Chapter 8 Momentum and Collision

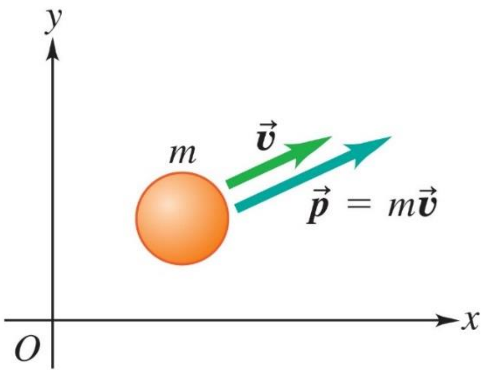
- Understand momentum
- Understand the concept of conservation of momentum.
- Understand impulse.
- Understand center of mass (c.o.m.)
- Understand how forces affect the motion of the c.o.m.

Courtesy of Wenhao Wu

8.1 Momentum

Components of Momentum: $p_x = mv_x$; $p_y = mv_y$

Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.



Another Way of Stating Newton's Second Law:

$$\sum \vec{F} = m\vec{a} = m \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta (m\vec{v})}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}$$

Multiple Particles (A, B, ….) in a System

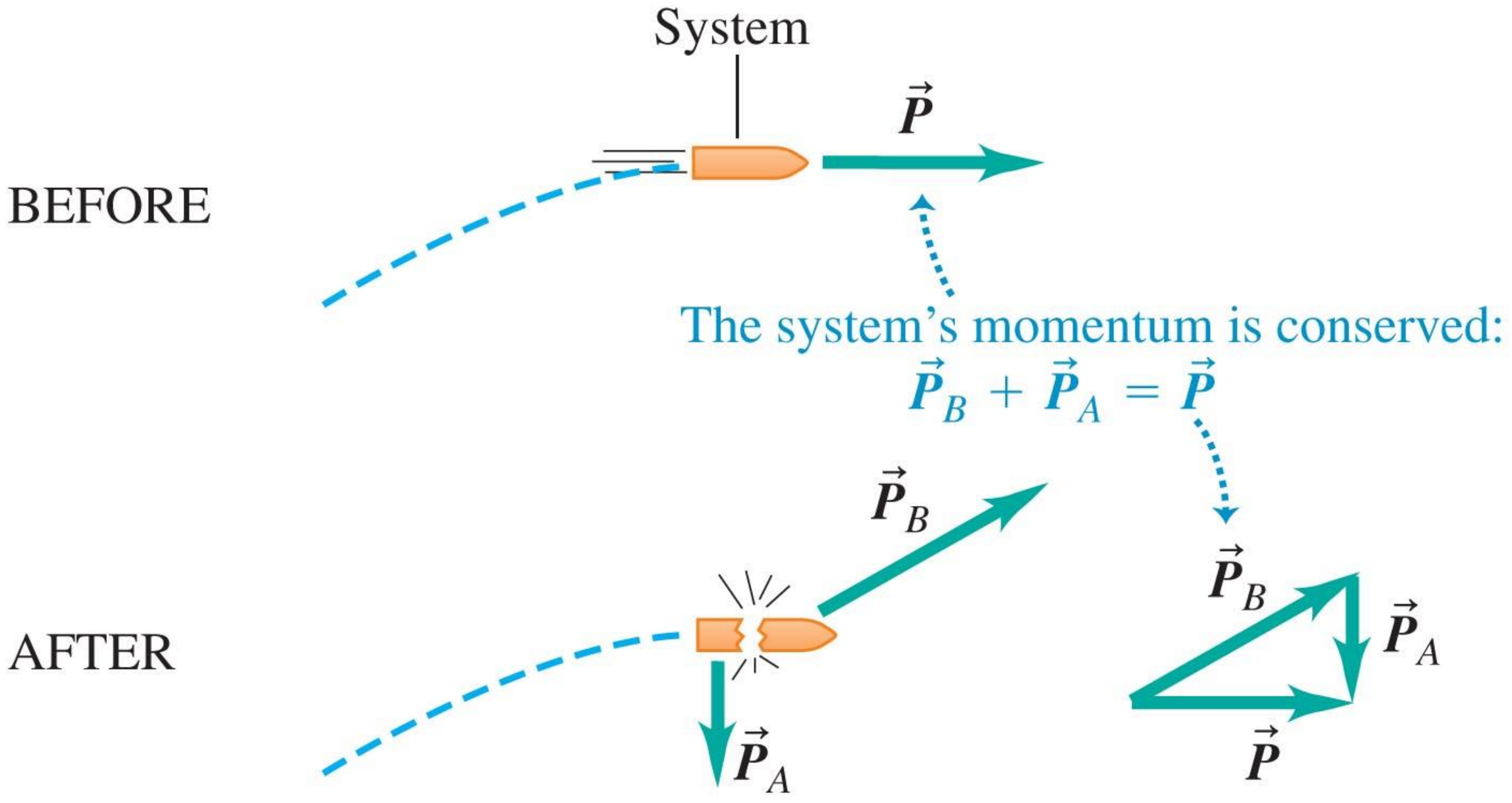
Net Momentum: $\vec{P} = \vec{p}_A + \vec{p}_B + \cdots$

Components: $P_x = p_{A,x} + p_{B,x} + \cdots$

$P_y = p_{A,y} + p_{B,y} + \cdots$

Unit = kg m/s

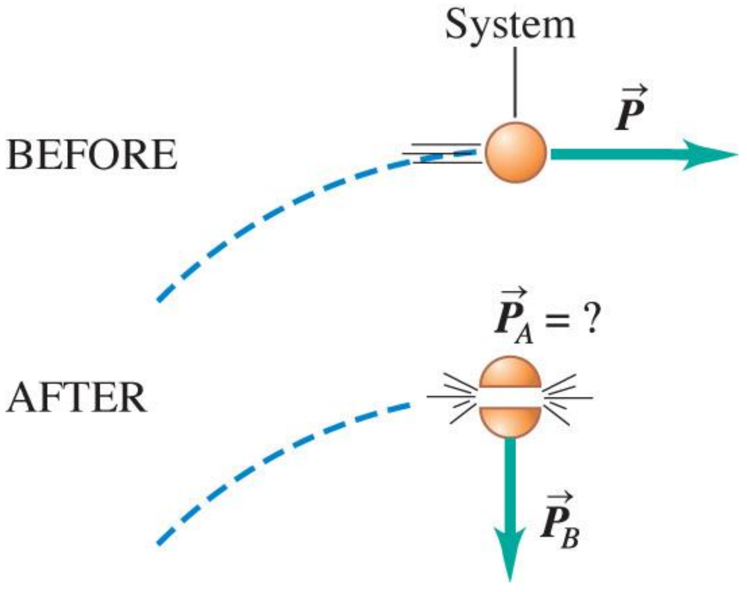
Exploding projectile after first stage burned out



Momentum is conserved

Clicker question

A space capsule follows a circular orbit around a planet. A shaped charge blows the capsule into two fragments. As shown in the figure, one fragment falls straight toward the planet's surface. How does the other fragment move right after the explosion?



a) It continues on the circular orbit of the original capsule.
b) It moves straight away from the planet.
c) It moves up and to the right.

Q-RT8.1

Clicker question

Rank the following objects in order of the *magnitude of the momentum* of the object, from largest to smallest.

$K = p^2/2m$ for the same magnitude of momentum p, the one with the smaller mass m has more kinetic energy K

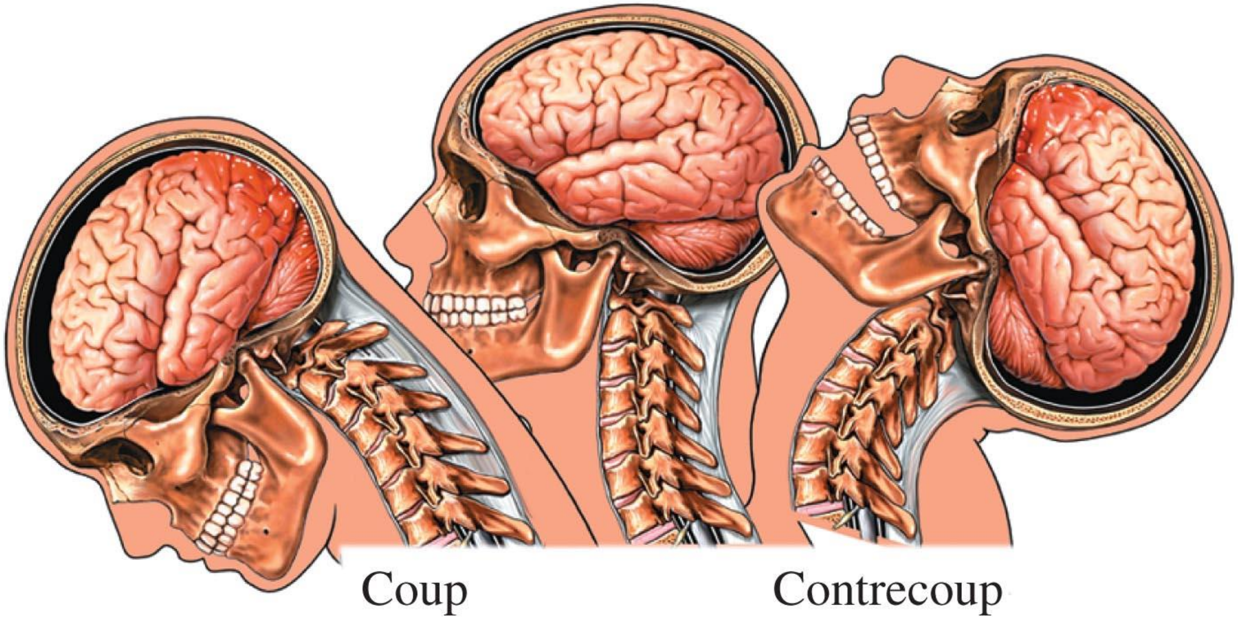
$K = p^2/2m = \frac{1}{2} mv^2$

A. Mass = 2.0 kg, kinetic energy = 2.0 J
B. Mass = 1.0 kg, kinetic energy = 2.0 J
C. Mass = 2.0 kg, kinetic energy = 4.0 J
D. Mass = 4.0 kg, kinetic energy = 4.0 J

a)ABCD b)DCBA c)DCAB d)ABDC

Momentum Can Cause Injury (Concussion)

- This is a frame of reference problem just like a passenger in a car. When the brain and skull are moving at the same velocity, there is no problem. If the skull changes abruptly and the brain does not, there is a possibility of an injury.



8.2 Conservation of Momentum

Consider a system consisting of a number of particles (A, B, …)

External Forces: Forces acting on objects in the system by objects outside the system.

Net External Force: the sum of the external forces.

Internal Forces: Forces acting on particles in the system by particles in the system.

Net Internal Force: the sum of the internal forces, which must be zero.

Conservation of Momentum:
If the net external force is zero, the momentum of the system is conserved:

$$\vec{p}_{A,i} + \vec{p}_{B,i} + \cdots = \vec{p}_{A,f} + \vec{p}_{B,f} + \cdots.$$

In the component form:

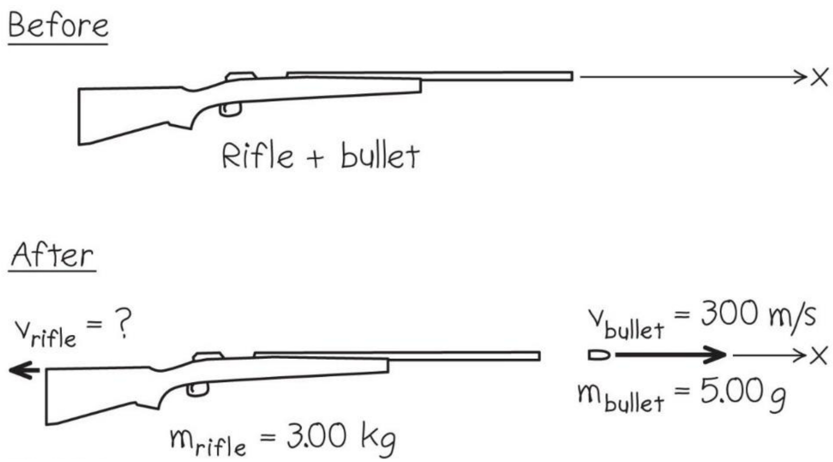
$$p_{A,i.x} + p_{B,i,x} + \cdots = p_{A,f,x} + p_{B,f,x} + \cdots$$

$$p_{A,i,y} + p_{B,i,y} + \cdots = p_{A,f,y} + p_{B,f,y} + \cdots$$

Examples 8.3 and 8.4 on page 226
Example 8.5 on page 227

Courtesy of Wenhao Wu

Rifle Recoil – Example 8.3 on page 228



Initial total momentum is zero.
Final total momentum is zero.

Conservation of Momentum:

$$p_{R,i} + p_{B,i} = p_{R,f} + p_{B,f}$$

$$0 + 0 = m_R v_{R,f} + m_B v_{B,f}$$

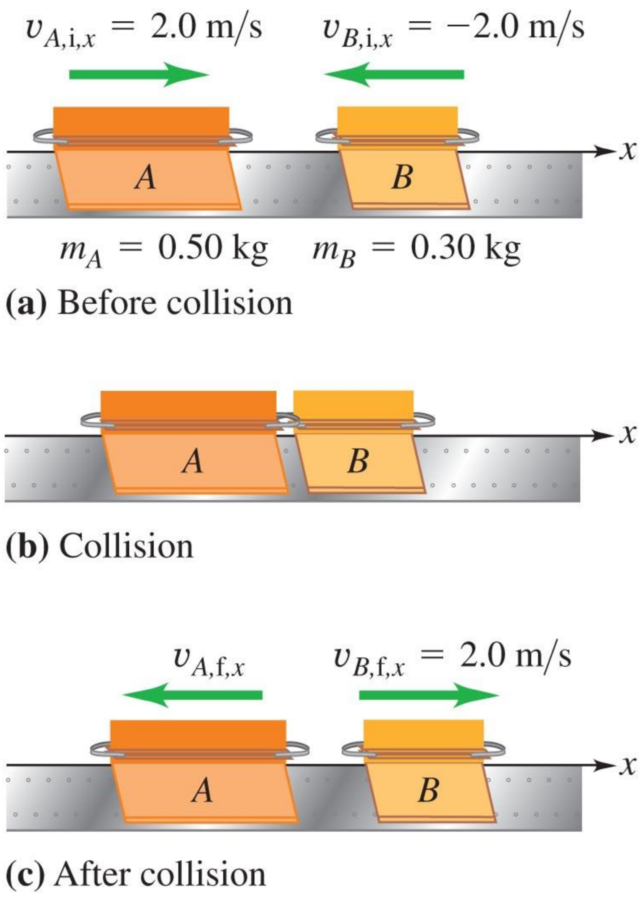
$$v_{R,f} = -\,(m_B v_{B,f})/m_R$$

$$K_{B,f} = \frac{1}{2} m_B v_{B,f}^2$$

$$K_{R,f} = \frac{1}{2} m_R v_{R,f}^2$$

Courtesy of Wenhao Wu

Head-on Collision – Example 8.4 on page 228



$$p_{A,i} + p_{B,i} = p_{A,f} + p_{B,f}$$

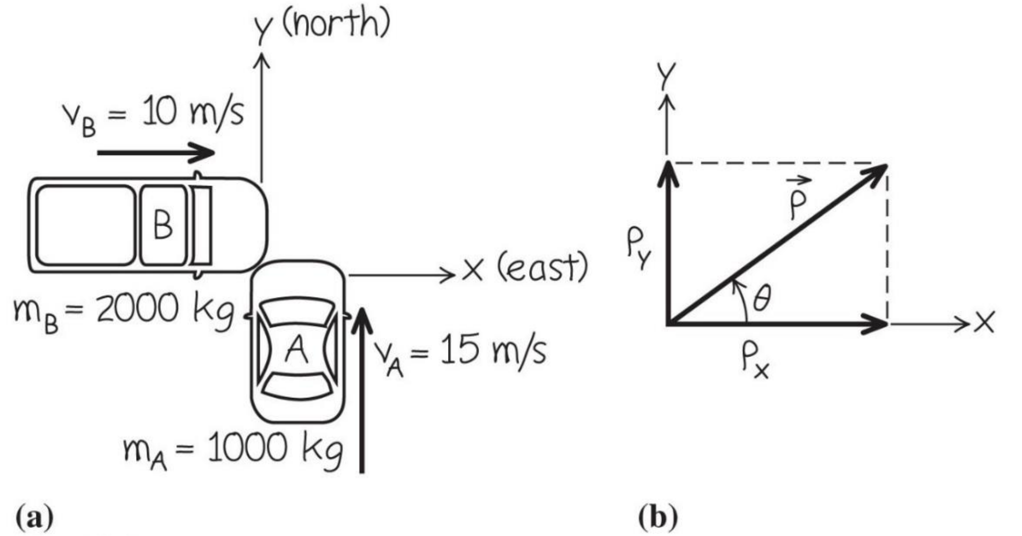
$$m_A v_{A,i} + m_B v_{B,i} = m_A v_{A,f} + m_B v_{B,f}$$

(0.50 kg)(2.0 m/s) + (0.30 kg)(– 2.0 m/s) =
(0.50 kg)($v_{A,f}$) + (0.30 kg)(2.0 m/s)

p_{ix}=(0.50 kg)(2.0 m/s) + (0.30 kg)(– 2.0 m/s)
p_{fx} = (0.50 kg)($v_{A,fx}$) + (0.30 kg)(2.0 m/s)

Conservation of x-component yields
$v_{A,fx}$ = – 0.40 m/s

Collision In a Horizontal Plane – Example 8.1 on page 225



The Initial State: Before Collision

$P_{i,x} = p_{A,i,x} + p_{B,i,x} = 0 + (2000\text{ kg})(10\text{ m/s}) = 2.0\times10^4$ kgm/s

$P_{i,y} = p_{A,i,y} + p_{B,i,y} = (1000\text{ kg})(15\text{ m/s}) + 0 = 1.5\times10^4$ kgm/s

The Final State: After Collision

$P_{f,x} = p\cos(\theta)$

$P_{f,y} = p\sin(\theta)$

Conservation of Momentum

2.0×10^4 kgm/s = pcos(θ)

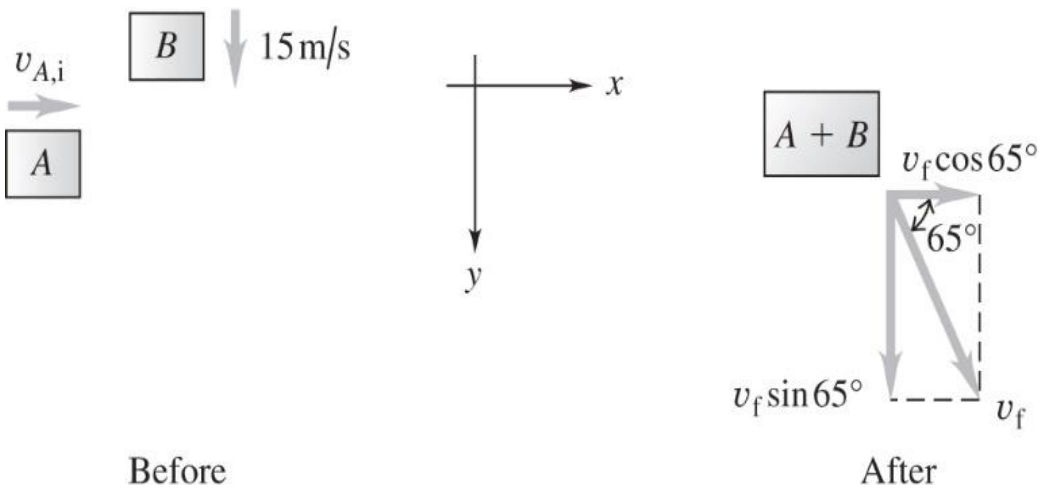
1.5×10^4 kgm/s = psin(θ)

$$\theta = \tan^{-1}\frac{1.5\times104}{2.0\times104} = 36.9°$$

$P = \sqrt{P_{f,x}^2 + P_{f,y}^2} = 2.5\times10^4$ kgm/s

8.27
A accident analysis

8.27. Set Up: Use coordinates where $+x$ is east and $+y$ is south. The system of two cars before and after the collision i sketched in the figure below. Neglect friction from the road during the collision. The enmeshed cars have mas $2000\text{ kg} + 1500\text{ kg} = 3500\text{ kg}$.



Solve: There are no external horizontal forces during the collision, so $P_{i,x} = P_{f,x}$ and $P_{i,y} = P_{f,y}$.

(a) $P_{i,x} = P_{f,x}$ gives $(1500\text{ kg})(15\text{ m/s}) = (3500\text{ kg})v_f \sin 65°$ and $v_f = 7.1\text{ m/s}$.

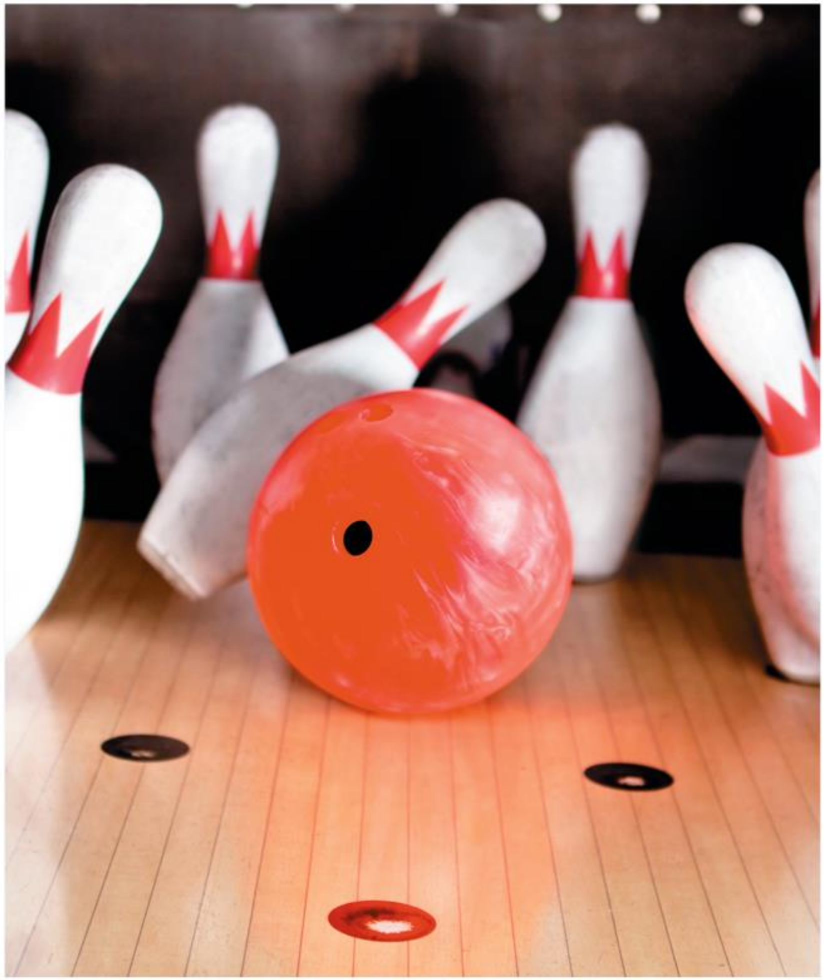
(b) $P_{i,y} = P_{f,y}$ gives $(2000\text{ kg})v_{A,i} = (3500\text{ kg})v_f \cos 65°$. And then with $v_f = 7.1\text{ m/s}$, $v_{A,i} = 5.2\text{ m/s}$.

Reflect: Momentum is a vector and we must treat each component separately.

Scoring a "Strike" Is Many Momentum Transfers at Once

Before the strike momentum is in the ball

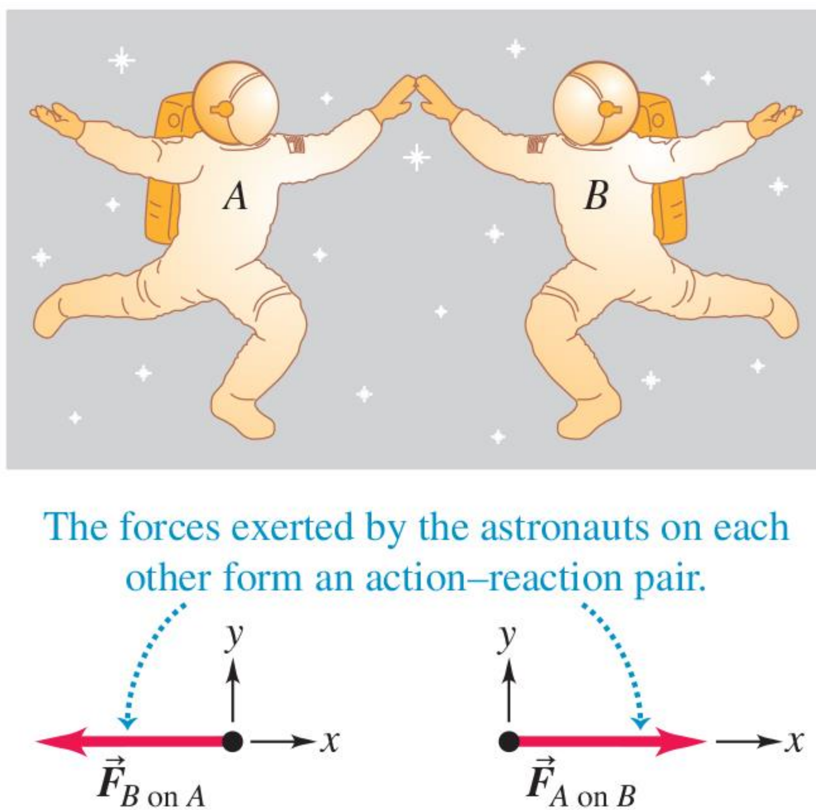
Afterwards in the ball and the pins



Momentum Is Conserved – Figure 8.3

- Astronauts provide excellent examples of momentum transfer.

Internal forces on the astronauts are the same



Pearson

8.71

Explosion with2 fragments

71. II A 7.0 kg shell at rest explodes into two fragments, one with a mass of 2.0 kg and the other with a mass of 5.0 kg. If the heavier fragment gains 100 J of kinetic energy from the explosion, how much kinetic energy does the lighter one gain?

8.71. Set Up: Call the fragments *A* and *B*, with $m_A = 2.0$ kg and $m_B = 5.0$ kg. After the explosion fragment *A* moves in the $+x$ direction with speed υ_A and fragment *B* moves in the $-x$ direction with speed υ_B.

Solve: $P_{i,x} = P_{f,x}$ gives $0 = m_A\upsilon_A + m_B(-\upsilon_B)$ and

$$\upsilon_A = \left(\frac{m_B}{m_A}\right)\upsilon_B = \left(\frac{5.0\text{ kg}}{2.0\text{ kg}}\right)\upsilon_B = 2.5\upsilon_B$$

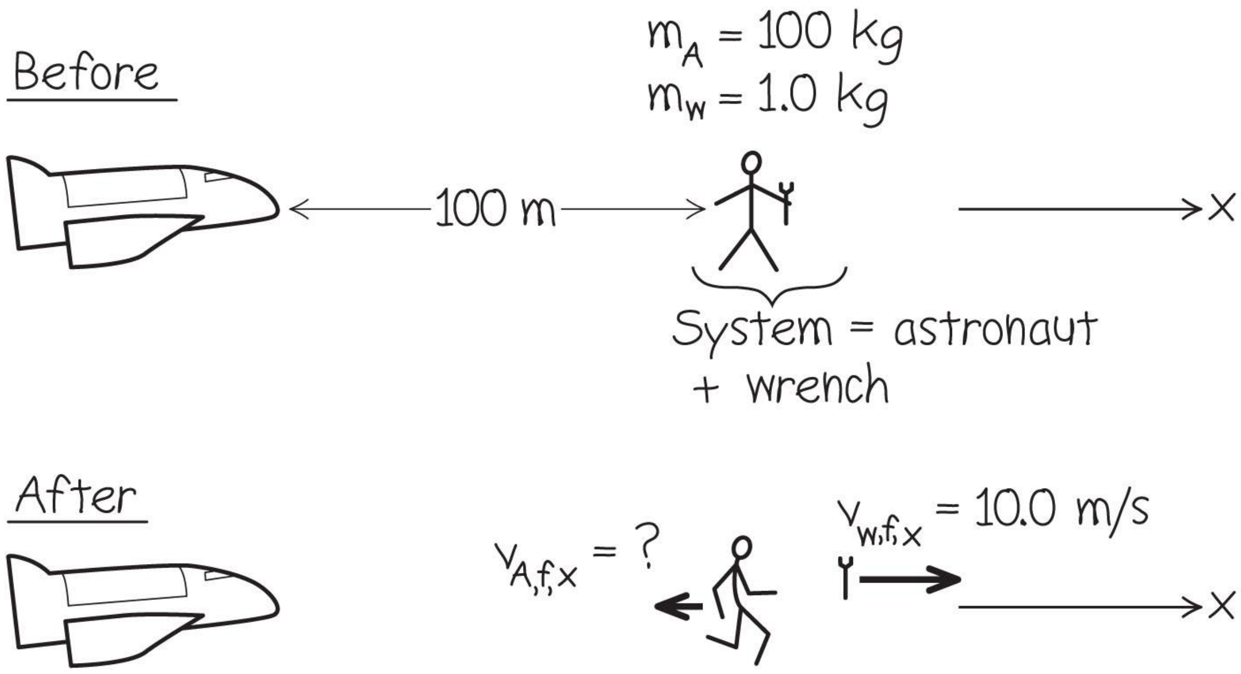
$$\frac{K_A}{K_B} = \frac{\frac{1}{2}m_A\upsilon_A^2}{\frac{1}{2}m_B\upsilon_B^2} = \frac{\frac{1}{2}(2.0\text{ kg})(2.5\upsilon_B)^2}{\frac{1}{2}(5.0\text{ kg})\upsilon_B^2} = \frac{12.5}{5.0} = 2.5$$

$K_A = 100$ J so $K_B = 250$ J

Reflect: In an explosion the lighter fragment receives the most of the liberated energy

An Astronaut Rescue – Example 8.2

- Refer to the worked example on page 227.



An Astronaut Rescue – Example 8.2

SOLVE The astronaut needs to acquire a velocity *toward* the spaceship. Because the total momentum (of her and the wrench) is zero, she should throw the wrench directly *away* from the ship.

All the vector quantities lie along the x axis, so we're concerned only with x components. We write an equation expressing the equality of the initial and final values of the total x component of momentum:

$$m_{\mathrm{A}}(v_{\mathrm{A,\,i,\,}x}) + m_{\mathrm{W}}(v_{\mathrm{W,\,i,\,}x}) = m_{\mathrm{A}}(v_{\mathrm{A,\,f,\,}x}) + m_{\mathrm{W}}(v_{\mathrm{W,\,f,\,}x}).$$

In this case, the initial velocity of each object is zero, so the left side of the equation is zero; that is, $m_{\mathrm{A}}(v_{\mathrm{A,\,i,\,}x}) + m_{\mathrm{W}}(v_{\mathrm{W,\,i,\,}x}) = 0$. Solving for $v_{\mathrm{A,\,f,\,}x}$, we get the astronaut's x component of velocity after she throws the wrench:

$$v_{\mathrm{A,\,f,\,}x} = \frac{-m_{\mathrm{W}}(v_{\mathrm{W,\,f,\,}x})}{m_{\mathrm{A}}} = \frac{-(1.0\ \mathrm{kg})(10.0\ \mathrm{m/s})}{(100\ \mathrm{kg})} = -0.10\ \mathrm{m/s}.$$

The negative sign indicates that the astronaut is moving toward the ship, opposite our chosen $+x$ direction.

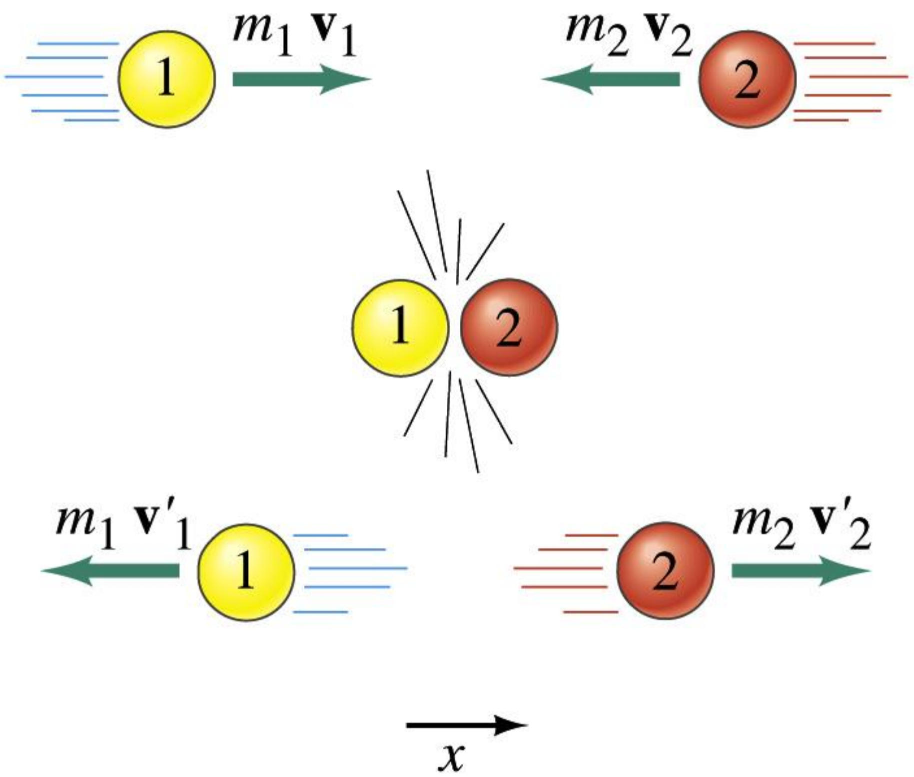
To find the total time required for the astronaut to travel 100 m (at constant velocity) to reach the ship, we use $x = vt$, or

$$t = \frac{100\ \mathrm{m}}{0.10\ \mathrm{m/s}} = 1.00 \times 10^{3}\ \mathrm{s} = 16\ \mathrm{min}\ 40\ \mathrm{s}.$$

REFLECT With a 20-minute air supply, she makes it back to the ship safely, with 3 min 20 s to spare.

ed

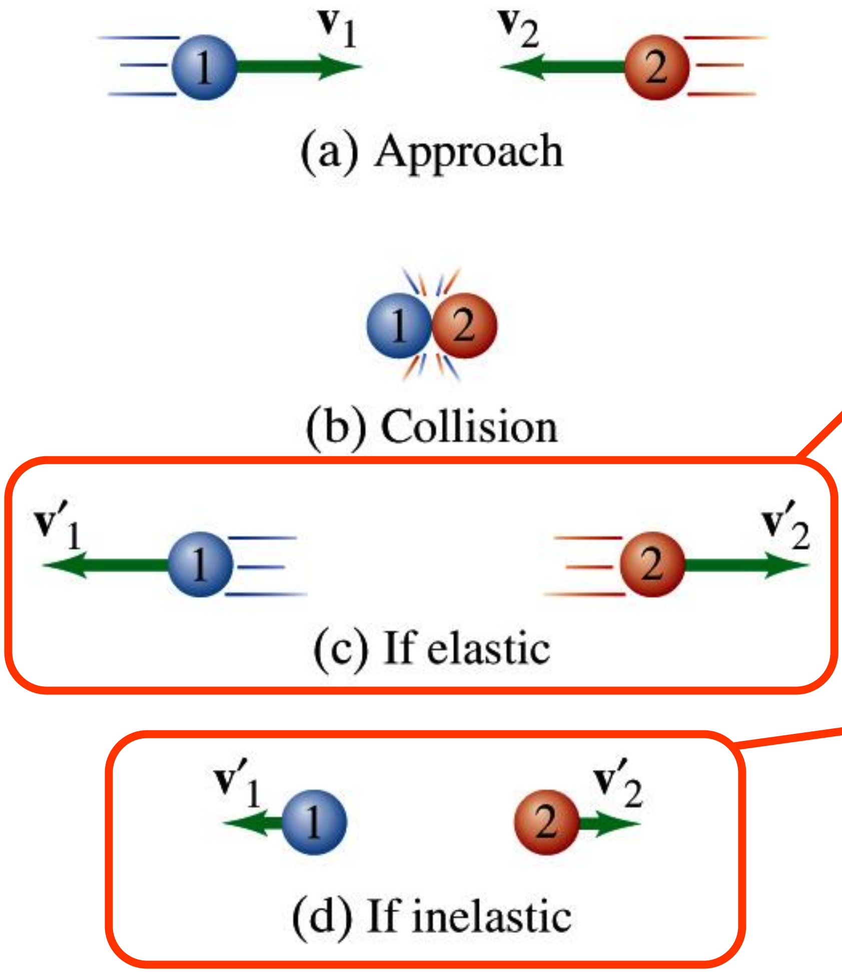
Elastic Collision



Momentum : $$m_1 v_1 + m_2 v_2 = m_1 v_1^{'} + m_2 v_2^{'}$$

Kinetic Energy : $$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1^{'\,2} + \frac{1}{2} m_2 v_2^{'\,2}$$

Energy Conservation



$$K_{1,i} + K_{2,i} = K_{1,f} + K_{2,f}$$

$$K_{1,i} + K_{2,i} = K_{1,f} + K_{2,f} + Q$$

Loss of energy as thermal and other forms of energy

Q8.7

Clicker question

Block *A* has mass 1.00 kg and block *B* has mass 3.00 kg. The blocks collide and stick together on a level, frictionless surface. After the collision, the kinetic energy (KE) of block *A* is

A. one-ninth the KE of block *B*.

B. one-third the KE of block *B*.

C. three times the KE of block *B*.

D. nine times the KE of block *B*.

E. the same as the KE of block *B*.

Clicker question

A 3.00-kg rifle fires a 0.00500-kg bullet at a speed of 300 m/s. Which force is greater in magnitude: the force that the *rifle* exerts on the *bullet* or the force that the *bullet* exerts on the *rifle?*

A. The force that the rifle exerts on the bullet is greater.

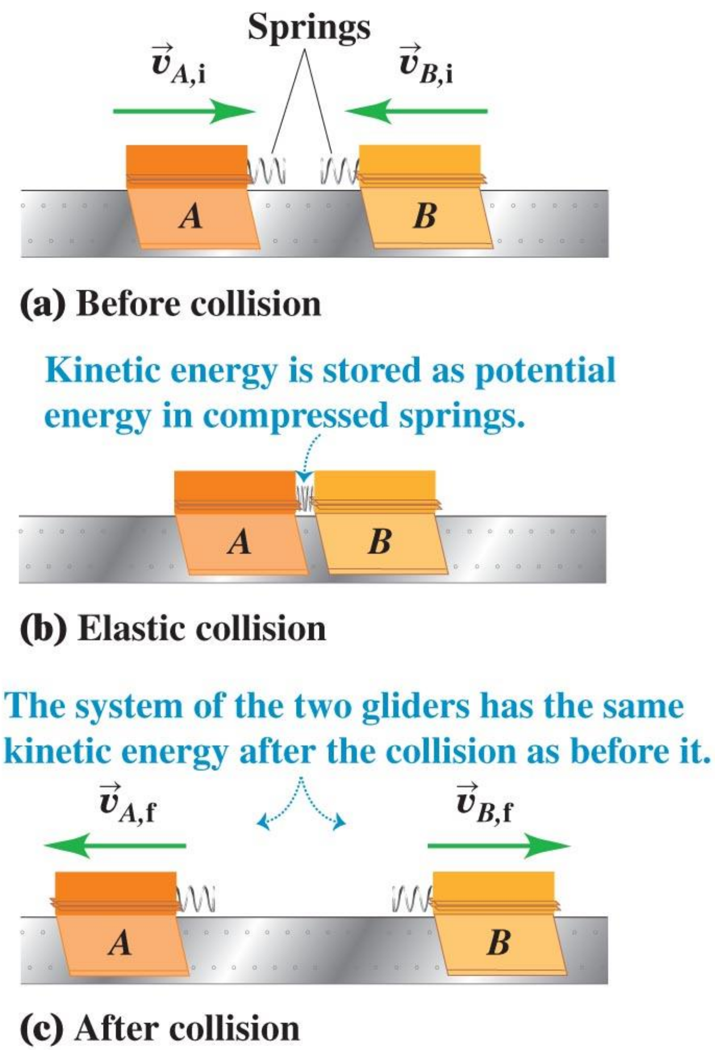
B. The force that the bullet exerts on the rifle is greater.

C. Both forces have the same magnitude.

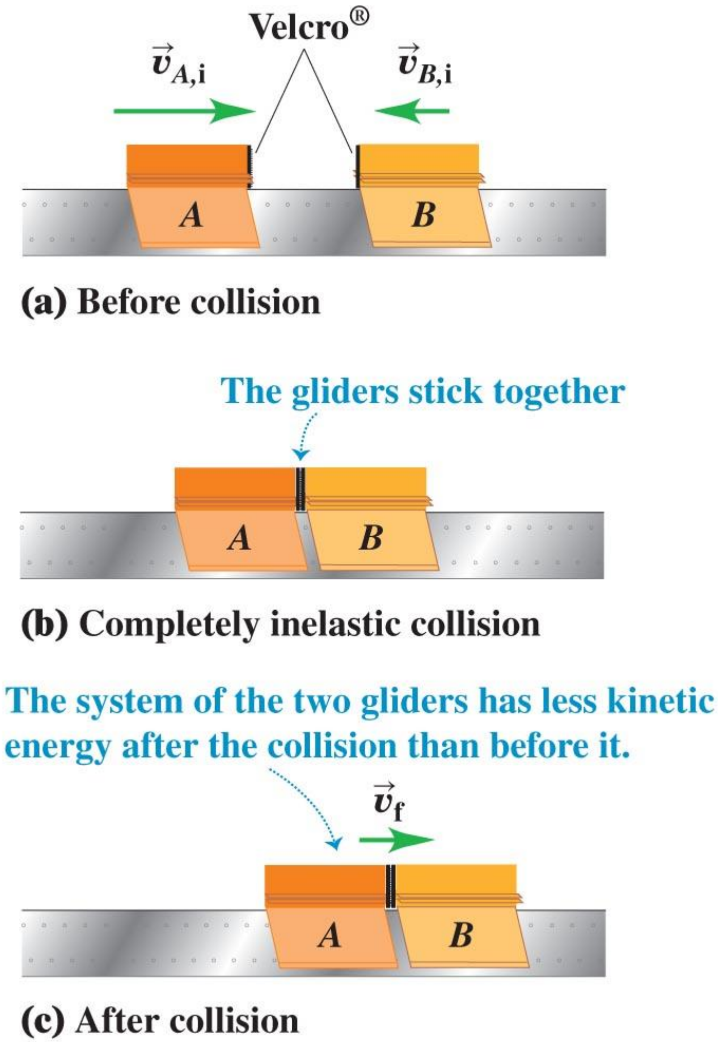
D. The answer depends on how the rifle is held.

E. The answer depends on how the rifle is held and on the inner workings of the rifle.

Elastic Collision



Inelastic Collision



8.4 Elastic Collision

Both the Momentum and the Kinetic Energy are conserved

Consider the elastic collision of two particles along a straight line.

Conservation of Momentum: $m_A v_{A,i} + m_B v_{B,i} = m_A v_{A,f} + m_B v_{B,f}$

Conservation of Kinetic Energy: $\frac{1}{2} m_A v_{A,i}^2 + \frac{1}{2} m_B v_{B,i}^2 = \frac{1}{2} m_A v_{A,i}^2 + \frac{1}{2} m_B v_{B,i}^2$

Head-on Collision with a Stationary Object ($v_{B,i} = 0$)

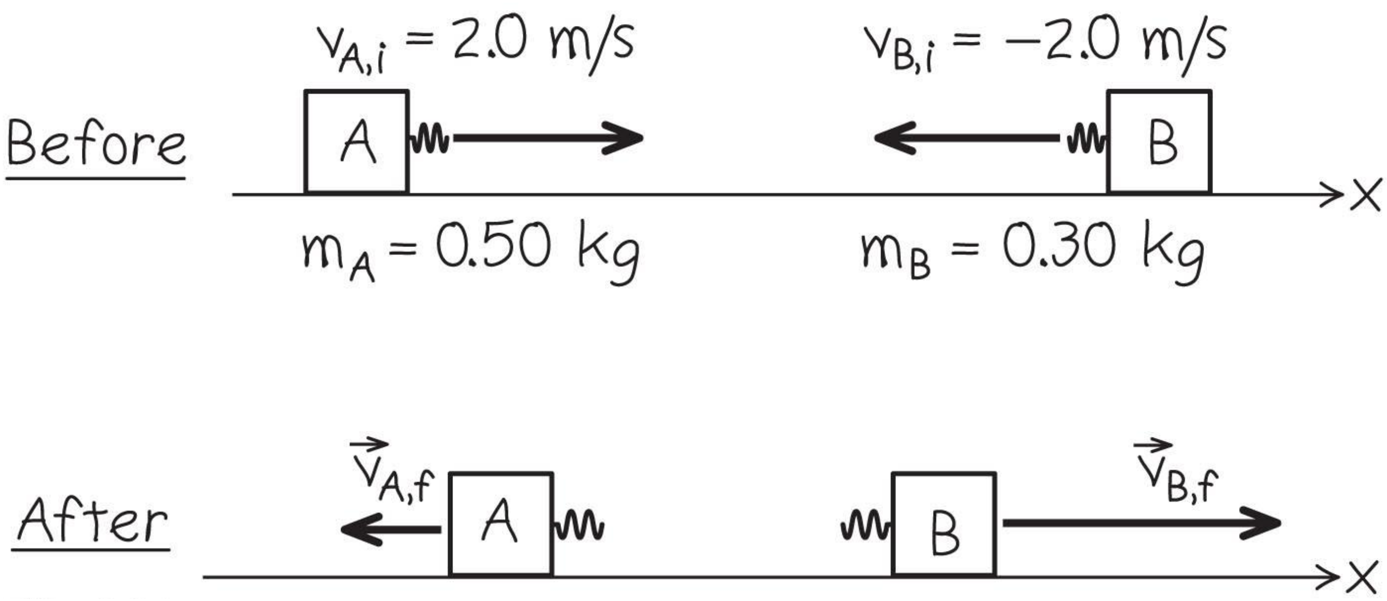
$$v_{A,f} = \frac{m_A - m_B}{m_A + m_B} v_{A,i}$$

$$v_{B,f} = \frac{2m_A}{m_A + m_B} v_{A,i}$$

Relative Velocity $v_{B,f} - v_{A,f} = v_{A,i}$

A Solved Air Track Problem – Example 8.9 (no friction)

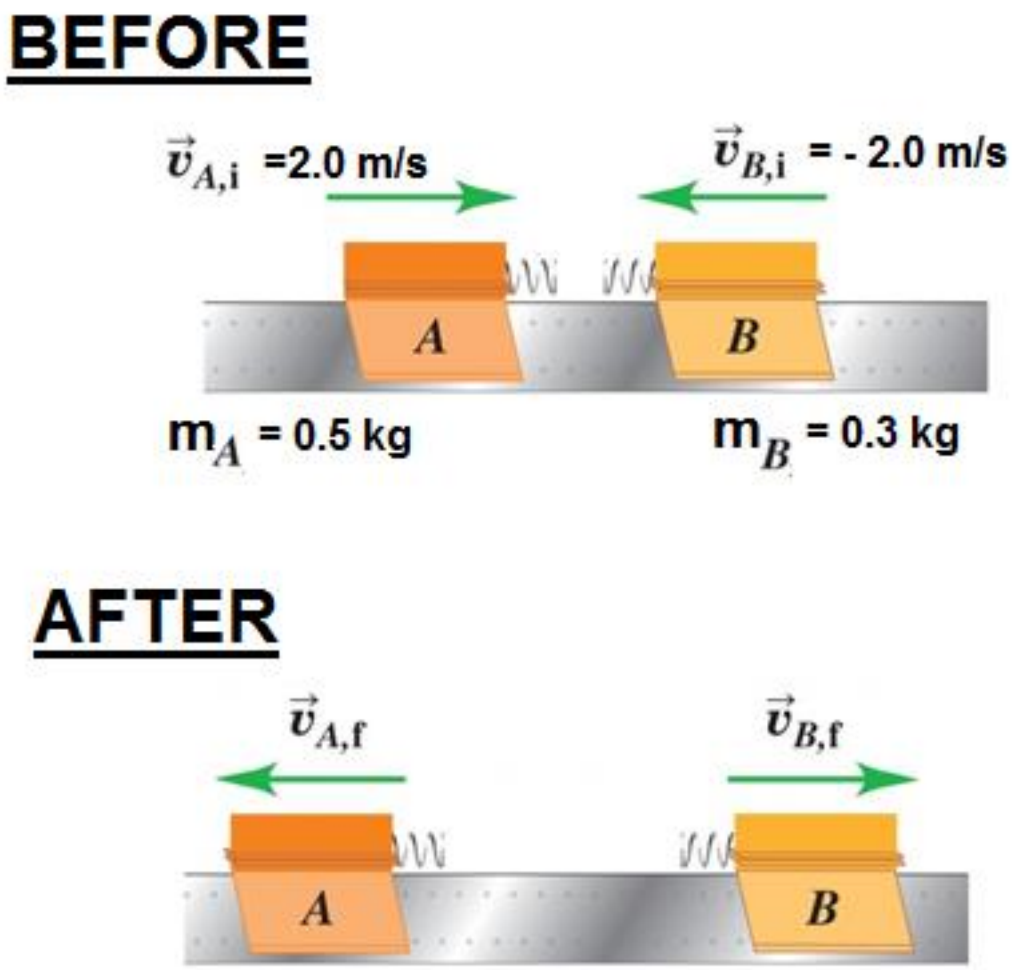
- See the example on page 236. An elastic collision



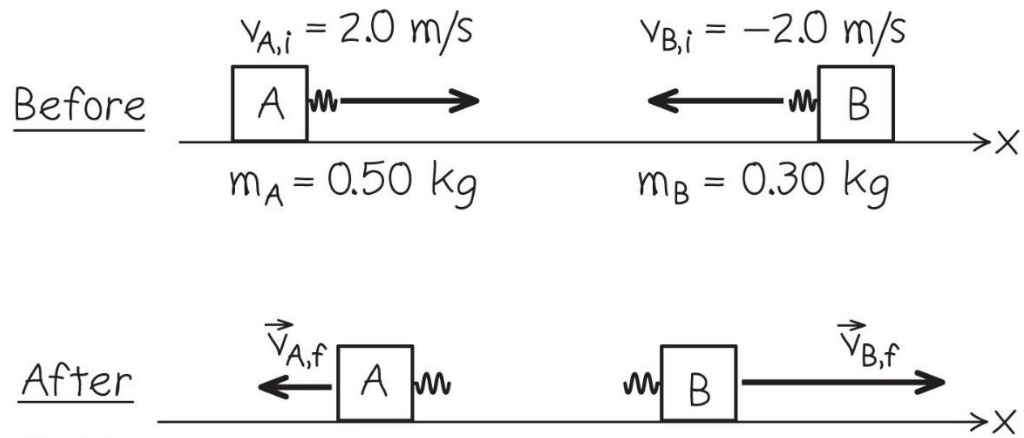
Special case : $m_A v_A = m_B v_B$ $K_f = K_A + K_B = U_f$

in the after situation when initially compressed and at rest

Elastic Collision on a air track



Elastic Collision on an Air Track – Example 8.9 on page 234



Conservation of Momentum: $m_A v_{A,i} + m_B v_{B,i} = m_A v_{A,f} + m_B v_{B,f}$ ……(1)

Conservation of Kinetic Energy: $\frac{1}{2} m_A v_{A,i}^2 + \frac{1}{2} m_B v_{B,i}^2 = \frac{1}{2} m_A v_{A,i}^2 + \frac{1}{2} m_B v_{B,i}^2$ ……(2)

Relative Velocities: $v_{B,f} - v_{A,f} = -(v_{B,i} - v_{A,i})$ ……(3)

From (3) $v_{B,f} = -(v_{B,i} - v_{A,i}) + v_{A,f}$ ……(4)

Substitute into (1) to solve for $v_{A,f}$. Then, calculate $v_{B,f}$ using (4).

Elastic Collision on an air track

Momentum conservation;

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

$$\rightarrow \quad 0.5kg * 2.0\frac{m}{s} + 0.3kg * \left(-2.0\frac{m}{s}\right) = (0.5kg)v_{Af} + (0.3kg)v_{Bf} = 0.4kg\frac{m}{s}$$

Note: Relative velocity changes sign in an elastic collision.

$$v_{Bf} - v_{Af} = -(v_{Bi} - v_{Ai}) = -\left(-2.0\frac{m}{s} - 2.0\frac{m}{s}\right) = 4.0\frac{m}{s}$$

Now;

$$(0.5)v_{Af} + (0.3)v_{Bf} = 0.4\frac{m}{s}$$

$$v_{Bf} - v_{Af} = 4.0\frac{m}{s} \qquad v_{Af} = v_{Bf} - 4.0\frac{m}{s}$$

$$0 + 0.8v_{Bf} = 2.4\frac{m}{s}$$

$$v_{Bf} = \frac{2.4}{0.8} = 3\frac{m}{s}$$

$$v_{Af} = -1\frac{m}{s}$$

Kinetic energy in elastic collision (energy conservation as expected);

$$K_i = \frac{1}{2}m_A v_{Ai}^2 + \frac{1}{2}m_B v_{Bi}^2 = \frac{1}{2}(0.5kg)\left(2\frac{m}{s}\right)^2 + \frac{1}{2}(0.3kg)\left(-2\frac{m}{s}\right)^2 = 1.6\,J$$

$$K_f = \frac{1}{2}m_A v_{Af}^2 + \frac{1}{2}m_B v_{Bf}^2 = \frac{1}{2}(0.5kg)\left(-1\frac{m}{s}\right)^2 + \frac{1}{2}(0.3kg)\left(3\frac{m}{s}\right)^2 = 1.6\,J$$

Elastic Collision between different mass balls

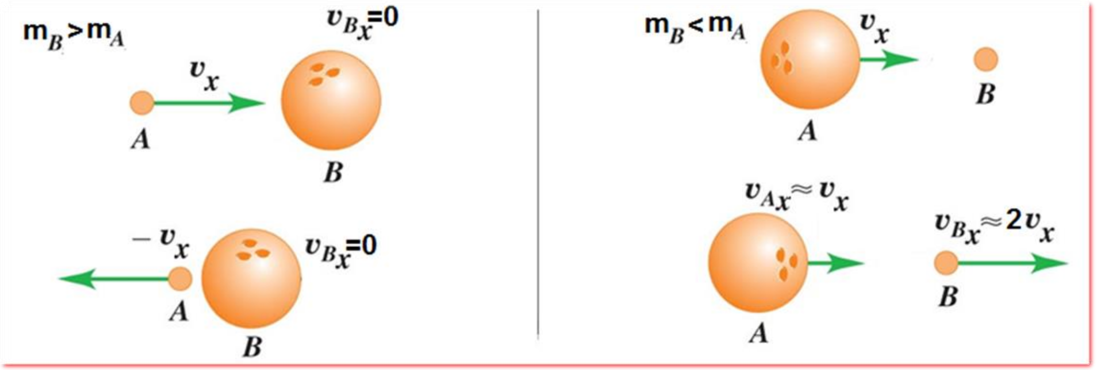
Kinetic energy conservation

$$\frac{1}{2}m_A v_x^2 + 0 = \frac{1}{2}m_A v_{Ax}^2 + \frac{1}{2}m_B v_{Bx}^2$$

Momentum conservation

$$m_A v_x = m_A v_{Ax} + m_B v_{Bx}$$

Note:single subscript before collision
double subscript after collision
special case: object B initially at rest



$$m_B v_{Bx}^2 = m_A\left(v_x^2 - v_{Ax}^2\right) = m_A(v_x + v_{Ax})(v_x - v_{Ax}) \qquad \textbf{(1)}$$

$$m_B v_{Bx} = m_A(v_x - v_{Ax}) \qquad \textbf{(2)}$$

Divide equation (1) and (2);

$$v_{Bx} = v_x + v_{Ax}$$

$$m_B(v_x + v_{Ax}) = m_A(v_x - v_{Ax})$$

in general $v_{Bf}\text{-}v_{Af} = -(v_{Bi}\text{ -}v_{Ai})$

$$v_{Ax} = \frac{m_A - m_B}{m_A + m_B} v_x$$

$$v_{Bx} = \frac{2m_A}{m_A + m_B} v_x$$

$v_{Bx}\text{-}v_{Ax} = v_x$

$m_B > m_A$; $v_{Ax} = \frac{m_A - m_B}{m_A + m_B} v_x \propto \frac{-m_B}{+m_B} v_x \approx -v_x$ and $v_{Bx} \approx 0$

$m_A > m_B$; $v_{Ax} = \frac{m_A - m_B}{m_A + m_B} v_x \propto \frac{m_A}{m_A} v_x \approx v_x$ when $v_{Bx} = \frac{2m_A}{m_A + m_B} v_x \approx \frac{2m_A}{m_A} v_x \approx 2v_x$

For $m_A = m_B$;

$v_{Ax} = 0$; $v_{Bx} = v_x$ (initial velocity) remember playing billiard (pool)

8.3 Inelastic Collision

Momentum is conserved but not the Kinetic Energy

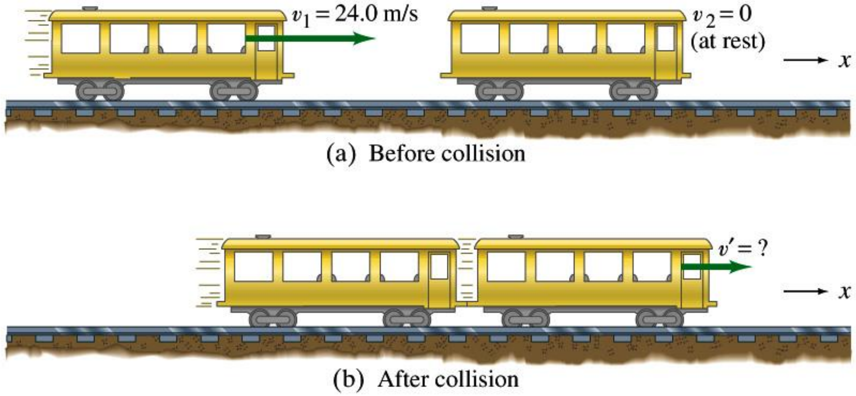
Completely inelastic collision along a straight line:

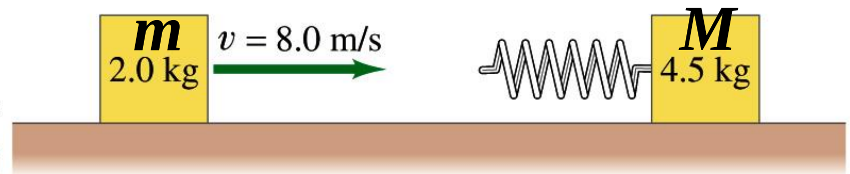
$$m_A v_{A,i,x} + m_B v_{B,i,x} = (m_A + m_B) v_{f,x}$$

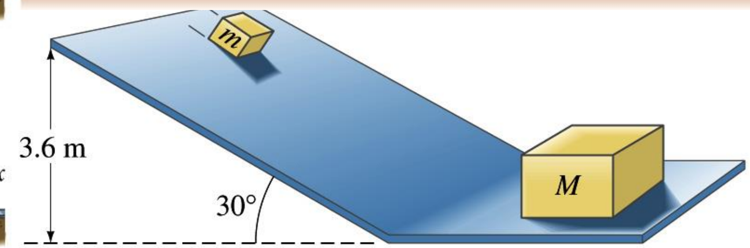
Example 8.6 on page 230
Example 8.8 on page 232
Example 8.7 on page 231, and other variations

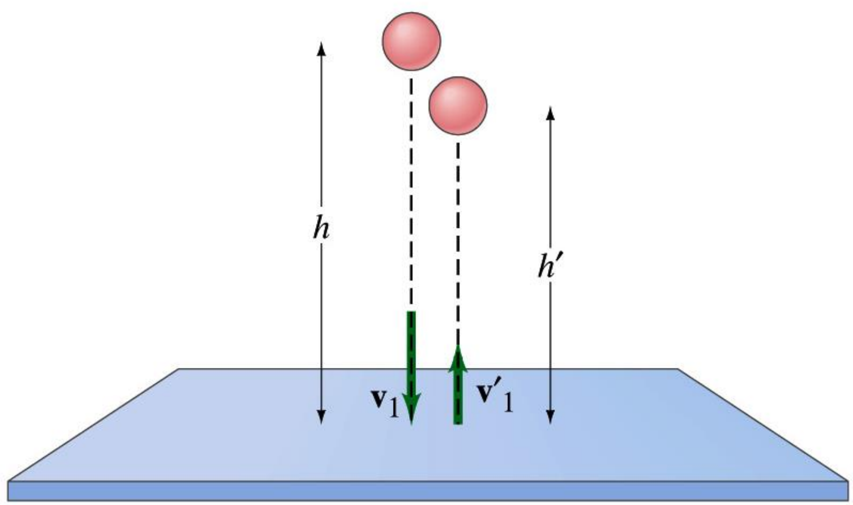
Courtesy of Wenhao Wu

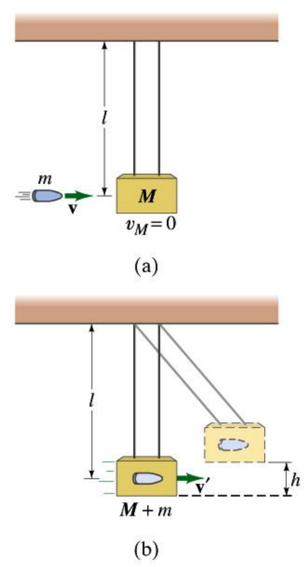
Examples of 1D Collisions Elastic and Inelastic

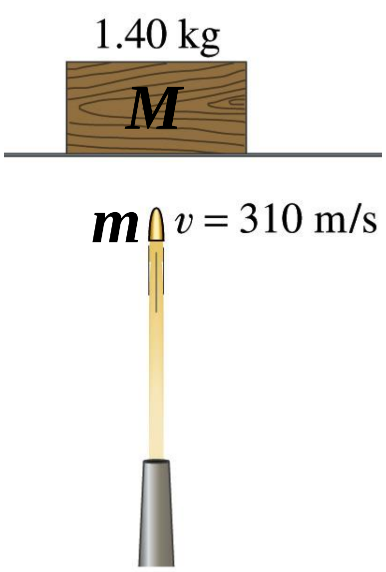






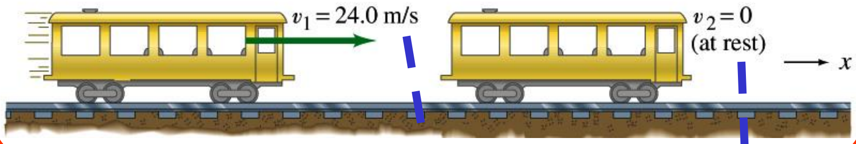




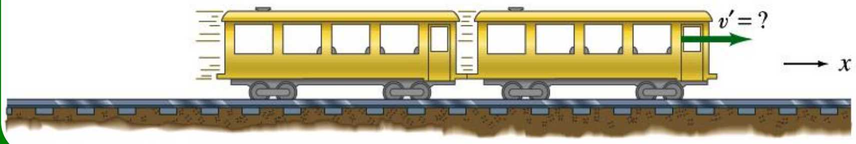


Example 2

Before collision



After collision



(totally inelastic collision)

$$m\,v_1 + m\,v_2 = m\,v_1' + m\,v_2'$$

$$v_1' = v_2'$$

Railroad cars, locking up after the collision

Find v'; when v_1=0.

$$m_1v_1 + m_2v_2 = (m_1+m_2)v'$$

$\rightarrow m_1v_1 + 0 = (m_1 + m_2)v' \rightarrow v' = \frac{m_1v_1}{m_1+m_2}$

case $m_1 = m_2$ $\quad v' = \frac{m_1v_1}{2m_1} = \frac{1}{2}v_1$

Kinetic energy in inelastic collision;

$\frac{1}{2}m_1{v_1}^2 + 0 \geq ? \frac{1}{2}(m_1 + m_2){v'}^2 \rightarrow \quad \frac{1}{2}m_1{v_1}^2 > ? \frac{1}{2}(2m_1)(\frac{1}{2}v_1)^2$

$\therefore \frac{1}{2}m_1{v_1}^2 > \frac{1}{4}m_1{v_1}^2$ Kinetic energy is reduced after the collision

How to fire a rifle to reduce recoil

What is the recoil of the rifle?

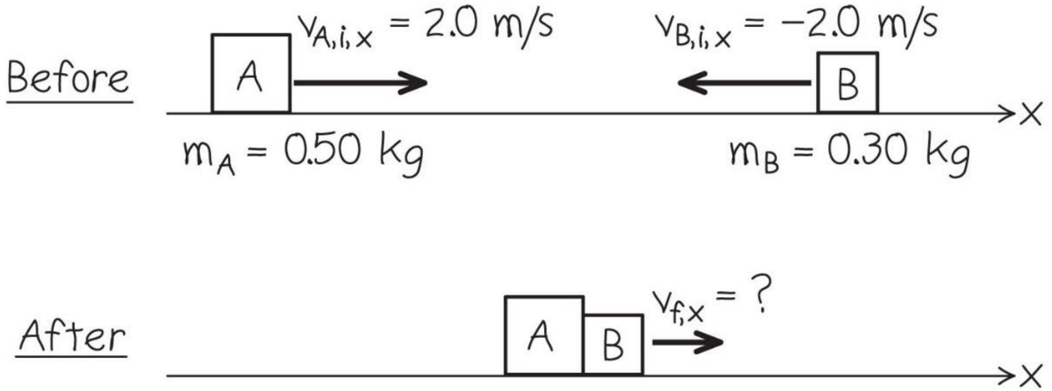
Given; $m_B = 5g$; $v_B = 300\frac{m}{s}$ and $m_R = 5kg$

$$m_Bv_B + m_Rv_R = 0 + 0 = m_Bv'_B + m_Rv'_R$$

$$\therefore v'_R = -\frac{m_Bv'_B}{m_R} = -1.2\frac{m}{s}$$

Inelastic Collision
on an Air Track

Example 8.6 on page 230



Completely inelastic collision along a straight line:

$$m_A v_{A,i,x} + m_B v_{B,i,x} = (m_A + m_B) v_{f,x}$$

(0.50 kg)(2.0 m/s) + (0.30 kg)(− 2.0 m/s) = (0.50 + 0.30 kg)$v_{f,x}$

$$v_{f,x} = 0.50 \text{ m/s}$$

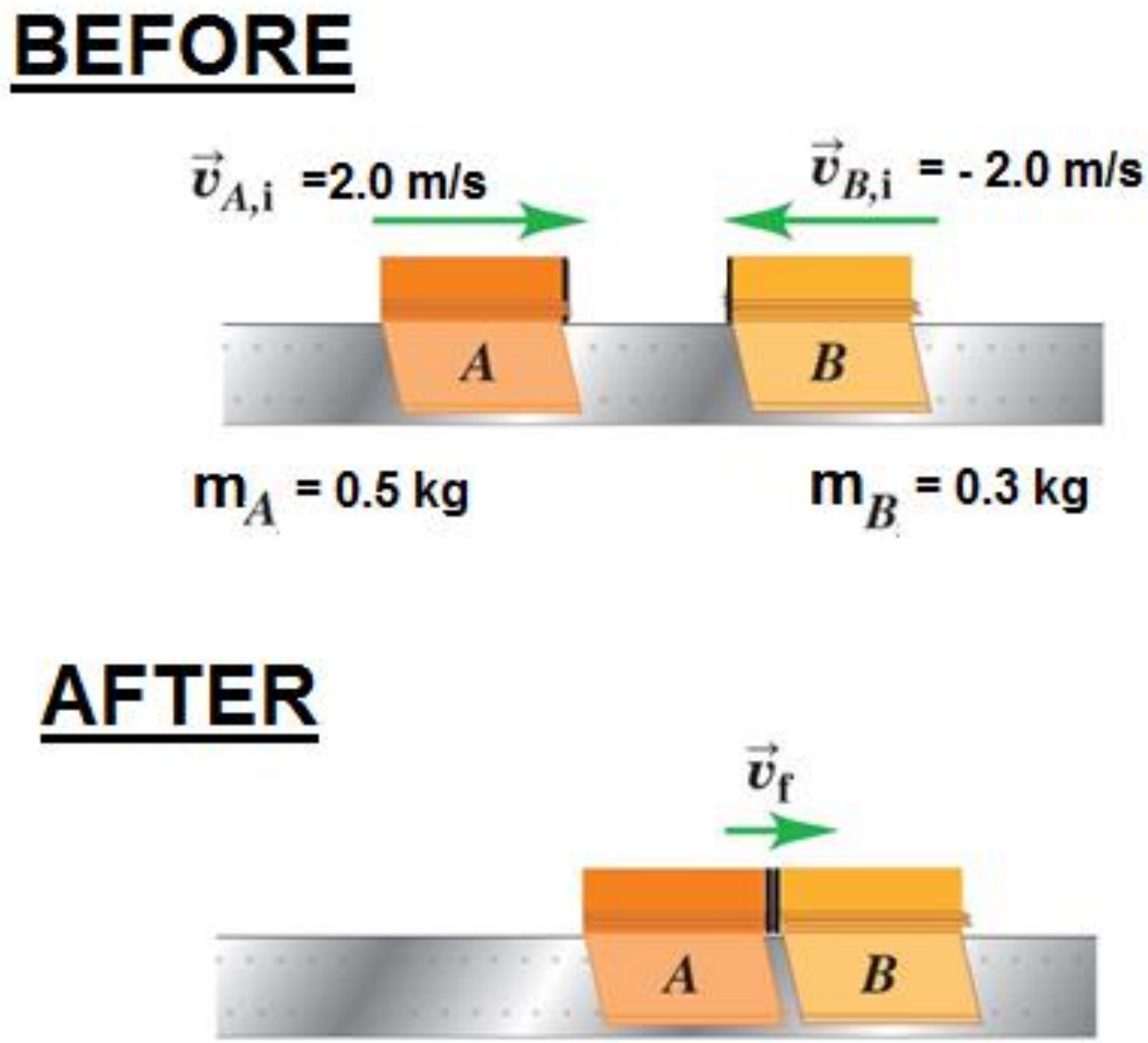
Initial kinetic energy $K_i = K_{A,i} + K_{B,i} = \frac{1}{2}(0.50\text{ kg})(2.0\text{ m/s})^2 + \frac{1}{2}(0.30\text{ kg})(-2.0\text{ m/s})^2$
$= 1.60$ J

Final kinetic energy $K_f = K_{A,f} + K_{B,f} = \frac{1}{2}(0.50 + 0.30\text{ kg})(0.50\text{ m/s})^2 = 0.10$ J

Kinetic energy decreased a lot in this inelastic collision.

Courtesy of Wenhao Wu

Inelastic Collision on an air track



Inelastic Collision on an air track

a) Find the final velocity of the joint on an air track.

$$m_A v_{Ai} + m_B v_{Bi} = (m_A + m_B) v_f$$

$$\rightarrow \quad 0.5kg * 2.0\frac{m}{s} + 0.3kg * \left(-2.0\frac{m}{s}\right) = (0.5kg + 0.3kg) v_f$$

$$\therefore v_f = 0.5\frac{m}{s}$$ (towards the right and together)

b) Find the total kinetic energy before the collision.

$$K_{Ai} = \frac{1}{2} m_A {v_{Ai}}^2 = \frac{1}{2} * 0.5 * 4 = 1.01\,J$$

$$K_{Bi} = \frac{1}{2} m_B {v_{Bi}}^2 = \frac{1}{2} * 0.3 * 4 = 0.06\,J$$

Total $E_{kin\,b}$ before collision is K_{Ai}+K_{Bi}=1.61 **J**

c) Find the total kinetic energy after the collision

$$E_{kin\,a} = K_f = \frac{1}{2}(m_A + m_B){v_f}^2 = \frac{1}{2} * (0.5kg + 0.3kg) * \left(0.5\frac{m}{s}\right)^2 = 0.10\,J$$

d) Compare the kinetic energies.

$$\frac{E_{kin\,b}}{E_{kin\,a}} = \frac{1.61}{0.1} \approx 16$$ **times larger than after the collision**

e) Where did this energy go?

Extra energies converted to the thermal energy after collision

Box A of mass m_A = 2.0 kg is sliding at 15 m/s on a horizontal surface. It collides with and sticks to Box B of mass m_A = 4.0 kg that is initially at rest on the horizontal surface. The combined object then slides up an incline. There is no friction for any of the surfaces.

Find:
(a) the speed of the combined object right after the collision;
(b) the height above the horizontal surface the combined object reaches before sliding down.

Solution:

(1) Completely inelastic collision $m_A v + 0 = (m_A + m_B)V$

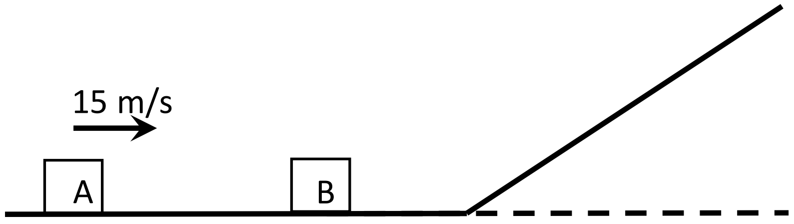
Solve for the speed of the combined object right after collision

$$V = m_A v/(m_A + m_B) = 2.0\times15/(2.0 + 4.0) = 5.0 \text{ m/s}$$

(2) Sliding up obeying the conservation of energy.

$$\frac{1}{2}(m_A + m_B)V^2 = (m_A + m_B)gh$$

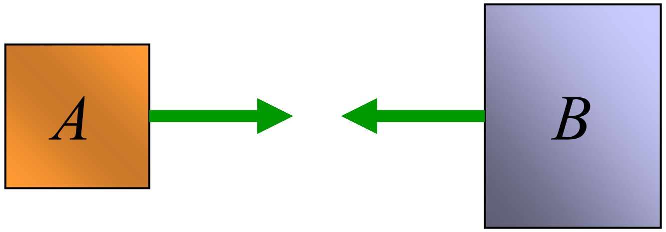
So, $h = \frac{V^2}{2g} = 25/(2\times9.8) = 1.3$ m



Courtesy of Wenhao Wu

Clicker question

Two objects with different masses collide with and *stick* to each other. Compared to *before* the collision, the system of two objects *after* the collision has



A. the same amount of total momentum and the same total kinetic energy.

B. the same amount of total momentum but less total kinetic energy.

C. less total momentum but the same amount of total kinetic energy.

D. less total momentum and less total kinetic energy.

E. Not enough information is given to decide.

Bullet shot into a block of wood attached to a spring

A bullet of unknown horizontal speed is shot into a block of wood attached to a spring which is initially at equilibrium. The bullet stuck in the wood block and together they compress the spring an amount L. Find the speed of the bullet before it strikes the wood block.

Given: m_B, m_W, L and spring constant k.
Find: v_B

Comment: Solve this in either the forward or the backward order.

Solution in the forward order:

(1) The bullet collides (strikes) the wood block. Energy is not conserved. Momentum is conserved.

$$m_B v_B + 0 = (m_B + m_W)V$$

Solve for the speed that the combined object gains

$V = m_B v_B/(m_B + m_W)$ or $v_B = (m_B + m_W)V/m_B$

(2) The compressing process obeys the conservation of energy for the combined object.

$$\frac{1}{2}(m_B + m_W)V^2 = \frac{1}{2}kL^2$$

Or

$$V = L\sqrt{\frac{k}{m_B+m_W}}$$

So,

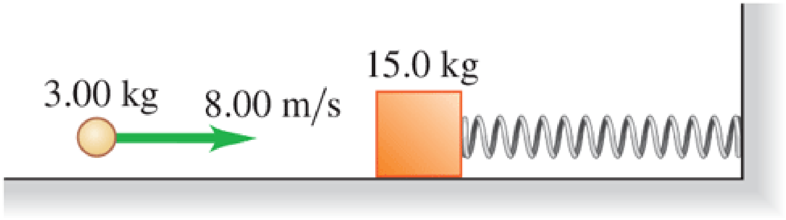
$$v_B = \frac{m_B+m_W}{m_B}V = \frac{m_B+m_W}{m_B}L\sqrt{\frac{k}{m_B+m_W}}$$

$$= \frac{L}{m_B}\sqrt{(m_B + m_W)k}$$

Courtesy of Wenhao Wu

8.19

II Combining conservation laws. A 15.0 kg block is attached to a very light horizontal spring of force constant k = 500 N/m and is resting on a frictionless horizontal table. (See Figure 8.40.) Suddenly it is struck by a 3.00 kg stone traveling horizontally at to the right, whereupon the stone rebounds at horizontally to the left. Find the maximum distance that the block will compress the spring after the collision. (Hint: Break this problem into two parts—the collision and the behavior after the collision—and apply the appropriate conservation law to each part.)



8.19. Set Up: The collision occurs over a short-time interval and the block moves very little during the collision, so the spring force during the collision can be neglected. Use coordinates where $+x$ is to the right.

Solve: *Collision*: There is no external horizontal force during the collision and $P_{i,x} = P_{f,x}$. This gives $(3.00\text{ kg})(8.00\text{ m/s}) = (15.0\text{ kg})\upsilon_{\text{block,f}} - (3.00\text{ kg})(2.00\text{ m/s})$ and $\upsilon_{\text{block,f}} = 2.00\text{ m/s}$.

Motion after the collision: When the spring has been compressed the maximum amount, all the initial kinetic energy of the block has been converted into potential energy $\frac{1}{2}kd^2$ that is stored in the compressed spring. Conservation of energy gives $\frac{1}{2}(15.0\text{ kg})(2.00\text{ m/s})^2 = \frac{1}{2}(500.0\text{ kg})d^2$ and $d = 0.346$ m.

Q8.8

Clicker question

Block A on the left has mass 1.00 kg. Block B on the right has mass 3.00 kg. The blocks are forced together, compressing the spring. Then the system is released from rest on a level, frictionless surface. After the blocks are released, how does p_A (the magnitude of momentum of block A) compare to p_B (the magnitude of momentum of block B)?

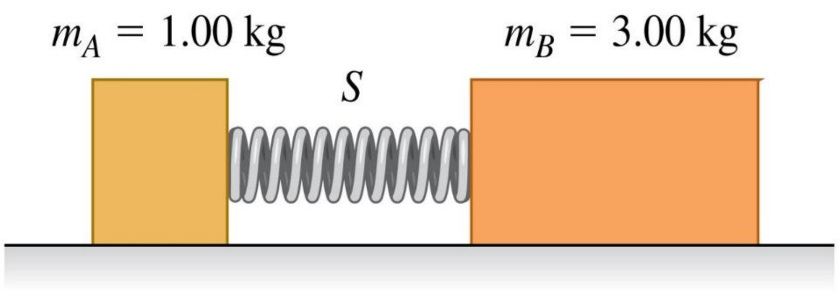
A. $p_A = p_B/9$

B. $p_A = p_B/3$

C. $p_A = p_B$

D. $p_A = 3p_B$

E. $p_A = 9p_B$



Q8.9

Clicker question

Block A on the left has mass 1.00 kg. Block B on the right has mass 3.00 kg. The blocks are forced together, compressing the spring. Then the system is released from rest on a level, frictionless surface. After the blocks are released, how does K_A (the kinetic energy of block A) compare to K_B (the kinetic energy of block B)?

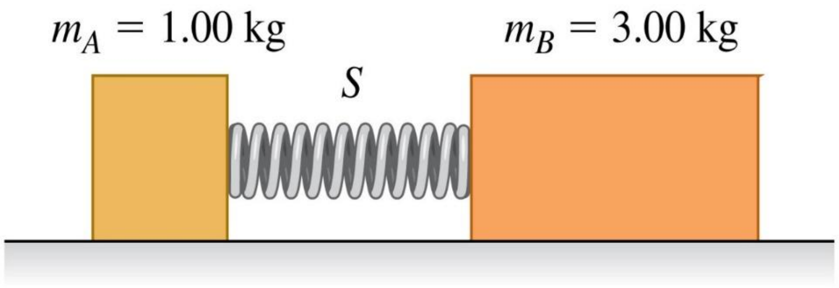
A. $K_A = K_B/9$

B. $K_A = K_B/3$

C. $K_A = K_B$

D. $K_A = 3K_B$

E. $K_A = 9K_B$



I Three identical boxcars are coupled together and are moving at a constant speed of 20 m/s on a level track. They collide with another identical boxcar that is initially at rest and couple to it, so that the four cars roll on as a unit. Friction is small enough to be ignored. (a) What is the speed of the four cars? (b) What percentage of the kinetic energy of the boxcars is dissipated in the collision? What happened to this energy?

8.20. Set Up: Let x be the direction of motion. Let each boxcar have mass m.

Solve: (a) $P_{i,x} = P_{f,x}$ says $(3m)(20.0 \text{ m/s}) = (4m)\upsilon_{f,x}$ and $\upsilon_{f,x} = 15.0 \text{ m/s}$.

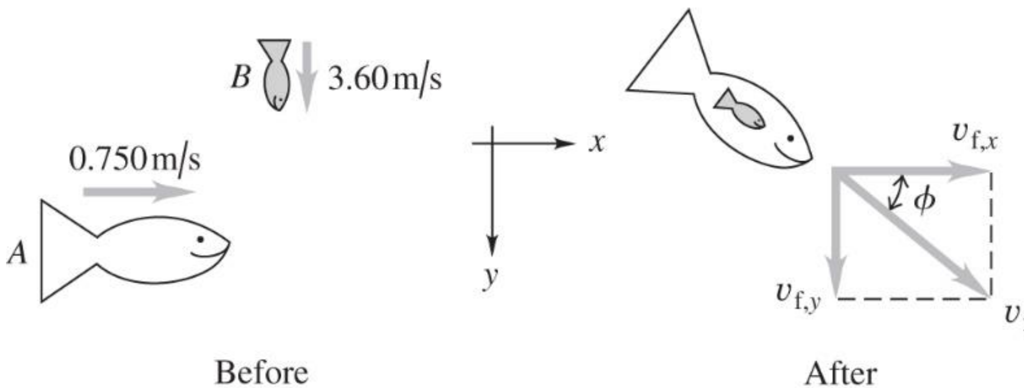
(b) $K_i = \frac{1}{2}(3m)(20.0 \text{ m/s})^2 = 600m \text{ J/kg}$; $K_f = \frac{1}{2}(4m)(15.0 \text{ m/s})^2 = 450m \text{ J/kg}$

$$\Delta K = -150m \text{ J/kg} \text{ and } \frac{\Delta K}{K_i} = \frac{-150m \text{ J/kg}}{600m \text{ J/kg}} = -0.250$$

25.0% of the original kinetic energy is dissipated. Kinetic energy is converted to other forms by work done by the forces during the collision.

II A hungry 11.5 kg predator fish is coasting from west to east at when it suddenly swallows a 1.25 kg fish swimming from north to south at Find the magnitude and direction of the velocity of the large fish just after it snapped up this meal. Ignore any effects due to the drag of the water.

8.25. Set Up: Use coordinates where $+x$ is east and $+y$ is south. Let the big fish be A and the small fish be B. The system of the two fish before and after the collision is sketched in the figure below.



Solve: There are no external forces on the fish so $P_{i,x} = P_{f,x}$ and $P_{i,y} = P_{f,y}$.

$$P_{i,x} = P_{f,x} \text{ gives } (11.5\text{ kg})(0.750\text{ m/s}) = (12.75\text{ kg})v_{f,x} \text{ so } v_{f,x} = 0.676\text{ m/s}$$

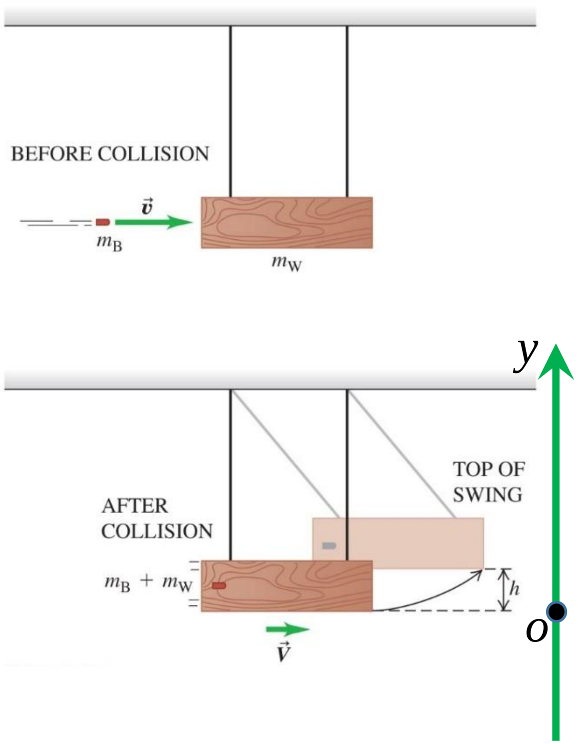
$$P_{i,y} = P_{f,y} \text{ gives } (1.25\text{ kg})(3.60\text{ m/s}) = (12.75\text{ kg})v_{f,y} \text{ so } v_{f,y} = 0.353\text{ m/s}$$

$v = \sqrt{v_{f,x}^2 + v_{f,y}^2} = 0.763\text{ m/s}$. $v_{f,x} = v_f \cos\phi$ so $\cos\phi = \dfrac{0.676\text{ m/s}}{0.763\text{ m/s}}$ and $\phi = 27.6°$. The large fish has velocity 0.763 m/s in a direction 27.6° south of east.

Reflect: Momentum is a vector and we must treat each component separately.

Momentum Conservation

The Ballistic Pendulum
– Example 8.7 on page 233



A bullet of unknown horizontal speed is shot into a block of wood suspended from the ceiling. The bullet stuck in the wood block and together they swing up by an amount *h*. Find the speed of the bullet before it strikes the wood block.

Given: m_B, m_W, and *h*. Find: *v*

Comment: Solve this in either the forward or the backward order.

Solution in the forward order:

(1) The bullet collides (strikes) the wood block. Energy is not conserved, but momentum is conserved.

$$m_B v + 0 = (m_B + m_W)V$$

Solve for the speed that the combined object gains

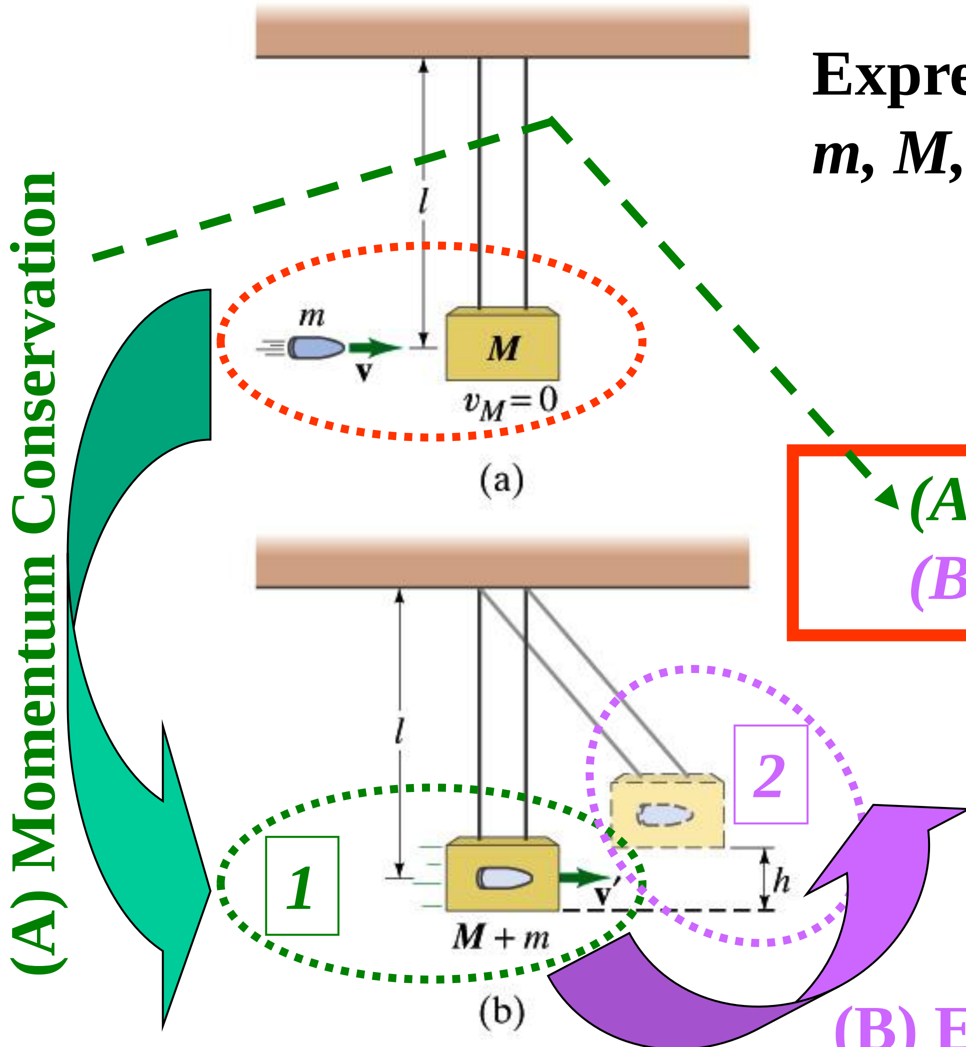
$$V = m_B v/(m_B + m_W) \quad \text{or} \quad v = (m_B + m_W)V/m_B$$

(2) The swing-up process obeys the conservation of energy for the combined object.

$$\frac{1}{2}(m_B + m_W)V^2 = (m_B + m_W)gh$$

So, $$v = \frac{m_B+m_W}{m_B}V = \frac{m_B+m_W}{m_B}\sqrt{2gh}$$

Ballistic Pendulum



Express *v and v′* in terms of *m*, *M*, *g*, and *h*.

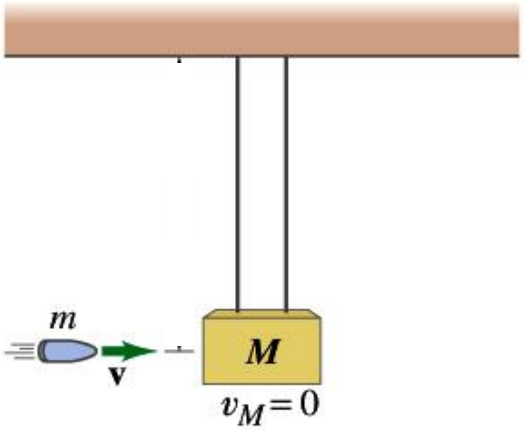
(A) $mv = (m+M)\,v'$

(B) $K_1 + U_{g1} = K_2 + U_{g2}$

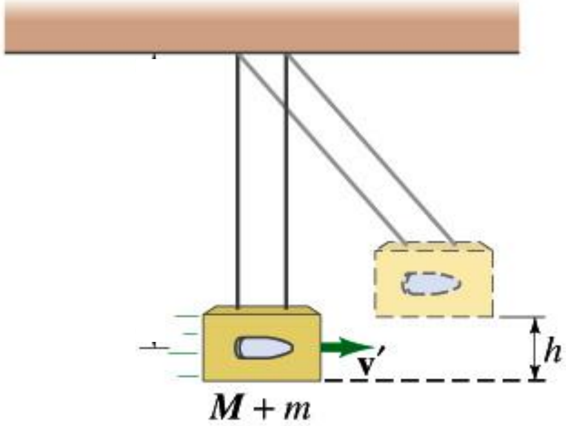
(B) Energy Conservation

Ballistic Pendulum (cont.)

- A bullet of mass *m* and velocity V_o plows into a block of wood with mass *M* which is part of a pendulum.
  - How high, *h*, does the block of wood go?
  - Is the collision elastic or inelastic?



(a)



(b)

Two parts: 1-collision (momentum is conserved)
2-from low point (after collision) to high point: conservation of energy

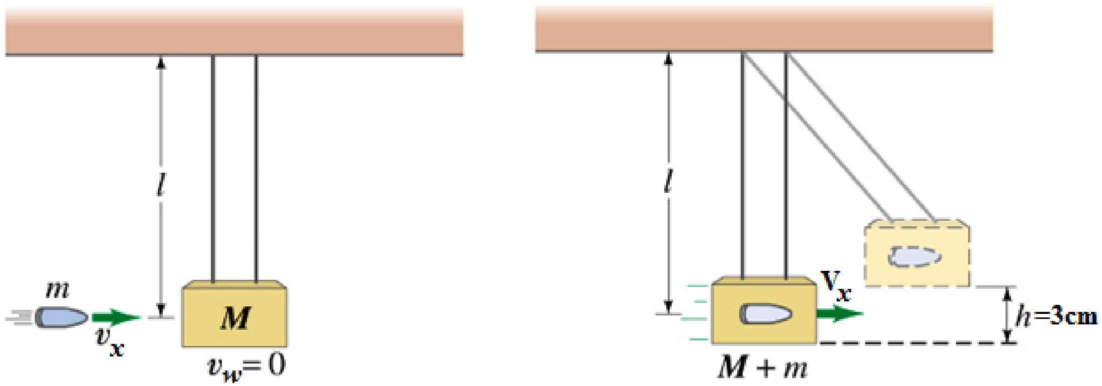
1st part:

$$x: \quad m\mathrm{v} + 0 = (M+m)\mathrm{v}'$$
$$y: \quad 0 + 0 = 0 + 0$$
$$\Rightarrow \quad \mathrm{v}' = \frac{m\mathrm{v}}{(M+m)}$$

2nd part:

$$E_{bottom} = E_{top}$$
$$\frac{1}{2}(M+m)(\mathrm{v}')^2 + 0 = 0 + (M+m)gh$$
$$\Rightarrow h = \frac{1}{2g}(\mathrm{v}')^2 = \frac{m^2\mathrm{v}^2}{2g(m+M)^2}$$

Ballistic Pendulum numerical example



Obtain v_x from the height of the ballistic pendulum swing?
(Unknowns are $\boldsymbol{v_x}$ and $\boldsymbol{V_x}$) (Given $\boldsymbol{m = 0.005kg}$ and $\boldsymbol{M = 2kg}$)

Momentum conservation. ($\boldsymbol{v_w = 0}$)

$$\boldsymbol{mv_x + Mv_w = (m + M)V_x}$$

$$\boldsymbol{\therefore v_x = \frac{m + M}{m} V_x}$$

Conservation of energy.

$$\boldsymbol{\frac{1}{2}(M + m){V_x}^2 = (m + M)gh}$$

$$\boldsymbol{\rightarrow V_x = \sqrt{2gh} = 0.767\frac{m}{s}}$$

$$\boldsymbol{\therefore v_x = \frac{m + M}{m}\sqrt{2gh} = 307\frac{m}{s}}$$

Energy of bullet;

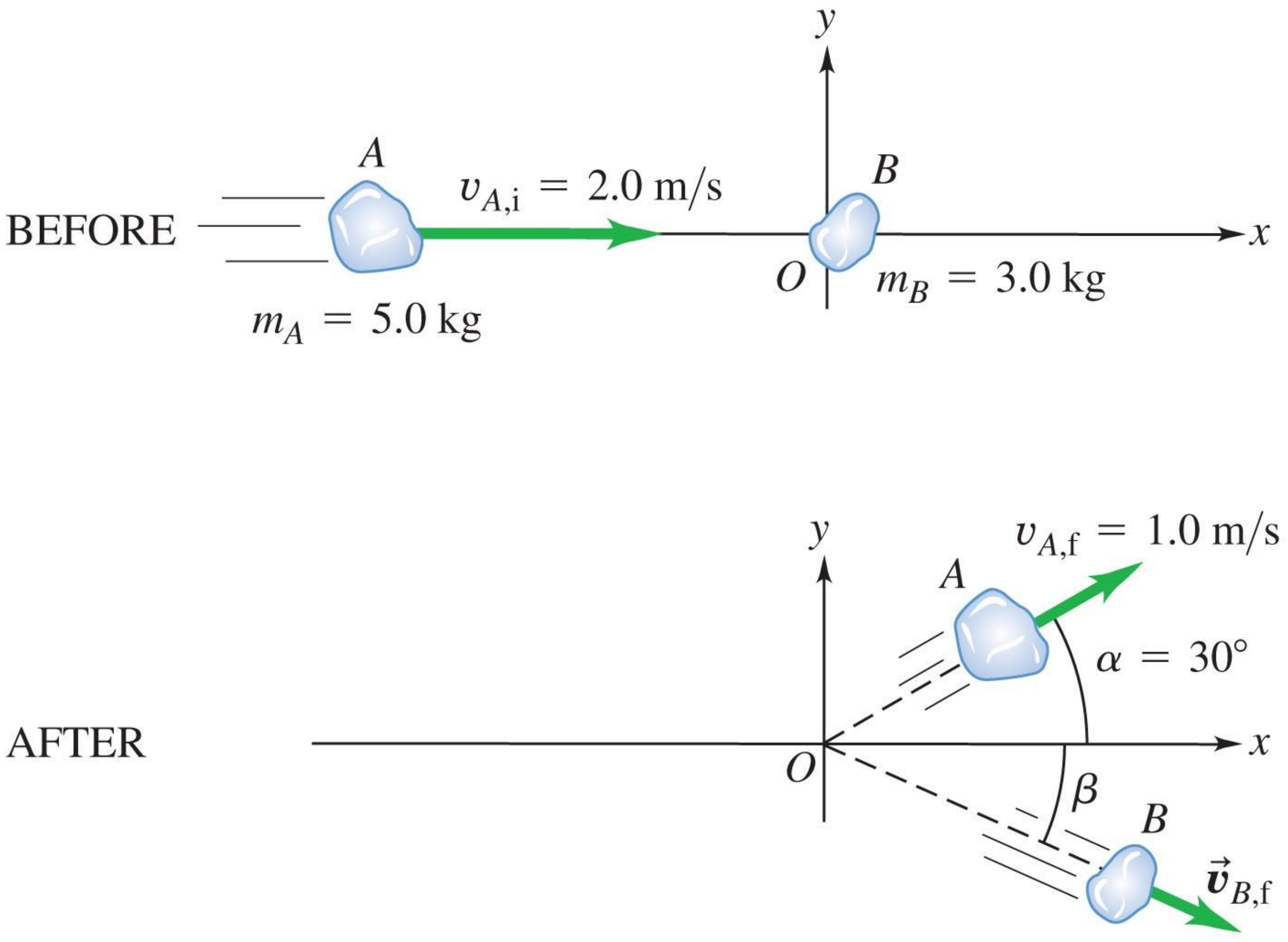
$$\boldsymbol{K_{bullet} = \frac{1}{2}m{v_x}^2 = 236J}$$

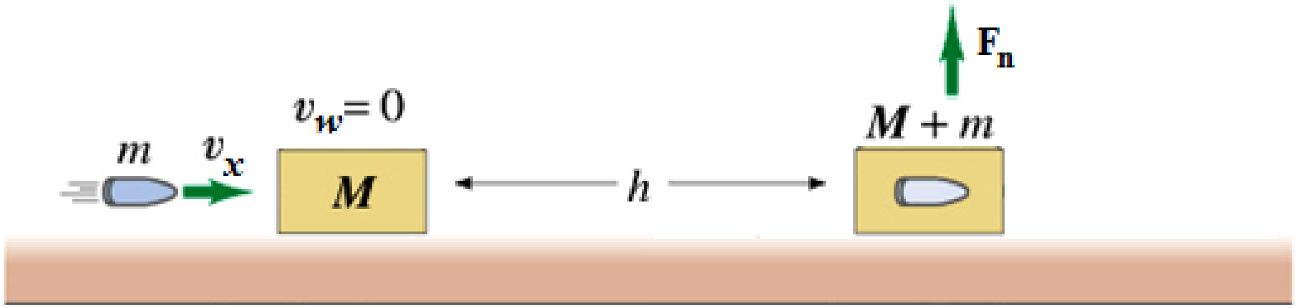
Energy of bullet and block;

$$\boldsymbol{K_{bullet} = \frac{1}{2}(m + M){V_x}^2 = 0.6J}$$

Collision In a Horizontal Plane – Example 8.5

- Refer to the worked example on page 227.





A bullet is fired into a wooden block, which slides to a certain distance and then comes to rest due to friction. What is the velocity of the bullet $\boldsymbol{v_x}$?
(Given $\boldsymbol{\mu = 0.2}$, $\boldsymbol{m = 0.005kg}$, $\boldsymbol{h = 0.23m}$ and $\boldsymbol{M = 1.2kg}$) $\boldsymbol{V_x}$=velocity of (M+m) right after the collision

Momentum conservation. ($\boldsymbol{v_w = 0}$)

$$\boldsymbol{mv_x + Mv_w = (m + M)V_x}$$

$$\therefore \boldsymbol{v_x = \frac{m+M}{m}V_x} \text{ and } \boldsymbol{F_N = \mu(m + M)g}$$

Conservation of energy.

$$\boldsymbol{\frac{1}{2}(M + m){V_x}^2 = F_N h = \mu(m + M)gh}$$

$$\rightarrow \boldsymbol{V_x = \sqrt{2\mu gh}}$$

$$\therefore \boldsymbol{v_x = \frac{m + M}{m}\sqrt{2\mu gh} = 229\frac{m}{s}}$$

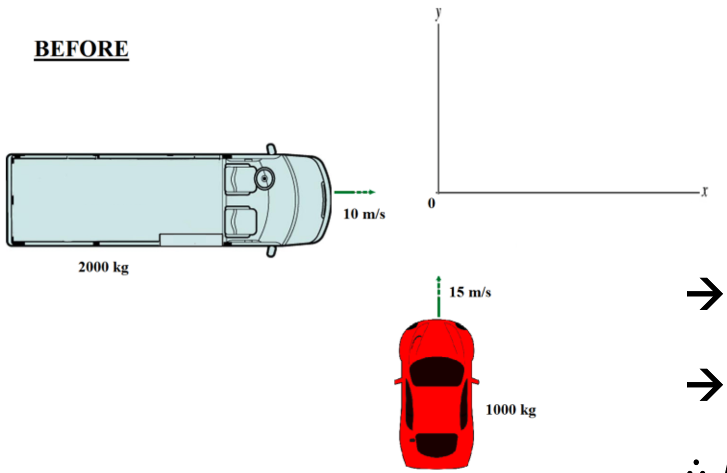
Example 8.8 Accident analysis

Conservation of momentum

(Given $m_A = 1000kg, m_B = 2000kg, v_{Ax} = 0$ and $v_{Bx} = 10\frac{m}{s}$)

$$P_x = P_{Ax} + P_{Bx} = m_A v_{Ax} + m_B v_{Bx} = 2x10^4 kg\frac{m}{s}$$

$$P_y = P_{Ay} + P_{By} = m_A v_{Ay} + m_B v_{By} = 1000 * 15 + 0 = 1.5x10^4 kg\frac{m}{s}$$

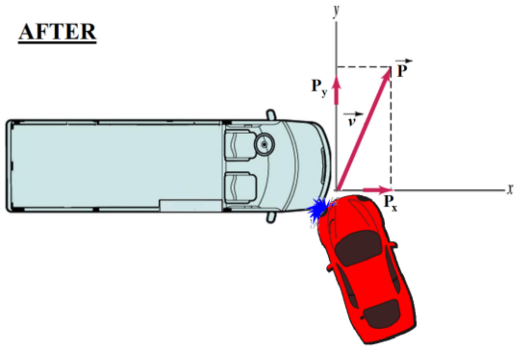


$\rightarrow P = \sqrt{(P_x)^2 + (P_y)^2} = 2.5x10^4 kg\frac{m}{s}$

$\rightarrow \tan\theta = \frac{P_y}{P_x} = \frac{1.5}{2.0} = 0.75$ and $\theta = 37^o$

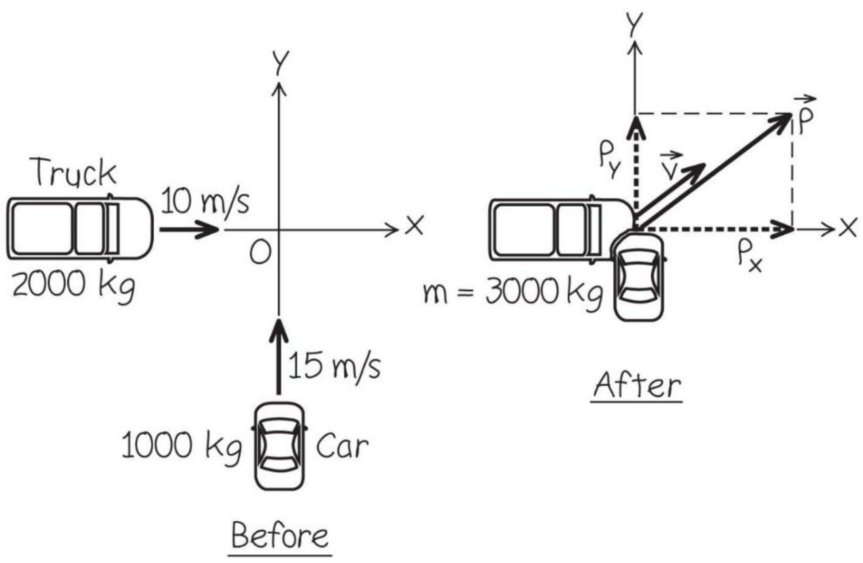
$\therefore P = (m_A + m_B)V \rightarrow V = \frac{P}{m_A + m_B} = 8.3\frac{m}{s}$

(Velocity of wreckage)



Different Masses – Example 8.8

- Incoming and outgoing velocities are very mass dependent.
- .



8.57. Set Up: Apply conservation of momentum to the collision between the two people. Apply conservation of energy to the motion of the stuntman before the collision and to the entwined people after the collision. For the motion of the stuntman, we have $y_1 - y_2 = 5.0$ m. Let υ_S be the magnitude of his horizontal velocity just before the collision. Let υ be the speed of the entwined people just after the collision. Let d be the distance they slide along the floor.

Solve: **(a)** Motion before the collision: $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$ and $\frac{1}{2}m\upsilon_S^2 = mg(y_1 - y_2)$. $\upsilon_S = \sqrt{2g(y_1 - y_2)} =$

$$\sqrt{2(9.80\ \text{m/s}^2)(5.0\ \text{m})} = 9.90\ \text{m/s}.$$

$$\text{Collision: } m_S v_S = m_{tot}\upsilon.\quad \upsilon = \frac{m_S}{m_{tot}}\upsilon_S = \left(\frac{80.0\ \text{kg}}{150.0\ \text{kg}}\right)(9.90\ \text{m/s}) = 5.3\ \text{m/s}$$

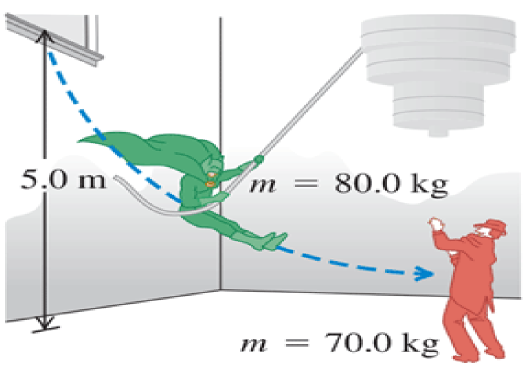
(b) Motion after the collision: $K_1 + U_1 + W_{other} = K_2 + U_2$ gives $\frac{1}{2}m_{tot}\upsilon^2 - \mu_k m_{tot} g d = 0$.

$$d = \frac{\upsilon^2}{2\mu_k g} = \frac{(5.28\ \text{m/s})^2}{2(0.250)(9.80\ \text{m/s}^2)} = 5.7\ \text{m}$$

Reflect: Mechanical energy is dissipated in the inelastic collision, so the kinetic energy just after the collision is less than the initial potential energy of the stuntman.

57. II A movie stuntman (mass 80.0 kg) stands on a window ledge 5.0 m above the floor (Figure 8.48). Grabbing a rope attached to a chandelier, he swings down to grapple with the movie's villain (mass 70.0 kg), who is standing directly under the chandelier. (Assume that the stuntman's center of mass moves downward 4.0 m. He releases the rope just as he reaches the villain.) (a) With what speed do the entwined foes start to slide across the floor? (b) If the coefficient of kinetic friction of their bodies with the floor is $\mu_k = 0.250$, how far do they slide?

Figure 8.48



Throwing a package overboard

(Given: $m_{package} = 5.4kg$, $v_{package} = 10\frac{m}{s}$, $m_{child} = 26kg$ and $m_{boat} = 55kg$)

$$0 = (m_{boat} + m_{child})v_{boat} + m_{package}v_{package} = (55 + 26) * v_{boat} + 5.4 * 10$$

$$\therefore v_{boat} = 0.667\frac{m}{s}$$

Find speed and rebound height

Find the speed of the blocks after collision. (assume elastic collision). How far up does the smaller block go (rebound height)? (Given $\theta = 30^o$, $m = 2.2kg$, $h = 3.6m$ and $M = 7kg$)

a) Energy and Momentum conservation ($\therefore v_2 = 0\frac{m}{s}$)

$$\frac{1}{2}mv_1{}^2 = mgh \rightarrow v_1 = \sqrt{2gh} = \sqrt{2 * 9.8 * 3.6} = 8.4\frac{m}{s}$$ (speed of small block before collision)

$$mv_1 + Mv_2 = mv_1' + Mv_2' \rightarrow 2.2 * 8.4 + 7 * 0 = 2.2 * v_1' + 7 * v_2'$$

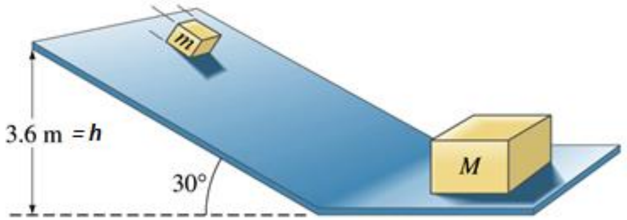
$$v_1 + v_2 = -(v_1' - v_2') \rightarrow 8.4 - 0 = v_2' - v_1'$$

$$\therefore v_2' = 4.02\frac{m}{s} \text{ and } v_1' = -4.38\frac{m}{s}$$

b) Rebound height

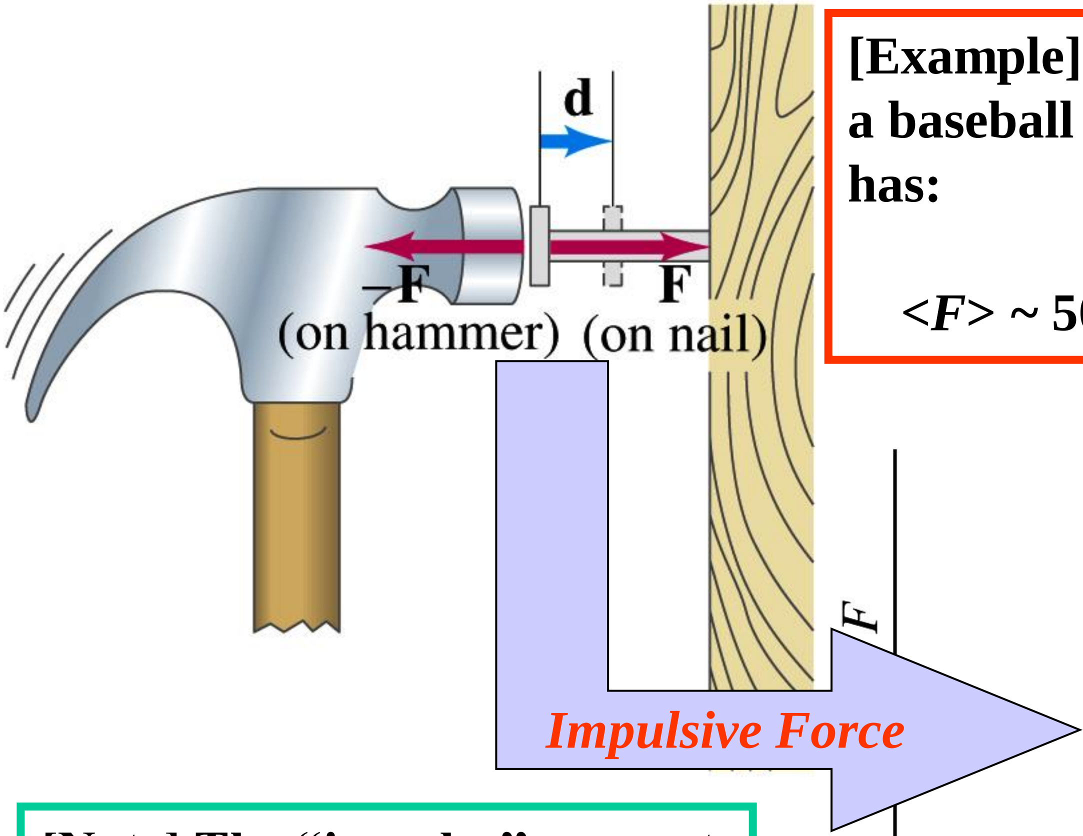
$$\therefore v_1'{}^2 = 2gh' \rightarrow (-4.38)^2 = 2 * 9.8 * h'$$

$$\rightarrow h' = 0.79\,m \quad \text{and} \quad d = \frac{h'}{\sin\theta} = 1.96m$$



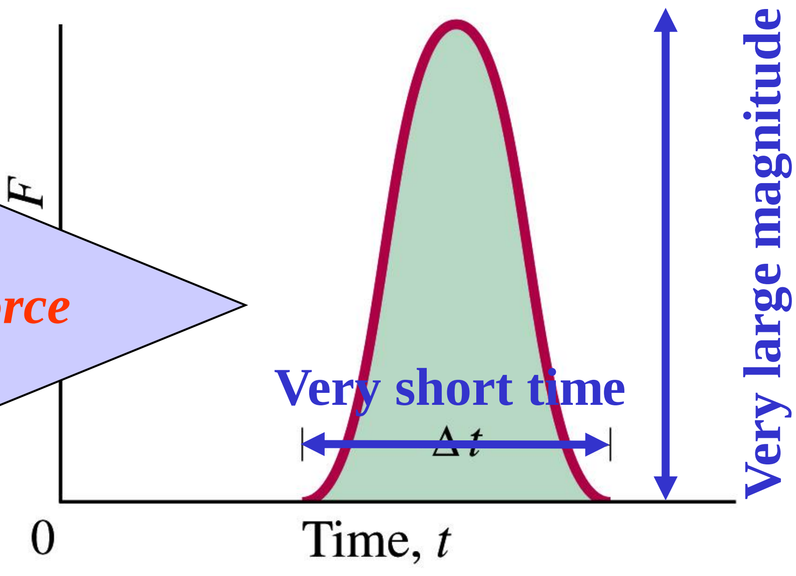
Momentum Conservation

Impulsive Force



[Example] an impulsive force on a baseball that is struck with a bat has:

$\langle F \rangle \sim 5000$ N & $\Delta t \sim 0.01$ s



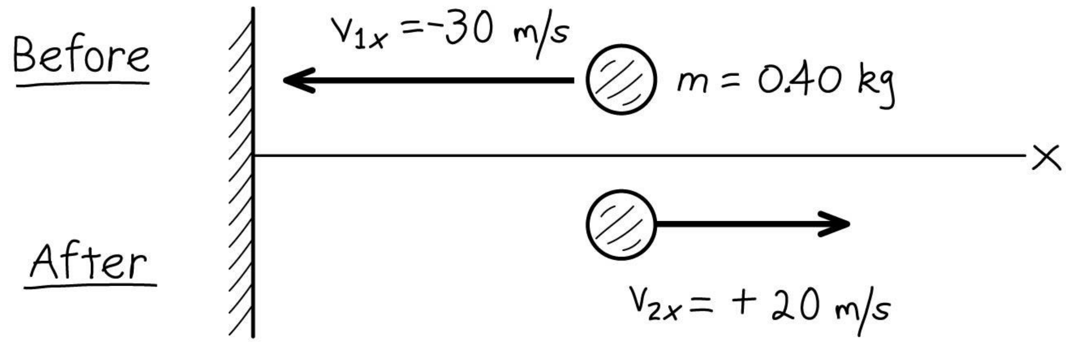
[Note] The "impulse" concept is most useful for impulsive forces.

A golf shot

- A large force applied for a very short duration, such as a golf shot, is often described as the club giving the ball an *impulse*.

Q8.1

A ball (mass 0.40 kg) is initially moving to the left at 30 m/s. After hitting the wall, the ball is moving to the right at 20 m/s. What is the impulse of the net force on the ball during its collision with the wall?



A. 20 kg • m/s to the right

B. 20 kg • m/s to the left

C. 4.0 kg • m/s to the right

D. 4.0 kg • m/s to the left

E. none of the above

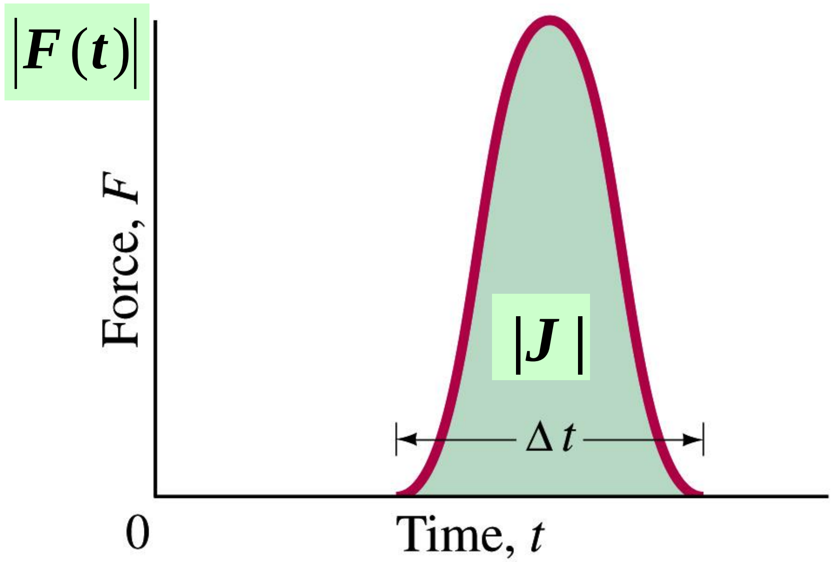
$$J_x = \Delta p_x = p_{f,x} - p_{i,x}$$
$$= mv_{f,x} - mv_{i,x}$$

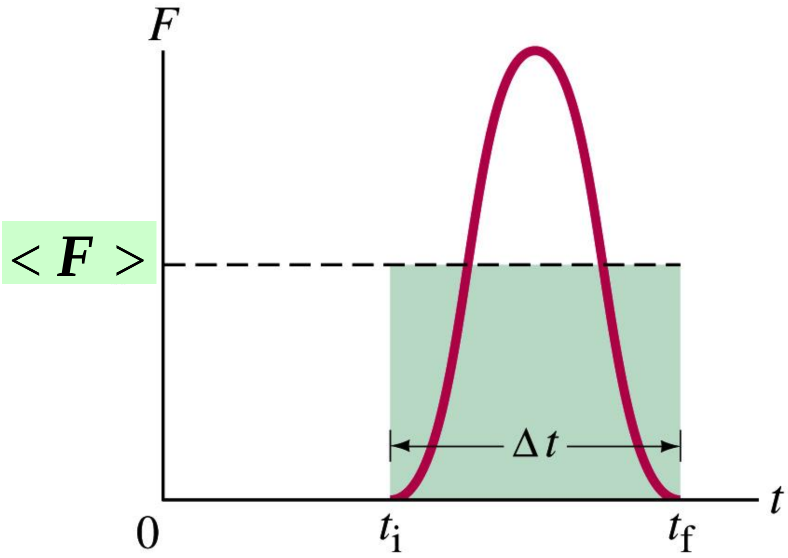
= 0.40 (20)-0.40(-30)

=8+12=20 kg m/s

to the right

Impulse-Momentum Theorem





$$< \vec{F} > = \frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{t_f - t_i}$$

$$J = < \vec{F} > (t_f - t_i) = \vec{p}_f - \vec{p}_i$$

8.5 Impulse

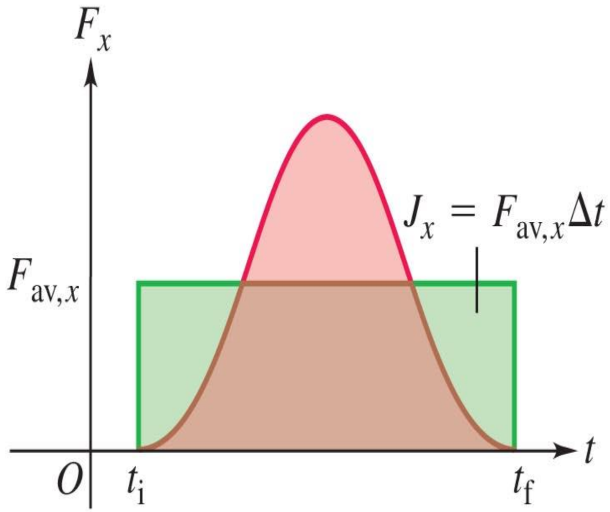
When a constant force $\vec{F}$ acts on an object over a period of time Δt, the impulse of the force is

$$\vec{J} = \vec{F}(t_f - t_i) = \vec{F}\Delta t$$

If the force is not a constant, then $\vec{J} = \vec{F}_{avg}\Delta t$

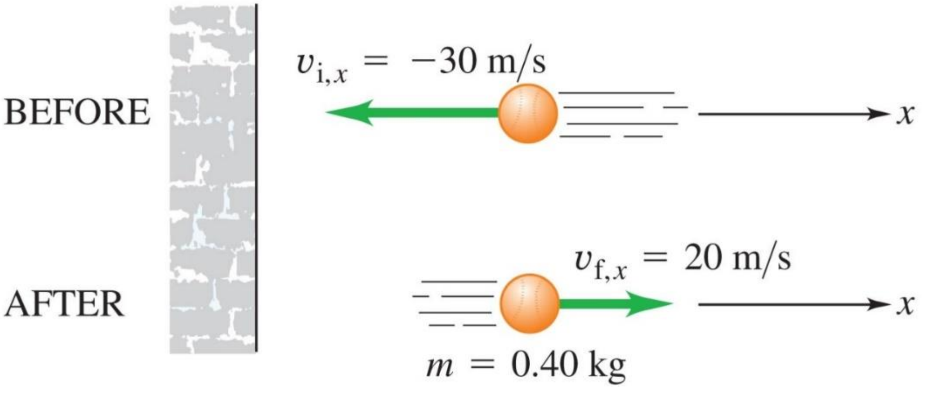
Since $\vec{F}_{avg} = m\vec{a}_{avg} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$,

we have $\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$



The impulse $\vec{J}$ of a force over a certain time interval is the area under the graph of force versus time. If the force varies with time, the impulse may be calculated by using the average force.

Example 8.11 on page 237
Impulse and Duration of the Impact



Given: $m = 0.40$ kg
$v_{i,x} = -30$ m/s
$v_{f,x} = 20$ m/s
contact time $\Delta t = 0.010$ s

Find: Average force F_{avg}.

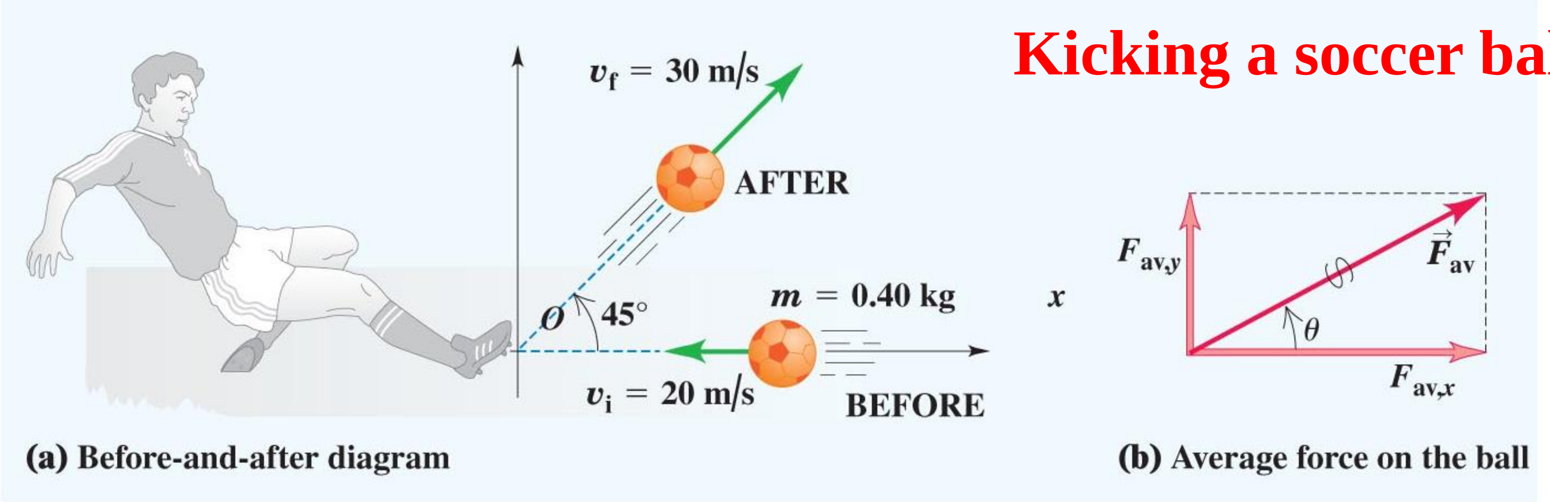
Solution:
Impulse is equal to change in momentum

$$J_x = \Delta p_x = p_{f,x} - p_{i,x}$$
$$= mv_{f,x} - mv_{i,x}$$
$$= 0.40(20) - 0.40(-30)$$
$$= 20 \text{ kgm/s}$$

Average force

$$F_{avg} = J_x/\Delta t = (20 \text{ kgm/s})/(0.010 \text{ s})$$
$$= 2000 \text{ N}$$

Kicking a soccer ball



Find the impulse and the average force on the ball, assuming a collision time $\Delta t = 0.01s = 10ms$

a) Given; $\boldsymbol{v_{ix} = -20\frac{m}{s}}$ and $\boldsymbol{v_{iy} = 0\frac{m}{s}}$

$$\boldsymbol{v_{fx} = v_{fy} = v_f \cos 45 = 30 * 0.7 = 21\frac{m}{s}}$$

$$\boldsymbol{J_x = m(v_{fx} + v_{ix}) = 0.4(21 - (-20)) = 16.4\ kg\frac{m}{s}}$$

red is for a 1D problem

$$\boldsymbol{J_y = m(v_{fy} + v_{iy}) = 0.4(21 - 0) = 8.4\ kg\frac{m}{s}}$$

b) The components of the average force on the ball

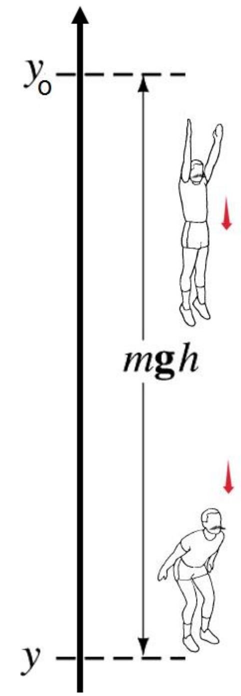
$$\boldsymbol{F_{av,x} = \frac{J_x}{\Delta t} = \frac{16.4}{10^{-2}} = 1640N}$$

$$\boldsymbol{F_{av,y} = \frac{J_y}{\Delta t} = \frac{8.4}{10^{-2}} = 840N}$$

$$\boldsymbol{\rightarrow F_{av} = \sqrt{1640^2 + 840^2} = 1.8x10^3N}$$

$$\boldsymbol{\therefore \theta = \tan^{-1}\frac{840}{1640} = 27^o}$$

Bend your legs while landing



Given; ***m=70kg*** and ***h=3m***

a) What is the velocity just before landing?

$$K_1 + U_1 = K_2 + U_2$$
$$K_1 - K_2 = U_2 - U_1$$

$\rightarrow \frac{1}{2}m{v_1}^2 - \frac{1}{2}m{v_2}^2 = \frac{1}{2}m{v_1}^2 - 0 = -mg(y - y_o) = mgh$ (v_1 is landing velocity)

$$\therefore v_1 = \sqrt{2gh} = \sqrt{2 * 9.8 * 3} = 7.7\frac{m}{s}$$

What is the impulse during landing?

$$J = F \cdot \Delta t = \Delta \mathrm{P} = \mathrm{P_f} - \mathrm{P_i} = -70\mathrm{kg} * 7.7\frac{\mathrm{m}}{\mathrm{s}} = -540\mathrm{N.s}$$

b) What is the average force while landing?

<u>Stiff- legged:</u> Total displacement of body during landing is *d*=1cm

$$\Delta t = \frac{d}{v_1} = \frac{10^{-2}}{7.7} = 1.3x10^{-3}s \text{ and } F = \frac{J}{\Delta t} = \frac{540}{1.3x10^{-3}} = 4.15x10^5 N$$

<u>Soft (bent) -legged:</u> Total displacement of body during landing is *d*=0.5m

$$\Delta t = \frac{d}{v_1} = \frac{0.5}{7.7} = 0.065\ s \text{ and } F = \frac{J}{\Delta t} = \frac{540}{0.065} = 8.32x10^3 N$$

The average force is about 50 times less for the bent landing.

8.39

Impulse and force

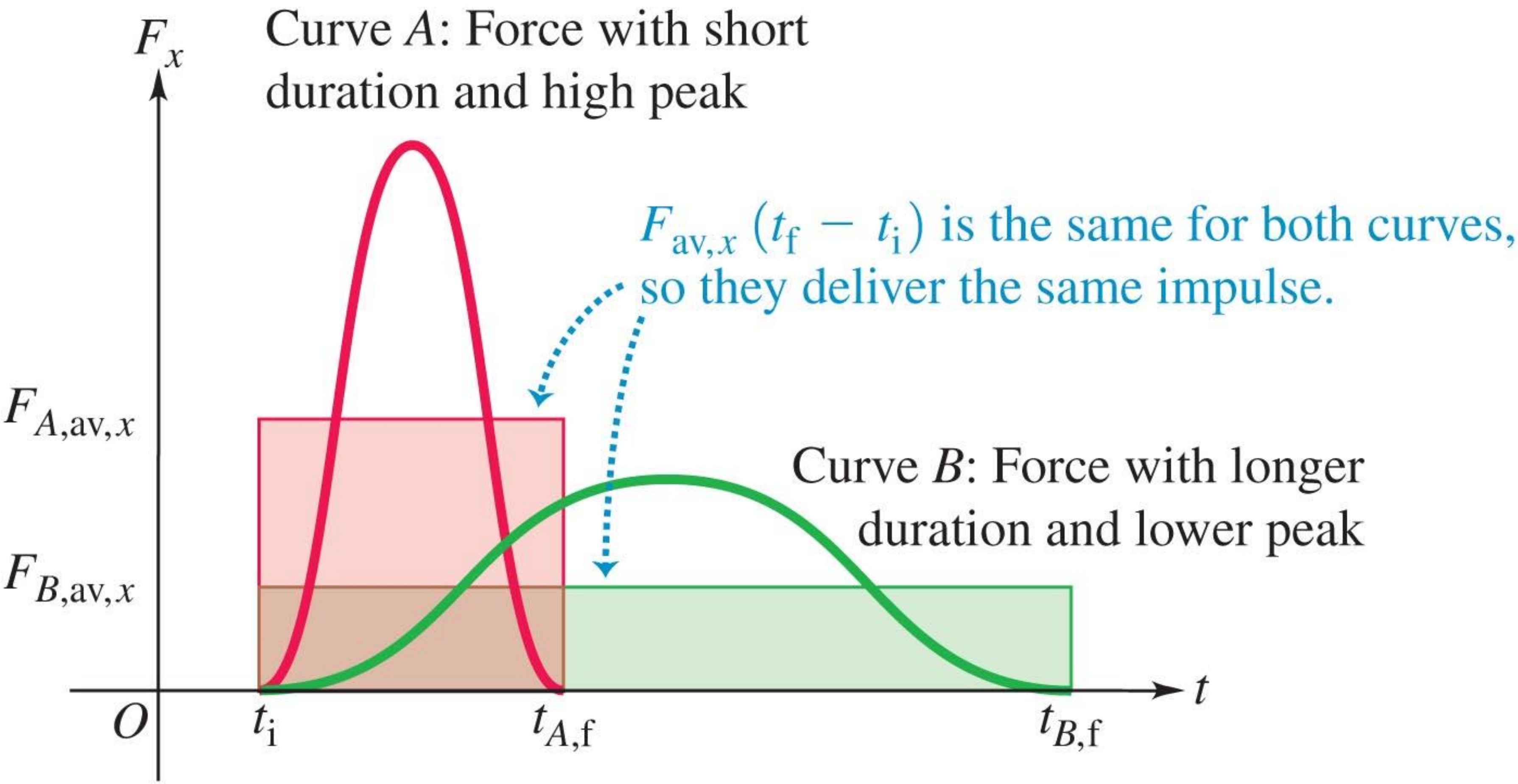
39. **II** Your little sister (mass 25.0 kg) is sitting in her little red wagon (mass 8.50 kg) at rest. You begin pulling her forward and continue accelerating her with a constant force for 2.35 s, at the end of which time she's moving at a speed of 1.80 m/s. (a) Calculate the impulse you imparted to the wagon and its passenger. (b) With what force did you pull on the wagon?

8.39. Set Up: Choose the positive *x* axis in the direction of motion. We know that the impulse applied to the wagon will be equal to the wagon's change in momentum: $J_x = \Delta p_x$. The total mass of your sister and her wagon is 33.5 kg.

Solve: (a) $\Delta p_x = mv_{\text{f},x} - mv_{\text{i},x} = (33.5\text{ kg})(1.80\text{ m/s}) - 0 = 60.3\text{ kg}\cdot\text{m/s}$. Thus, we have $J_x = 60.3\text{ kg}\cdot\text{m/s} = 60.3\text{ N}\cdot\text{s}$.

(b) Since $J_x = F_x t$, we have $F_x = \dfrac{J_x}{t} = \dfrac{60.3\text{ N}\cdot\text{s}}{2.35\text{ s}} = 25.7\text{ N}$.

Reflect: The force calculated in part (b) is the net force acting on the wagon—which is the force that you must apply, assuming that no other forces act on the wagon.


F_x
Curve A: Force with short duration and high peak
$F_{av,x}\,(t_f - t_i)$ is the same for both curves, so they deliver the same impulse.
$F_{A,av,x}$
Curve B: Force with longer duration and lower peak
$F_{B,av,x}$
O
t_i
$t_{A,f}$
$t_{B,f}$
t

Q8.2

Clicker question

You are testing a new car using crash test dummies. Consider two ways to slow the car from 90 km/h (56 mi/h) to a complete stop:

(i) You let the car slam into a wall, bringing it to a sudden stop.

(ii) You let the car plow into a giant tub of gelatin so that it comes to a gradual halt.

In which case is there a greater *impulse* of the net force on the car?

A. In case (i).

B. In case (ii).

C. The impulse is the same in both cases.

D. The answer depends on how rigid the front of the car is.

E. The answer depends on how rigid the front of the car is and on the mass of the car.

Clicker question

When you catch a softball, you reduce the impact by letting your hand move backward during the catch. This works mainly because

a) you increase the time over which the ball slows.
b) you reduce the ball's speed relative to your glove.
c) you reduce the impulse delivered to your glove.

The term "impulse" is also used to refer to a fast-acting force or impact.

Center of Mass (CM)

What is the *"Center of Mass?"*

- More importantly *"Why do we care?"*
- This is a special point in space where *"it's as if the object could be replaced by all the mass at that one little point"*

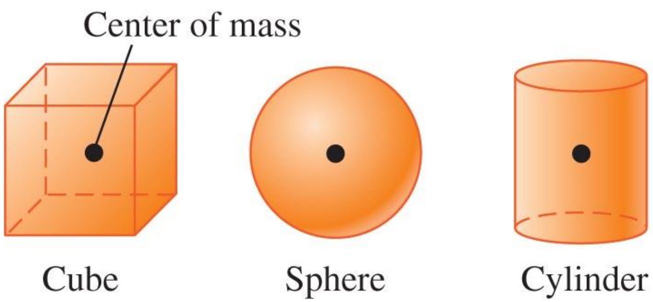
How do you calculate CM?

1. Pick an origin
2. Look at each "piece of mass" and figure out how much mass it has and how far it is (vector displacement) from the origin. Take mass times position
3. Add them all up and divide out by the sum of the masses

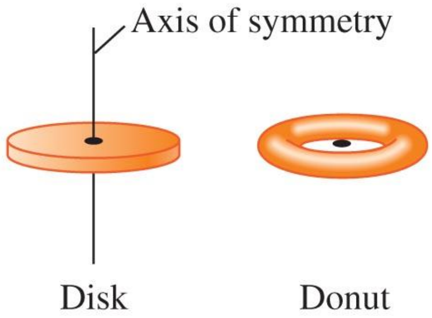
The center of mass is a displacement vector *"relative to some origin"*

8.6 Center of Mass

● For a shape with a simple symmetry, we can easily understand where the center of the object is. This is also the center of mass for the shape.



If a homogeneous object has a geometric center, that is where the center of mass is located.



If an object has an axis of symmetry, the center of mass lies along it. As in the case of the donut, the center of mass may not be within the object.

Center of Mass Position

- General definition of the center of mass position:

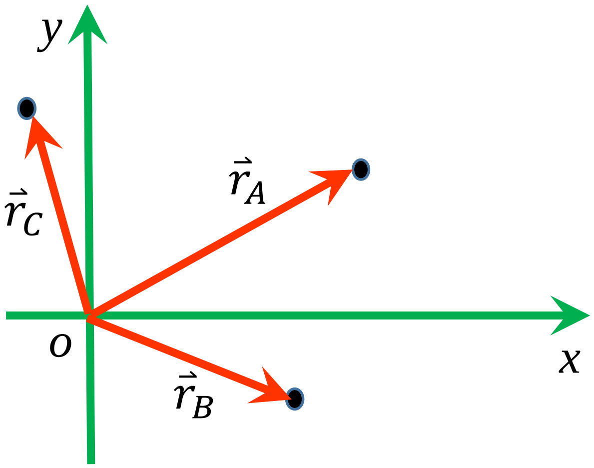
Vector form

$$\vec{r}_{cm} = \frac{m_A\vec{r}_A + m_B\vec{r}_B + m_C\vec{r}_C + \cdots}{m_A + m_B + m_C + \cdots}$$

Component form

$$x_{cm} = \frac{m_A x_A + m_B x_B + m_C x_C + \cdots}{m_A + m_B + m_C + \cdots}$$

$$y_{cm} = \frac{m_A y_A + m_B y_B + m_C y_C + \cdots}{m_A + m_B + m_C + \cdots}$$



Center of Mass Velocity

Vector form $$\vec{v}_{cm} = \frac{m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C + \cdots}{m_A + m_B + m_C + \cdots}$$

Component form $$v_{cm} = \frac{m_A v_{A,x} + m_B v_{B,x} + m_C v_{C,x} + \cdots}{m_A + m_B + m_C + \cdots}$$

$$v_{cm} = \frac{m_A v_{A,y} + m_B v_{B,y} + m_C v_{C,y} + \cdots}{m_A + m_B + m_C + \cdots}$$

Total Momentum of the System

$$\vec{P} = m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C + \cdots$$

$$= (m_A + m_B + m_C + \cdots)\frac{m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C + \cdots}{m_A + m_B + m_C + \cdots}$$

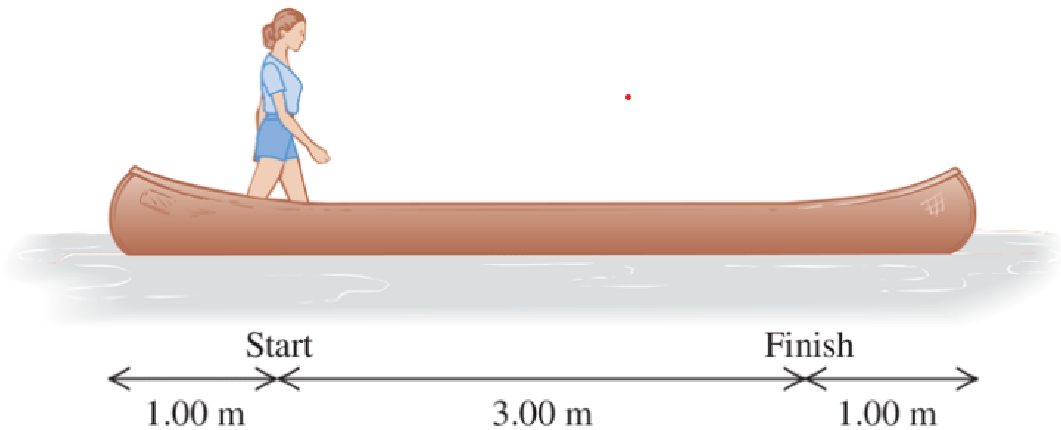
$$= M\vec{v}_{cm}$$

Courtesy of Wenhao Wu

47. II Walking in a boat. A 45.0 kg woman stands up in a 60.0 kg canoe of length 5.00 m. She walks from a point 1.00 m from one end to a point 1.00 m from the other end. (See Figure 8.47.) If the resistance of the water is negligible, how far does the canoe move during this process?

Motion of center of mass

Figure 8.47



8.47. Set Up: Let $+x$ be to the right, with the origin at the initial position of the left-hand end of the canoe. $m_A = 45.0$ kg, $m_B = 60.0$ kg. The center of mass of the canoe is at its center.

Solve: There is no net horizontal external force so υ_{cm} is constant. Initially, $\upsilon_{cm} = 0$, so the center of mass doesn't move. Initially,

$$x_{cm,i} = \frac{m_A x_{A,i} + m_B x_{B,i}}{m_A + m_B}$$

After she walks,

$$x_{cm,f} = \frac{m_A x_{A,f} + m_B x_{B,f}}{m_A + m_B}$$

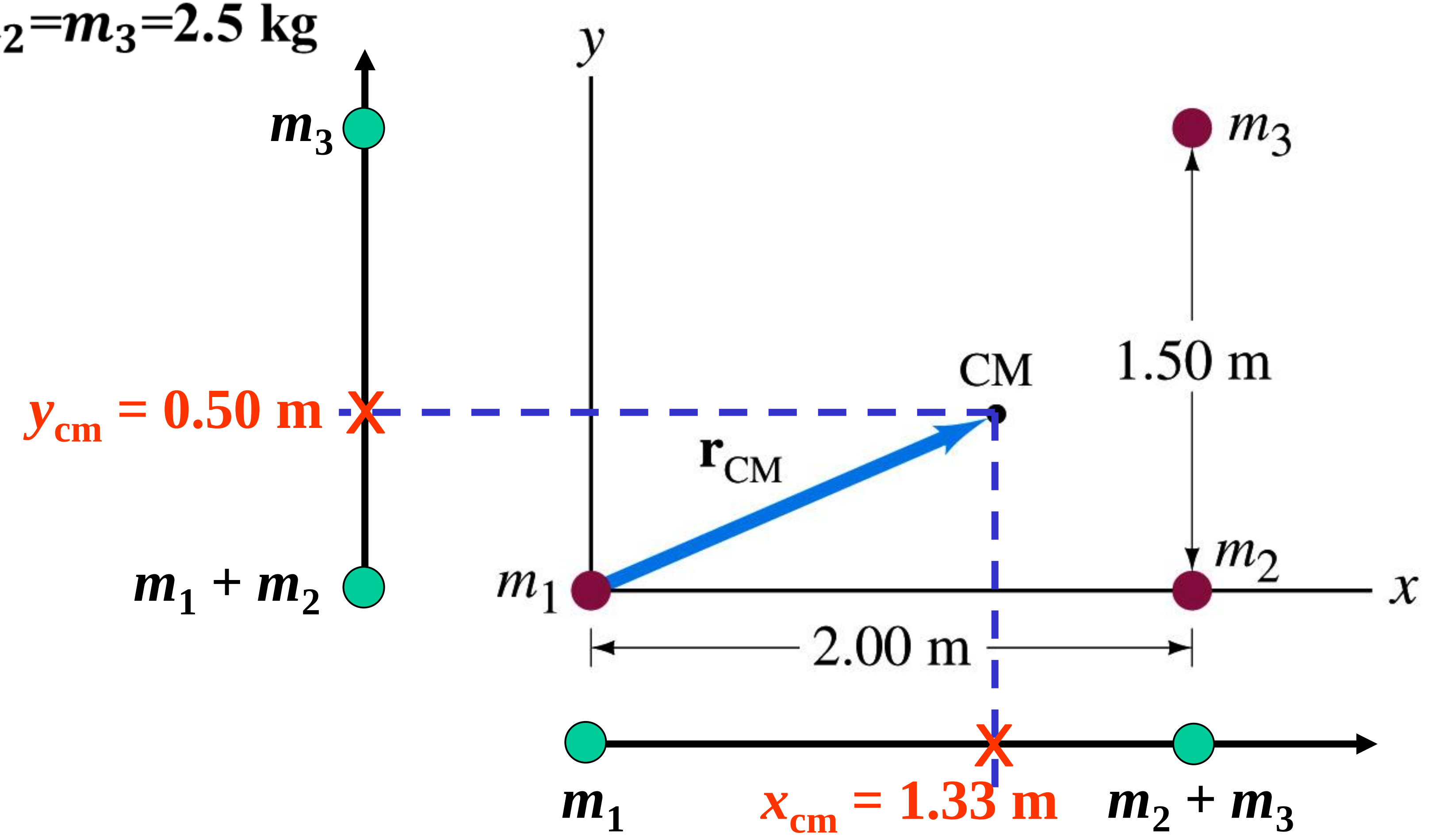
$x_{cm,i} = x_{cm,f}$ gives $m_A x_{A,i} + m_B x_{B,i} = m_A x_{A,f} + m_B x_{B,f}$. She walks to a point 1.00 m from the right-hand end of the canoe, so she is 1.50 m to the right of the center of mass of the canoe and $x_{A,f} = x_{B,f} + 1.50$ m.

$$(45.0 \text{ kg})(1.00 \text{ m}) + (60.0 \text{ kg})(2.50 \text{ m}) = (45.0 \text{ kg})(x_{B,f} + 1.50 \text{ m}) + (60.0 \text{ kg})x_{B,f}$$

$(105.0 \text{ kg})x_{B,f} = 127.5 \text{ kg} \cdot \text{m}$ and $x_{B,f} = 1.21$ m. $x_{B,f} - x_{B,i} = 1.21 \text{ m} - 2.50 \text{ m} = -1.29$ m. The canoe moves 1.29 m to the left.

CM Position (2D)

$m_1 = m_2 = m_3 = 2.5$ kg

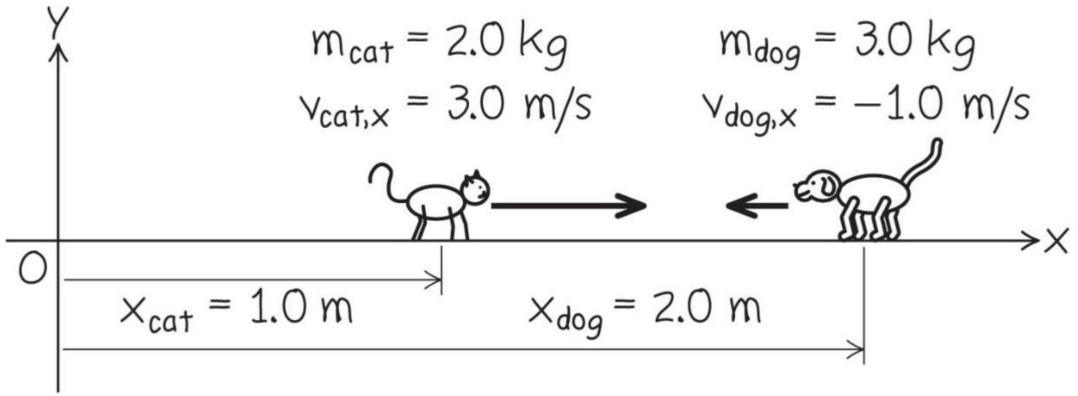


$$x_{cm} = \frac{2.5kg(0m) + 2.5kg(2m) + 2.5kg(2m)}{3(2.5kg)} = 1.33\text{m}$$

$$y_{cm} = \frac{2.5kg(0m) + 2.5kg(0m) + 2.5kg(1.5m)}{7.5kg} = 0.5\text{m}$$

Quarreling pets

Example 8.13
on page 242



$$x_{cm} = \frac{m_c x_c + m_d x_d}{m_c + m_d} = \frac{2.0\times1.0+3.0\times2.0}{2.0+3.0} = 1.6\,m$$

$$v_{cm,x} = \frac{m_c v_c + m_d v_d}{m_c + m_d} = \frac{2.0\times3.0+3.0\times(-1.0)}{2.0+3.0} = 0.6\,m/s$$

$$P_x = M v_{cm,x} = (m_c + m_d) v_{cm,x} = (2.0 + 3.0) \times 0.6 = 3.0\,kgm/s$$

or

$$P_x = m_c v_c + m_d v_d = 2.0 \times 3.0 + 3.0 \times (-1.0) = 3.0 kgm/s$$

Coutesy of Wenhao Wu

8.7 Motion of the Center of Mass

Center of Mass Acceleration $$\vec{a}_{cm} = \frac{m_A\vec{a}_A + m_B\vec{a}_B + m_C\vec{a}_C + \cdots}{m_A + m_B + m_C + \cdots}$$

$$M\vec{a}_{cm} = m_A\vec{a}_A + m_B\vec{a}_B + m_C\vec{a}_C + \cdots$$
$$= (\vec{F}_{A,ext} + \vec{F}_{A,int}) + (\vec{F}_{B,ext} + \vec{F}_{B,int}) + (\vec{F}_{C,ext} + \vec{F}_{C,int}) + \cdots$$
$$= \sum \vec{F}_{ext} + \sum \vec{F}_{int} = \sum \vec{F}_{ext}$$

Newton's Second Law applied to the center of mass motion:

$$\sum \vec{F}_{ext} = M\vec{a}_{cm}$$

Since $\vec{P} = M\vec{v}_{cm}$, if $\sum \vec{F}_{ext} = 0$, then $\vec{a}_{cm} = 0$, or, $\vec{P} = M\vec{v}_{cm} = \text{constant}$

Conservation of Momentum

Courtesy of Wenhao Wu

Center of mass

Center of Mass (c.m. or CM)

The overall motion of a mechanical system can be described in terms of a special point called "center of mass" of the system:

$$\vec{F}_{system} = M_{system}\, \vec{a}_{cm}$$

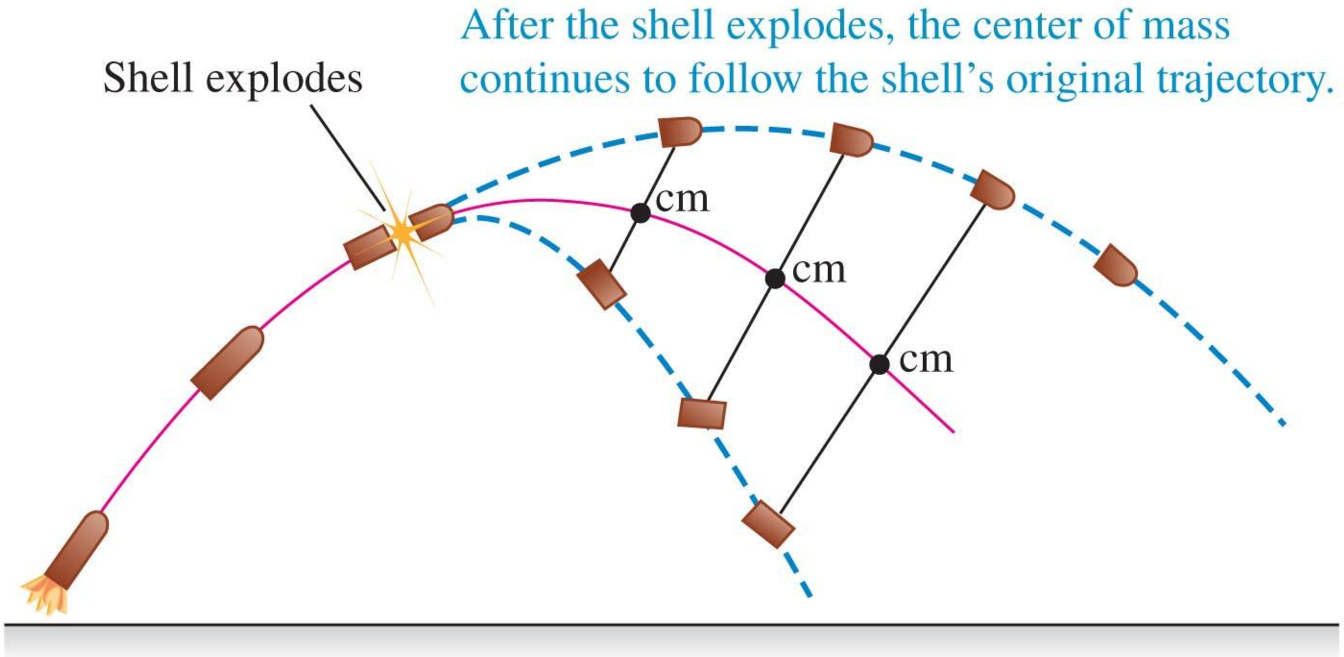
where $\vec{F}_{system}$ is the vector sum of all the forces exerted on the system.

(a)

(b)

The center of mass is marked with a white dot

If the sum of the external forces is zero, then the acceleration of the CM is zero


Shell explodes
After the shell explodes, the center of mass continues to follow the shell's original trajectory.
cm
cm
cm

Total momentum in terms of mass

$$M\vec{v}_{cm} = m_a\vec{v}_a + m_b\vec{v}_b + m_c\vec{v}_c + \ldots = \vec{p}$$

Motion of center of mass

$$M\vec{a}_{cm} = m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + \ldots = \sum \vec{F}_{ext}$$

Summary of chapter 8

Momentum $\vec{p} = m\vec{v}$ → $[kg\frac{m}{s}]$ or **[N.s]**

Impulse $\Delta p = F\Delta t$ =F (t_f -t_i)= J

Force-momentum relation $\frac{d}{dt}\vec{P} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}$

Elastic Collision $K_{1i} + K_{2i} = K_{1f} + K_{2f}$

Inelastic Collision $K_{1i} + K_{2i} = K_{1f} + K_{2f} + Q$

Here; Q is other forms of energy

Both elastic and inelastic collisions conserve momentum

Elastic Collision $\overrightarrow{P_1} + \overrightarrow{P_2} = \overrightarrow{P'_1} + \overrightarrow{P'_2} \rightarrow m_1v_1 + m_2v_2 = m'_1v'_1 + m'_2v'_2$

Inelastic Collision $\overrightarrow{P_1} + \overrightarrow{P_2} = \overrightarrow{P'_1} + \overrightarrow{P'_2} \rightarrow m_1v_1 + m_2v_2 = m'_1v'_1 + m'_2v'_2$

Kinetic energy is not conserved in an inelastic collisions

Galilean Cannon

When balls are stacked on top of each and dropped, the linear momentum of the bottom balls are transferred to the lightest ball on top, which will reach a maximum height many times from which it was dropped.

Momentum Conservation