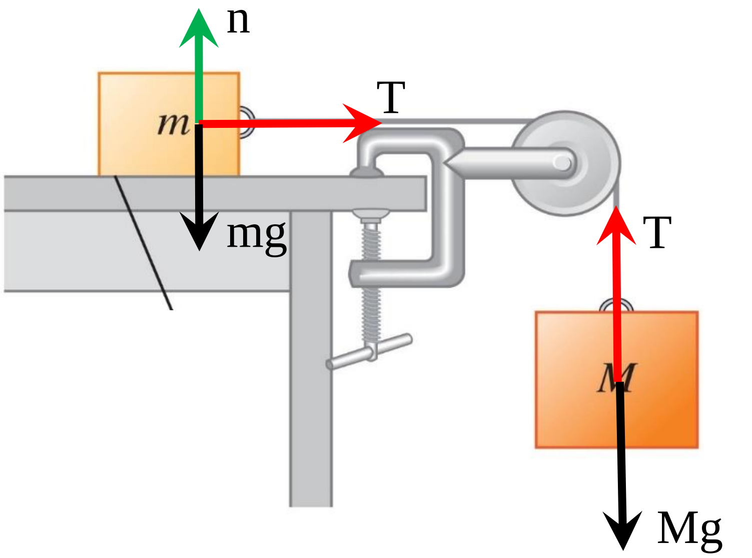
(b) The scale stretched by a known mass

(c) Free-body diagram for the fish

Example with massless and non-stretchable rope & massless and frictionless pulley.

Given: m and M.

Answer the questions for the following two cases.



(a) If the table top is frictionless.

Question 1: what is the condition under which M accelerates downward?

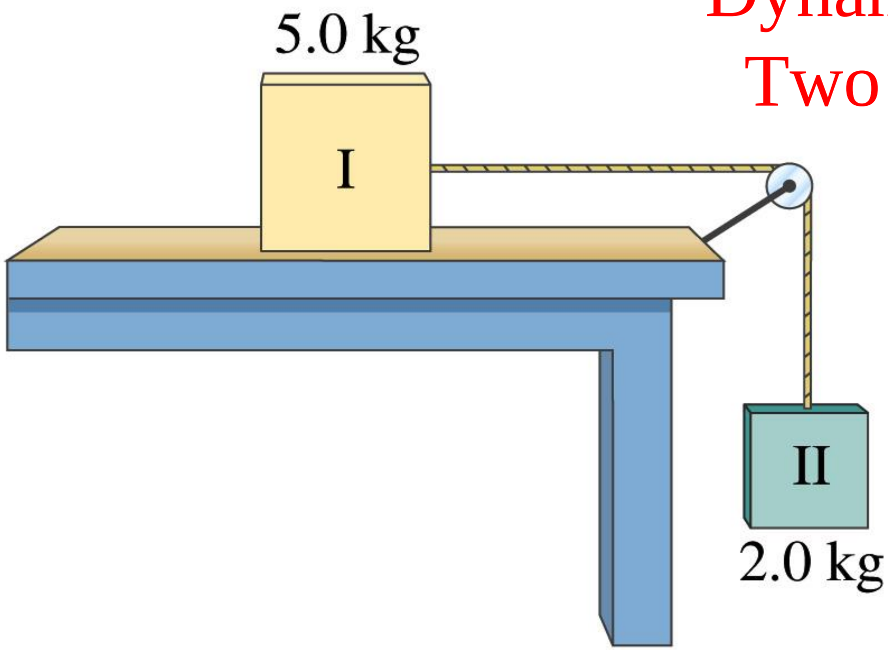
Question 2: If M accelerates downward, calculate the acceleration a and the tension in the rope T.

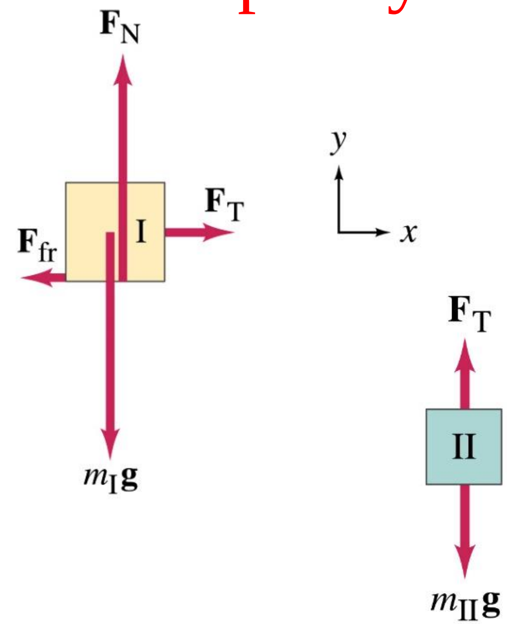
(b) Assume the coefficient of friction between m and the table top to be to be $\mu_s = \mu_k = \mu$.

Question 1: what is the condition under which M accelerates downward?

Question 2: If M accelerates downward, calculate the acceleration a and the tension in the rope T.

Dynamic motion problem: Two boxes and a pulley





Two boxes and a pulley with friction $\mu_k = 0.2$

(a) What is the tension in the rope?

(b) What is the tension in the rope?

Y: $F_N = m_1 g = 49N$ and $F_{fr} = \mu_k F_N = 0.2 * 49 = 9.8N$

$$F_T - F_{fr} = m_1 a \rightarrow F_T = F_{fr} + m_1 a \qquad \textit{(box I)}$$

$$m_2 g - F_T = m_2 a \rightarrow F_T = m_2(g - a) \qquad \text{(box II)}$$

→ $m_1 a + m_2 a = m_2 g - F_{fr}$

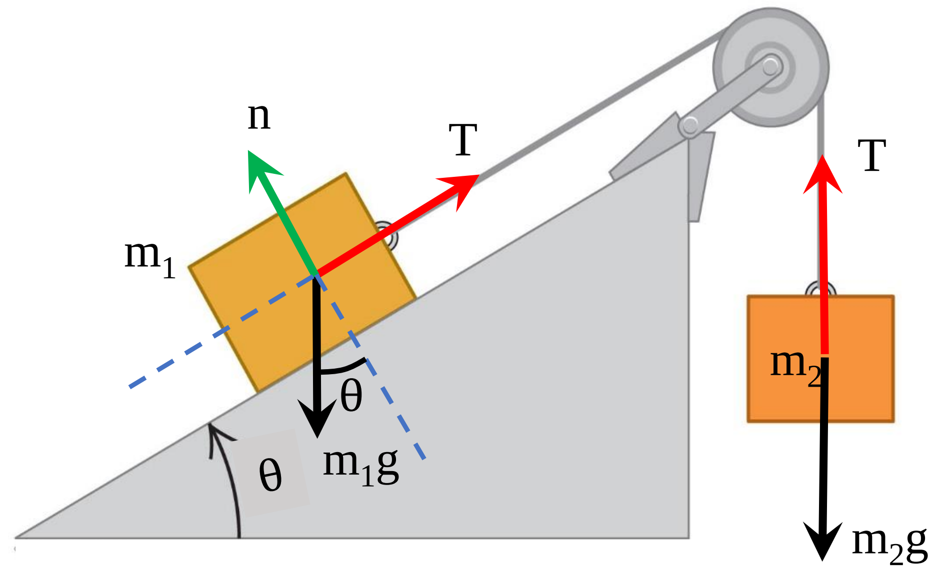
$$(a)\ a = \frac{m_2 g - F_{fr}}{m_1 + m_2} = 1.4\,\frac{m}{s^2}$$

$$\text{(b) } F_T = m_2(g - a) = 17N$$

Example with massless and non-stretchable rope & massless and frictionless pulley.

Given: m_1, m_2, and θ.

Answer the questions for the following two cases.



(a) If the incline is frictionless.

Question 1: what is the condition under which m_2 accelerates downward?

Question 2: If m_2 accelerates downward, calculate the acceleration *a* and the tension in the rope T.

(b) Assume the coefficient of friction between m_1 and the incline to be $\mu_s = \mu_k = \mu$.

Question 1: what is the condition under which m_2 accelerates downward?

Question 2: If m_2 accelerates downward, calculate the acceleration *a* and the tension in the rope T.

A box rests on a frozen pond (assume it to be frictionless and horizontal). If a fisherman applies a horizontal force with a magnitude 48N to the box and produces an acceleration of 3 m/s^2 , what is the box's mass?

Solution: This problem is a an application of Newton's second law, F = ma. Here we are given the force (48N) and the acceleration (3 m/s^2). To find the mass we re-arrange the equation to solve for m.

$$\frac{F}{a} = \frac{48}{3} = 16 = m$$

m= 16 kg

A table with a weight equivalent to 800N is slid across a floor with friction. The pushing force has a magnitude of 100N. If the table moves with constant speed, the friction force must be…

Solution: Because we are told the speed is constant, this means that there is no acceleration (or more specifically, the acceleration is 0). This would indicate that the pushing force and friction force have canceled out completely, thus telling use that the friction force is equal in magnitude and opposite in direction to the pushing force. Thus, the friction force is <u>100N</u>.

Two boxes connected by a rope are pulled across a horizontal floor. There is friction between the floor and the boxes. Which way would the force due to friction point in each of the box's free-body diagrams?

Solution: Friction always points opposite the direction of motion.

Two figure skaters, one weighing 625N and the other 725N, push off against each other on frictionless ice. If the lighter skater travels at 1.74 m/s, how fast will the heavier one travel?

Solution: There are many ways to solve this problem, but the simplest way is to use conservation of energy & Newton's 3rd law. If the skaters were initially at rest, and then moved by pushing off against each other (over a frictionless surface), their kinetic energies will be equal to each other. Knowing their masses (indirectly from their weight) and one of the skater's velocity, we can then solve for the other skater's velocity.

$$\frac{1}{2}\left(\frac{\omega_1}{g}\right)v_1^2 = \frac{1}{2}\left(\frac{\omega_2}{g}\right)v_2^2$$

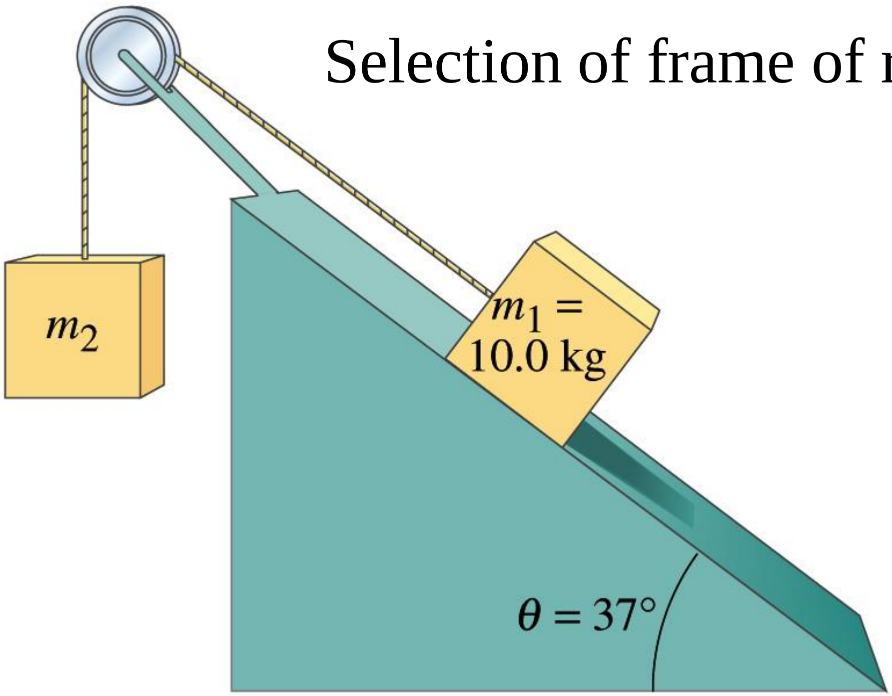
Canceling the common terms (such as the ½ and g), and then solving for v_2, we get:

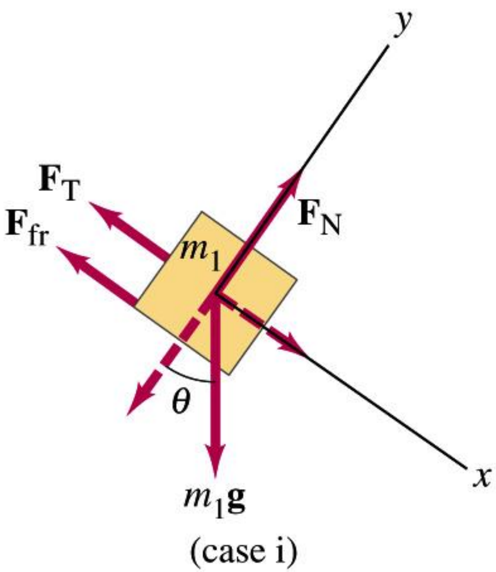
$$\sqrt{\frac{w_1 v_1^2}{w_2}} = \sqrt{\frac{625(1.74)^2}{725}} = 1.6 = v_2 \qquad v_2\text{=1.6 m/s}$$

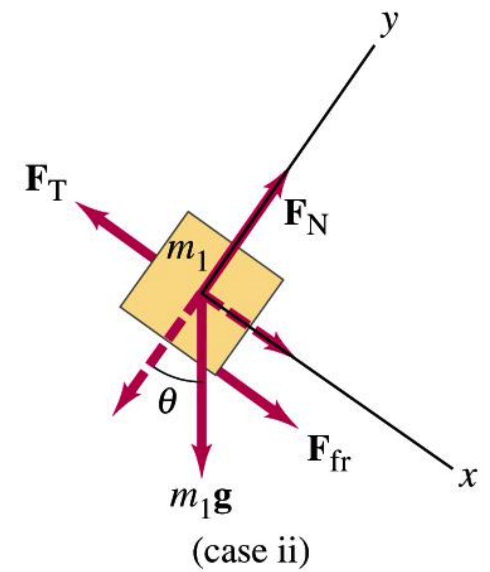
Thanks for your attention

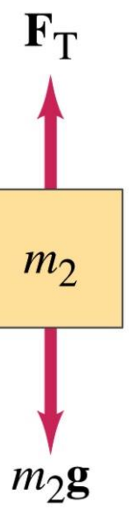
A plane, a pulley and two boxes

Selection of frame of reference for each object – up to us!



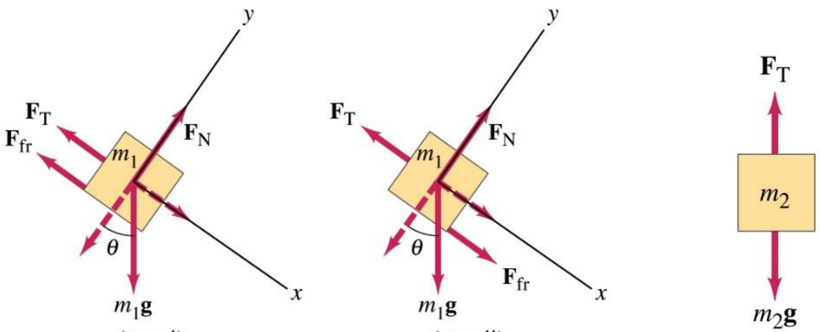




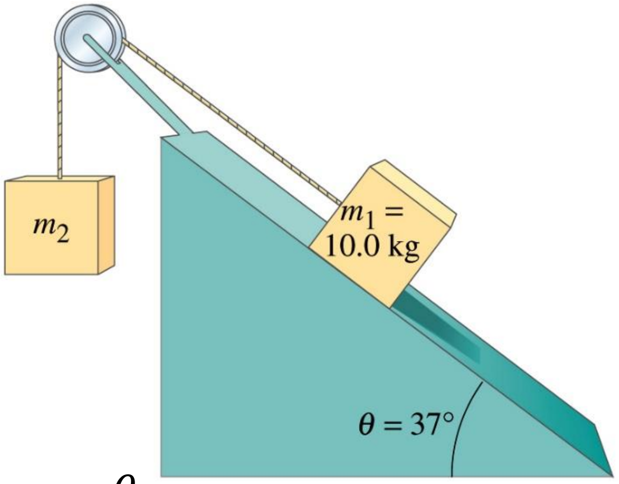


What can be the mass of m_2 for this system, so that it doesn't move?

Inclined plane with two boxes



(case i) (case ii)



Two boxes and a pulley with friction $\mu_s = 0.4$ and $\theta = 37^o$

<u>y-motion:</u> $F_N - m_1 g cos\theta = m_1 a_y = 0 \rightarrow F_N = m_1 g cos\theta$

$$F_{fr} = \mu_s F_N = 0.4 * m_1 g cos37$$

<u>x-motion:</u> $m_1 g sin\theta - F_T - F_{fr} = m_1 a_x = 0 \quad \because a_y = 0 \;\; F_T = m_2 g$

Static case → $F_T = m_1 g sin\theta - F_{fr} = m_2 g$

$m_1 g sin\theta - \mu_s m_1 g cos\theta = m_2 g,$ divide by g

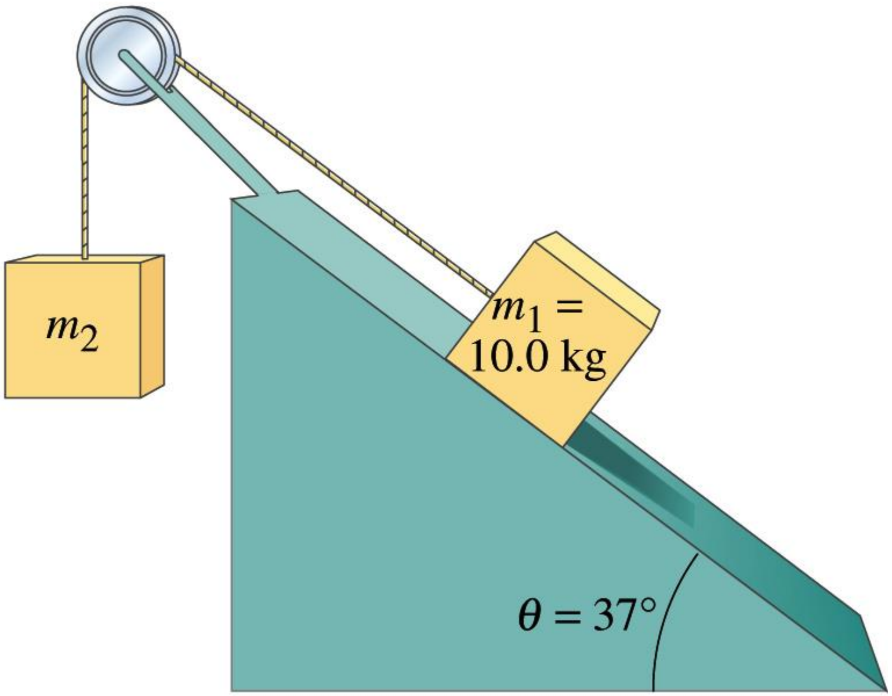
$$\rightarrow m_2 = m_1 sin\theta - \mu_s m_1 cos\theta = 2.8kg$$

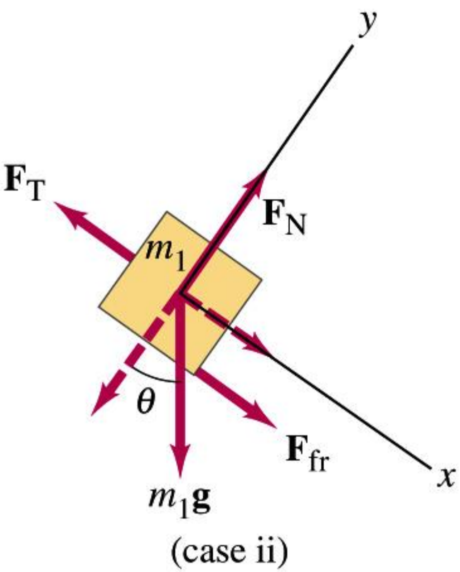
$$m_1 g sin\theta - F_T + F_{fr} = m_1 a_x = 0 \quad \because a_x = 0$$

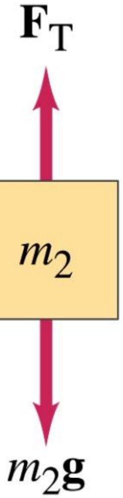
Static case → $F_T = m_1 g sin\theta + \mu_s m_1 g cos\theta = m_2 g$

$\because m_2 = \frac{F_T}{g} = 9.2kg$ $\quad\quad \because 2.8kg < m_2 < 9.2kg$

A plane, a pulley and two boxes







$\mu_k = 0.3, m_2 =$10kg and $\theta = 37^o$

m_2 will fall and m_1 will rise

Two boxes and an Inclined plane with kinetic friction

Given that $\sum F = ma;$

$$F_T - m_1 g sin\theta - \mu_k F_N = m_1 a \rightarrow$$

$$since\ m_2\ also\ accelerates \qquad F_T - m_2 g = -m_2 a$$

$$F_T = m_2(g - a)$$

$$m_1 a = m_2(g - a) - m_1 g sin\theta - \mu_k F_N \ \rightarrow F_N = m_1 g cos\theta$$

$$\because a = \frac{m_2 g - m_1 g sin\theta - \mu_k m_1 g cos\theta}{m_1 + m_2} = 0.079g = 0.78\frac{m}{s^2}$$

m2>>m1 a->g

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