Chapter 6 Circular Motion and Gravitation

- To understand the dynamics of circular motion.
- To study the application of circular motion as it applies to Newton's law of gravitation.
- To examine the idea of weight and relate it to mass and Newton's law of gravitation.
- To study the motion of objects in orbit (satellites) as a special application of Newton's law of gravitation.

Courtesy of Wenhao Wu

In Section 3.4

- We studied the kinematics of circular motion.
  - Centripetal acceleration
  - Changing velocity vector
  - Uniform circular motion
- We acquire new terminology.
  - Radian
  - Period (*T*)
  - Frequency (*f*)

How many degrees are in one radian ?

$\boldsymbol{\theta = \frac{S}{r}} \rightarrow$ ratio of two lengths (dimensionless)

$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi \, rad \cong 360°$$

$$1 \, rad \cong \frac{360°}{2\pi} = \frac{360°}{6.28} = 57° \therefore$$

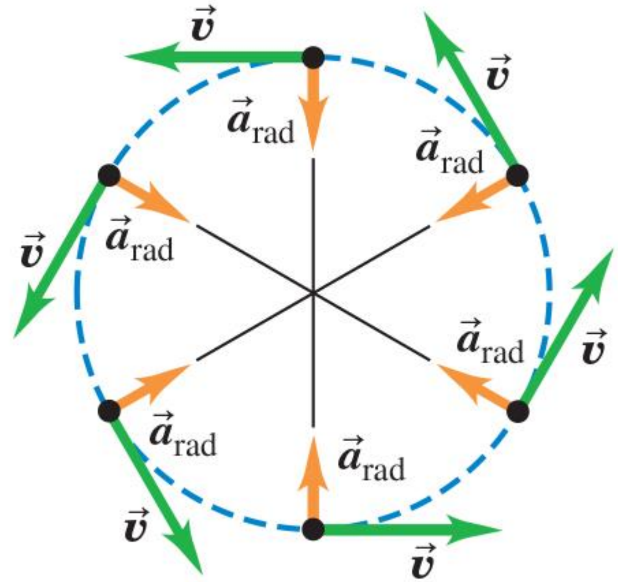
Factors of unity $\frac{1\,rad}{57°}$ or $\frac{57°}{1\,rad}$

1 radian is the angle subtended at the center of a circle by an arc with length equal to the radius.

Velocity Changing from the Influence of a_{rad} – Figure 6.1

- A review of the relationship between υ and $\boldsymbol{a}_{rad}$.
- The velocity changes direction, not magnitude.
- The magnitude of the centripetal acceleration is:

$$a_{rad} = \frac{\upsilon^2}{R}$$



- In terms of the speed and period (time to make one complete revolution)

$$\upsilon = \frac{2\pi R}{T} \quad \Rightarrow \quad a_{rad} = \frac{4\pi^2 R}{T^2}$$

A wheel with radius 0.5m is rotating at a constant angular speed of 3 rad/s. What is the linear speed of a point on the rim of the wheel?

Solution: In this question the relationship between angular velocity and linear velocity must be known. From the definition of angular speed, it is the number of radians per second. This is given as:

$$\omega = \frac{2\pi}{T}$$

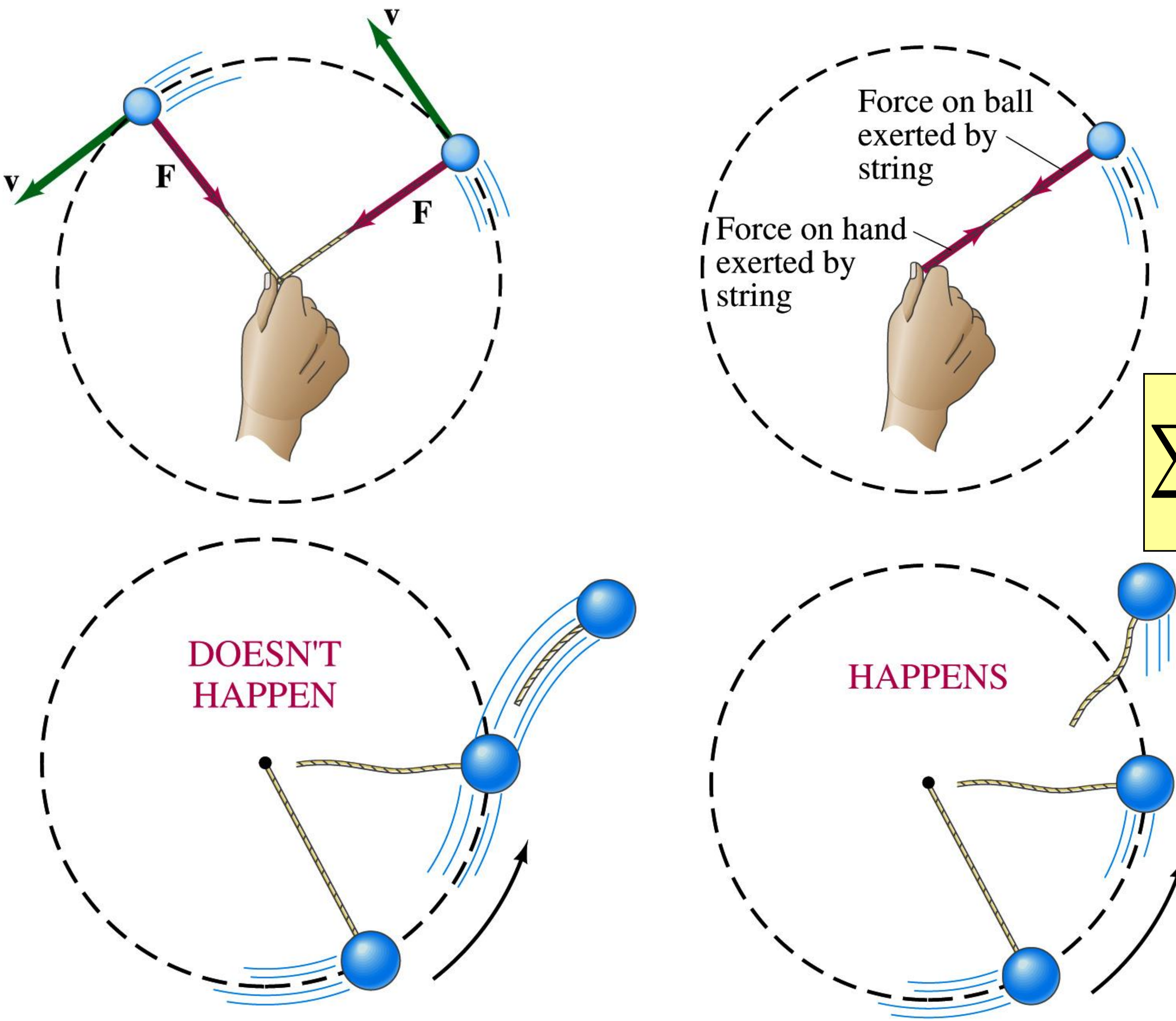
Where T is the period of rotation (i.e. the amount of time it takes for one full rotation). We see from the formula sheet that the relation for linear velocity has these values in it, giving the relationship & answer:

$v = \frac{2\pi R}{T} = \omega R = (3)(0.5) = 1.5$ $\qquad$ $v = 1.5$ m/s

1 rad=360^0 / 2π =57.3^0

$1\ rev/s = 2\pi$ rad/s

Circular motion and Gravitation



$$\sum F_R = ma_R = m\frac{v^2}{R}$$

A Review of Uniform Circular Motion----Section 3.4

Velocity: tangent to the circle with constant magnitude $v_1 = v_2 = v$

Acceleration: $a_{\text{rad}} = \frac{v^2}{R}$, always pointing toward the center of the circle.

known as the centripetal acceleration

<u>A new concept</u>

Period (T, in s): Time for the object to complete one circle.

<u>New relationships</u> between v, R, and T:

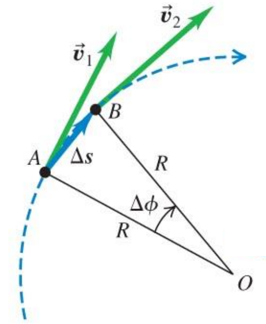
Velocity $$v = \frac{2\pi R}{T}$$

Acceleration: $$a_{\text{rad}} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$$

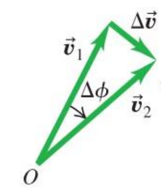
<u>Another new concept</u>

Frequency (f, in s^{-1}, or, Hz): revolutions per second.

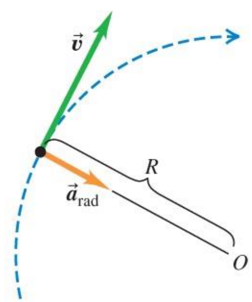
Relationship between T and f: $f = 1/T$



(a) A point moves a distance Δs at constant speed along a circular path.



(b) The corresponding change in velocity



(c) The acceleration in uniform circular motio
always points toward the center of the circle.

6.1 Force in Circular Motion

Question: How can an object of mass *m* maintain its centripetal acceleration?

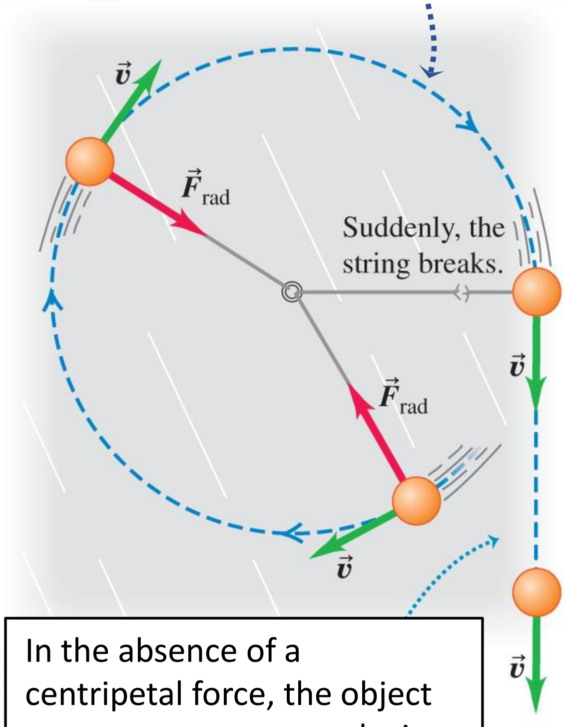
Answer: A centripetal force (pointing to the center of the circle) must act on the object to maintain the centripetal accelertion.

The magnitude of the net centripetal force:

$$F_{net} = F_{rad} = m\frac{v^2}{R}$$

Note: $m\frac{v^2}{R}$ itself is not a force. It is equal to F_{rad}.

With a centripetal force provided by a string, the object moves along a circular orbit



In the absence of a centripetal force, the object moves at a constant velocity

Courtesy of Wenhao Wu

Example 6.1 Model Airplane on a String

Given: mass m = 0.500 kg
radius R = 5.00 m
period T = 4.00 s.

Find: Tension force in the string, F_T.

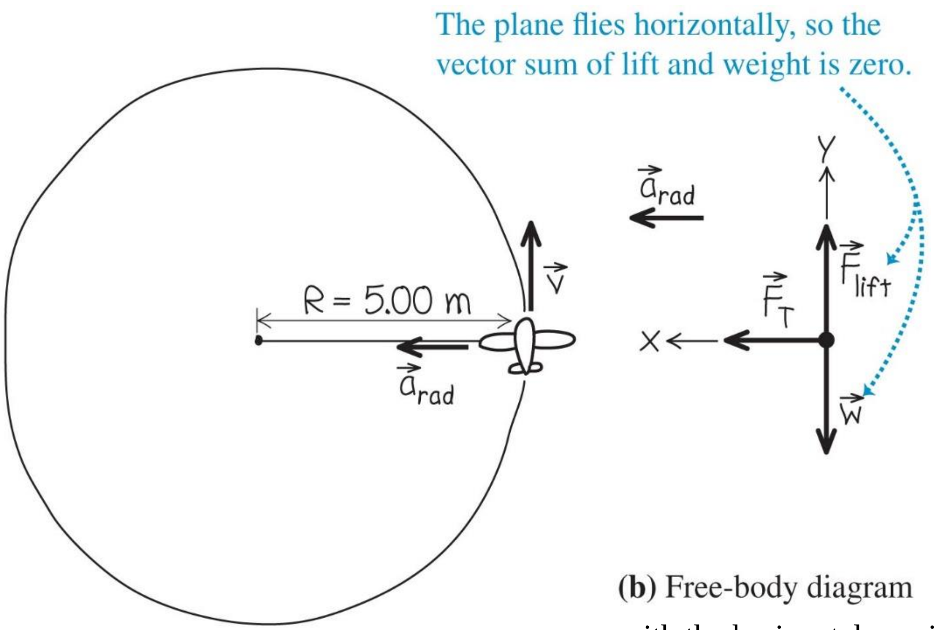
$$\sum F_x = ma_{rad},$$

$$F_T = m\frac{v^2}{R}$$

$$\sum F_y = 0,$$

$$F_{lift} + (-mg) = 0$$

with $$v = \frac{2\pi R}{T}$$



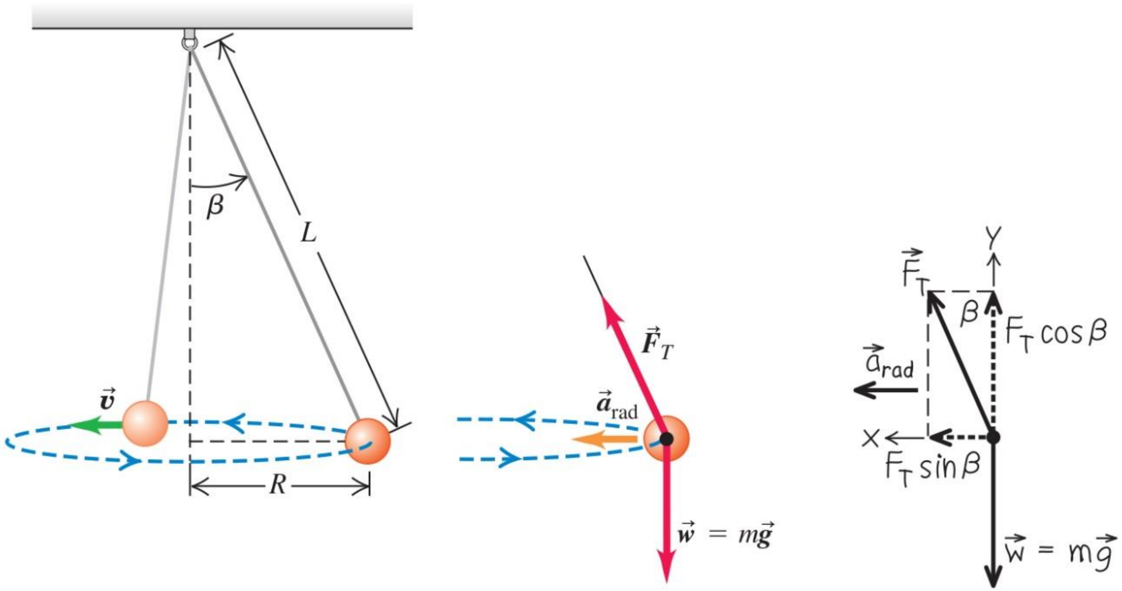
(a) Sketch (from above)

(b) Free-body diagram with the horizontal x-axis pointing toward the center of the circle and the y-axis pointing up vertically

Courtesy of Wenhao Wu

A Tether Ball Problem – Example 6.2

- Refer to the worked example on page 156.



(a) The situation **(b)** The forces on the ball **(c)** Free-body diagram of the ball

$$\upsilon = \frac{2\pi R}{T}$$

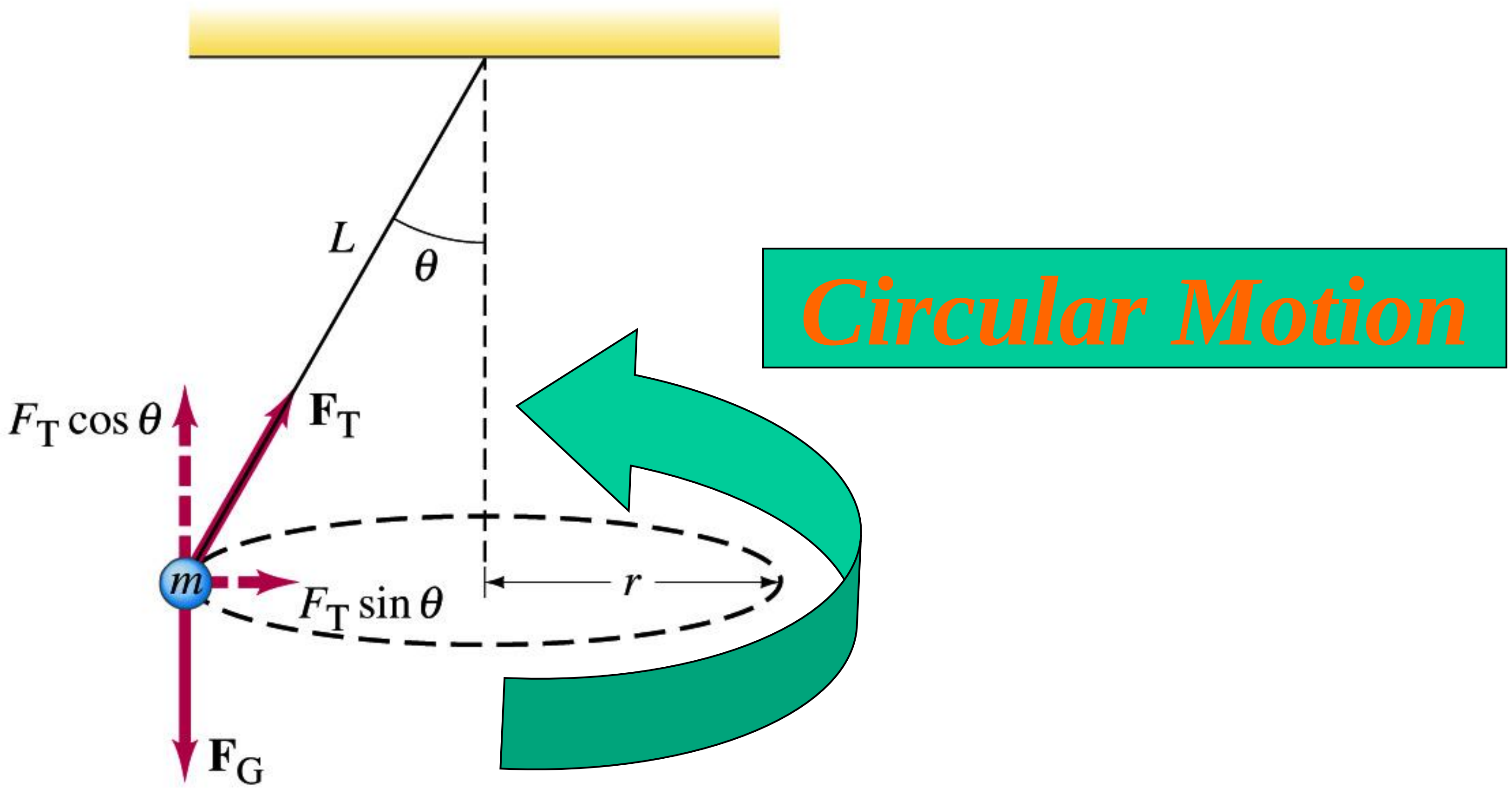
$$R = L\sin\beta$$

$$\begin{cases} \sum F_x = ma_{rad}, & F_T \sin\beta = m\frac{\upsilon^2}{R} = m\frac{4\pi^2}{T^2} \\ \sum F_y = 0, & F_T\cos\beta + (-mg) = 0 \end{cases}$$

Conical Pendulum Tether Ball

Problem – Example 6.2

Center-seeking Force: Tension



Example 6.2 A Tether Ball Problem

Given: mass m,
length of string L
period T

Find: Tension in the string, F_T.
Angle with vertical, β

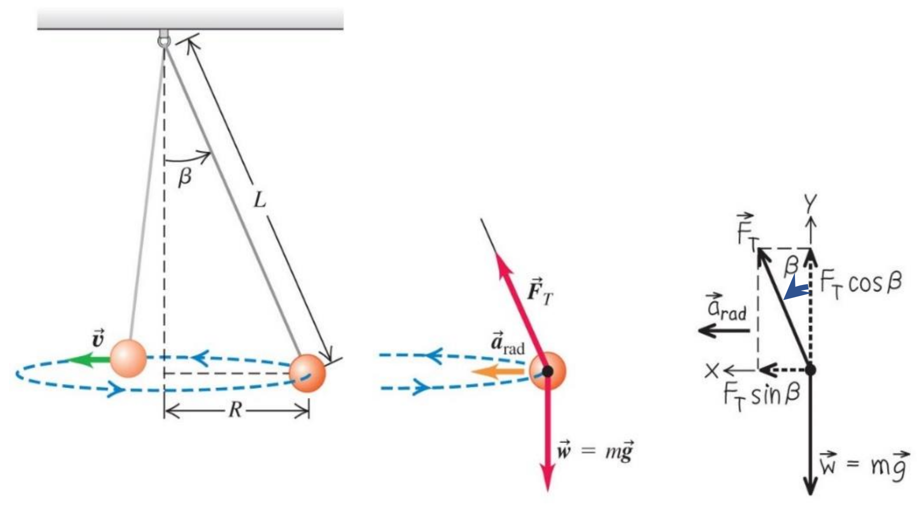
$\sum F_x = ma_{rad}$, $F_T sin\beta = m\frac{v^2}{R} = m\frac{4\pi^2}{T^2}$

$\sum F_y = 0$, $F_T cos\beta + (-mg) = 0$

with $v = \frac{2\pi R}{T}$

and

$R = Lsin\beta$

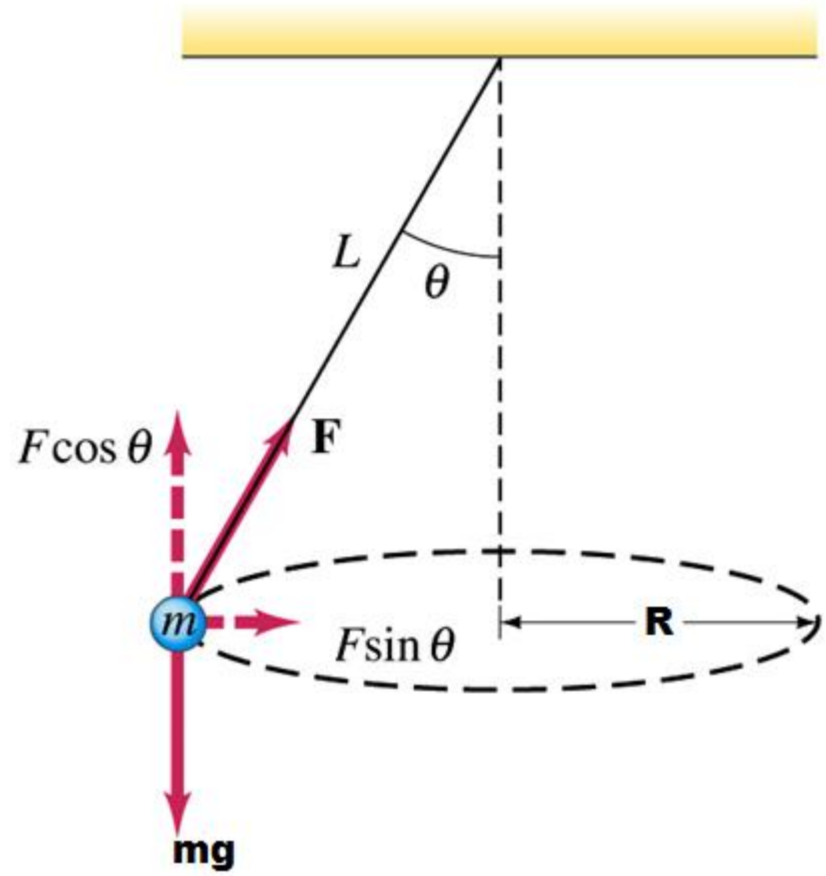


(a) The situation (b) The forces on the ball (c) Free-body diagram of the ball

$$F_T sin\beta = m\frac{v^2}{R} = m\frac{4\pi^2 R}{T^2} = m\frac{4\pi^2 Lsin\beta}{T^2}$$

$F_T = m\frac{4\pi^2 L}{T^2}$ and $cos\beta = \frac{gT^2}{4\pi^2 L}$

Conical Pendulum



What is the period of this conical pendulum ?

$$a_r = \frac{v^2}{R}, \; R = L\sin\theta \text{ and } T = \frac{2\pi R}{v}$$

So; $a_r = (\frac{2\pi R}{T})^2 \frac{1}{R} = \frac{4\pi^2 R}{T^2}$

$$\sum F_y = ma_y = F\cos\theta - mg = 0$$

$$\rightarrow F\cos\theta = mg \rightarrow F = \frac{mg}{\cos\theta}$$

$$\sum F_x = ma_x = ma_r = F\sin\theta = \frac{mv^2}{R} = \frac{mg\sin\theta}{\cos\theta}$$

$$\therefore a_r = g\tan\theta$$

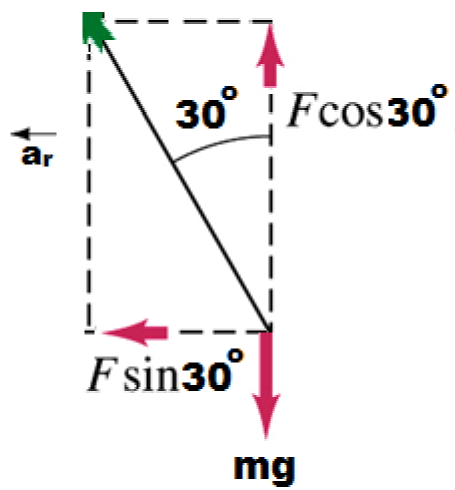
So; $a_r = \frac{g\sin\theta}{\cos\theta} = \frac{4\pi^2 R}{T^2} \rightarrow \tan\theta = \frac{4\pi^2 R}{T^2 g} = \frac{4\pi^2 L\sin\theta}{T^2 g}$

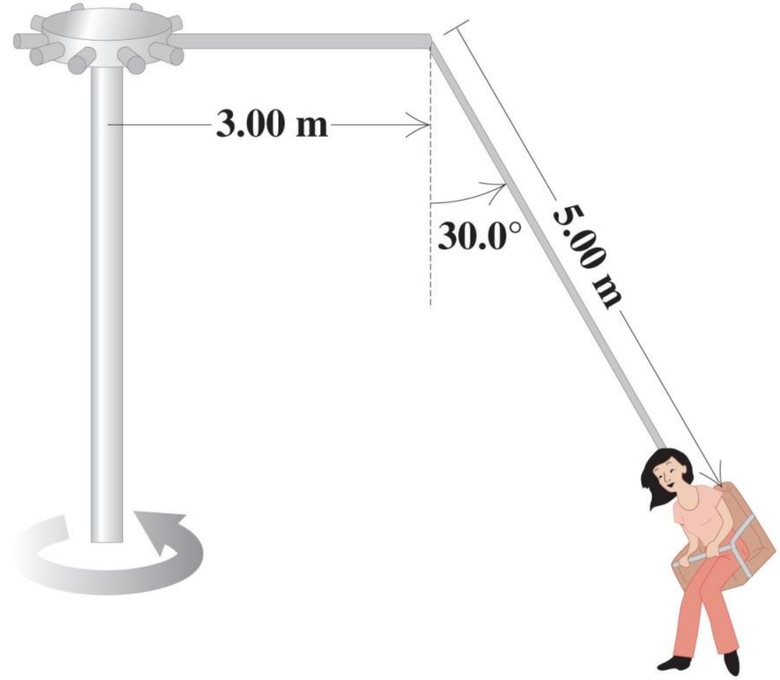
$\therefore T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$ (the period is independent of mass)

All information is in the equations in red

6.5

From a Brazos county fair





<u>The "Giant Swing":</u>

a) Make a free body diagram of the seat including the person on it.
b) Find the time for one revolution for the indicated angle of 30°
c) Does the angle depend on the weight of the passenger?

a) The person moves on a radius of **R=3+5sin30=5.5m**

$$a_r = \frac{v^2}{R} \text{ and } T = \frac{2\pi R}{v}$$

b) $\sum F_y = ma_y \rightarrow F\cos 30 = mg \rightarrow F = \frac{mg}{\cos 30}$

$$\sum F_x = ma_x \rightarrow F\sin 30 = \frac{mv^2}{R} = \frac{mg\sin 30}{\cos 30}$$

$$v = \sqrt{Rg\tan 30} = \sqrt{5.5 * 9.8 * tan30} = 5.58\frac{m}{s}$$

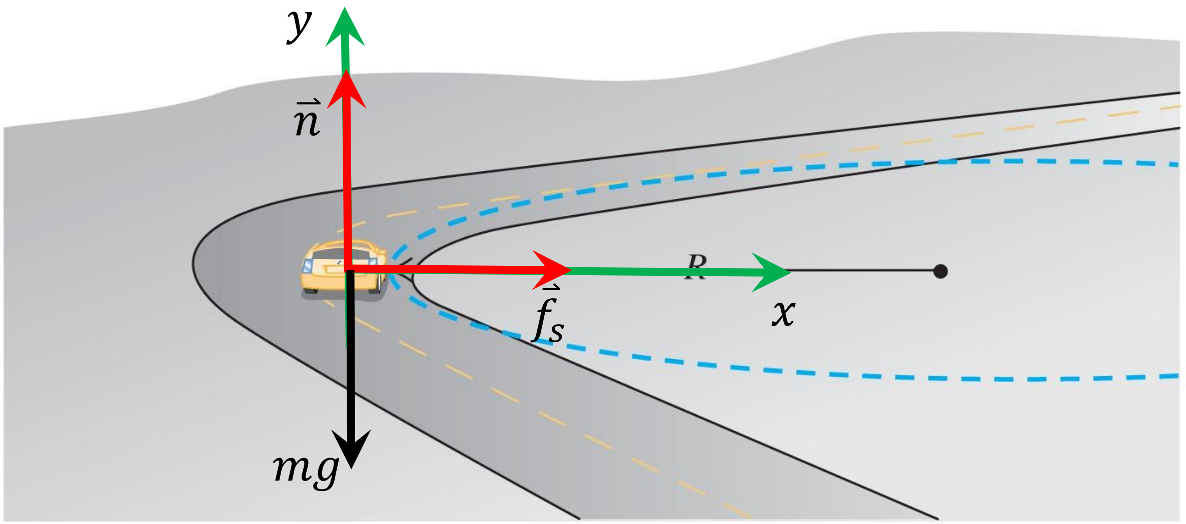
$$T = \frac{2\pi 5.5}{5.58} = 6.19\ s$$

c) The net force is proportional to mass that divides out in $\vec{F} = m\vec{a}$ .The angle is independent of mass.

Example 6.3
Rounding a Flat Curve, page 157

Given: radius $R = 250$ m
$\mu_s = 0.90$

Find: v_{max}



Solution:

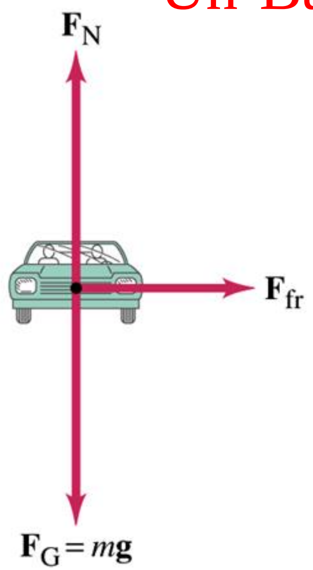
The source of centripetal force is static friction force provided by the tires.

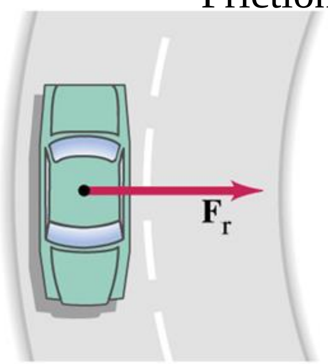
$$\left.\begin{aligned} \sum F_x &= ma_{rad}, & f_s &= m\frac{v^2}{R} \\ \sum F_y &= 0, & n + (-mg) &= 0 \end{aligned}\right\} \quad \text{with } f_{s,max} = \mu_s n = \mu_s (mg)$$

$$\Rightarrow \quad v_{max} = \sqrt{\mu_s g R}$$

Equalize the maximum friction force to the force required by circular motion $(m/R)\, v_{max}{}^2$

Un-Banked Curve





Given: $r = 50\,m$, m= $1000\,kg$, $v = 14\frac{m}{s}$ and $\mu_s = 0.6$

Friction force is larger than radial force

$$F_{fr} > F_r \text{ (no skidding)}$$

Friction force is smaller than radial force

$$F_{fr} < F_r \text{ (skidding)}$$

$$F_N = |-F_G| = mg = 1000 * 9.8 = 9800\,N$$
$$F_{fr} = \mu_s F_N = 0.6 * 9800 = 5900\,N$$
$$F_r = ma_r = \text{m}\frac{v^2}{r} = 1000 * \frac{14^2}{50} = 3900\,N$$

So, no skidding at $\mu_s = 0.6$

By changing friction to $\mu_s = 0.25$

$F_{fr} = \mu_s F_N = 0.25 * 9800 = 2500\,N < F_r$ (Skidding)

The maximum safe velocity

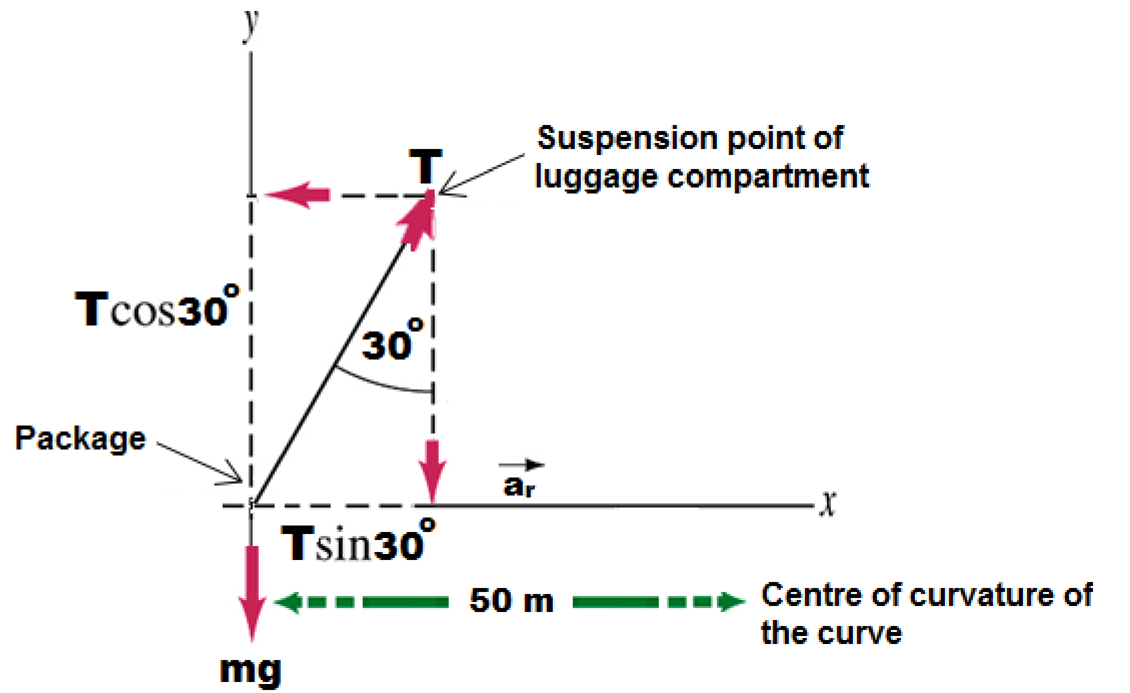
$$\sum F_y = ma_y = F_N = mg = 0 \rightarrow F_N = mg$$
$$\sum F_x = ma_x = ma_r = \text{m}\frac{v^2}{r}$$
$$F_{fr,max} = \mu_s F_N = \mu_s mg = m\frac{v_{max}^2}{r}$$
$$\therefore v_{max}^{safe} = \sqrt{\mu_s g r}$$

6-52 . As the bus rounds a flat curve at constant speed, a package suspended from the luggage rack on a string makes an angle with the vertical as shown.

What is the velocity of the bus?

What is the radial acceleration?

What is the radius of the curve?



$$\sum F_y = ma_y = 0 \rightarrow T\cos 30 - mg = 0$$

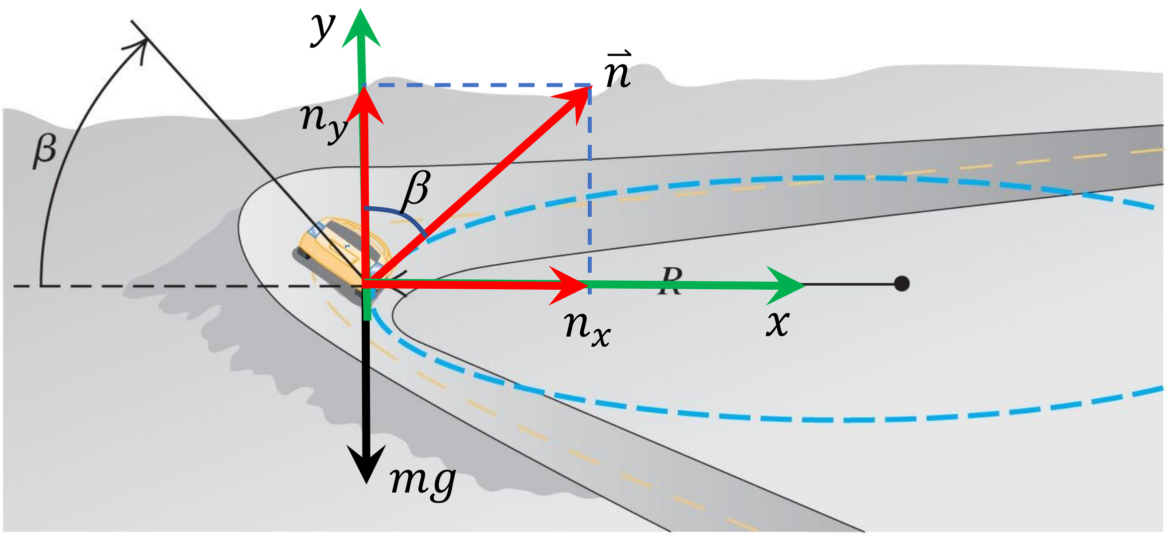
$$\therefore T = \frac{mg}{\cos 30}$$

$$\sum F_x = ma_x \rightarrow T\sin 30 = \mathrm{m}\frac{v^2}{r}$$

$$\therefore v = \sqrt{\frac{rT\sin 30}{m}} = \sqrt{\frac{mg}{\cos 30} * \frac{r\sin 30}{m}} = \sqrt{gr\tan 30}$$

$$\rightarrow v = \sqrt{9.8 * 50 * tan30} = 16.8\frac{m}{s}$$

Example 6.4 Rounding a Banked Curve, page 158



No need to rely on friction. The horizontal component of the normal force is the source of the centripetal force.

Given: radius R = 250 m

Design: v_{max} = 25 m/s

Find: β

$\sum F_x = ma_{rad}$, $n\sin\beta = m\frac{v^2}{R}$

$\sum F_y = 0$, $n\cos\beta + (-mg) = 0$

→

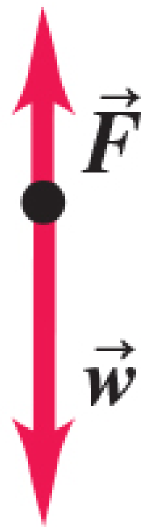
$n\sin\beta = m\frac{v^2}{R}$

$n\cos\beta = mg$

→ $\tan\beta = \frac{v^2}{gR}$

Rounding a Banked Curve – Example 6.4

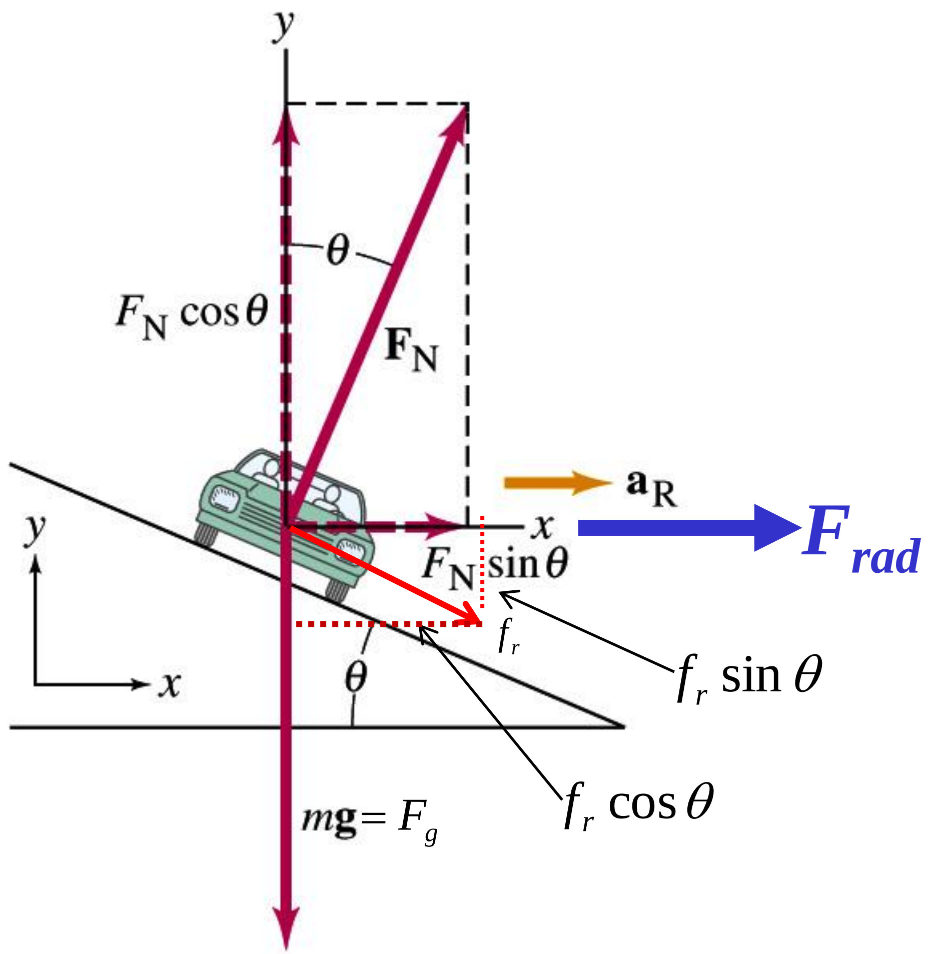
- The centripetal force comes only from a component of normal force



$$\left.\begin{aligned} \sum F_x &= ma_{\text{rad}}, \qquad n\sin\beta = m\frac{v^2}{R} \\ \sum F_y &= 0, \qquad n\cos\beta + (-mg) = 0 \end{aligned}\right\} \quad \tan\beta = \frac{v^2}{gR}$$

Pearson

No Skidding on Banked Curve



No Skidding on Banked Curve

The key to this problem is to realize that the net force F_{net} causes the car to move along the curve.

$$F_N \sin\theta + f_r \cos\theta = F_{net}$$

$$F_N \cos\theta - F_g - f_r \sin\theta = 0 \;\therefore F_g = mg \text{ and } f_r = \mu_s F_N$$

$$F_{net} = F_{rad} = ma_{rad} = \mathrm{m}\frac{v^2}{r}$$

Use;

$$\sum F_y = ma_y = 0$$

$$\sum F_x = ma_x = ma_{rad} = \mathrm{m}\frac{v^2}{r}$$

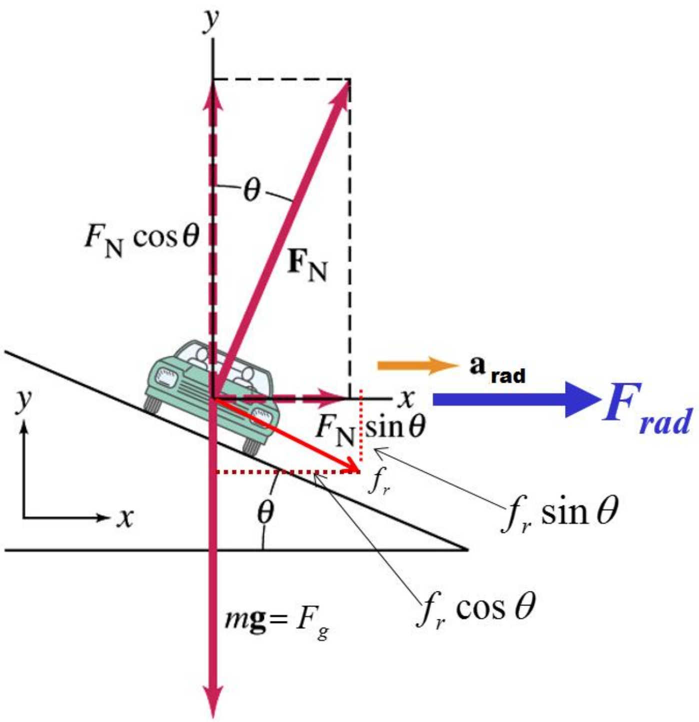
$$F_N \sin\theta + \mu_s F_N \cos\theta = F_{net} = \mathrm{m}\frac{v^2}{r} = F_N(\sin\theta + \mu_s\cos\theta)\text{....}(1)$$

$$F_N \cos\theta - \mu_s F_N \sin\theta = F_g = mg = F_N(\cos\theta - \mu_s\sin\theta)\text{..........}(2)$$

Divide equation (1) by equation (2);

$$\frac{(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{v^2}{gr}$$

$$v = \sqrt{gr\frac{(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}}$$



Note:

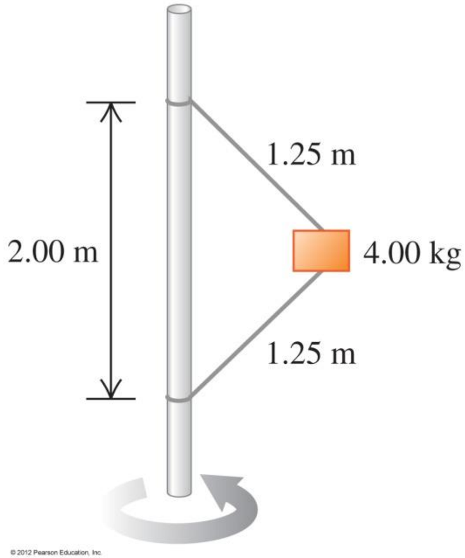
For special case μ_s=0;

$$v_{design} = \sqrt{gr\tan\theta} \text{ and } \tan\theta = \frac{v^2}{gr}$$

For special case θ=0°;

$$\therefore v_{design} = \sqrt{\mu_s gr} \quad \text{(unbanked curve)}$$

6-51: When the system rotates about the rod the strings are extended as shown. (The tension in the upper string T_H is 80 N)



a) The block moves in a circle of radius

$$r = \sqrt{(1.25m)^2 - (1.00m)^2} = 0.75m$$

Each string makes an angle θ with the vertical pole

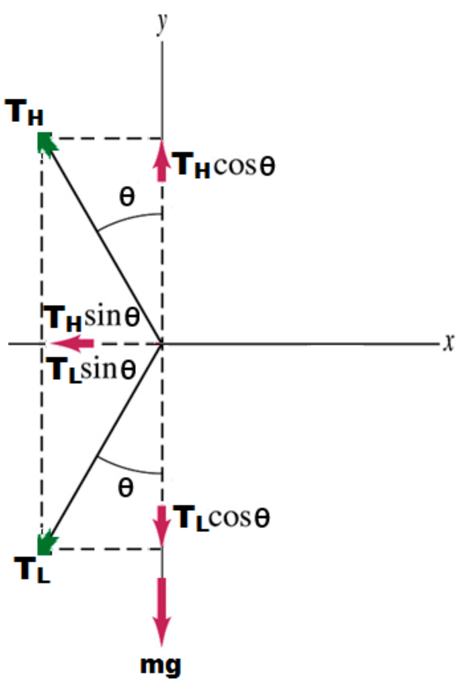
$$\cos\theta = \frac{1.00}{1.25} \rightarrow \theta = 36.9°$$

This block has an acceleration of $a_r = \frac{v^2}{r}$

b) What is the tension in the lower string?

$$\sum F_y = ma_y = 0 \rightarrow T_H\cos\theta - T_L\cos\theta - mg = 0$$

$$\therefore T_L = T_H - \frac{mg}{\cos\theta} = 80N - \frac{4.00kg * 9.8\frac{m}{s^2}}{cos36.9} = 31N$$



c) What is the speed of the block?

$$\sum F_x = ma_x \rightarrow (T_H + T_L)\sin\theta = \mathrm{m}\frac{v^2}{r}$$

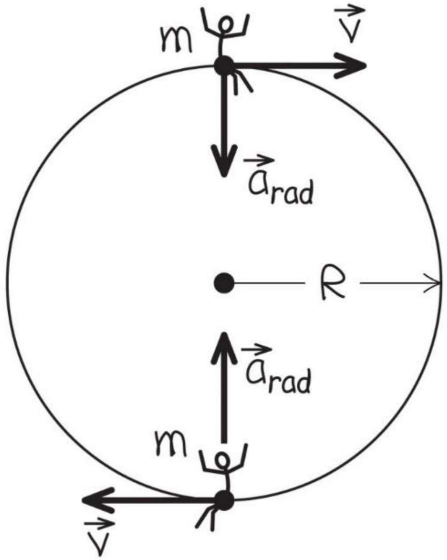
$$\therefore v = \sqrt{\frac{r(T_H+T_L)\sin\theta}{m}} = \sqrt{\frac{0.75(80+31)sin36.9}{4}} = 3.53\frac{m}{s}$$

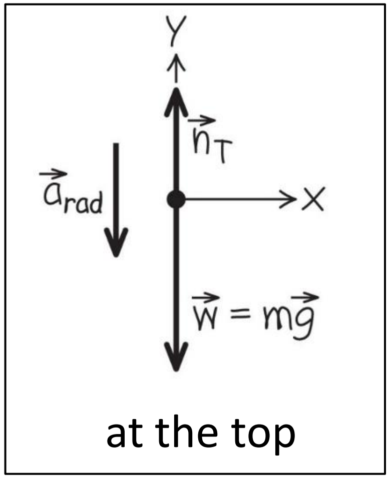
6.2 Motion in a Vertical Circle

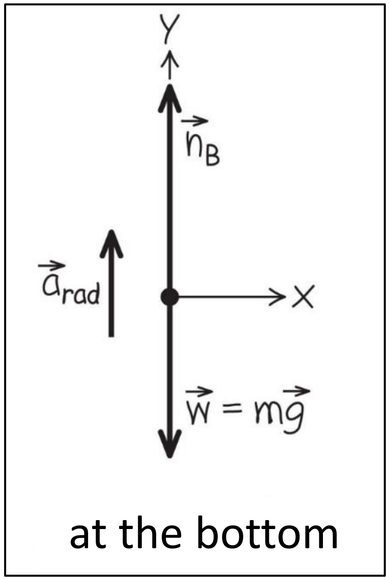
Example 6.5 Dynamics of a Ferris wheel at a constant speed, page 158

Given: m = 60.0 kg
R = 8.00 m
T = 10.0 s ($v = 2\pi R/T$)

Find the normal force:
(a) at the top (n_T)
(b) at the bottom (n_B)







(a) At the top

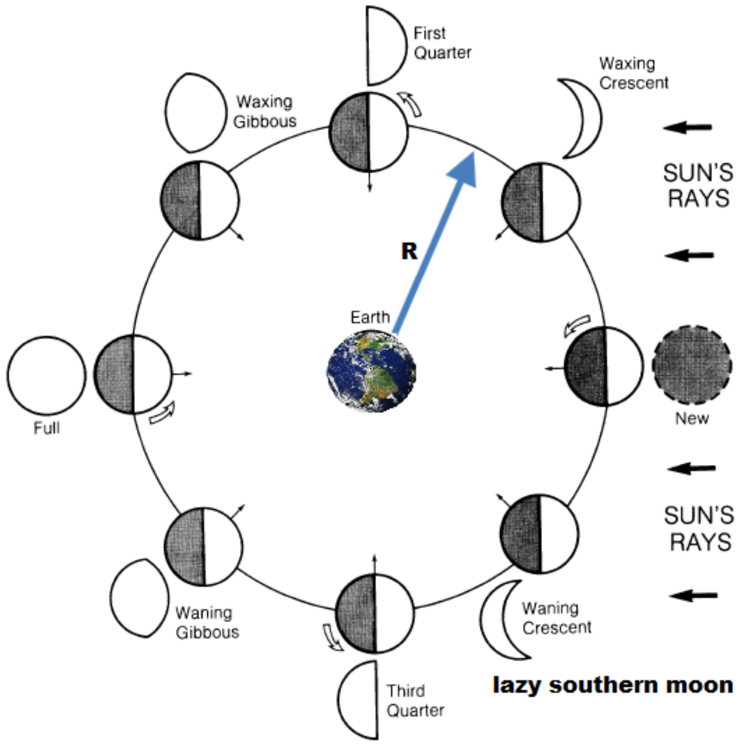
$$n_T - mg = ma_T = m(-\frac{v^2}{R}) \qquad n_T = mg - m\frac{v^2}{R} = m(g - \frac{v^2}{R})$$

(b) At the bottom

$$n_B - mg = ma_B = m(\frac{v^2}{R}) \qquad n_T = mg + m\frac{v^2}{R} = m(g + \frac{v^2}{R})$$

Courtesy of Wenhao Wu

Work out the radial acceleration of the moon around the earth.

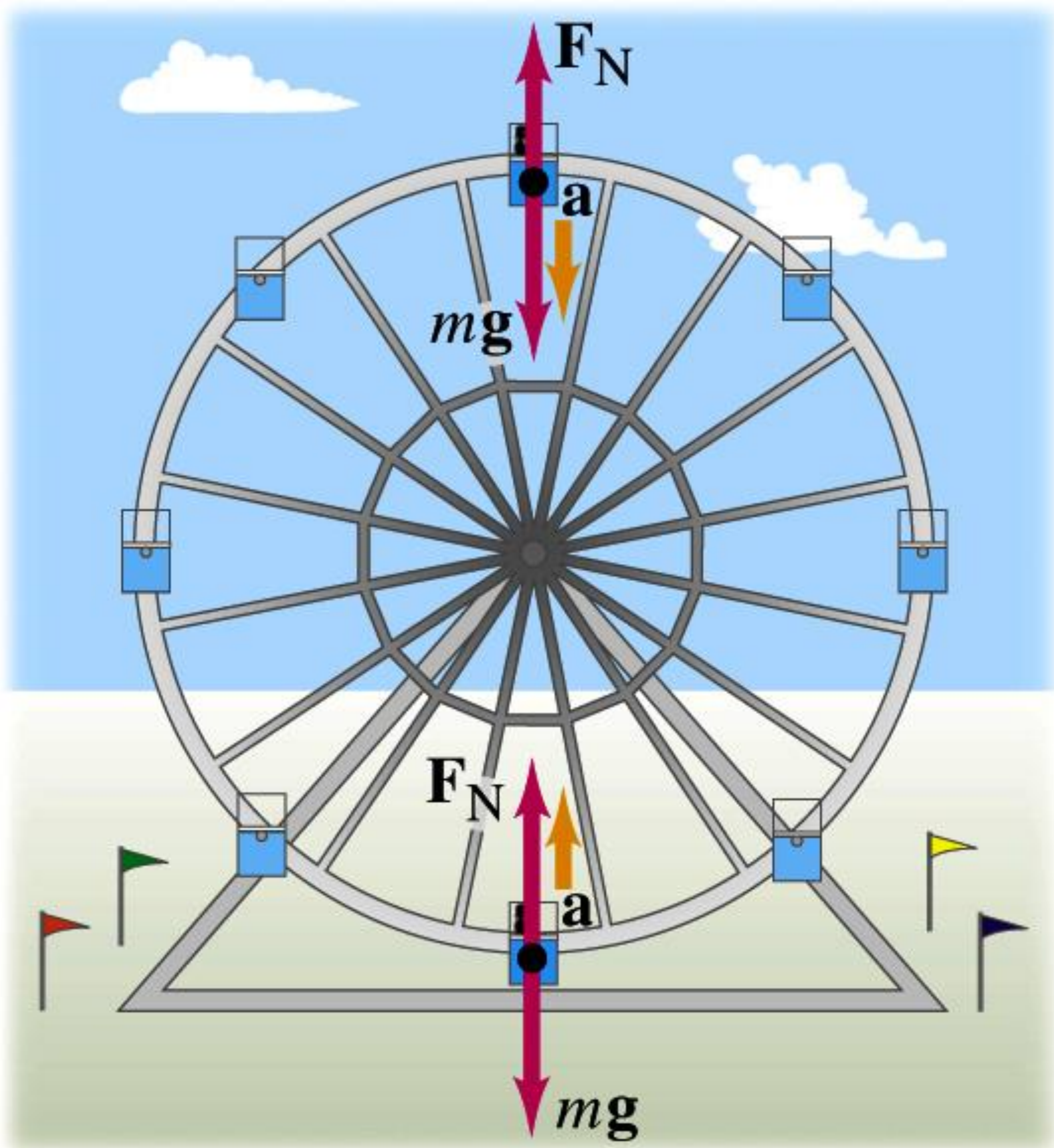


$a_r = \frac{v^2}{R}$ and $v = \frac{2\pi R}{T}$ g=9.8 m/s^2

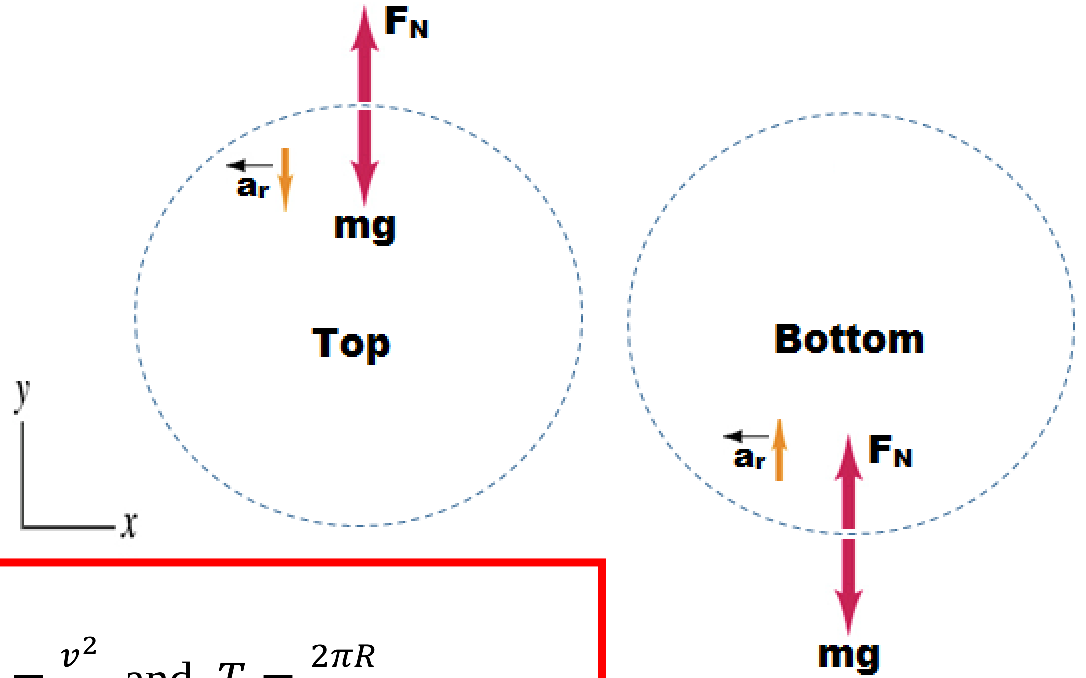
$T = 27.3\ d * \frac{24\ h}{1\ d} * \frac{3600\ s}{1h} = 2.36x10^6 s$ and $R = 3.84x10^8 m$

So; $v = 10.22 * 10^2 \frac{m}{s}$ $a_r = 27.2 * 10^{-4} m/s^2 \approx 3 * 10^{-4} \frac{m}{s\text{^}2}$ g

Ferris wheel



Ferris wheel



Top:

$$a_r = \frac{v^2}{R} \text{ and } T = \frac{2\pi R}{v}$$

$$\sum F_y = ma_y = F_N - mg = -\frac{mv^2}{R}$$

$$\rightarrow F_N = m(g - \frac{v^2}{R})$$

The force which the seat applies to the passenger is smaller than his weight.

For, $\frac{v^2}{R} = g$ passenger is starting to fly off.

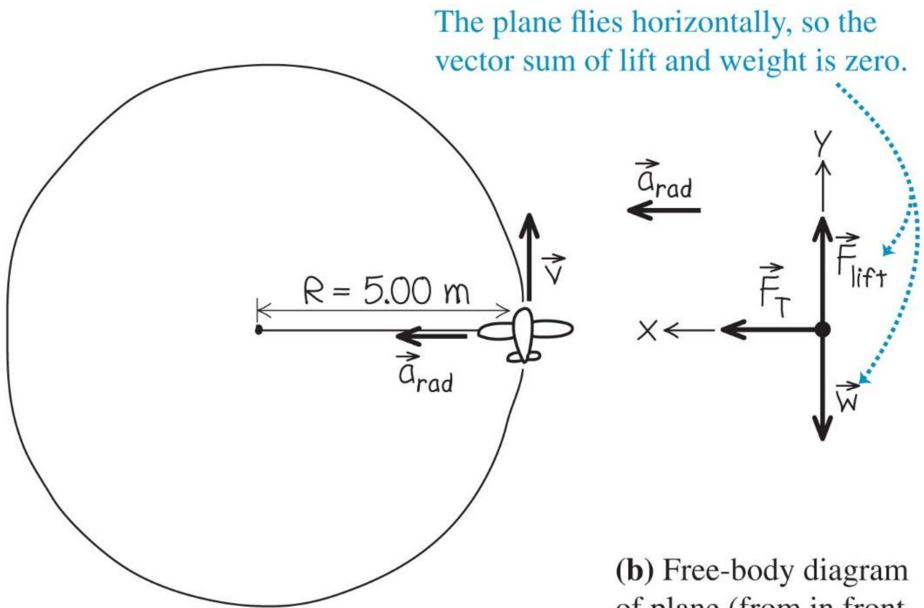
Bottom:

$$F_N - mg = \frac{mv^2}{R} \rightarrow F_N = m(g + \frac{v^2}{R})$$

Passenger in Ferris wheel is pressed into the seat.

Model Airplane on a String – Example 6.1

- How hard must you pull on the string to keep the airplane flying in a circle?
- T=4s m=0.5 kg



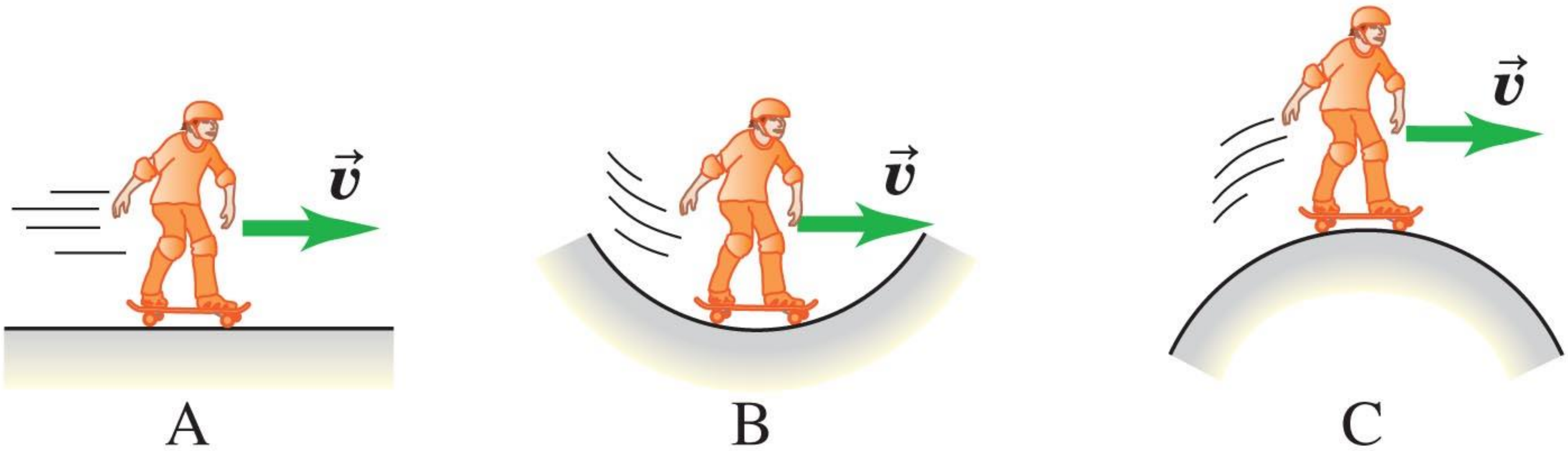
(a) Sketch (from above)

(b) Free-body diagram of plane (from in front or behind)

$$v = \frac{2\pi R}{T} \qquad \begin{cases} \sum F_x = ma_{rad}, & F_T = m\dfrac{v^2}{R} \\ \sum F_y = 0, & F_{lift} + (-mg) = 0 \end{cases}$$

$v^2 = (T R / m)^2$

snowboarding

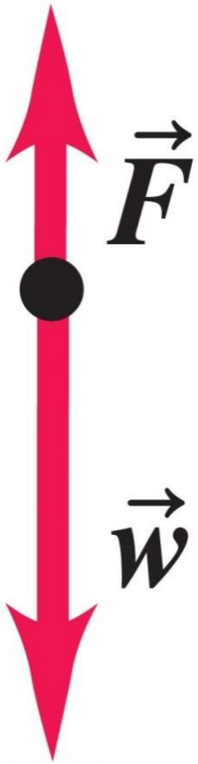


The net force F always down, but a=0 in A. a=positive in B, a=negative in C

C. Just like elevator accelerating down F-mg = m (-a) F = m (g-a)

Clicker question

You're snowboarding down a slope. The free-body diagram in the figure represents the forces on you as you



a) go over the top of a mogul.
b) go through the bottom of a hollow between moguls.
c) go along a horizontal stretch.
d) go along a horizontal stretch or over the top of a mogul.

Clicker question

You whirl a ball of mass *m* in a fast vertical circle on a string of length *R*. At the bottom of the circle, the tension in the string is five times the ball's weight. The ball's speed at this point is given by

a) $\sqrt{gR}$

b) $\sqrt{4gR}$

c) $\sqrt{6gR}$

d) $6\sqrt{gR}$

$F_T = 5mg = mg + (m/R)\, v^2$

Clicker question

You whirl a ball of mass *m* in a fast vertical circle on a string of length *R*. At the top of the circle, the tension in the string is five times the ball's weight. The ball's speed at this point is given by

a) $\sqrt{gR}$

b) $\sqrt{4gR}$

c) $\sqrt{6gR}$

d) $6\sqrt{gR}$

$$F_T = 5mg = -mg + (m/R)\, v^2$$

6.3 Newton's Law of Gravitation

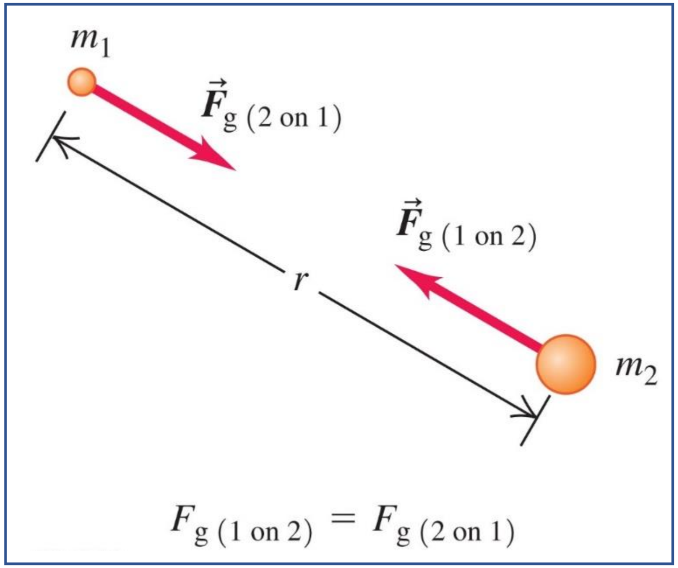
Properties of Gravitation Forces

- Always attractive.
- Directly proportional to both the masses involved.
- Inversely proportional to the square of the center-to-center distance between the two masses.
- Magnitude of force is given by:

$$F_g = G\frac{m_1 m_2}{r^2}$$

- G is the gravitational constant:

$$G = 6.674\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$$



Courtesy of Wenhao Wu

Gravitation

Newton's Law of Gravitation

$$F_g = \frac{Gm_1 m_2}{r^2}$$

G=gravitational constant = $6.673(10)\times 10^{-11} Nm^2/kg^2$

Note: The weight ω of a body of mass m on the earth's surface with radius R_E *is*

$$\omega = mg = \frac{Gm_E \cdot m}{R_E^2} \quad \text{or} \quad g = \frac{Gm_E}{R_E^2}$$

Clicker question

Compared to the earth, planet X has twice the mass and twice the radius. This means that compared to the earth's surface gravity, the surface gravity on Planet X is

A. four times as much.

B. twice as much.

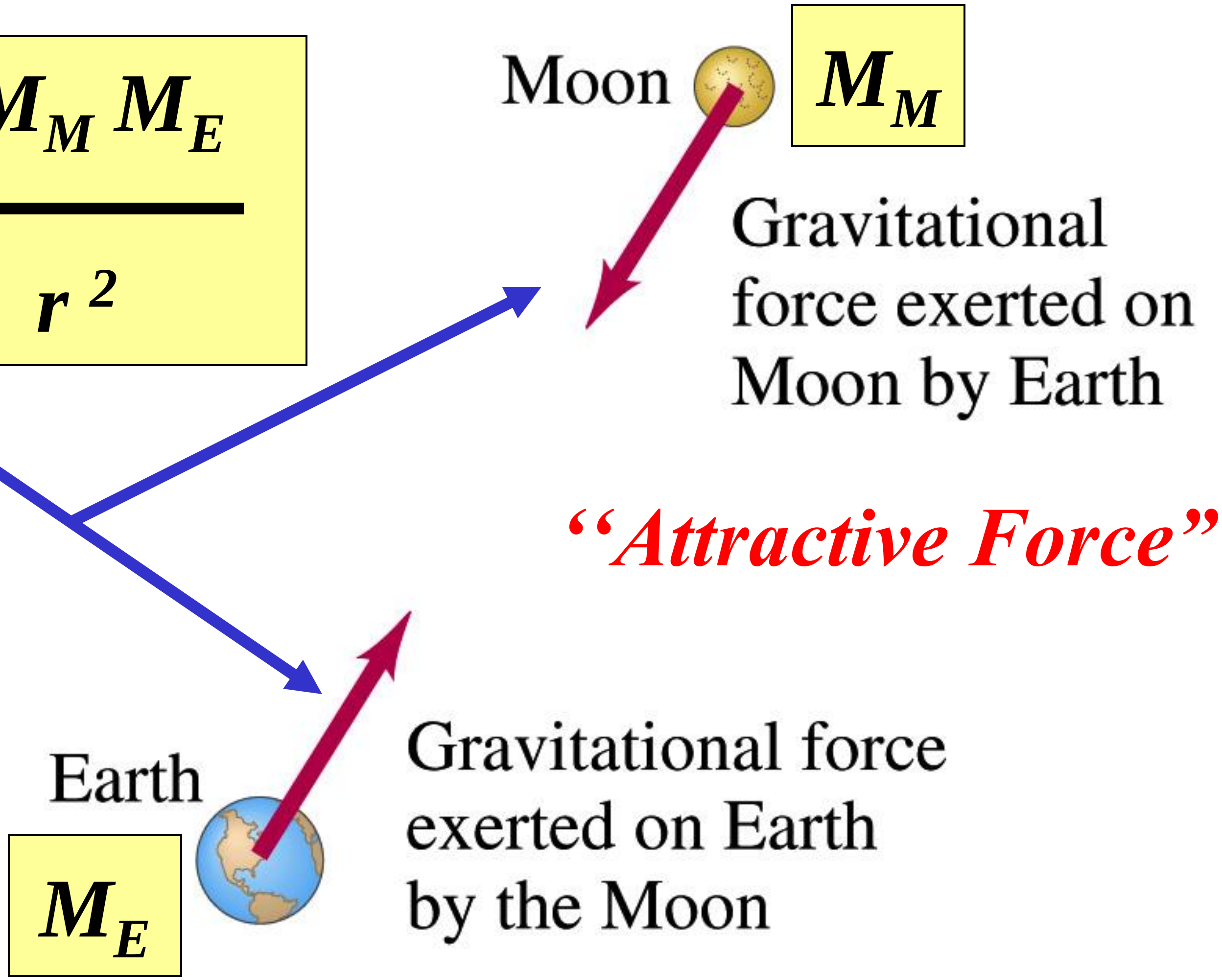
C. the same.

D. half as much.

E. one-quarter as much.

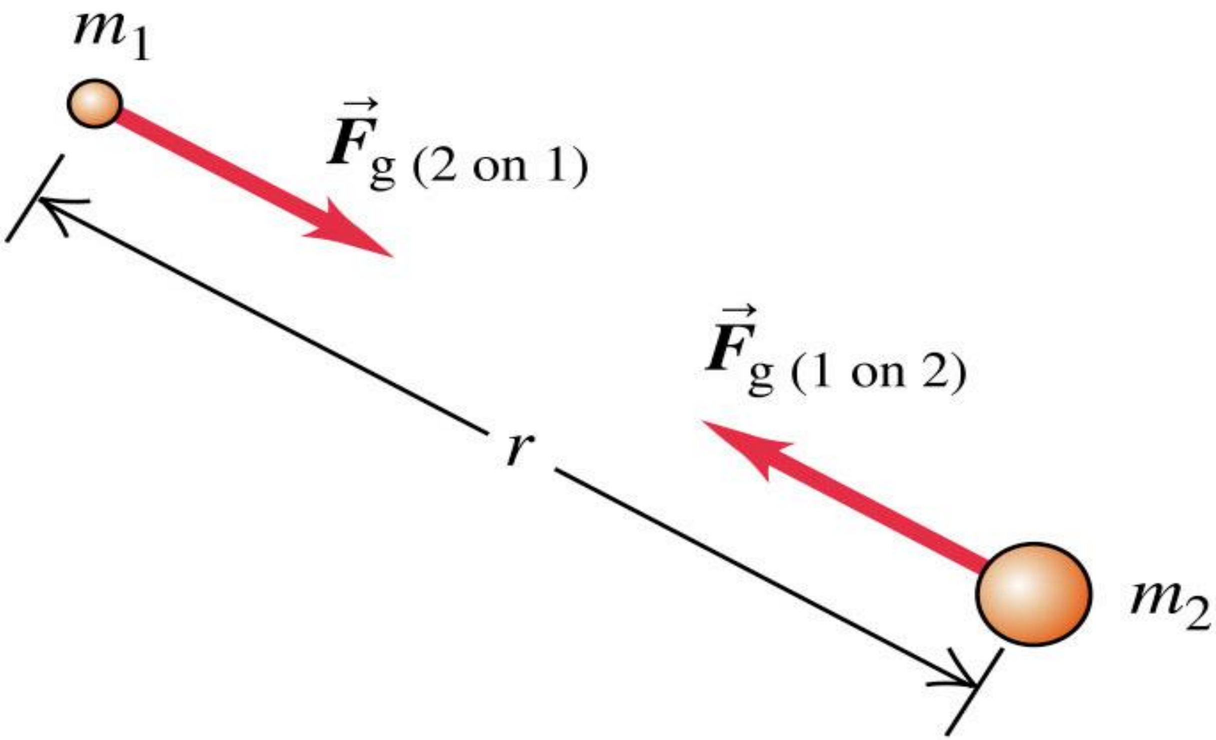
Gravitational Forces (I)

$$F_G = G \frac{M_M M_E}{r^2}$$



"Attractive Force"

Gravitational attraction



$$F_{g\,(1\text{ on }2)} = F_{g\,(2\text{ on }1)}$$

Note: Two particles of different mass exert equally strong gravitational force on each other

Clicker question

The mass of the moon is 1/81 of the mass of the earth. Compared to the gravitational force that the earth exerts on the moon, the gravitational force that the moon exerts on the earth is

A. $81^2 = 6561$ times greater.

B. 81 times greater.

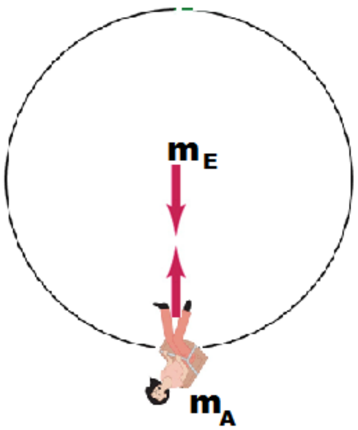
C. equally strong.

D. 1/81 as great.

E. $(1/81)^2 = 1/6561$ as great.

Why is the Aggie not falling off the earth?

Remember there is equally strong attraction between the earth and the Aggie and vice versa



Compare the acceleration of the Aggie to the acceleration of the Earth

$$F = G\frac{m_A m_E}{r_E^2} = m_A g$$

$$\rightarrow g = G\frac{m_E}{r_E^2}$$

Forces are equal between the Aggie and the Earth

$$F = G\frac{m_A m_E}{r_E^2} = m_A a_A = m_E a_E \qquad a_E\text{=g}$$

$$\rightarrow \frac{a_A}{a_E} = \frac{m_E}{m_A} \approx 10^{23} \text{ with } m_E = 6x10^{24} kg$$

$$m_A = 60\ kg \text{ (Aggie's mass)} \qquad a_E\text{=}10^{-23}\text{g}$$

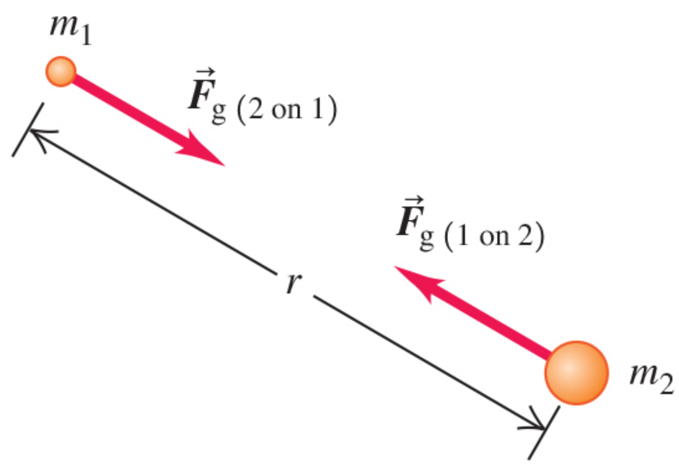
Newton's Law of Gravitation – Figure 6.12

- Always attractive.
- Directly proportional to the masses involved.
- Inversely proportional to the square of the separation between the masses.
- Magnitude of force is given by:

$$F_g = G\frac{m_1 m_2}{r^2}$$

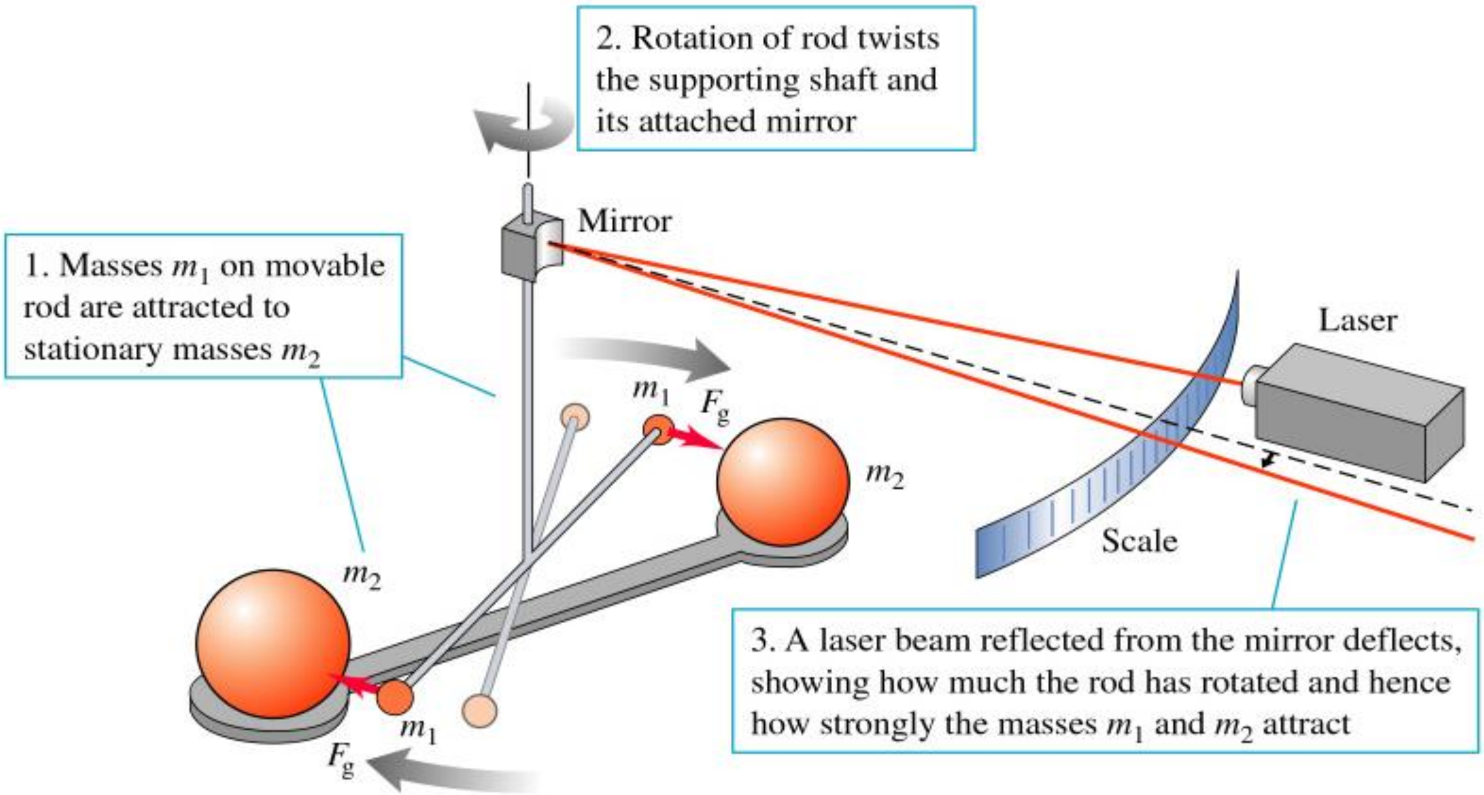
- *G* is gravitational constant:

$$G = 6.674 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$$



$F_{g\,(1\text{ on }2)} = F_{g\,(2\text{ on }1)}$

Cavendish balance (1798)



Cavendish(1798) announced that he has weighted the earth

Cavendish Tension balance (1798)

Air current in the room is negligible to the gravitational attraction force

$$F = G\frac{Mm}{r^2} = 1.33x10^{-10}N \text{ (Torsion force)}$$

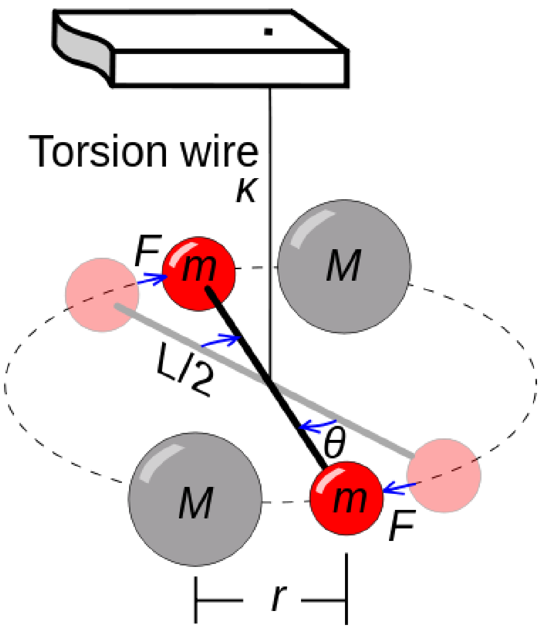
and $M = 0.5\ kg; m = 0.01\ kg\ and\ r = 0.05m$

When torsion and gravitational forces are in equilibrium;

$$1.33x10^{-10} = G\frac{0.5*0.01}{0.05^2}$$

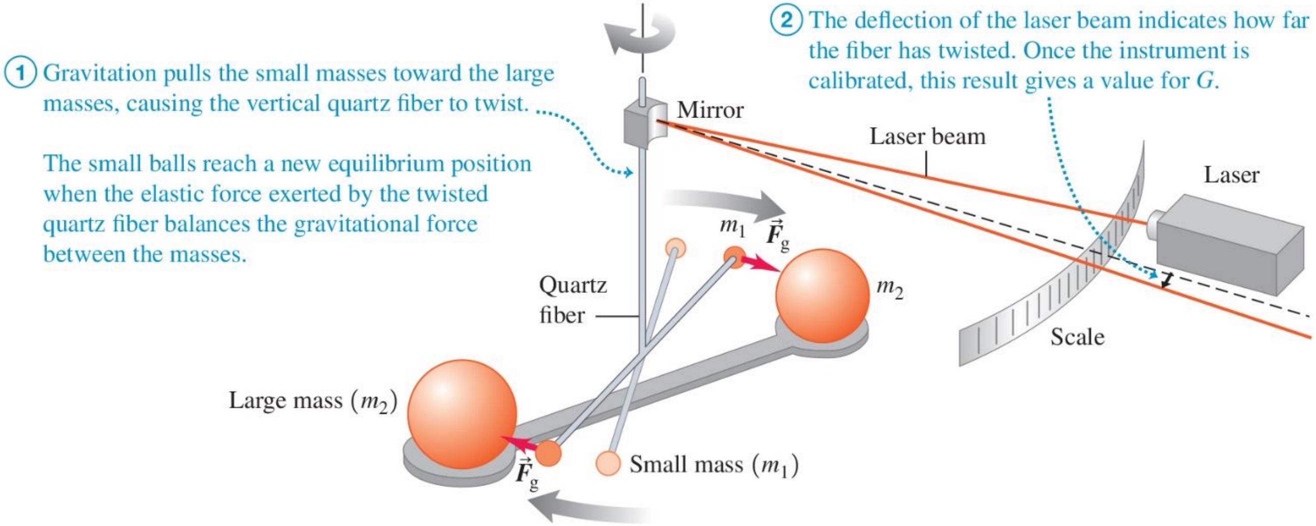
$$\rightarrow G = 6.6x10^{-11}\frac{m^2N}{kg^2}$$

Molecular motors (kinetics); F $= 1.33x10^{-12}N$



This May Be Done in a Lab – Cavendish Experiment (1798)

- The slight attraction of the masses causes a nearly imperceptible rotation of the string supporting the masses connected to the mirror. → use this to calculate *G.*
- Use of the laser allows a point many meters away to move through measurable distances as the angle allows the initial and final positions to diverge.



6.4 Weight and Gravitation Acceleration near the surface of the Earth

- The weight of an object near the surface of the earth is:

$$m_1 g = w = F_{\mathrm{g,\ earth\ surface}} = G\frac{m_1 m_{\mathrm{E}}}{R_{\mathrm{E}}^2}$$

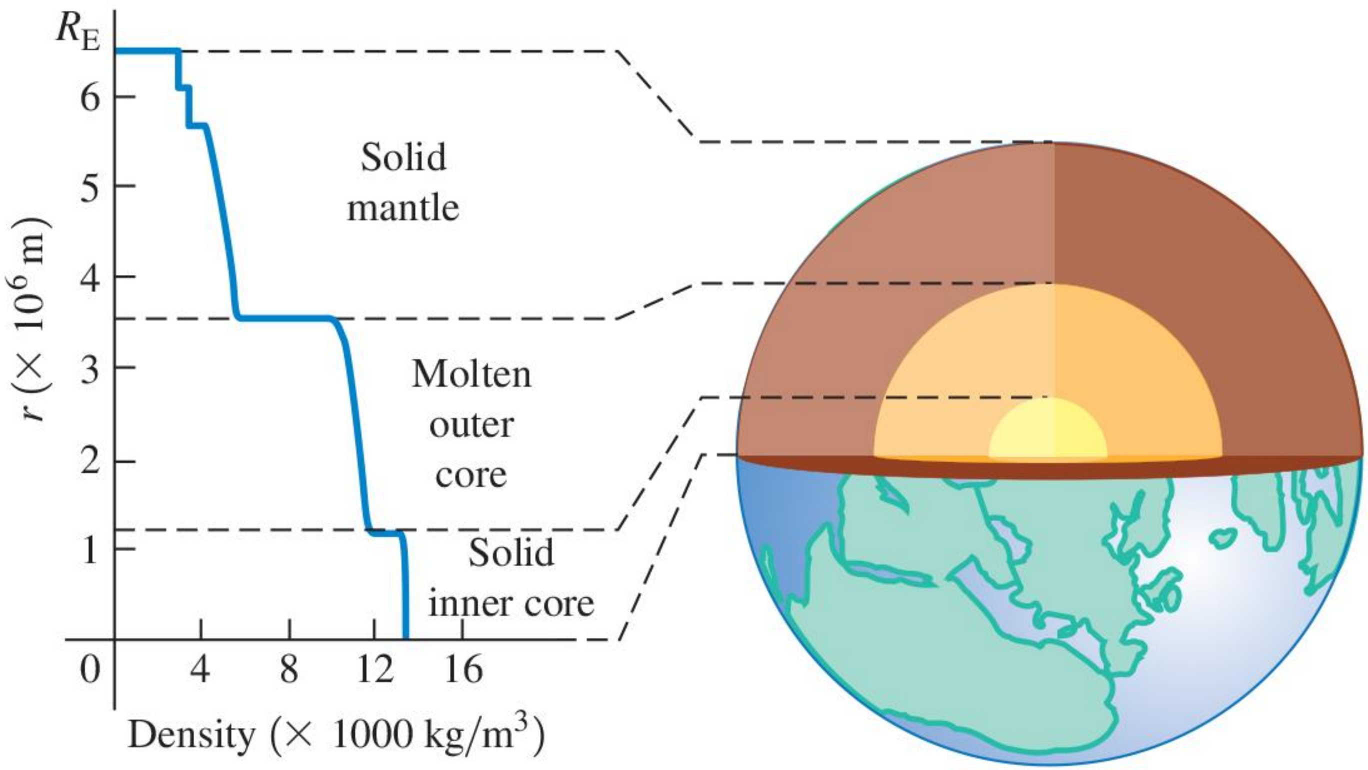
- With this we find that the acceleration due to gravity near the earth's surface is:

$$g = G\frac{m_{\mathrm{E}}}{R_{\mathrm{E}}^2} = 9.8 \ \ \mathrm{m/s^2} \text{ at surface of Earth}$$

Courtesy of Wenhao Wu

Even Within the Earth Itself, Gravity Varies – Figure 6.17

- Distances from the center of rotation and different densities allow for interesting increase in F_g.



Average Density of the Earth

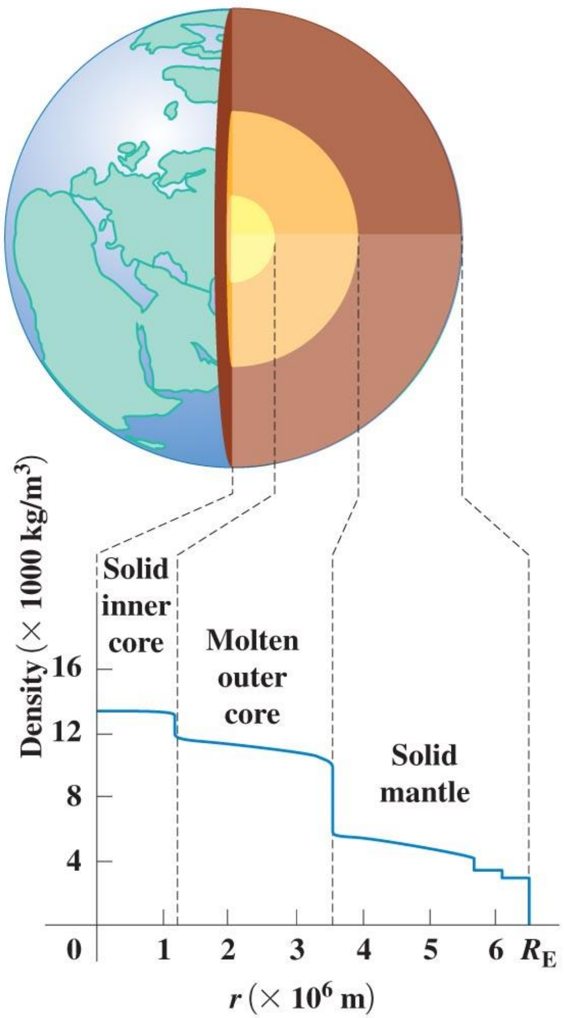
$g = 9.80 \text{ m/s}^2$

$R_E = 6.37 \times 10^6 \text{ m}$

$M_E = 5.96 \times 10^{24} \text{ kg}$

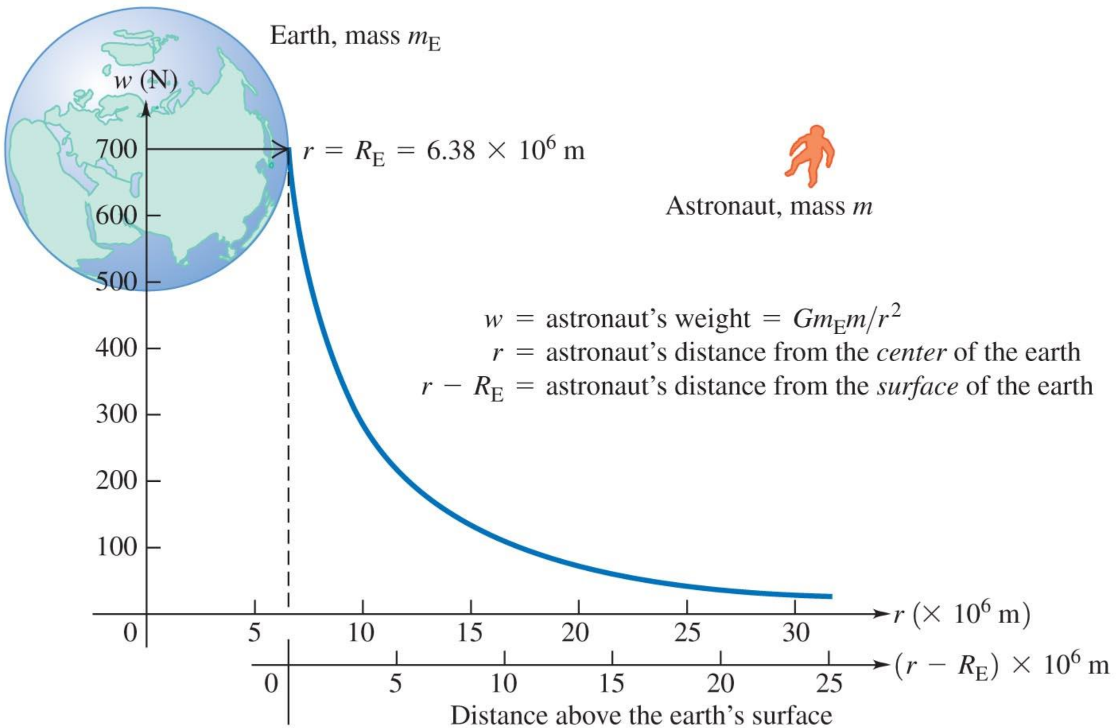
$\rightarrow \rho_E = 5.50 \times 10^3 \text{ kg/m}^3$

$= 5.50 \text{ g/cm}^3 \sim 2 \times \rho_{Rock}$

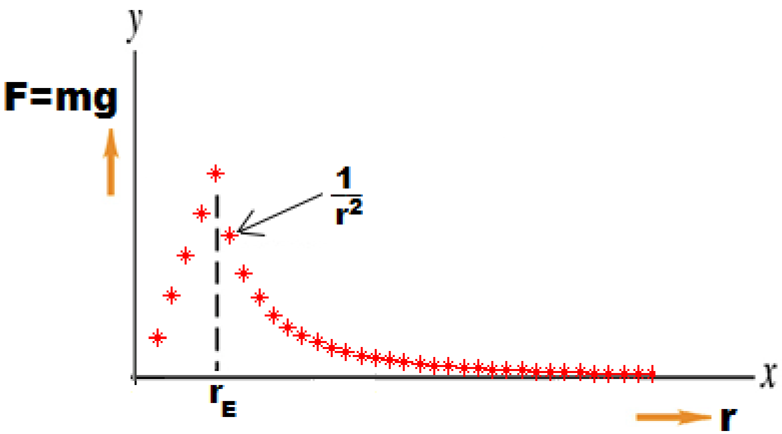


Gravitational Force Falls off Quickly – Figure 6.15

- The gravitational force is proportional to $1/r^2$, and thus the weight of an object decreases inversely with the square of the distance from the earth's center (not distance from the surface of the earth).



What is the magnitude of the gravitational force inside, on the surface, and outside the earth??



Earth mass $M_E = 6x10^{24}kg$ and radius $R_E = 6.37x10^6 m$

$$F = G\frac{M_E m}{{R_E}^2} = mg$$

$$\rightarrow M_E = \frac{g{R_E}^2}{G}$$

When radius is variable like r with variable mass m_{inside} of Earth.

Then;

$$F = G\frac{m_{inside}\, m}{r^2} \text{ and } m_{inside} = \frac{M_E \frac{4}{3}\pi\, r^3}{\frac{4}{3}\pi\, R_E^3} = \frac{M_E\, r^3}{R_E^3}$$

$$\therefore F = G\frac{M_E m}{{R_E}^3} r$$

At the center r=0 and F=?

Gravitation Applies Elsewhere – Figure 6.18

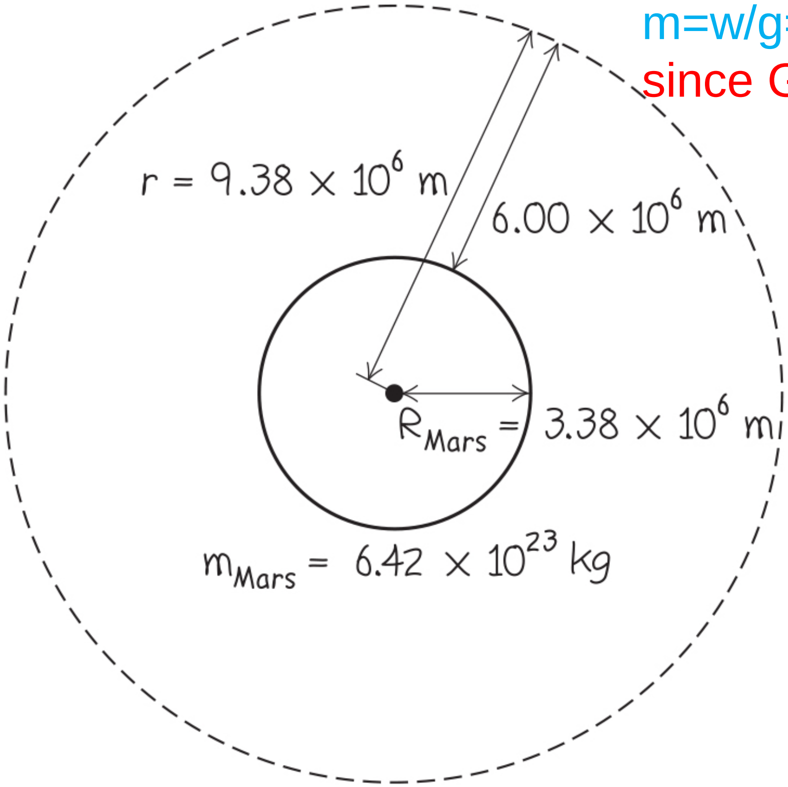
Example Mars

- Mars calculate the weight on the surface
- See the worked example on pages 166–167.

$m = w/g = 3.92 \times 10^4$ N / 9.8 m/s

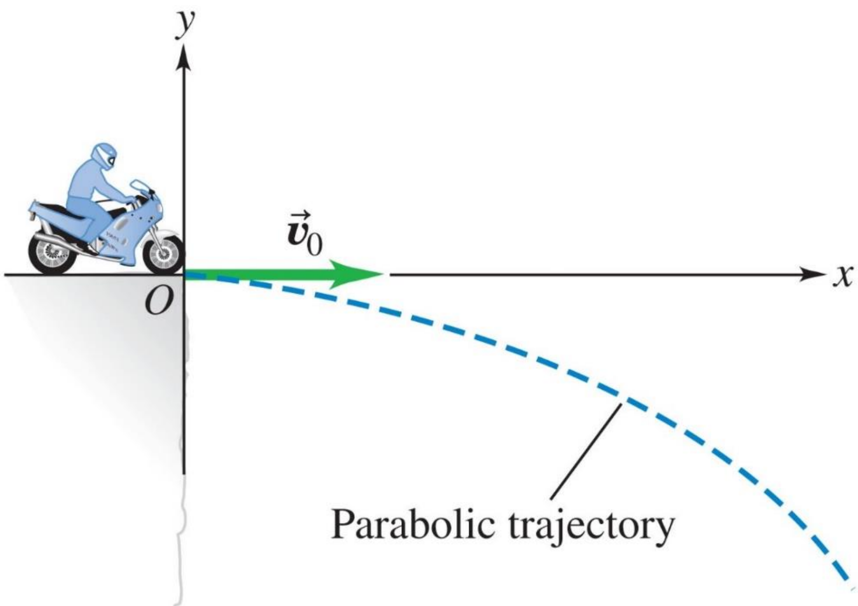
since G is the same everywhere

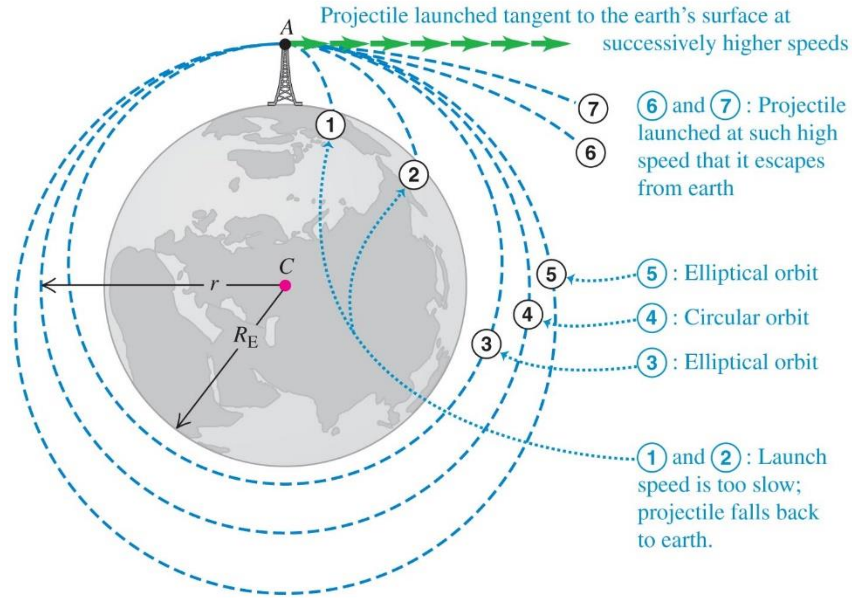
Earth weight of mars lander=39,200 N



6.5	Satellite Motion
	What happens when the velocity increases?

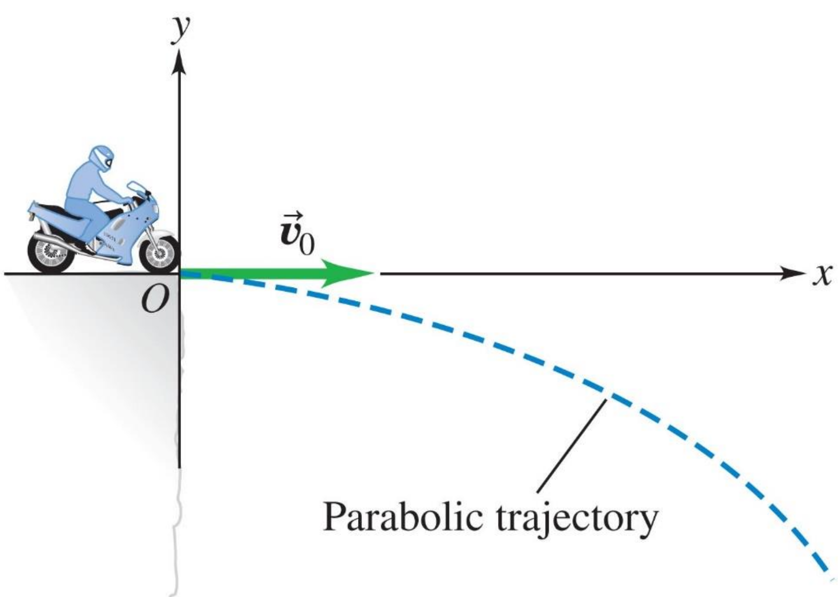
- When v is not large enough, you fall back onto the earth.
- Eventually, F_g balances and you have an orbit.
- When v is large enough, you achieve escape velocity.

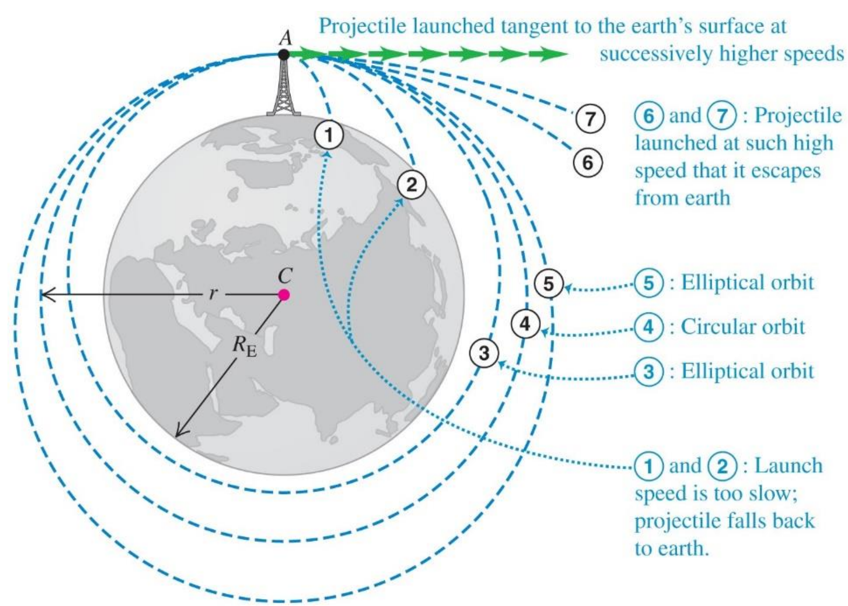




Satellite Motion: What Happens When Velocity Rises?

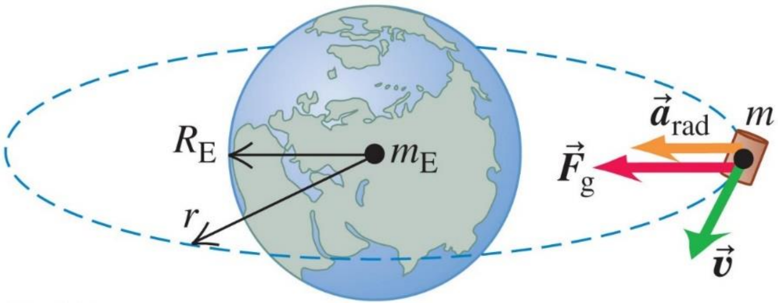
- Eventually, F_g balances and you have orbit.
- When v is large enough, you achieve escape velocity.
- An orbit is not fundamentally different from familiar trajectories on earth. If you launch it slowly, it falls back. If you launch it fast enough, the earth curves away from it as it falls, and it goes into orbit.





Circular Satellite Orbit

- If a satellite is in a circular orbit with speed v_{orbit}, the gravitational force provides the centripetal force needed to keep it moving in a circular path.



The orbital speed of a satellite

$$G\frac{mm_E}{r^2} = F_g = F_{rad} = m\frac{v^2}{r}$$

$$\rightarrow \quad v_{orbit} = \sqrt{\frac{Gm_E}{r}}$$

The period of a satellite

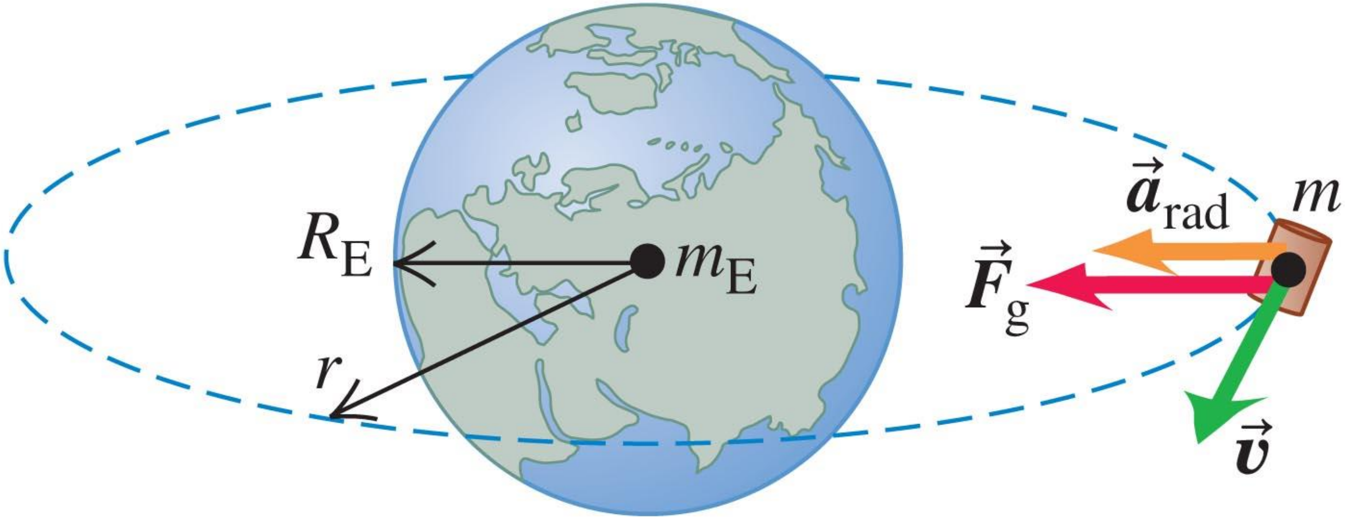
$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$

Courtesy of Wenhao Wu

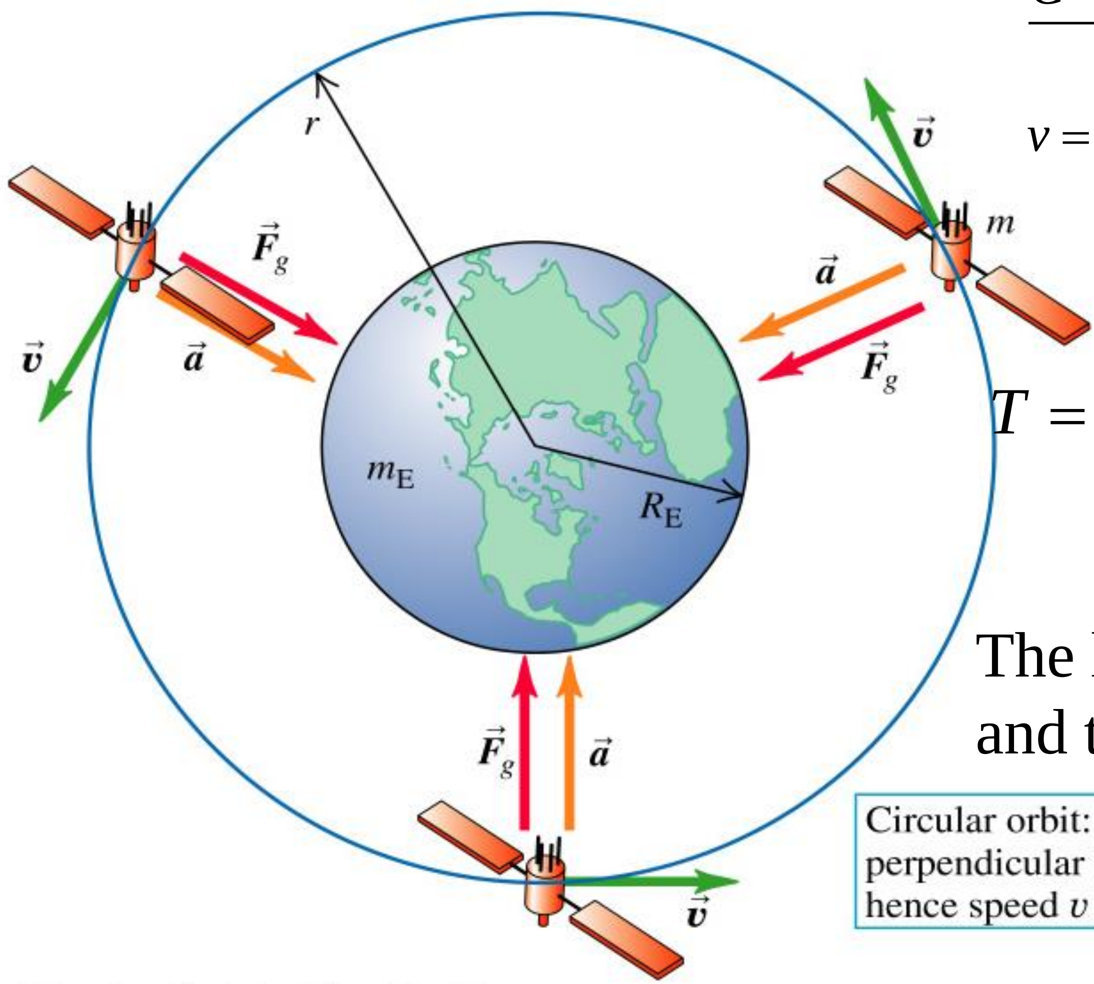
Circular Satellite Orbit Velocity

- If a satellite is in a perfect circular orbit with speed v_{orbit}, the gravitational force provides the centripetal force needed to keep it moving in a circular path.



$$\frac{Gm_{\text{sat}}m_E}{r^2} = F_g = F_{\text{rad}} = m\frac{v^2}{R} \quad \Rightarrow \quad v_{\text{orbit}} = \sqrt{\frac{Gm_E}{r}}$$

Circular orbit period



$$\frac{G \cdot m_E \cdot m}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{Gm_E / r}$$

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = \frac{2\pi r^{2/3}}{\sqrt{Gm_E}}$$

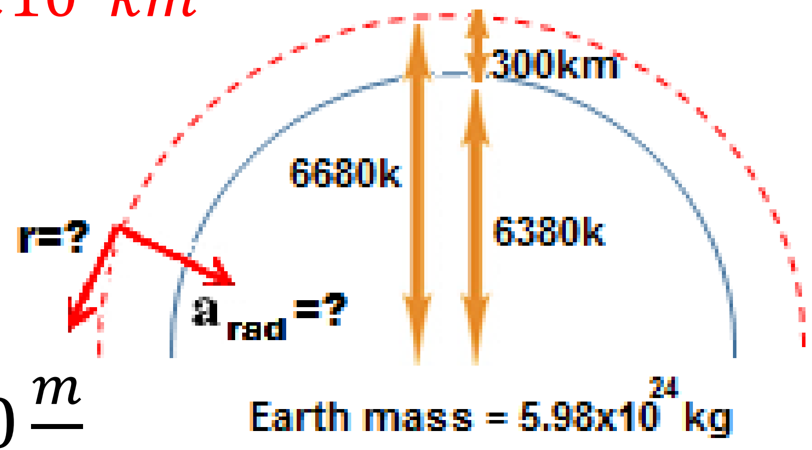
The larger r then slower the speed and the larger the period

Weather Satellite

Example 6.10:

Earth mass $M_E = 5.98x10^{24}kg$ and radius $R_E = 6380km$

$$r = 6380km + 300km = 6.68x10^6 km$$



a) What is the speed?

$$\frac{mv^2}{r} = G\frac{M_E m}{r^2}$$

$$\rightarrow v = \sqrt{G\frac{M_E}{r}} = 7730\frac{m}{s}$$

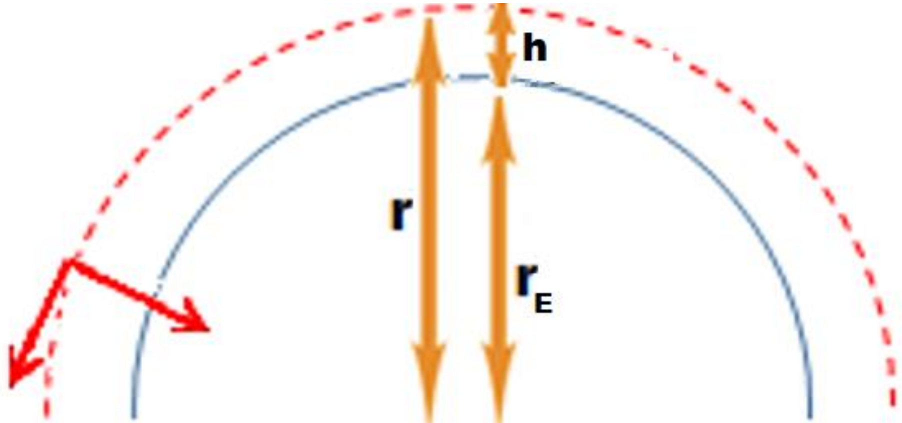
b) When is the period?

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.68x10^6)}{7730} = 5430\ s$$

c) What is the radial acceleration?

$$a_{rad} = \frac{v^2}{r} = \frac{(7730)^2}{6.68x10^6} = 8.95\frac{m}{s^2}$$

Geo-synchronous Satellite (at the equator of Earth)



Not to scale

a) Height above the surface of Earth.

$$h = r - r_E$$

Earth mass $M_E = 6x10^{24}kg$ and radius $r_E = 6380km$

$$\frac{m_s v^2}{r} = G\frac{M_E m_s}{r^2} \rightarrow v = \frac{2\pi r}{T} \text{ and } T = 1\ day = 86400\ sec$$

$$\frac{m_s(2\pi r)^2}{rT^2} = G\frac{M_E m_s}{r^2} \rightarrow r^3 = \frac{GM_E T^2}{4\pi^2} = 7.54x10^{22}m^3$$

$$\therefore r = 4.23x10^7 m$$

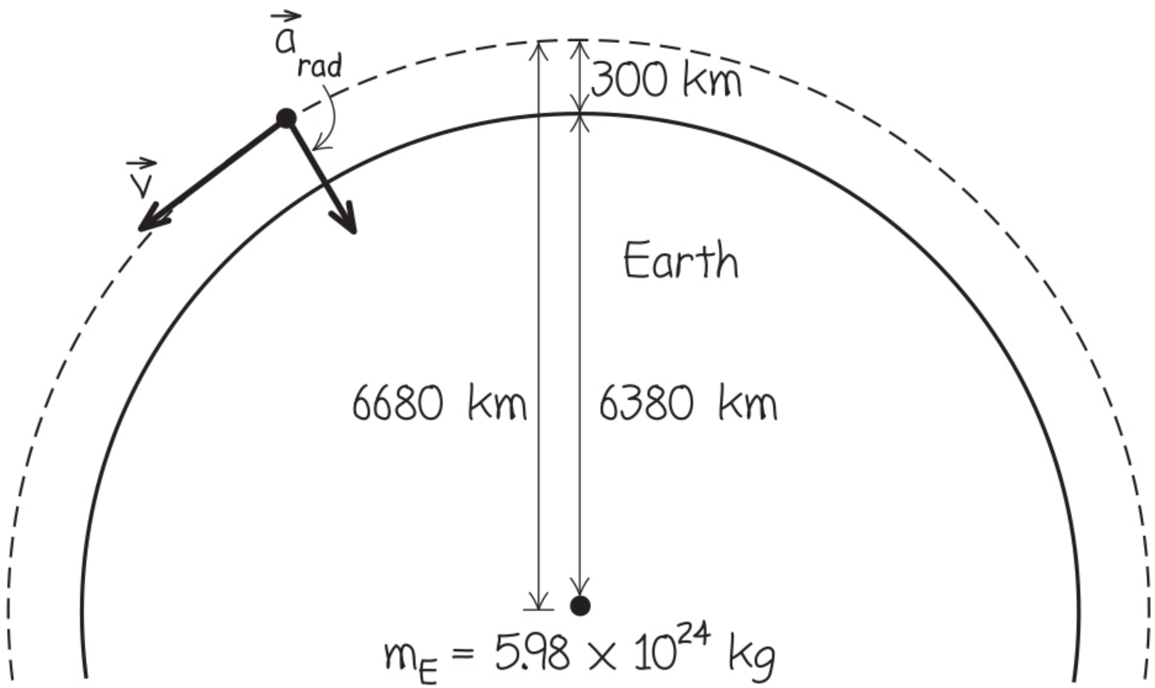
$$\rightarrow h = r - r_E = 36000 \text{ km} \approx 6r_E$$

b) What is the velocity?

$$\rightarrow v = \sqrt{G\frac{M_E}{r}} = 3070\frac{m}{s}$$

Calculations of Satellite Motion – Example 6.10 (not Geo-synchronous)

- Work on an example of a relay designed to stay in orbit permanently.
- See the worked example on page 169.



Pearson

Satellite motion

- **An artificial satellite is orbiting the earth (M earth = 5.97E+24 kg and radius = 38E+6 m) in a circular orbit. If the orbital speed of the satellite is 4000 m/s, what is the radius of the satellite's orbit (measured from the center of the earth)?**
- Solution: Here we use combine two equations given to us. The first is the relationship between linear velocity and the radius & period of rotation of an object in circular motion:
- $v = \frac{2\pi r}{T}$
- The second equation is the period of orbit of a satellite:
- $T = \frac{2\pi r^{3/2}}{\sqrt{GM_e}}$
- If we arrange this second equation, we find that we can substitute in the linear velocity:
- $\frac{T}{2\pi r} = \frac{r^{\frac{1}{2}}}{\sqrt{GM_e}} \Rightarrow \frac{1}{v} = \sqrt{\frac{r}{GM_e}}$
- We are given G from the formula sheet (6.67E-11 $N*m^2*kg^{-2}$), and the values of M_e (5.97E+24 kg) and v (4000 m/s) in the problem. We can re-arrange the equation to solve for r, and we get:
- $\frac{GM_e}{v^2} = \frac{(6.67\times10^{-11})(5.97\times10^{24})}{(4000)^2} \approx 2.5 \times 10^7$ r= 2.5×10^7 m

If an Object is Massive, Even Photons Cannot Escape

- A "black hole" is a collapsed sun of immense density such that a tiny radius contains all the former mass of a star.
- The radius to prevent light from escaping is termed the "Schwarzschild Radius."
- The edge of this radius has even entered pop culture in films. This radius for light is called the "event horizon."

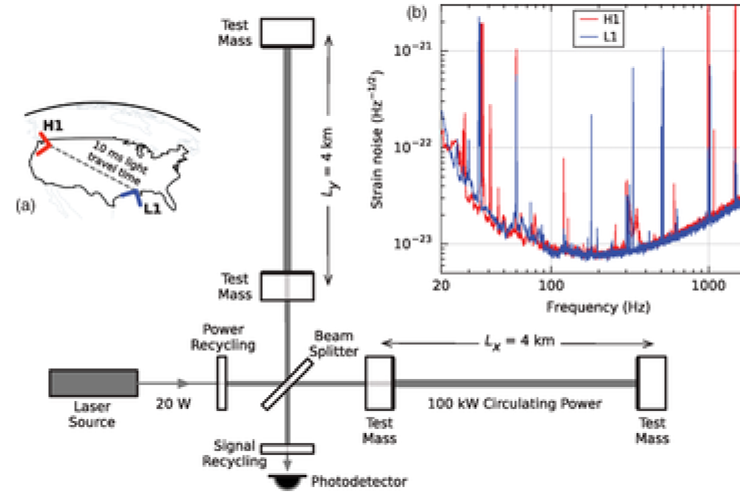
Hawking @ ranch

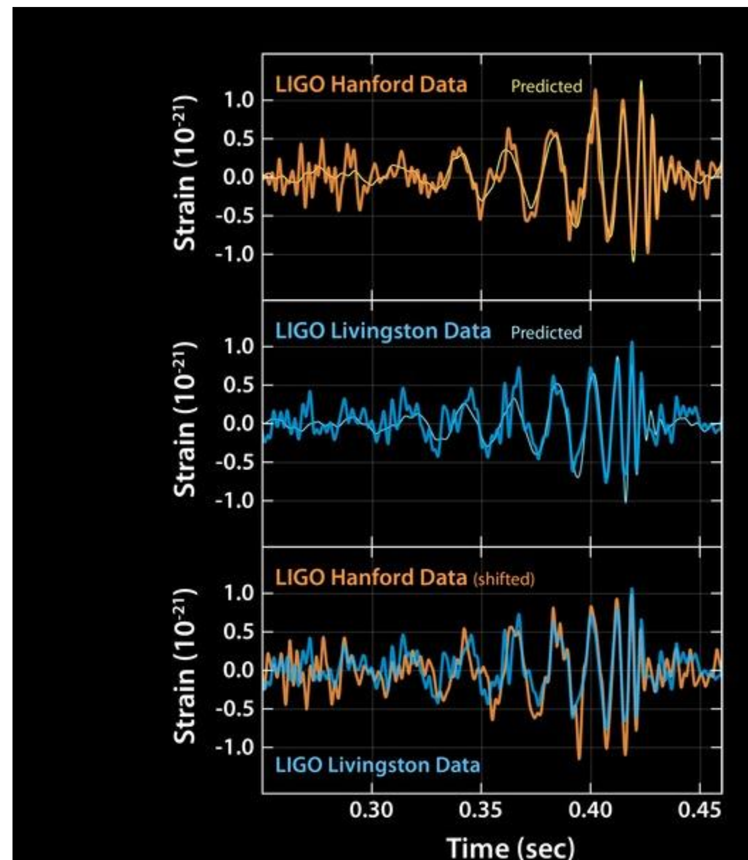
SIBOR
Stored Ion & Bio-Optics Research

Laser Interferometer Gravitational-Wave Observatory (LIGO)

operates two gravitational wave observatories in unison: the LIGO Livingston Observatory in Livingston, Louisiana, and the LIGO Hanford Observatory, on the DOE Hanford Site ,located near Richland, Washington. These sites are separated by 3,002 kilometers (1,865 miles)

Collison of two black holes 1.3 billion years ago, each black hole was about 30 times mass of the Sun, and 3 solar mass were converted to gravitational waves.





Sun properties

Sun mass $M_S = 1.99x10^{30}kg$ and radius $R = 6.96x10^{8}m$
Average density of Sun;

$$\rho = \frac{M_S}{V} = \frac{M_S}{\frac{4}{3}\pi R^3} = \frac{1.99x10^{30}}{\frac{4}{3}\pi(6.96x10^{8})^3} = 1.41\,\frac{g}{cm^3}$$

→ 40% denser than water

Temperature: 5800° K at surface and $(1.5x10^{7})$° K in the interior of Sun. (highly ionize plasma gas)

Steven Hawkins is associated with the department of Physics and Astronomy at TAMU

Clicker question

A Gravitational wave was created in a collision of two black holes 1.3 billion years ago, each black hole was about 30 times mass of the Sun, and 3 solar mass were converted to gravitational waves

A. In this process total energy was conserved
B. In this process the gravitational acceleration was g
C. In this process also light from the merger reached LIGO

$Critical\ radius\ for\ ligt\ emission\ R_s = 2G\frac{M_s}{c^2}$

(Schwarzschild radius)

For $R > R_s$ → light can be emitted

For $R < R_s$ → no light can be emitted (Black hole)

To what fraction of sun's current radius would the sun have to be compressed to become a black hole?

$$R_s = 2G\frac{M_s}{c^2} = \frac{2x6.67x10^{-11}x1.99x10^{30}}{(3x10^8)^2} = 2.95\ km$$

$$\rightarrow \frac{R_s}{R} = \frac{2.95x10^3}{6.96x10^8} = 4.2x10^{-6}$$

Example: Problem 7, Exam II, Fall 2016

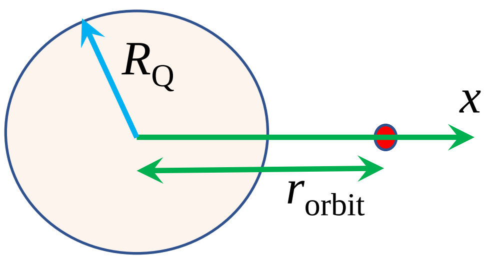
(a) A satellite of mass 80.0 kg is in a circular orbit around a spherical planet Q of radius 3.00×10^6 m. The satellite has a speed 5000 m/s in an orbit of radius 8.00×10^6 m. What is the mass of the planet Q?

(b) Imagine that you release a small rock from rest at a distance of 20.0 m above the surface of the planet. What is the speed of the rock just before it reaches the surface?

Given:

- About the satellite ($m_s = 80.0$ kg, $r_{orbit} = 8.00\times10^6$ m, $v = 5000$ m/s)
- About the planet Q($R_Q = 3.00\times10^6$ m)

Find: (a) The mass of the planet Q (m_Q)
(b) Speed of a rock after falling $h = 20.0$ m.



(a)
$$G\frac{m_s m_Q}{r_{orbit}^2} = F_g = F_{rad} = m_s\frac{v^2}{r_{orbit}}$$
$$m_Q = \frac{r_{orbit}v^2}{G}$$

(b) First, find the gravitational acceleration g_Q near the surface of the planet Q.

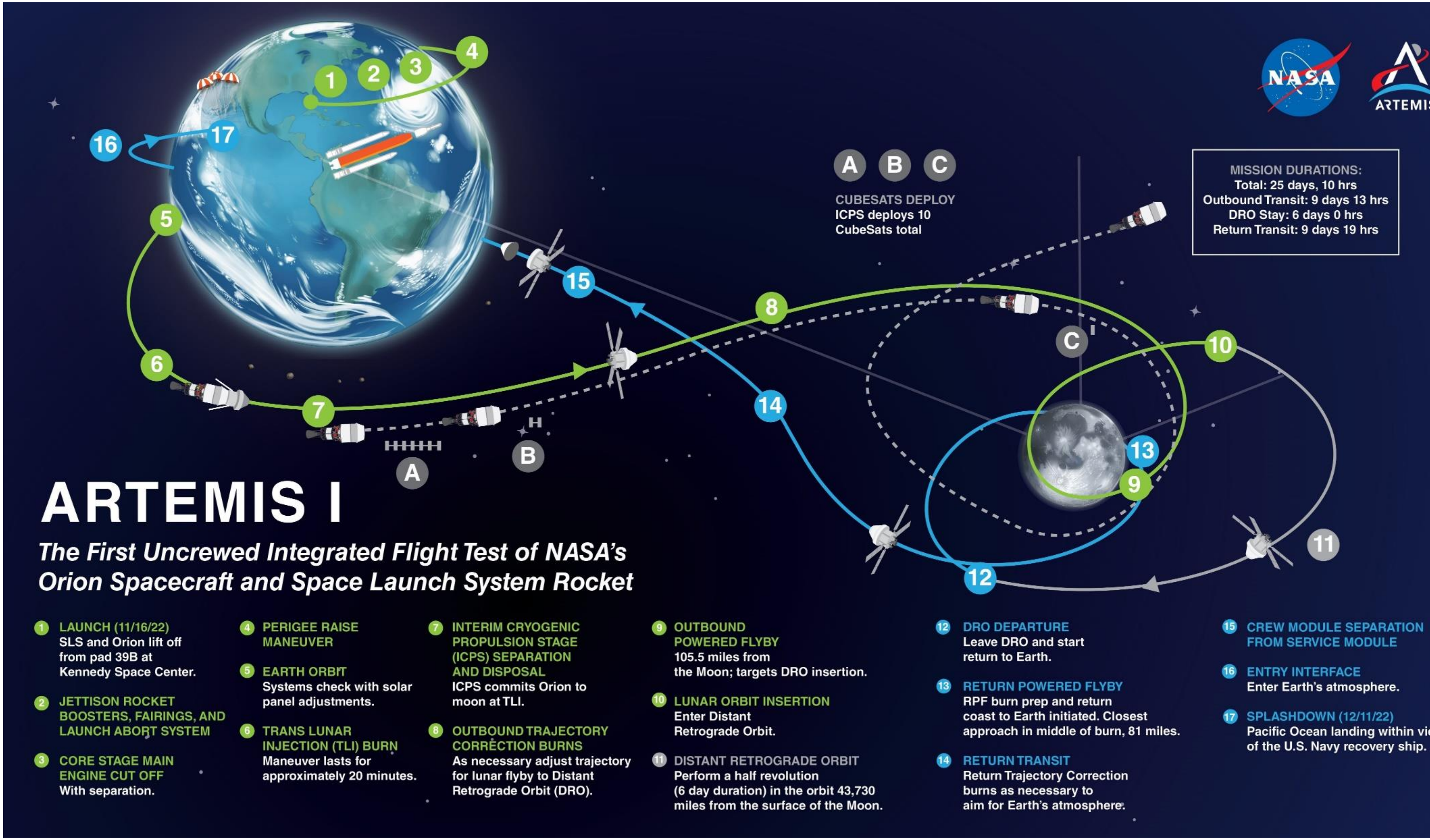
$$m_s g_Q = G\frac{m_s m_Q}{R_Q^2} \qquad g_Q = G\frac{m_Q}{R_Q^2}$$

Then, apply the kinematic equation

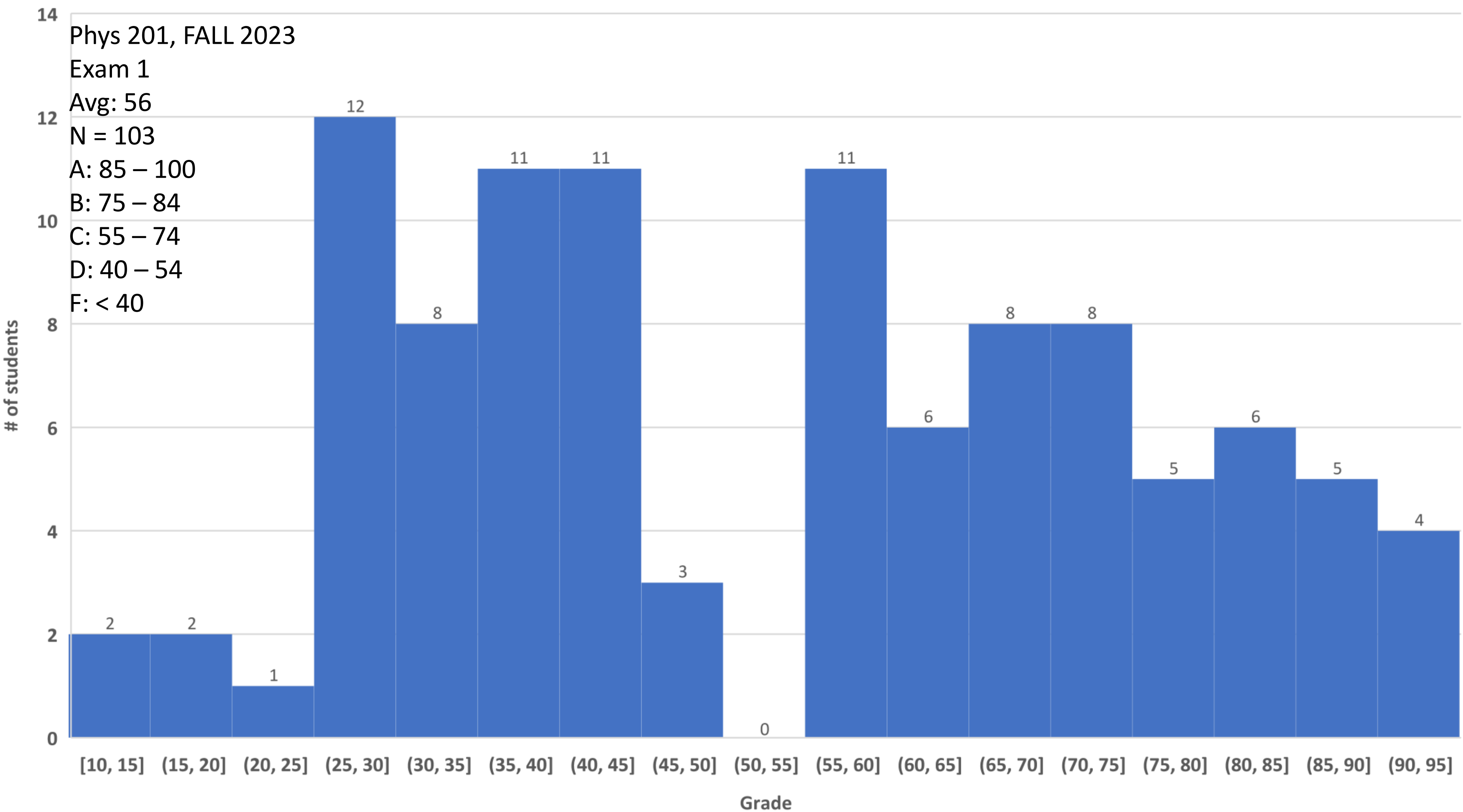
$$v_2^2 = v_1^2 + 2g_Q h$$

to v_2 find with $v_1 = 0$.

Courtesy of Wenhao Wu



ARTEMIS I
The First Uncrewed Integrated Flight Test of NASA's Orion Spacecraft and Space Launch System Rocket
MISSION DURATIONS:
Total: 25 days, 10 hrs
Outbound Transit: 9 days 13 hrs
DRO Stay: 6 days 0 hrs
Return Transit: 9 days 19 hrs
A B C
CUBESATS DEPLOY
ICPS deploys 10 CubeSats total
1 LAUNCH (11/16/22)
SLS and Orion lift off from pad 39B at Kennedy Space Center.
2 JETTISON ROCKET BOOSTERS, FAIRINGS, AND LAUNCH ABORT SYSTEM
3 CORE STAGE MAIN ENGINE CUT OFF
With separation.
4 PERIGEE RAISE MANEUVER
5 EARTH ORBIT
Systems check with solar panel adjustments.
6 TRANS LUNAR INJECTION (TLI) BURN
Maneuver lasts for approximately 20 minutes.
7 INTERIM CRYOGENIC PROPULSION STAGE (ICPS) SEPARATION AND DISPOSAL
ICPS commits Orion to moon at TLI.
8 OUTBOUND TRAJECTORY CORRECTION BURNS
As necessary adjust trajectory for lunar flyby to Distant Retrograde Orbit (DRO).
9 OUTBOUND POWERED FLYBY
105.5 miles from the Moon; targets DRO insertion.
10 LUNAR ORBIT INSERTION
Enter Distant Retrograde Orbit.
11 DISTANT RETROGRADE ORBIT
Perform a half revolution (6 day duration) in the orbit 43,730 miles from the surface of the Moon.
12 DRO DEPARTURE
Leave DRO and start return to Earth.
13 RETURN POWERED FLYBY
RPF burn prep and return coast to Earth initiated. Closest approach in middle of burn, 81 miles.
14 RETURN TRANSIT
Return Trajectory Correction burns as necessary to aim for Earth's atmosphere.
15 CREW MODULE SEPARATION FROM SERVICE MODULE
16 ENTRY INTERFACE
Enter Earth's atmosphere.
17 SPLASHDOWN (12/11/22)
Pacific Ocean landing within
of the U.S. Navy recovery ship.


Phys 201, FALL 2023
Exam 1
Avg: 56
N = 103
A: 85 – 100
B: 75 – 84
C: 55 – 74
D: 40 – 54
F: < 40
# of students
14
12
10
8
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Grade