Exam III Details:

Exam III will take place in rooms HECC-108 on November 2nd , 2023 from 7:00pm – 9:00pm. It is a common exam, all sections of PHYS 201 will take the same exam at this given time. Please arrive at least 15 minutes early to get settled; we would like to begin exactly on time. If you have any questions during the exam, you may call over one of the proctors monitoring the exam.

Chapters to be covered on exam 3: 9 – 11

Items to bring:

- Pencil and eraser
- Scientific Calculator (up to a Ti-84 is allowed)
- Yourself

Items *not* to bring:

- Your own formula sheet (one will be provided on exam)
- Scantron (one will be provided on exam)
- Computerized Calculator (example: Ti-nspire with touchpad keyboard)
- A laptop, tablet, or phone (all computer devices should be kept powered off and left in your bag for the duration of the exam)
- Any other prohibited item

Concepts to review:

- Rotational Mechanics
    - Kinematics & Dynamics
        - Angular position
        - Angular velocity
        - Angular acceleration
    - Moments of inertia
        - Distribution of mass from axis of rotation
        - $I = mr^2$
        - Varies with shape
    - Rotational Kinetic Energy
        - $K = \frac{1}{2}I\omega^2$
    - Relationship between angular and linear
    - Torque
        - T = r x F
        - Forms a corner with the force and radius (think of the floating bike wheel demonstration)
    - Angular momentum
        - L = r x p

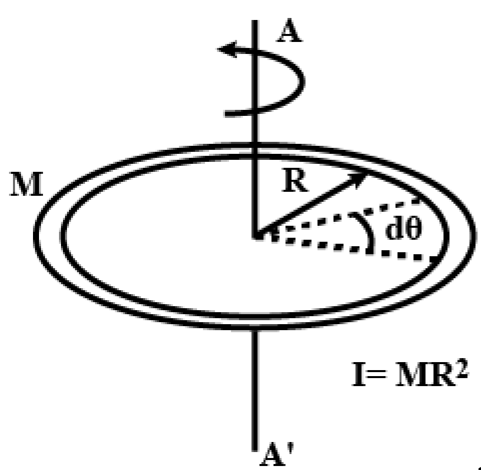
        - Always conserved
- Elasticity
    - Stress
        - Force per area
    - Strain
        - Deformation due to stress
    - Shear
        - Stressing forces that move sidesways to the area
- Periodic Motion
    - Simple Harmonic Motion
        - Repetitive sine (or cosine) motion
            - x = ACos(ωt + ϕ)
            - A: amplitude
            - ω: frequency
            - ϕ: phase (a.k.a. starting position/angle)
    - Examples
        - Rigid pendulums
        - Springs

<u>Useful review:</u>

<u>Determining Moment of Inertia (for things not on the formula sheet)</u>

Moment of inertia <u>only</u> depends on radius of mass from the central axis, it does not matter how much the mass lies along the axis. Examples

Moment of inertia of a thin ring can be the same as the moment of inertia of a hollow thin cylinder:



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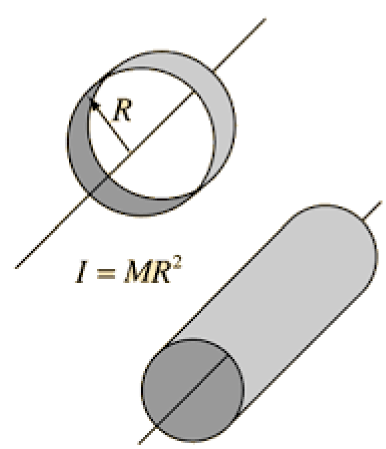
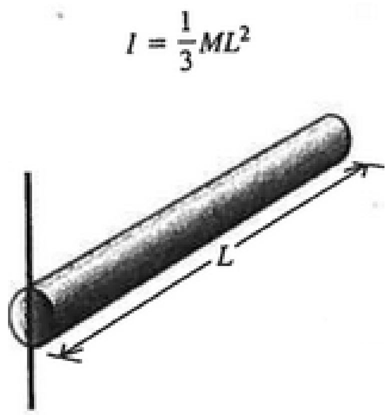


Moment of inertia of a rod can be the same as the moment of inertia of a plate:



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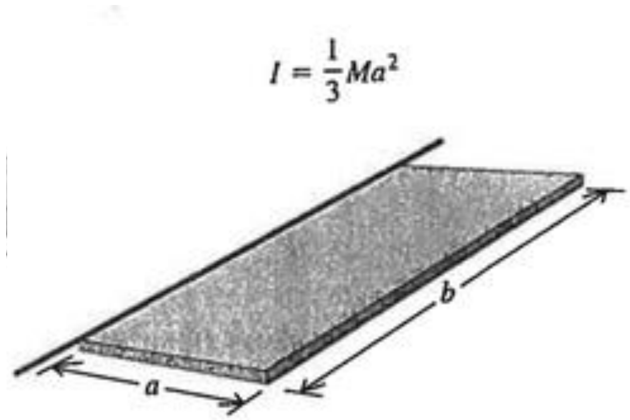
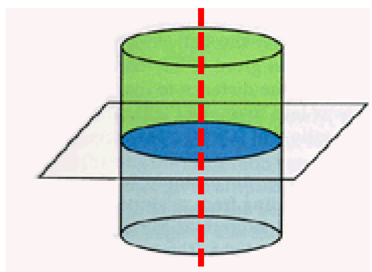


Why does this occur? If the rotation axes are aligned, the simpler/smaller shape is simply a slice of the large shape. More importantly, the slice has the same mass distribution around the axis, which means that the moment of inertia will be the same.

As a final example, we can see how a solid disk can be sliced out from a solid cylinder in the below image. As long as the axis the slice is taken from is the same, the disk will have the same moment of inertia as the cylinder.



Translating from linear quantities to angular quantities

One important thing to know is how to translate from linear quantities of motion to angular quantities of motion (e.g., position, velocity, acceleration, momentum, kinetic energy, etc.).

The direct equations are given in the formula sheet for the exam, as seen below:

$s = r\theta \qquad v = r\omega \qquad a_{tan} = r\alpha \qquad a_{rad} = v^2 / r = r\omega^2$

$K = \frac{1}{2} I\omega^2 \qquad I = m_A r_A^2 + m_B r_B^2 + \ldots. \qquad U = Mgy_{cm}$

$L = mvl$

However, just knowing the equations will not tell you when to use the equations. As an example, take the following problem. “A CD disk (mass = 0.03kg, outer radius = 0.06m, inner radius = 0.008m, assume thin disk) falls off a table and rolls along the floor with a velocity of +10 m/s. What is the total kinetic energy of the disk?”

In this problem, the simple approach would be to multiply the square of the velocity by half of the mass (i.e. $K = \frac{1}{2}mv^2$) which would given the linear kinetic energy. However, because the disk is rolling, we must also think about rotational kinetic energy ($K_r = \frac{1}{2}I\omega^2$), i.e. the energy from the disk spinning.

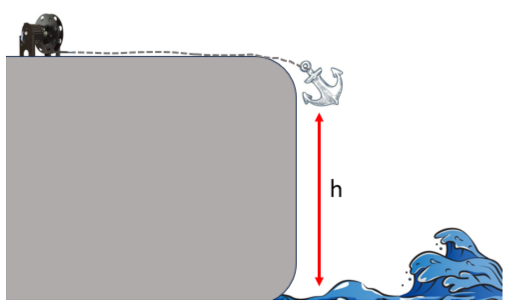
We can find the rotational speed ω from the linear speed v by the equation v=rω. The moment of inertia *I* is given by using the moment of inertia of a hollow cylinder, $I = \frac{1}{2}m\left(r_i^2 + r_o^2\right)$ (see the prior review point for why this can be done) where r_i is the inner radius and r_o is the outer radius. From there is straight forward (if a bit mess) to plug in the values we have for the radii, mass, and velocity to find *I* and ω to obtain the rotational kinetic energy.

The total kinetic energy will be the sum of both the linear and rotational kinetic energy.

<u>Potential to Kinetic Energy</u>

Earlier in the course, it was mentioned that the conservation laws (both energy and momentum) were powerful tools for solving problems. This is also true for rotational motion. In particular, using a conservation law might be the only way to solve for rotational quantities of only linear quantities are known (or vice versa).

As an example, imagine the following problem "An anchor is dropped off the side of a 5 meter tall ship. The anchor is attached to a chain (whose mass is negligible compared to the anchor at such a short length) which connects to a well-balanced spool with a 0.1 meter radius. If the anchor starts at rest a height h above the waterline and is allowed to fall freely, what will the angular velocity of the spool be when the anchor hits the water?"



In this problem, we have to find a way of turning the linear motion of one object into rotational motion of another object. Fortunately, this can be down with conservation of energy. Because the anchor is attached to a chain, which in turn is connected to the edge of the spool, the *velocity of the anchor will be equal to the linear velocity of a point on the spool*. Thus, by finding the velocity of the anchor, we will know the linear velocity of the spool and therefore can find the angular velocity of the spool. At the start of the problem, the anchor is at rest (i.e. no initial velocity) and has potential energy mgh (assuming we set our origin point at the water line). When the anchor falls, it will have fully converted all of its potential energy into kinetic energy at the bottom, right before it hits the water. Thus, we can find the velocity of the falling anchor by setting the kinetic energy equal to the potential energy and solving for velocity.

$$K = U \Rightarrow \frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh}$$

From there, we can use the relationship between linear velocity to find the angular velocity.

$$v = r\omega \Rightarrow \omega = \frac{v}{r} = \frac{\sqrt{2gh}}{r}$$

Plugging in the values for g, h, and r, gives an angular velocity of close to 100 rad/s. (This is about 16 rotations a second, so that chain spool is turning very fast! As a side note, in real life there would be some disruptive effects from friction, but the large weight of anchors and chains, and the relatively smooth surfaces they slide on, means run away anchors can be very fast moving and destructive.)

<u>Example of formula sheet provided on exam:</u>

PHYS 201 Formula Sheet Chapters 9—11 (Exam 3)

For constant α:

$$\omega = \omega_0 + \alpha t \qquad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad \theta - \theta_0 = \left(\frac{\omega + \omega_0}{2}\right)t$$

$$s = r\theta \qquad v = r\omega \qquad a_{tan} = r\alpha \qquad a_{rad} = v^2/r = r\omega^2$$

$$K = \tfrac{1}{2}I\omega^2 \qquad I = m_A r_A^2 + m_B r_B^2 + \ldots \qquad U = Mgy_{cm}$$

$$K_{total} = \tfrac{1}{2}Mv_{cm}^2 + \tfrac{1}{2}I_{cm}\omega^2$$

$$\tau = Fl \qquad \sum \tau = I\alpha \qquad \Delta W = \tau\Delta\theta \qquad P = \tau\omega \qquad L = I\omega$$

$$\sum \tau = \frac{\Delta L}{\Delta t} \qquad L = mvl$$

first and second conditions for equilibrium:
$\sum F_x = 0$, $\sum F_y = 0$ and $\sum \tau = 0$ (any axis)

$$Y = \frac{F_\perp / A}{\Delta l / l_0} \qquad B = -\frac{\Delta p}{\Delta V / V_0} \qquad S = \frac{F_\parallel / A}{x / h} = \frac{F/A}{\phi}$$

$$F_x = -kx \qquad a_x = -\frac{k}{m}x \qquad \omega = 2\pi f \qquad f = \frac{1}{T}$$

$$U_{el} = \tfrac{1}{2}kx^2 \qquad K = \tfrac{1}{2}mv^2$$

$$x = A\cos\omega t \qquad v_x = -\omega A\sin\omega t \qquad \omega = \sqrt{\frac{k}{m}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

TABLE 9.2 Moments of inertia for various bodies