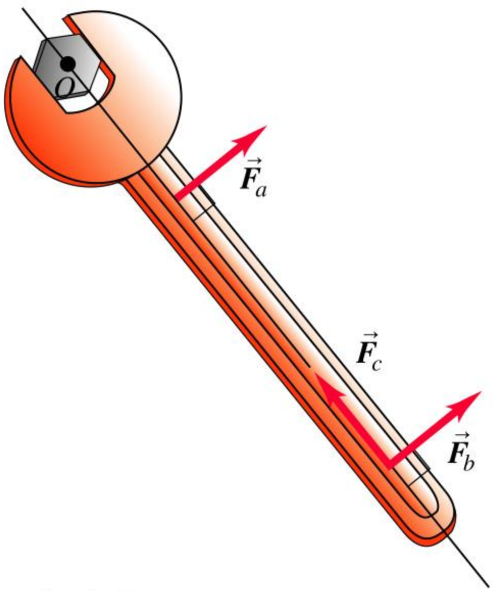
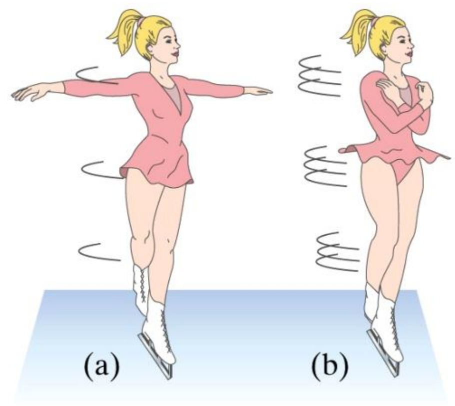
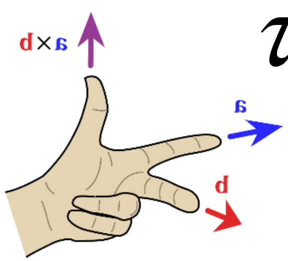
Chapter 10 Dynamics of Rotational Motion





$$\vec{L} = \vec{r} \times \vec{P}$$

$$\frac{d\vec{L}}{dt} = 0$$



$$\vec{\tau} = \vec{r} \times \vec{F}$$

(Conservation of angular momentum)

$$|\boldsymbol{a}\ \ \boldsymbol{b}| = |\boldsymbol{a}|\,|\boldsymbol{b}|\boldsymbol{sin\theta}$$

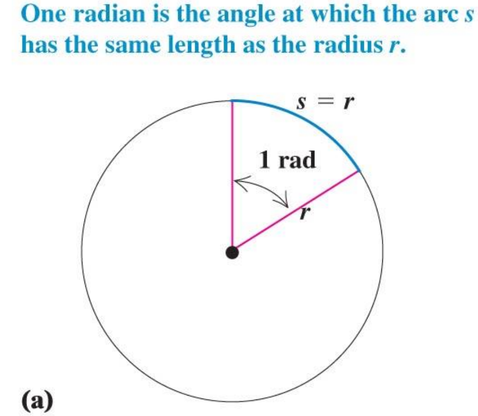
Concepts of rotational motion

3 word dictionary

Acceleration $\vec{a}$ → angular acceleration $\vec{\propto}$
Distance d → angle θ
Velocity $\vec{v}$ → angular velocity $\vec{w}$

Euclidean geometry
325-265 BC

circumference
/diameter=3.141593



(a)

Angle θ

How many degrees are in one radian ? (rad is the unit if choice for rotational motion)

$$\theta = \frac{s}{r} \rightarrow \text{ratio of two lengths (dimensionless)}$$

$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi\, rad \cong 360°$$

$$1\, rad \cong \frac{360°}{2\pi} = \frac{360°}{6.28} = 57° \;\therefore \text{Factors of unity } \frac{1\, rad}{57°} \text{ or } \frac{57°}{1\, rad}$$

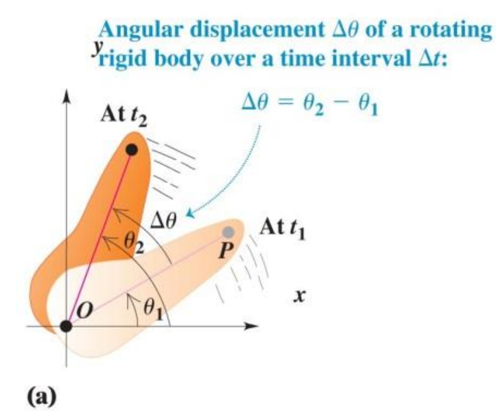
1 radian is the angle subtended at the center of a circle by an arc with length equal to the radius.

Angular velocity $\vec{w}$

$$w_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \left[\frac{rad}{s}\right] \rightarrow w = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \left[\frac{rad}{s}\right]$$

Other units are;

$$1\frac{rev}{s} = \frac{2\pi\, rad}{s} \qquad \therefore 1\frac{rev}{min} = 1\, rpm = \frac{2\pi}{60}\frac{rad}{s}$$



(a)

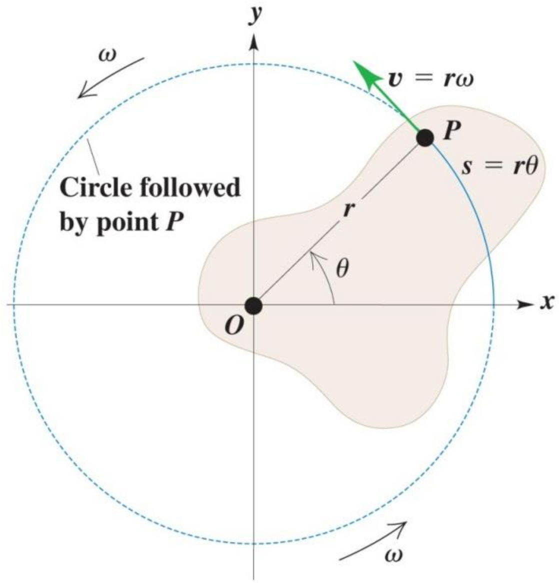
Angular acceleration $\vec{\propto}$

$$\propto_{av} = \frac{w_2 - w_1}{t_2 - t_1} = \frac{\Delta w}{\Delta t} \quad \left[\frac{rad}{s^2}\right] \quad \rightarrow \propto = \lim_{\Delta t \rightarrow 0} \frac{\Delta w}{\Delta t} \quad \left[\frac{rad}{s^2}\right]$$

Relationship between linear and angular quantities

$$s = \theta r \rightarrow v_{av} = \frac{\Delta s}{\Delta r} = r\frac{\Delta \theta}{\Delta t} = rw_{av}$$

$$\therefore \Delta t \rightarrow 0 \quad gives \quad v = rw$$



Chapter 10 Dynamics of Rotational Motion

- To understand the concept of torque.
- To relate angular acceleration and torque.
- To work and power in rotational motion.
- To understand angular momentum.
- To understand the conservation of angular momentum.
- To study how torques add a new variable to equilibrium.
- To see the vector nature of angular quantities.

Ultimate goal is to derive a rotational version of Newtons second law

Rotational Dynamics: *I*

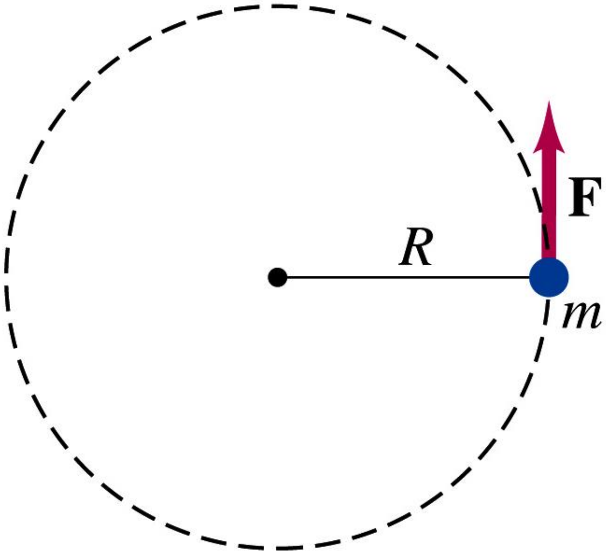
$$F = m\,a = m\,(R\,\alpha) \rightarrow \tau\,(\text{torque}) = F \cdot R = (m\,R^2)\,\alpha$$

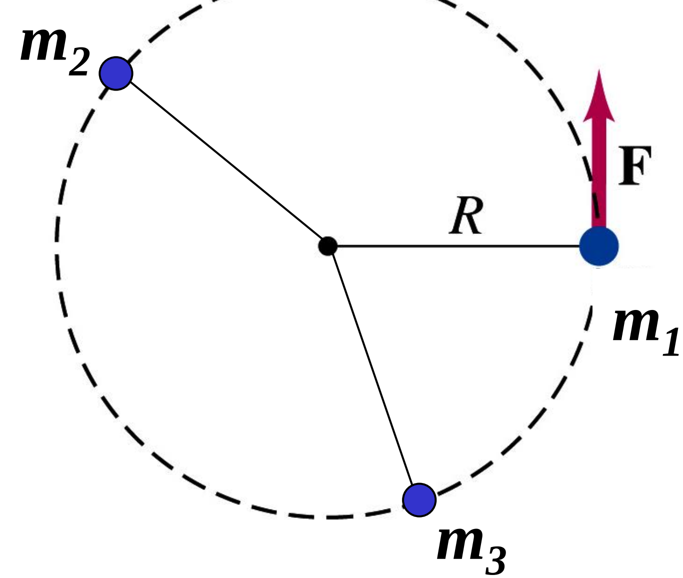
(a) $\therefore I = m\,R^2$

(moment of inertia for single particle)

(b) $\therefore I = \sum m_i R_i^{\,2}$

(moment of inertia for a group of particles)

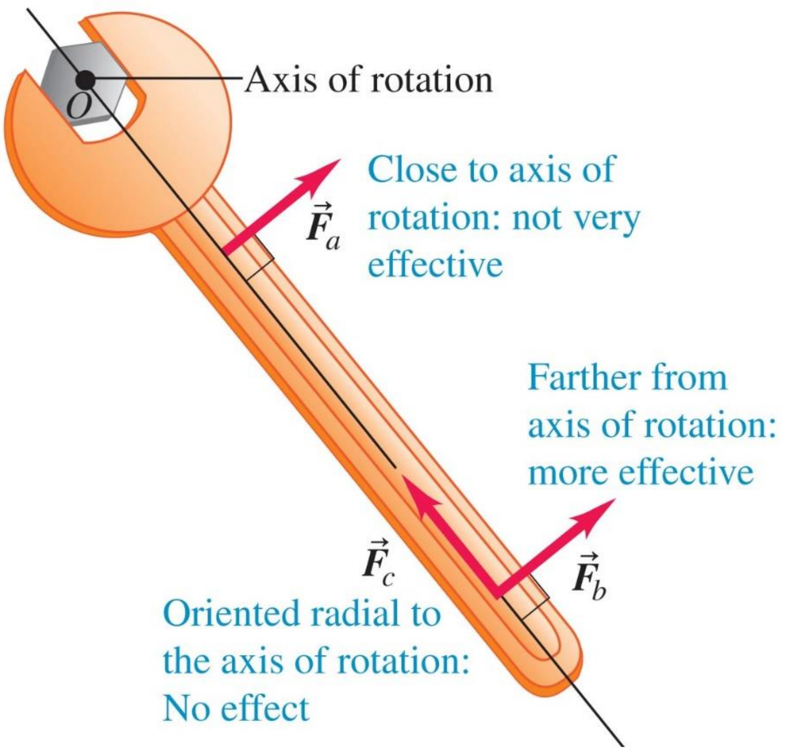




What is torque? When you apply a force to rotate an object about a pivot or an axis, the effectiveness of your action depends not only on the magnitude of the force you apply, but also on a quantity known as the moment arm.

What is the moment arm? It is the perpendicular distance from the pivot or the axis to the line of action of the applied force.

Torque = (Magnitude of Force) × (Moment Arm) $\tau = Fl$ Units: N·m



Torque is a Vector

The Magnitude and the Direction of a Torque

Using the examples in the figure on the right:

(a) Torque by $\vec{F}_1$

$$\tau_1 = F_1 l_1$$

counter-clockwise, positive

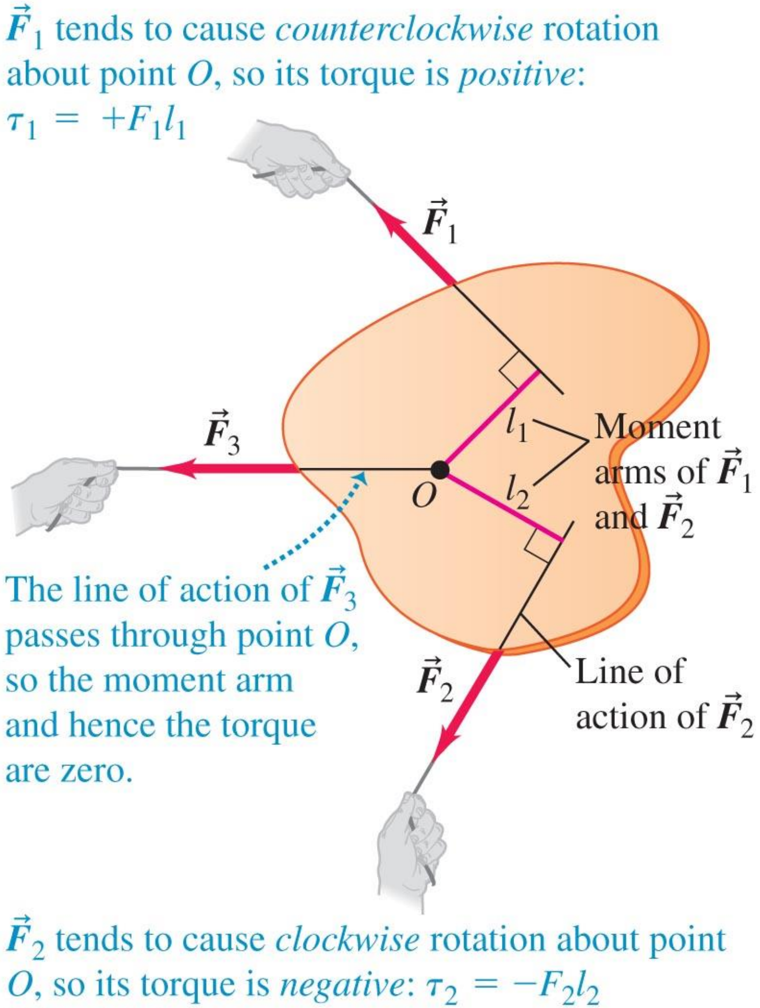
(b) Torque by $\vec{F}_2$

$$\tau_2 = -F_2 l_2$$

clockwise, negative

(c) Torque by $\vec{F}_3$

$$\tau_3 = F_3 l_3 = 0$$



10.2 Torque and Angular Acceleration

Again, cut the rigid body into many small pieces, A, B, C,…. The force acting on piece A is $\vec{F}_A$.

Consider the motion of piece (particle) A. According to Newton's Second Law,

$$F_{A,tan} = m_A a_{A,tan} = m_A r_A \alpha$$
$$\tau_A = r_A F_{A,tan} = m_A r_A^2 \alpha$$

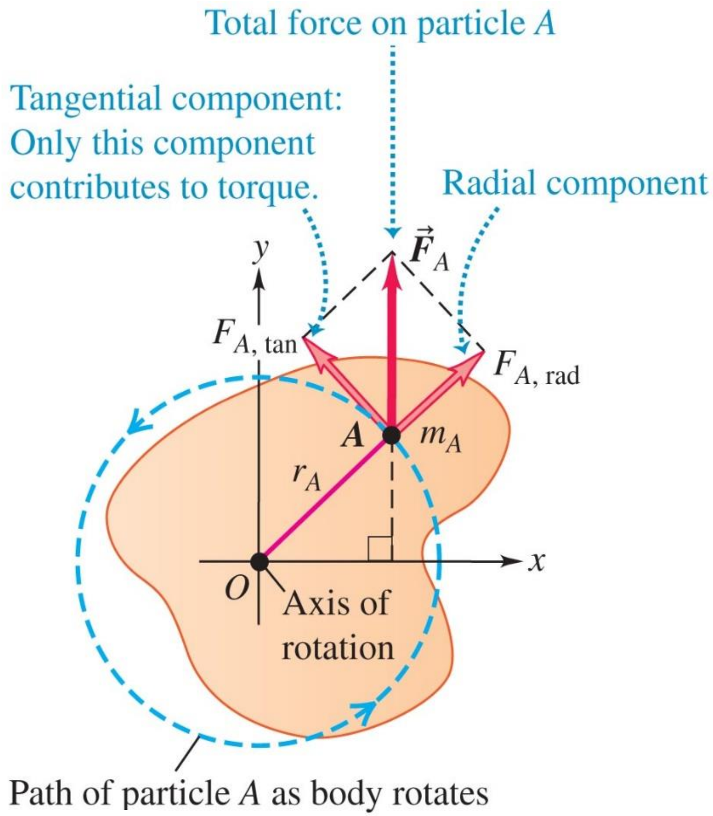
Sum over the torques for all the pieces:

$$\tau_A + \tau_B + \tau_C \ldots = m_A r_A^2 \alpha + m_B r_B^2 \alpha + m_C r_C^2 \alpha + \cdots$$
$$= (m_A r_A^2 + m_B r_B^2 + m_C r_C^2 + \cdots)\alpha$$

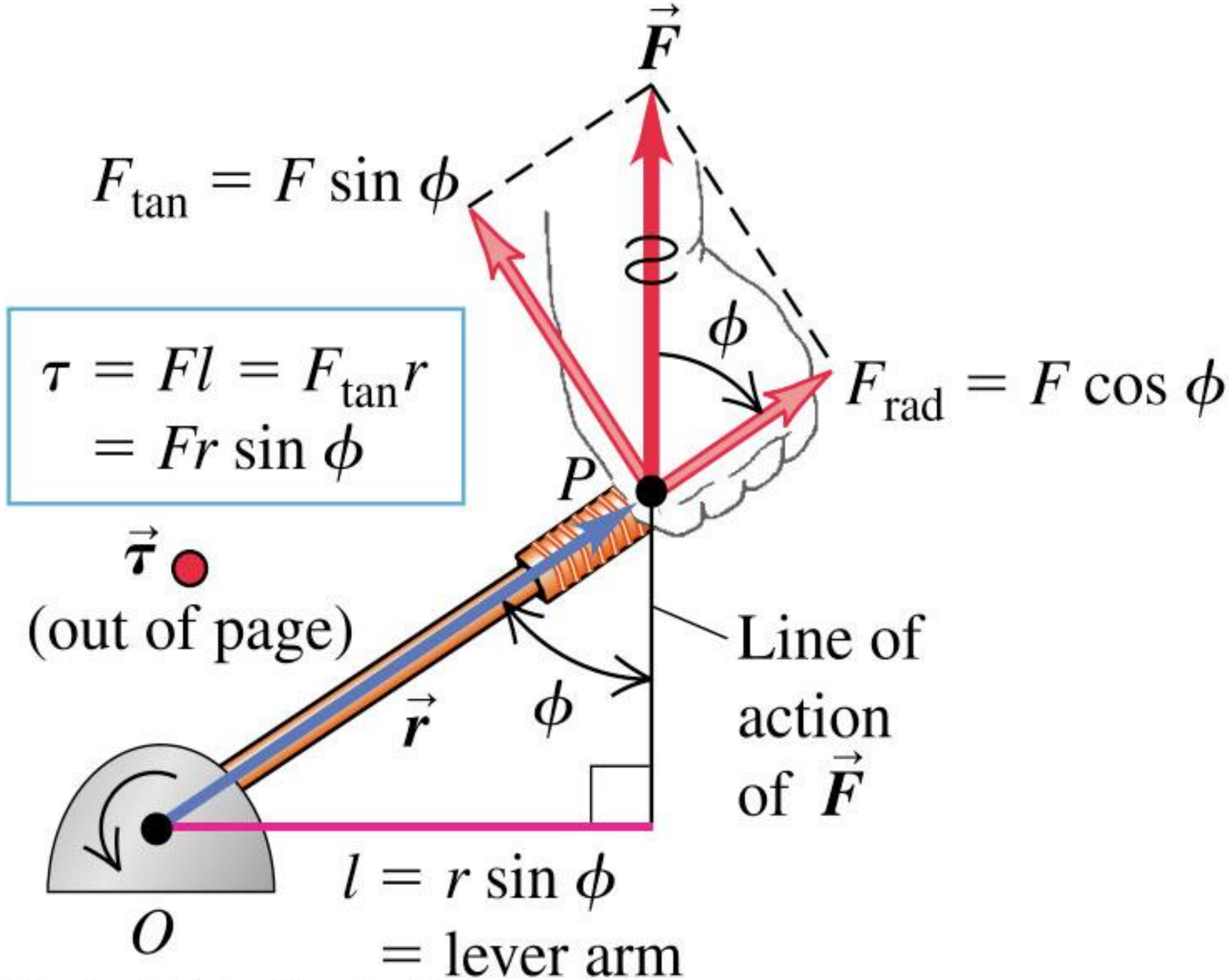
or

$$\sum \tau = I\alpha$$

This is also known as Newton's Second Law for rotational motion.

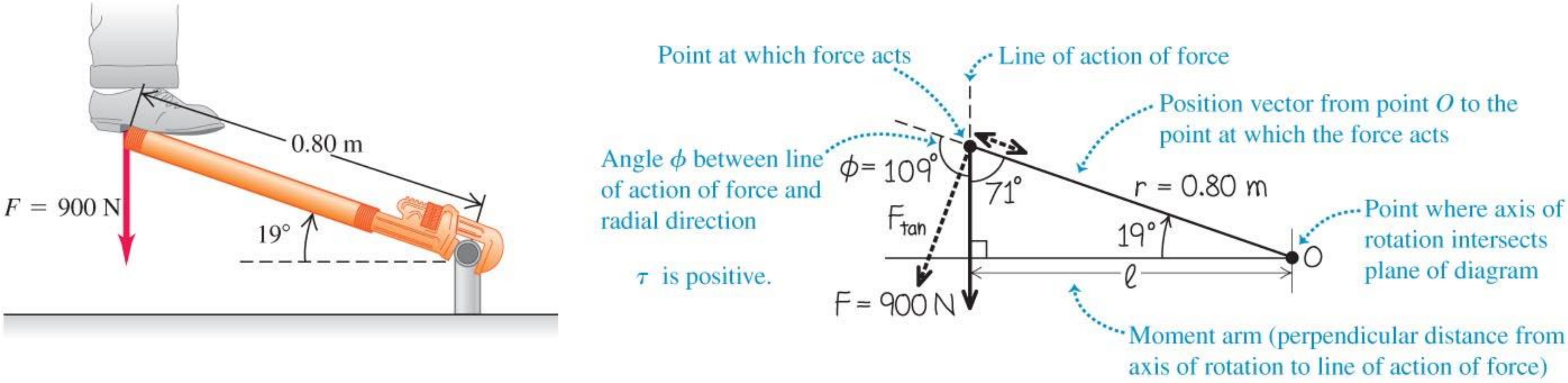


Torque $\vec{\tau} = \vec{r} \times \vec{F}$



Note: F(rad)has no torque with respect to O

A Plumbing Problem to Solve – Example 10.1



(a) Diagram of situation

(b) Free-body diagram

The angle between r and F is $\phi = 109°$

$l = 0.80 sin 71° = 0.76m$

$\tau = Fl = 900N \cdot (0.76) = 680Nm$ (magnitude)

Next-Time Question

James finds it difficult to muster enough torque to turn the stubborn bolt with the wrench. He wishes he had a pipe handy to effectively lengthen the wrench handle, but doesn't. He does, however, have a piece of rope. Will torque be increased if he pulls as hard on the rope as shown?

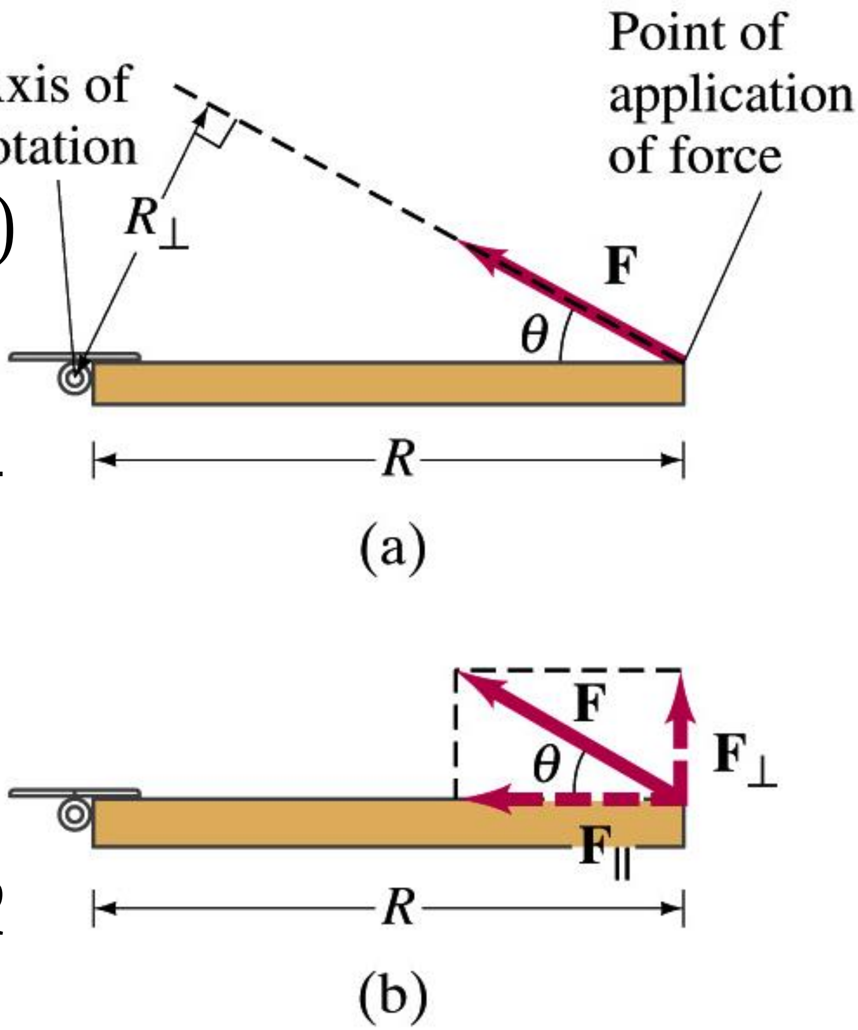
no

Note: $\tau = F\,R\sin\theta$

$\textit{Lever arm} : l \text{ or } R_{\perp}$

(Perpendicular distance ...)

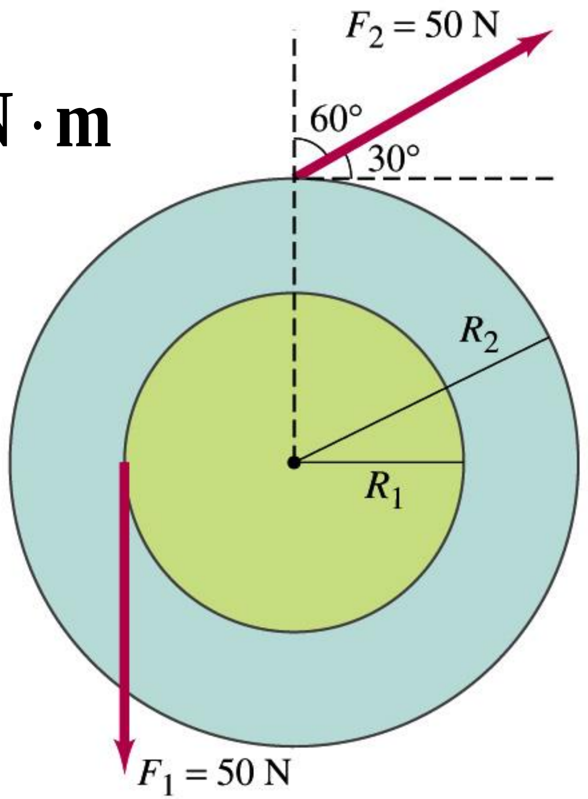
$$\tau = F\,(R\sin\theta) = F \cdot R_{\perp}$$



$$\tau = (F\sin\theta)\,R = F_{\perp} \cdot R$$

Note: sign of τ

$$\tau_1 = F_1\,(R_1 \sin 90^\circ)$$
$$= (50.0\ \mathrm{N})(0.300\ \mathrm{m}) = 15.0\ \mathrm{N\cdot m}$$
$$\tau_2 = F_2\,(R_2 \sin 60^\circ)$$
$$= (50.0\ \mathrm{N})(0.500\ \mathrm{m})(0.866)$$
$$= 21.7\ \mathrm{N\cdot m}$$
$$\tau_{net} = \tau_1(\mathrm{c.c.w}) + \tau_2(\mathrm{c.w.})$$
$$= \tau_1(+1) + \tau_2(-1)$$
$$= (15.0\ \mathrm{N\cdot m}) - (21.7\ \mathrm{N\cdot m})$$
$$= -6.7\ \mathrm{N\cdot m} \rightarrow 6.7\ \mathrm{N\cdot m}\ (\mathrm{c.w.})$$



$R_1 = 0.300$ m $R_2 = 0.500$ m

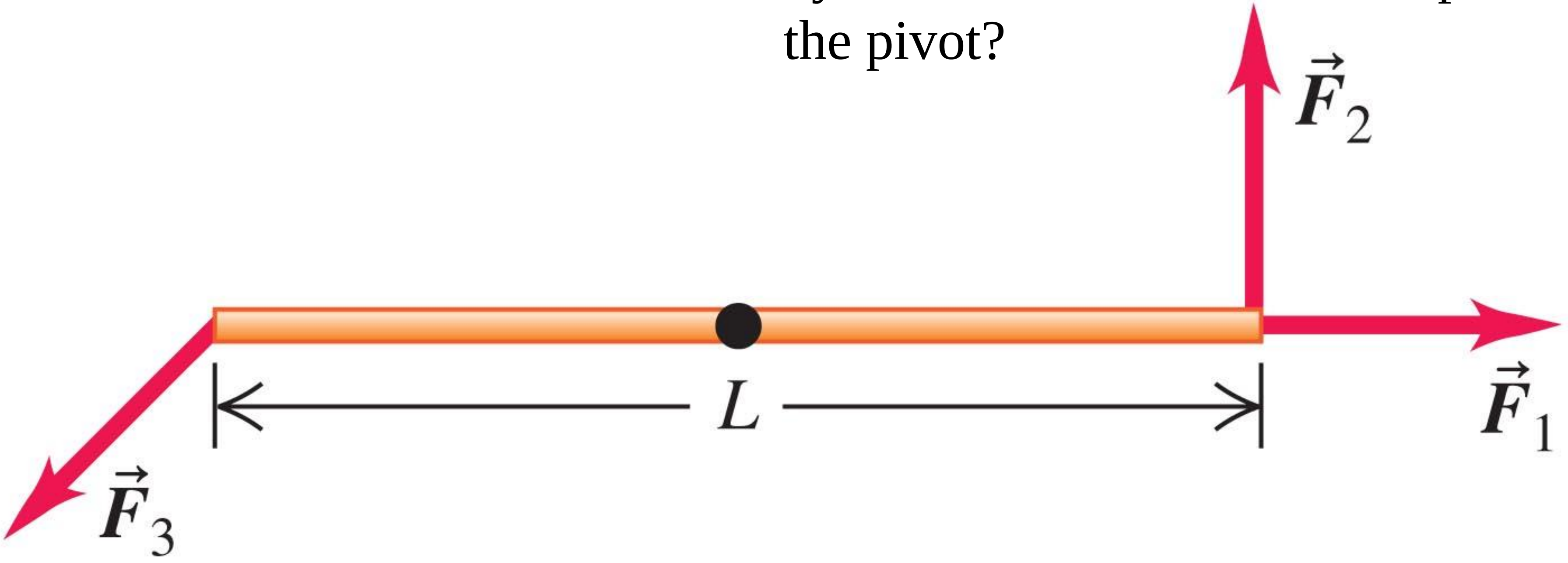
Clicker question

Torques on a Rod

- a) $\tau_1 > \tau_2 > \tau_3$
- $b)$ $\tau_1 > \tau_3 > \tau_2$
- $c)$ $\tau_2 > \tau_3 > \tau_1$

The 3 forces have equal magnitude

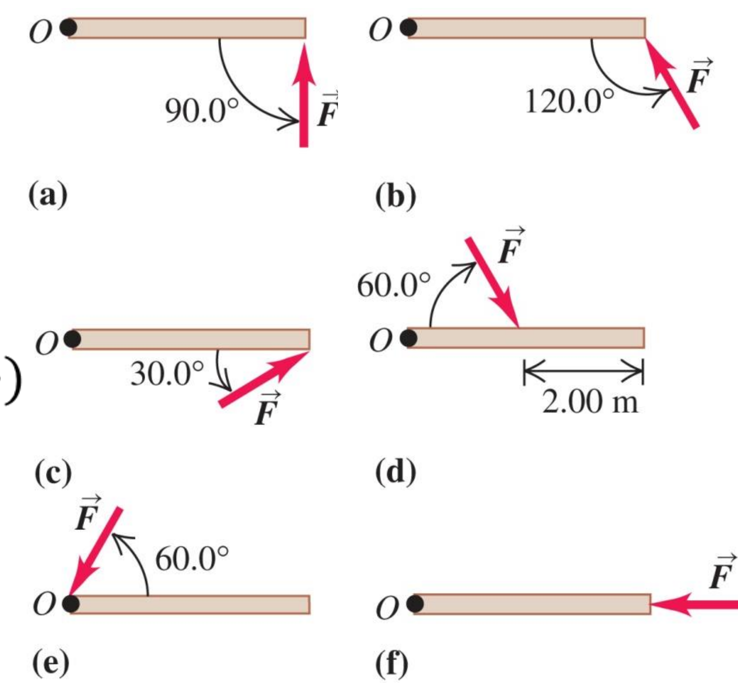
A rod is pivoted at its center. Three forces of equal magnitude are applied as shown. Which of the following statements correctly describe the order in the magnitudes of the torques by these three forces with respect to the pivot?

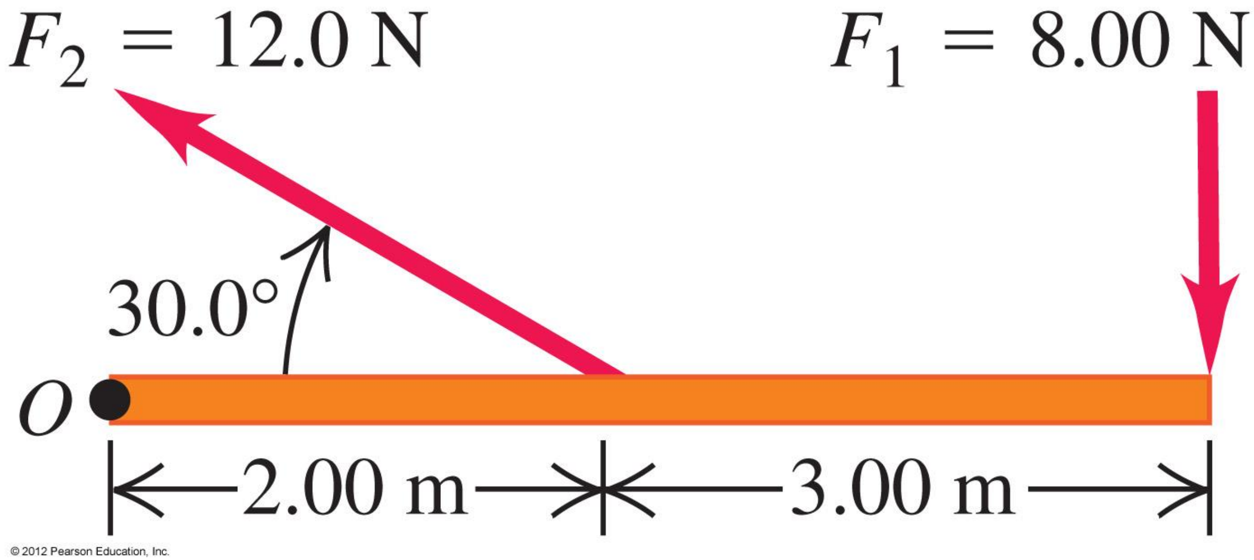


<u>Problem 10.1</u>: Calculate the torque about point **O** due to the force $\vec{F}$ for each situation shown. The rod has a length **4m** and the force a magnitude of **10N**

Let counterclockwise torques be positive $\tau = F \cdot l = F \cdot r \sin\varphi$

a) $\tau = +10N \cdot (4m) \sin 90° = 40Nm \; (cc = +)$
b) $\tau = +10N \cdot (4m) \sin 60° = 34Nm \; (cc = +)$
c) $\tau = +10N \cdot (4m) \sin 30° = 20Nm \; (cc = +)$
d) $\tau = -10N \cdot (2m) \sin 60° = -17.3Nm \; (cw = -)$
e) $\tau = 0N$ since force acts on axis
f) $\tau = 0N$ since force has line of action through the focus





$$\tau = F \cdot l = F \cdot r \sin\varphi$$

$$\tau_1 = -F_1 \cdot l_1 = -8N \cdot 5m = -40Nm$$

$$\tau_2 = F_2 \cdot l_2 = 12N \cdot 2m \sin 30° = +12Nm$$

$$\sum \tau_i = \tau_1 + \tau_2 = -28Nm \ (cw = -)$$

In order not to fall the acrobat keeps the her center of gravity directly above the rope. She creates proper torques in this balancing act.

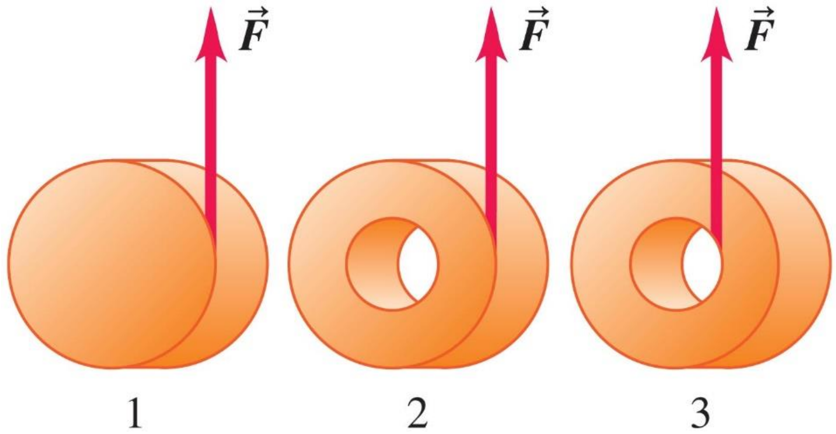
Why Do Acrobats Carry Long Bars?

Clicker Question

Rotating Cylinders

Relationship of torque and angular acceleration

$\Sigma\tau = I\alpha$ rotational analog of Newton's law $\Sigma F = ma$



Which choice correctly ranks the magnitudes of the angular accelerations of the three cylinders?

A. $\alpha_3 > \alpha_2 > \alpha_1$

B. $\alpha_1 > \alpha_2 > \alpha_3$

C. $\alpha_2 > \alpha_1 > \alpha_3$

D. $\alpha_1 = \alpha_2 = \alpha_3$

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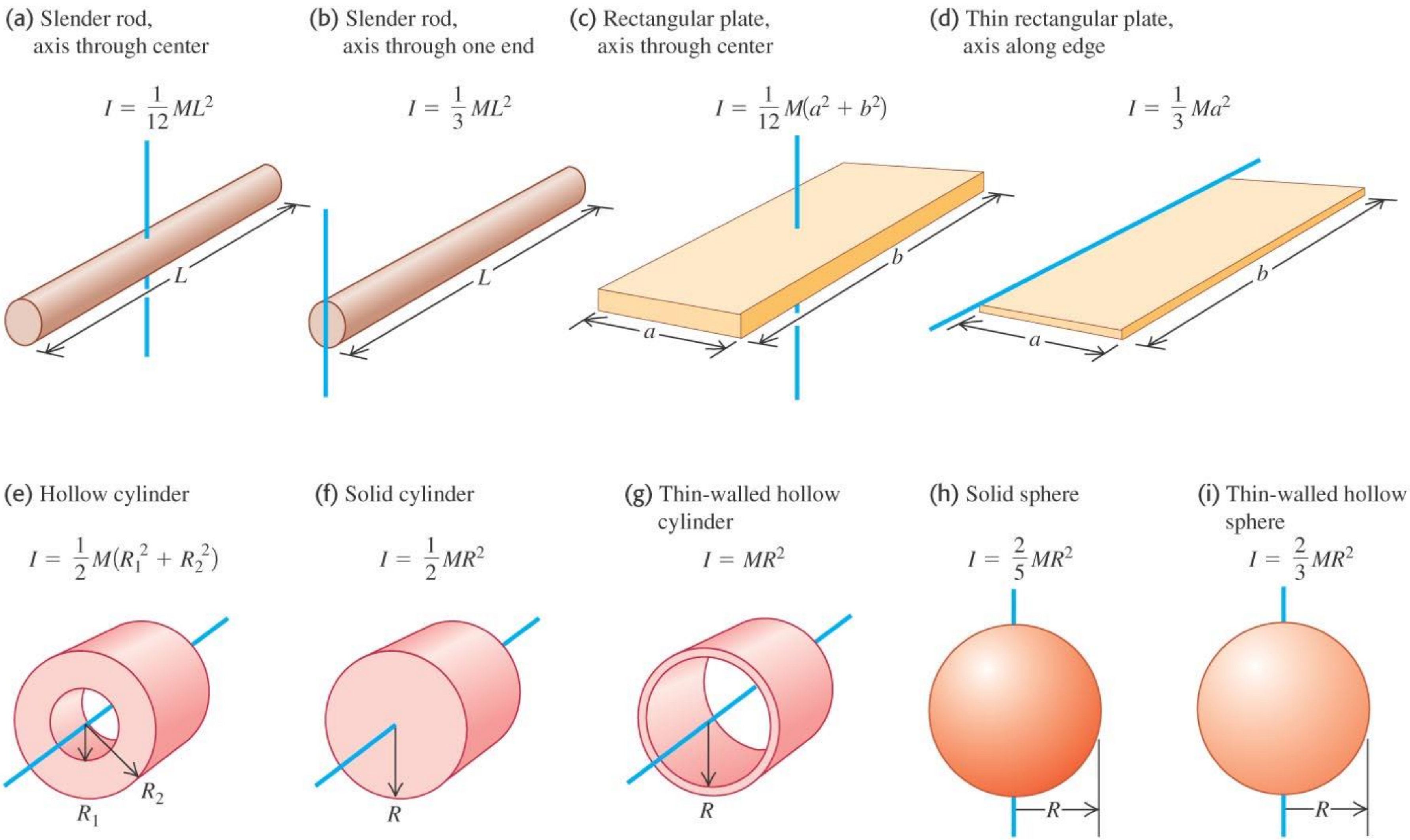
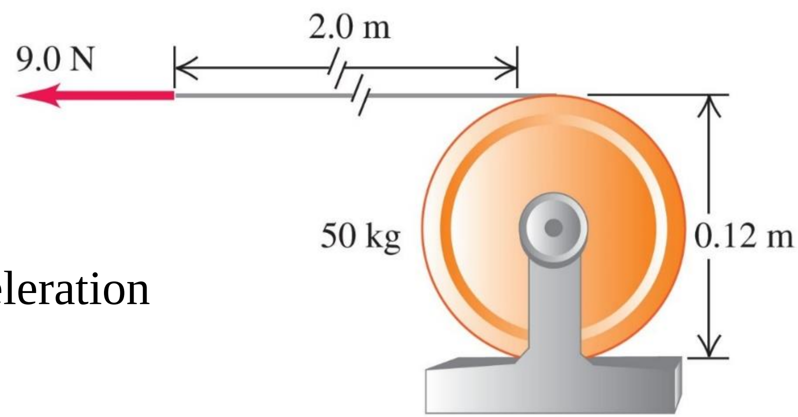
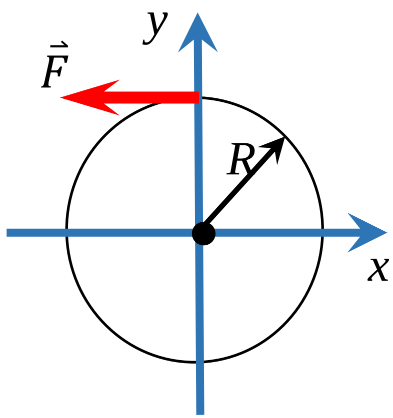
Finding the moment of inertia for common shapes



Re-visit an Earlier Problem





Example 10.2
A cable unwinding from a winch.

Find:
(a) Magnitude of the angular acceleration
(b) Final angular velocity
(c) Final speed of cable

Solution:

The torque by the tension force $\tau = Fl = (9.0\ \text{N}) \times (0.06\ \text{m}) = 0.54\ \text{N•m}.$

The moment of inertia about the given axis $I = \frac{1}{2}MR^2 = \frac{1}{2}(50\ \text{kg})(0.06\ \text{m})^2 = 0.090\ \text{kg•m}^2$

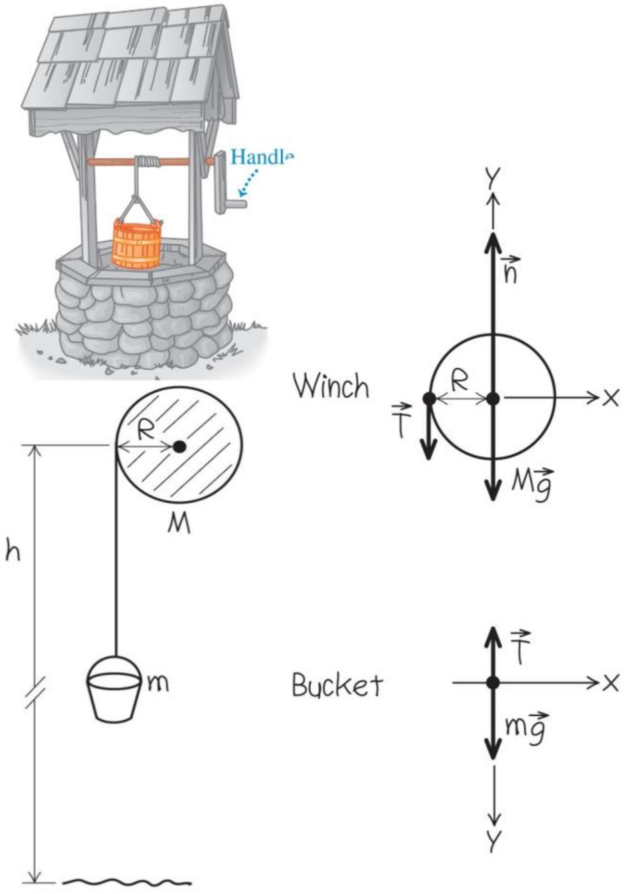
Therefore, the angular acceleration $\alpha = \tau/I = (0.54\ \text{N•m})/(0.090\ \text{kg•m}^2) = 6.0\ \text{rad/s}^2$

Angular Displacement $\theta - \theta_0 = s/r = (2.0\ \text{m})/(0.06\ \text{m}) = 33\ \text{rad}$

Final angular velocity $\omega = \sqrt{\omega_0^2 + 2\alpha(\theta - \theta_0)} = \sqrt{2\alpha(\theta - \theta_0)} = 20\ \text{rad/s}$

Final speed of cable $v = r\omega = (0.06\ \text{m})(20\ \text{rad/s}) = 1.2\ \text{m/s}$

Re-visit Another Problem



Example 10.3 Given: M, R, m, and h

Find: (a) α for winch, a for bucket, and tension force.
(b) v for the bucket just before hitting the water
(c) ω for the winch just before hitting the water

Solution:

Linear motion of the bucket $mg - T = ma$ ………(1)

Rotational motion of the winch

$$\tau = I\alpha \quad \text{or} \quad TR = (\tfrac{1}{2}MR^2)\alpha \text{ ………(2)}$$

From (2): $T = \frac{1}{2}MR\alpha = \frac{1}{2}Ma$ ……………………(3)

Substitute (3) into (1): $mg - \frac{1}{2}Ma = ma$

Linear acceleration: $a = mg/(m + M/2) = g/(1 + M/2m)$

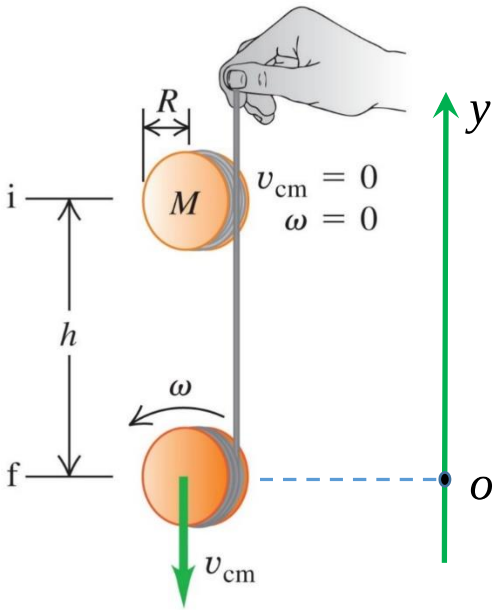
Tension force $T = Ma/2 = Mg/(2 + M/m)$

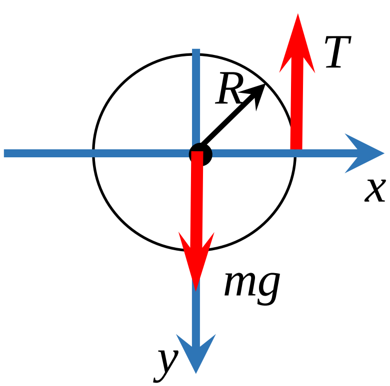
Angular acceleration: $\alpha = a/R = g/[R(1 + M/2m)$

Final v and ω calculated using the kinematic equations:

$$v = \sqrt{\frac{2gh}{1+M/2m}} \quad \text{and} \quad \omega = \frac{1}{R}\sqrt{\frac{2gh}{1+M/2m}}$$

Re-visit One more Problem





Example 10.4 Given: Given M, R, and h

Find: (a) Center of mass acceleration and angular acceleration

(b) Tension force

(c) Velocity of the center of mass v_{cm}

Solution:

Apply Newton's Second Law for linear motion

$$Mg - T = Ma \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots(1)$$

Apply Newton's Second Law for rotational motion

$$\tau = I\alpha \quad \text{or} \quad TR = (\tfrac{1}{2}MR^2)\alpha \dots\dots\dots(2)$$

From (2): $T = \frac{1}{2}MR\alpha = \frac{1}{2}Ma \quad \dots\dots\dots\dots\dots\dots\dots(3)$

Substitute (3) into (1): $Mg - \frac{1}{2}Ma = Ma$

Linear acceleration: $a = 2g/3$

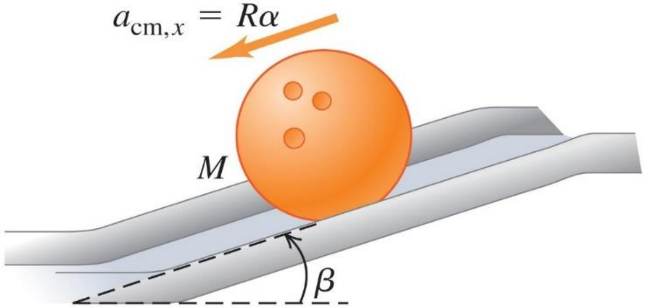
Angular acceleration: $\alpha = a/R = 2g/3R$

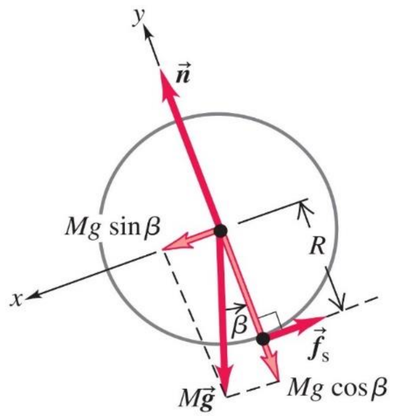
Tension force $T = Mg/3$

Final v_{cm} using the kinematic equations

$$v_{cm} = \sqrt{2ah} = \sqrt{4gh/3}$$

Example 10.5 on page 291: Rolling without slipping
What is the acceleration of a rolling bowling ball?





Given: Given M, R, and β

Find: (a) a_{cm} and α (also, f_s, required minimum μ_s)

Solution:

Apply Newton's Second Law for linear motion

$$Mg\sin\beta - f_s = Ma_{cm} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Apply Newton's Second Law for rotational motion

$$\tau = I\alpha \quad \text{or} \quad f_s R = (\tfrac{2}{5}MR^2)\alpha \quad \ldots\ldots\ldots(2)$$

From (2): $$f_s = \tfrac{2}{5}MR\alpha = \tfrac{2}{5}Ma_{cm} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Substitute (3) into (1): $$Mg\sin\beta - \tfrac{2}{5}Ma_{cm} = Ma_{cm}$$

Linear acceleration: $$a_{cm} = \frac{5}{7}g\sin\beta$$

Angular acceleration: $$\alpha = \frac{a_{cm}}{R} = \frac{5}{7R}g\sin\beta$$

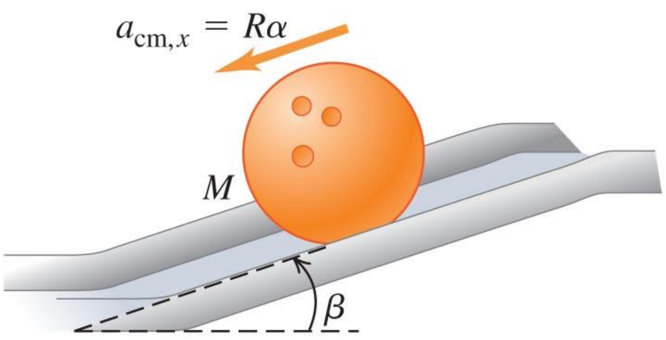
Static friction force $$f_s = \tfrac{2}{5}MR\alpha = \frac{2}{7}Mg\sin\beta$$

Minimum static coefficient of friction

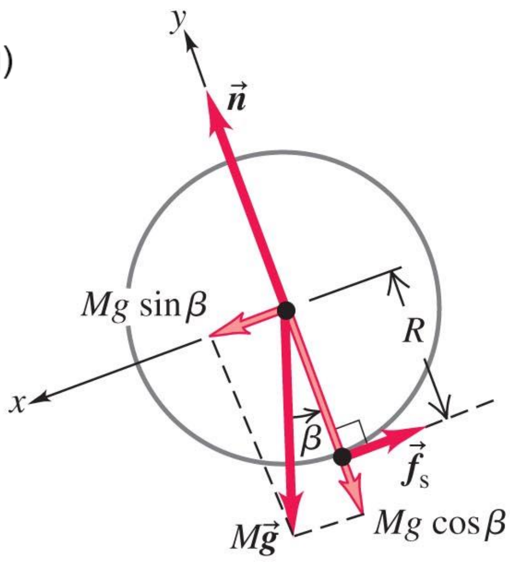
$$\mu_s = f_s/n = \left(\frac{2Mg\sin\beta}{7}\right)/Mg\text{con}\beta$$
$$= \frac{2}{7}\tan\beta$$

A Bowling Ball Rotates on a Moving Axis – Example 10.5

a. What is the ball's linear acceleration $a_{cm,x}$?
b. What is the friction force f_s? (note: it is static friction = not slipping)



(a)



(b)

Translation: $\Sigma F_x = Mg sin\beta - f_s$ (1)

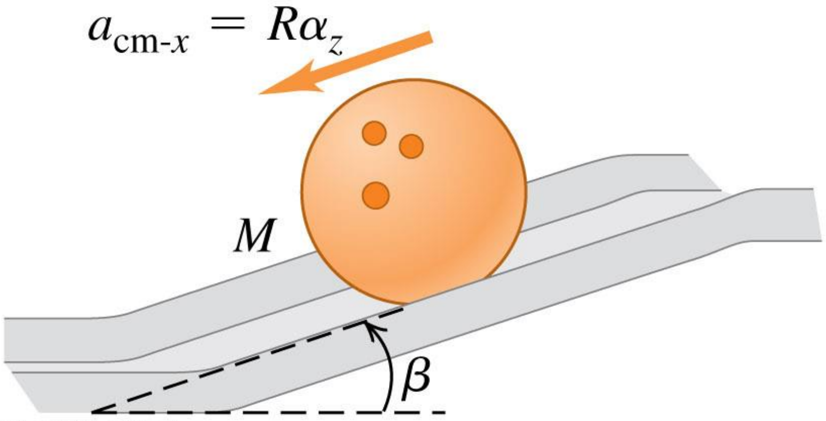
Rotations: $\Sigma\tau = f_s R = I_{cm}\alpha = \left(\frac{2}{5}MR^2\right)\alpha$ (2)

$a_{cm,x} = R\alpha$ $f_s R = \frac{2}{5}MR^2\left(\frac{a_{cm,x}}{R}\right)$ $\boldsymbol{f_s = \frac{2}{5}M a_{cm,x}}$

From(1) $Mg sin\beta - \frac{2}{5}Ma_{cm,x} = Ma_{cmx}$ $\boldsymbol{a_{cm,x} = \frac{5}{7}g sin\beta}$

Clicker question

A solid bowling ball rolls down a ramp. Which of the following forces exerts a torque on the bowling ball about its center?



A. the weight of the ball

B. the normal force exerted by the ramp

C. the friction force exerted by the ramp

D. more than one of the above

E. depends on whether the ball rolls without slipping

10.3 Work and Power in Rotational Motion

Again, cut the rigid body into many small pieces, A, B, C,…. The force acting on piece A is $\vec{F}_A$.

If the angular position of A changes by $\Delta\theta$, the work done by a constant force $\vec{F}_A$ acting on A is:

$$W_A = F_{A,tan}(R\Delta\theta) = \tau_A \Delta\theta$$

Sum over contributions from all the pieces, the total work

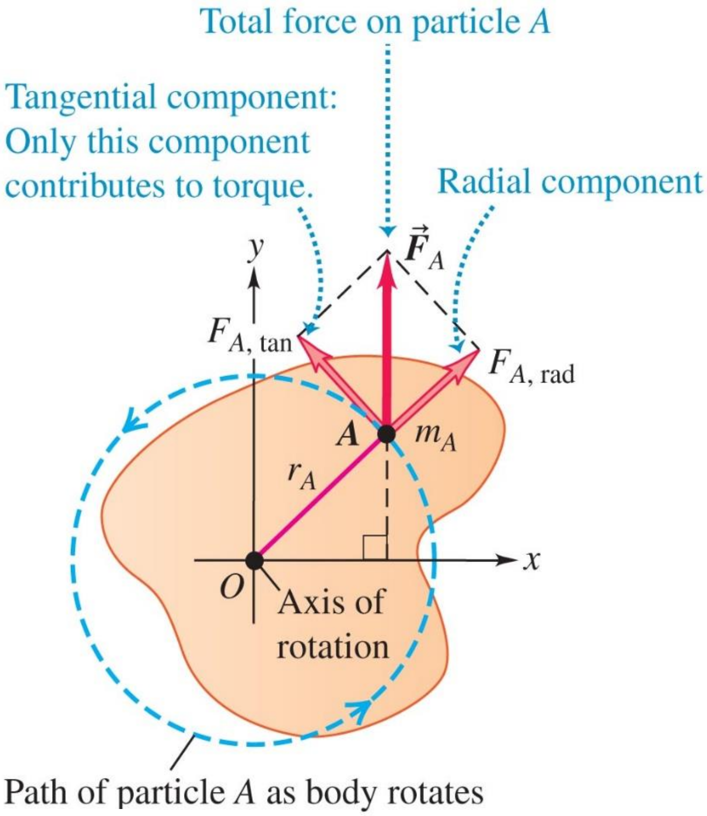
$$W = \tau\Delta\theta = \tau(\theta_2 - \theta_1)$$

A few notes: (a) Applicable for constant torque τ.
(b) Work has units N·m.

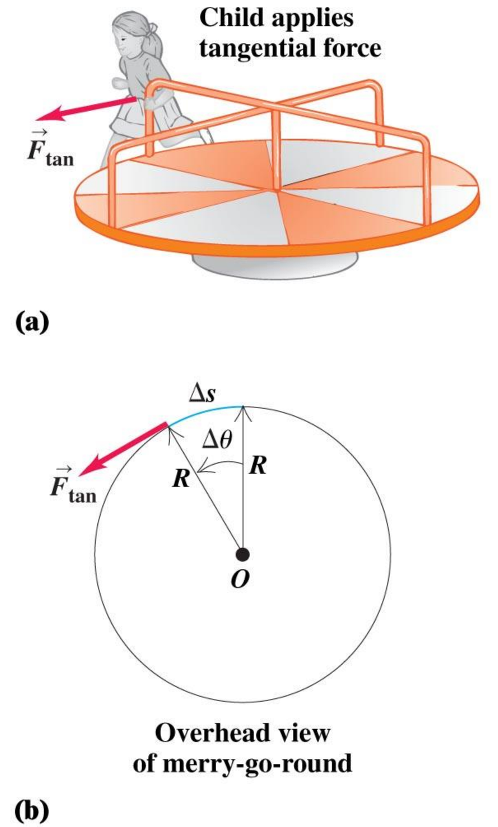
Power is the rate at which work is done:

$$P = \frac{\Delta W}{\Delta t} = \tau\frac{\Delta\theta}{\Delta t} = \tau\omega$$

(Compare with linear motion in which case $P = Fv$)



Work and power in rotational motion



$$\Delta W = F_{tan} \cdot \Delta s = (F_{tan} R)\Delta\theta = \tau \cdot \Delta\theta$$

($F_{tan} R = \tau$; $\Delta s = R\Delta\theta$)

if the torque $\tau = const$ while the object rotates

$$\Delta\theta = (\theta_2 - \theta_1)$$

Work: $W = \tau(\theta_2 - \theta_1) = \tau\Delta\theta$ Joule

(τ in Nm; $\theta_2 - \theta_1$ in radians)

power $\frac{\Delta W}{\Delta t} = \tau \frac{\Delta\theta}{\Delta t}$

$$P = \tau\,\omega \quad \text{watt}$$

(τ in N·m; ω in rad/s)

10.4 Angular Momentum

In Chapter 8 we defined the momentum of a particle as $\vec{p} = m\vec{v}$, we could state Newton's Second Law as $\vec{F} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}$.

Here we define the angular momentum of a rigid body:

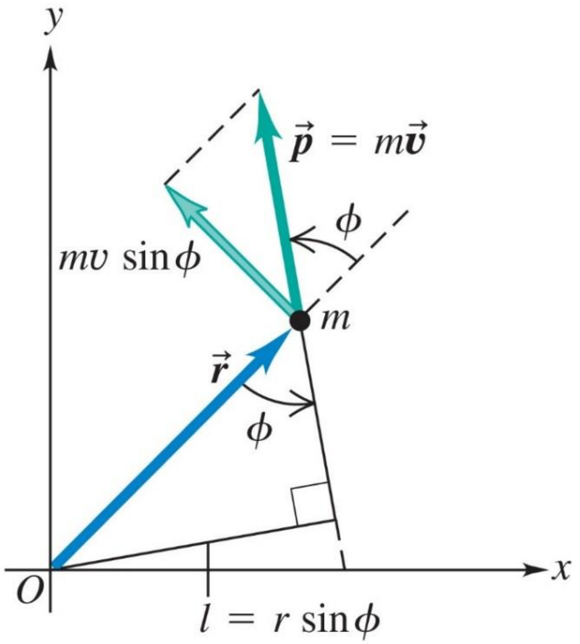
$$L = I\omega$$

Notes: It is also a vector, same as ω.
Units: kg·m^2/s

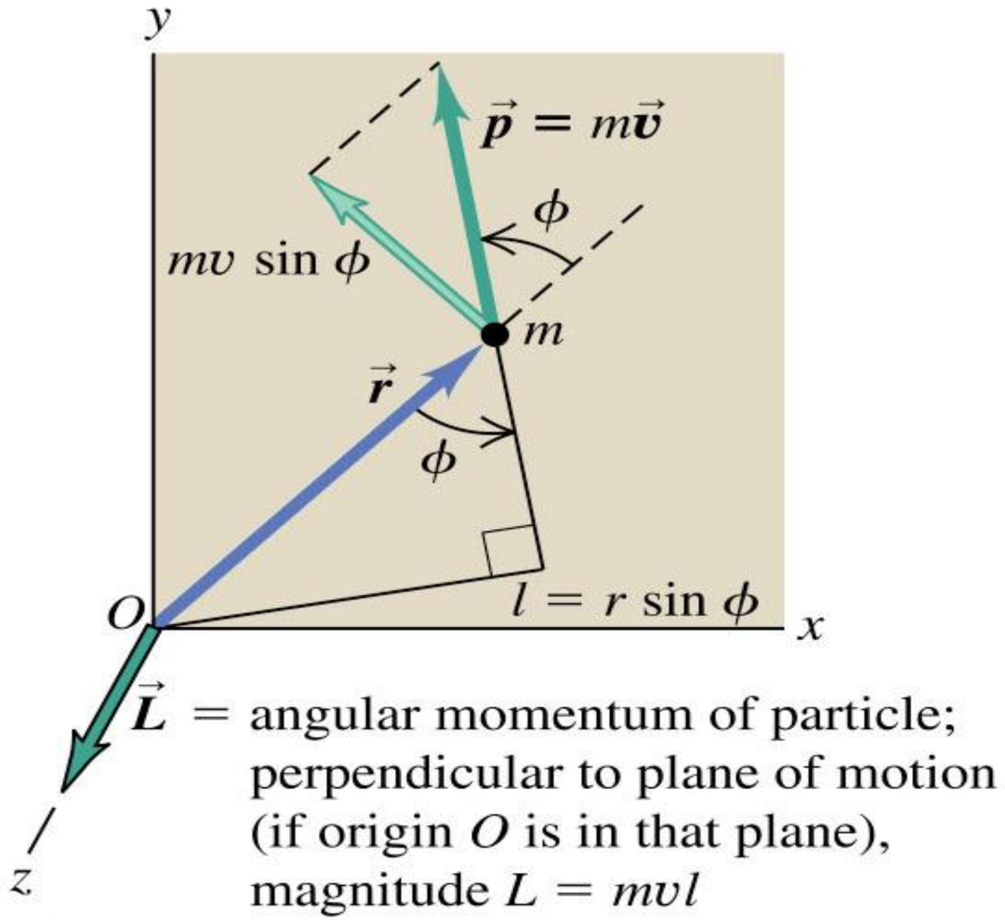
Angular momentum of a point particle:

$$L = mvl \qquad (\text{kg·m}^2/\text{s})$$

Notes: (a) l is effective the "moment arm"
(b) a particle moving along a straight line can still have an angular momentum about a pivot or rotational axis



Angular momentum $\vec{L} = \vec{r} \times \vec{p}$



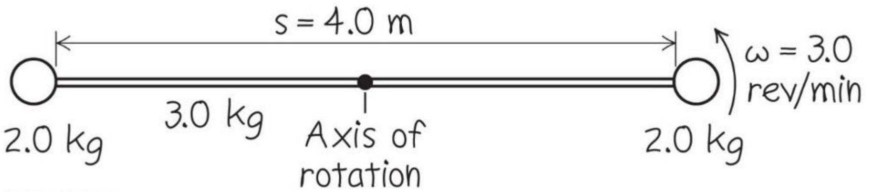
Example 10.7: A kinetic sculpture

Given: Two small metal sphere of masses $m_1 = m_2 = 2.0$ kg

Uniform metal rod of mass $M = 3.0$ kg and length $s = 4.0$ m

Angular velocity 3.0 rev/minutes about a vertical axis through the middle

Find: Angular momentum and Kinetic energy



Solution:

(a) Angular momentum

Total moment of inertia $I_{total} = I_{sphere\ 1} + I_{sphere\ 2} + I_{rod} = m_1(s/2)^2 + m_2(s/2)^2 + (1/12)Ms^2$

$= 20\ \text{kg}\cdot\text{m}^2$

Angular momentum $L = I\omega = (20\ \text{kg}\cdot\text{m}^2)(3.0\cdot 2\pi/60\ \text{s}^{-1}) = 6.2\ \text{kg}\cdot\text{m}^2/\text{s}$

(b) Kinetic energy $K = (1/2)I\omega^2 = 0.96$ J

Calculate the angular momentum and kinetic energy of a solid uniform sphere with a radius of 0.12 m and a mass of 14.0 kg if it is rotating at 6.00 rad/s about an axis through its center

$$I = \frac{2}{5}MR^2 = \frac{2}{5} * (14kg)(0.12m)^2 = 0.0806\ kgm^2$$

$$L = Iw = 0.0806\ kgm^2\ * 6.0\ \text{rad} = 0.484\ \text{kg}\frac{m^2}{s}$$

$$K = \frac{1}{2}Iw^2 = \frac{1}{2}Lw = \frac{1}{2}\left(0.484\ \text{kg}\frac{m^2}{s}\right) * 6\ \text{rad} = 1.45\ \text{J}$$

10.5 Conservation of Angular Momentum

The Relationship Between Torque and Angular Momentum

Since $$\sum \tau = I\alpha = I \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \lim_{\Delta t \to 0} \frac{I \Delta \omega}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta (I\omega)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta L}{\Delta t}$$

We can state Newton's Second Law for rotational motion as

$$\sum \tau = \lim_{\Delta t \to 0} \frac{\Delta L}{\Delta t}$$

Conservation of Angular Momentum

If $\sum \tau = 0,$

Then $L = \text{constant}$

Angular Momentum Is Conserved – Figure 10.19

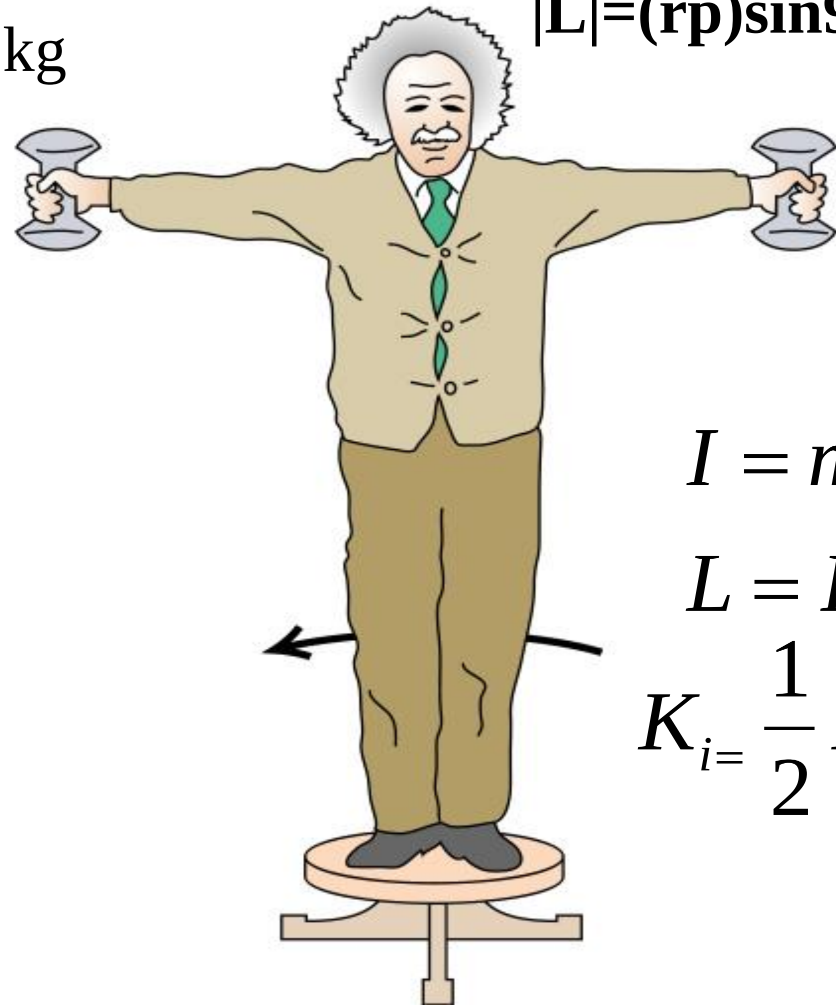
- The first figure shows the figure skater with a large moment of inertia.
- In the second figure, she has made the moment much smaller by bringing her arms in.
- Since L is constant, ω must increase.

Conservation of angular momentum

L=r x **p**

|L|=(rp)sin90=(r)(mv)

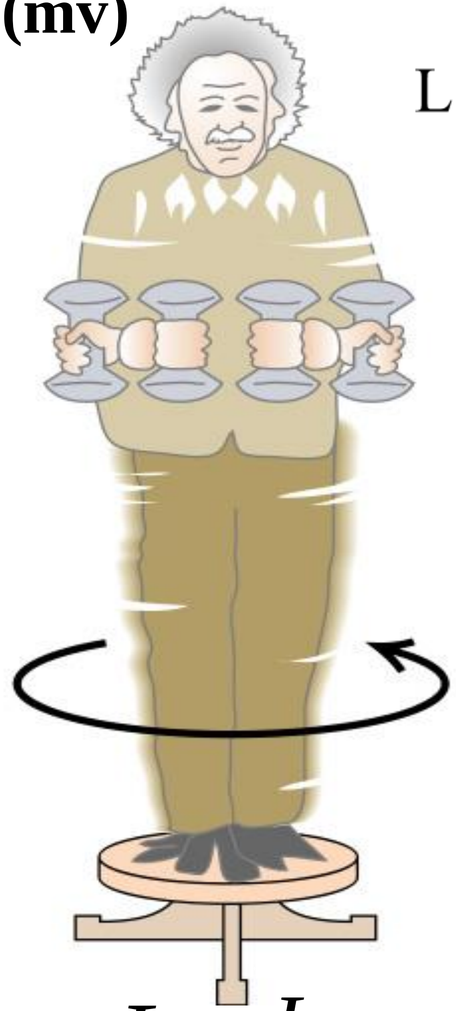
5 kg



v=rω

$L = (r^2 m)\omega$

$= I\omega$



$$I = mr^2$$

$$L = I\omega$$

$$K_i = \frac{1}{2} I_i \omega_i^2$$

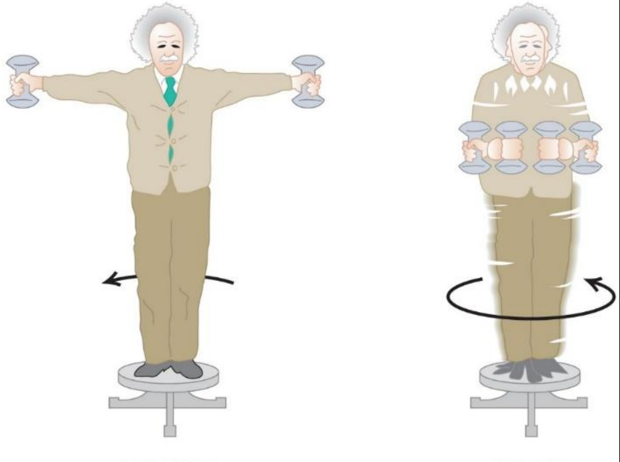
$$L_1 = I_1\omega_1$$

$$L_2 = I_2\omega_2$$

$I_1 > I_2 \longrightarrow \omega_1 < \omega_2$

Conservation of L

Example 10.9:



Given: $I_{body,\,i} = 3.0$ kg•m^2 $I_{body,\,f} = 2.2$ kg•m^2

$m_{dumbbell} = 5.0$ kg each

$R_{dumbbell,\,i} = 1.0$ m $R_{dumbbell,\,f} = 0.20$ m

$\omega_i = 1$ rev in 2 s = $2\pi/2$ rad/s = π rad/s

Find: (a) ω_f (b) compare K_f with K_i

Solution: (a) for ω_f

Total initial moment of inertia

$$I_{total,i} = I_{body,\,i} + 2I_{dumbbell,i} = (3.0\ \text{kg•m}^2) + 2[m_{dumbbell}(R_{dumbbell,\,i})^2] = 13\ \text{kg•m}^2$$

Total final moment of inertia

$$I_{total,f} = I_{body,\,f} + 2I_{dumbbell,f} = (2.2\ \text{kg•m}^2) + 2[m_{dumbbell}(R_{dumbbell,\,f})^2] = 2.6\ \text{kg•m}^2$$

Apply the conservation of angular momentum $I_{total,i}\,\omega_i = I_{total,f}\,\omega_f$

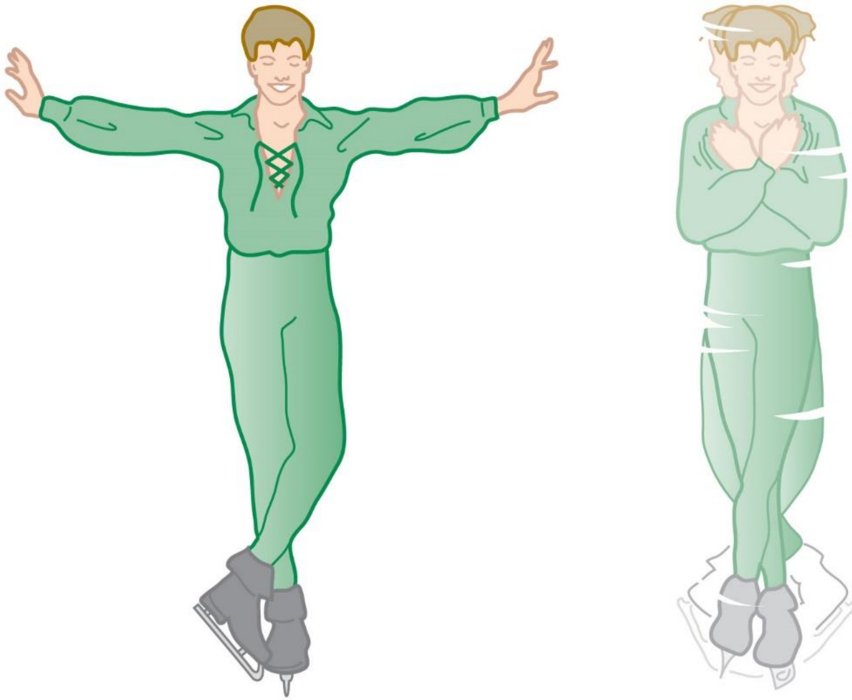
so that $\omega_f = I_{total,i}\,\omega_i / I_{total,f} = 5.0\pi$ rad/s = 2.5 rev/s

(b) compare K_f with K_i

$K_i = (1/2) I_{total,i}\,\omega_i^{\,2} = 64$ J $K_f = (1/2) I_{total,f}\,\omega_f^{\,2} = 320$ J

Q10.12

A spinning figure skater pulls his arms in as he rotates on the ice. As he pulls his arms in, what happens to his angular momentum L and kinetic energy K?



A. L and K both increase.

B. L stays the same; K increases.

C. L increases; K stays the same.

D. L and K both stay the same.

E. None of the above.

10.6 Equilibrium of a Rigid Body

The equilibrium of a point particle is determined by the conditions of

$$\sum F_x = 0 \qquad \text{and} \qquad \sum F_y = 0.$$

The Equilibrium of a Rigid Body

For the equilibrium of a rigid body, both its linear motion and rotational motion must be considered. Therefore, in addition to

$$\sum F_x = 0 \qquad \text{and} \qquad \sum F_y = 0,$$

We must add another condition regarding its rotational motion

$$\sum \tau = 0.$$

This third condition can be set up about any chosen axis.

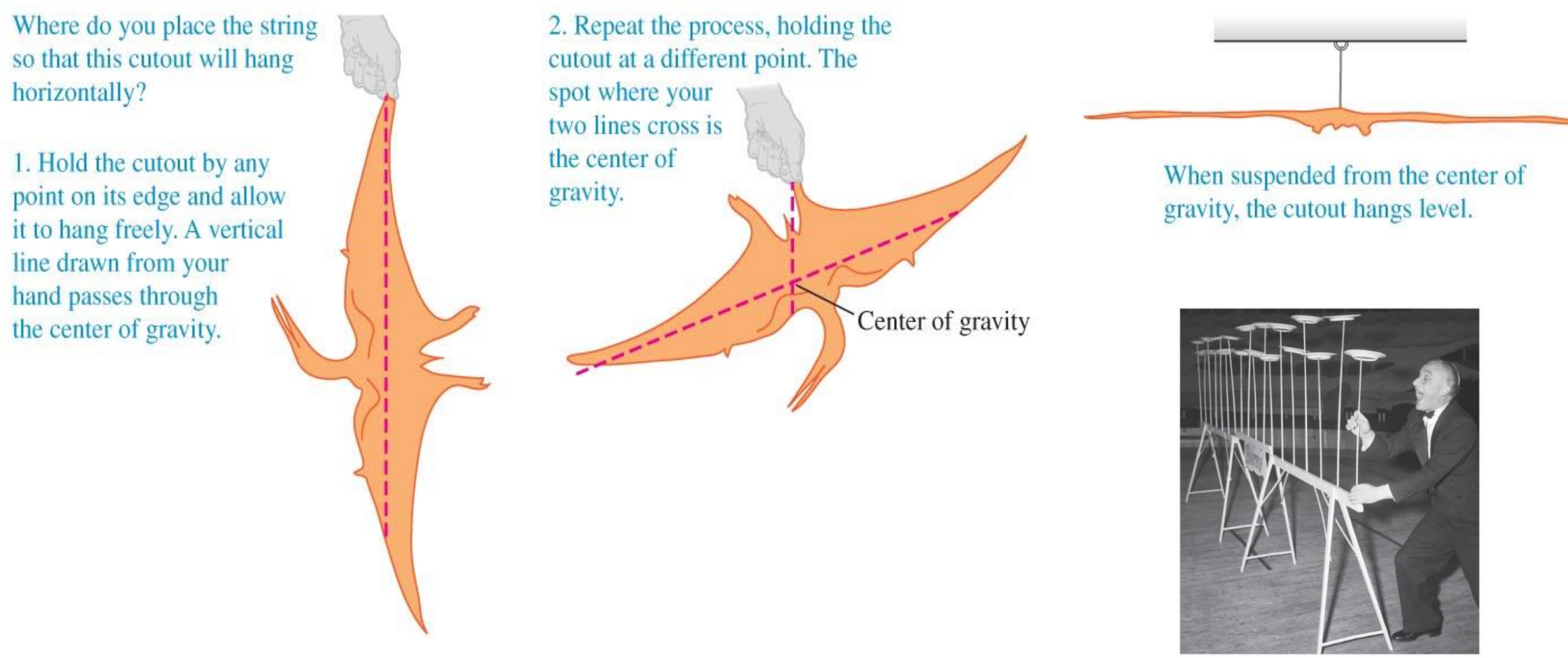
Strategy for Solving Rigid Body Equilibrium Problems

General principle: The net force must be zero.
The net torque about any axis must be zero.

- Draw a diagram according to the physical situation.
- Analyze all the forces acting on each part of a rigid body.
- Sketch all the relevant forces acting on the rigid body.
- Based on the force analysis, set up a most convenient *x*-*y* coordinate system.
- Break each force into components using this coordinates.
- Sum up all the *x*-components of the forces to an equation: $\sum F_x = 0$.
- Sum up all the *y*-components of the forces to an equation: $\sum F_y = 0$.
- Based on the force analysis, set up a most convenient rotational axis.
- Calculate the torque by each force about this rotational axis.
- Sum up all the torques by the forces to an equation: $\sum \tau = 0$.
- Use the above three equations to solve for unknown quantities.

A New Equilibrium Condition – Figure 10.24

- Now, in addition to $\Sigma F_x = 0$ and $\Sigma F_y = 0$, we also must add $\Sigma \tau = 0$.

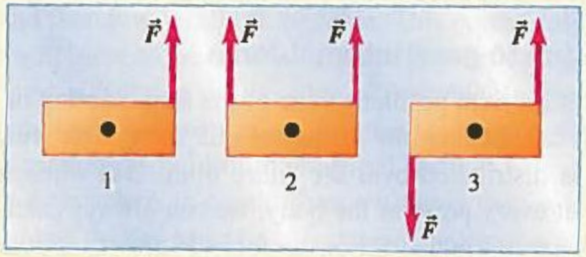


CONCEPTUAL ANALYSIS 10.5

Blocks in equilibrium

Three rectangular blocks rest on a horizontal sheet of ice. The blocks are free to both slide along the ice and rotate about their centers of mass as they slide. Figure 10.25 shows how one or more forces, all with a magnitude of F, act along the short end of each block. Which block is in equilibrium?

A. Block 1
B. Block 2
C. Block 3
D. None of the blocks is in equilibrium.



▲ FIGURE 10.25

SOLUTION To be in equilibrium, both the net force and the net torque must be zero. Because only one force acts at the edge of block 1, neither the net force nor the net torque can be zero. On block 2, the torques produced by the two forces are in opposite directions, so the net torque is zero. Because both forces point in the same direction, however, the net force acting on block 2 cannot be zero. On block 3, the forces point in opposite directions, so the net force is zero. But both forces produce a torque in the counterclockwise direction, so the net torque on block 3 cannot be zero. Therefore, none of the blocks is in equilibrium. The correct answer is D.

Balancing on a Teeter-Totter – Figure 10.26

- The heavier child must sit closer to balance the torque from the smaller child.

Your torque is $\tau_{you} = +(90\ kg)gx$

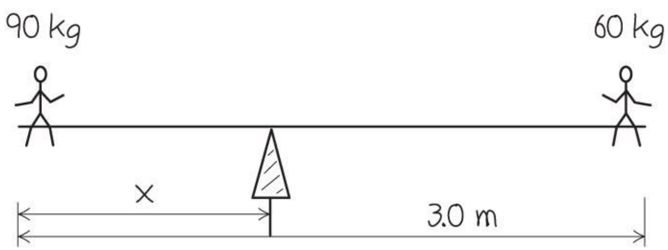
Your friend is $3.0\ \mathrm{m} - x$ from the pivot, and her torque is: $\tau_{friend} = -(60\ kg) \times g(3.0\ \mathrm{m} - x)$.

For equilibrium, $\Sigma\tau = 0$.

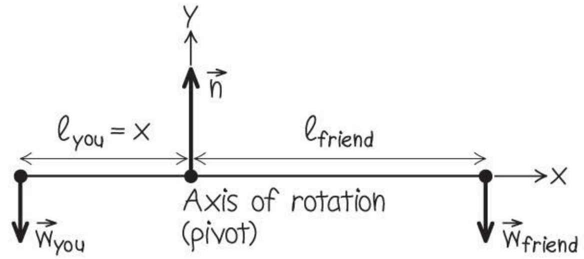
$$\tau_{you} + \tau_{friend} = 0$$

$$+(90\ \mathrm{kg})gx - (60\ \mathrm{kg})g(3.0\ \mathrm{m} - x) = 0$$

$$\boldsymbol{x = 1.2\ \mathrm{m}}$$



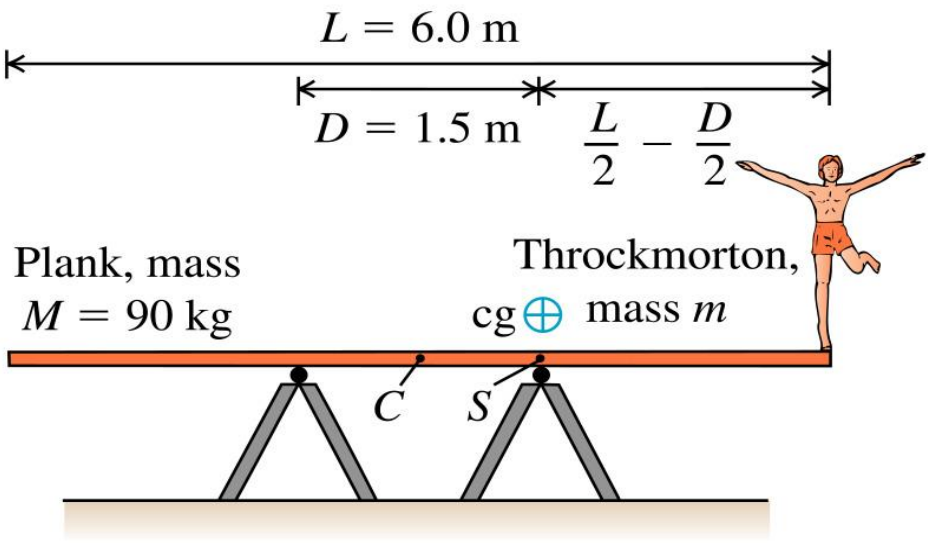
(a) Sketch of physical situation



(b) Free-body diagram

Walking the plank

In equilibrium board does not tilt



Given; $L = 6m, D = 15m, M = 90kg$

How heavy is Mr.Throckmorton?

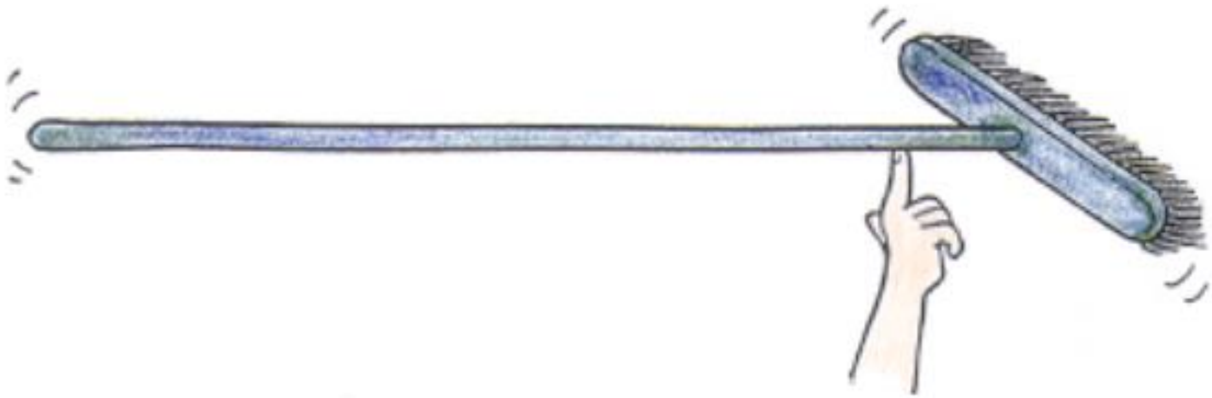
$$\tau_{throckmorton} = m\left(\frac{L}{2} - \frac{D}{2}\right)g$$

$$\tau_{plank} = M\left(\frac{D}{2}\right)g$$

$$\sum \tau_i = 0 \rightarrow m\left(\frac{L}{2} - \frac{D}{2}\right) = M\left(\frac{D}{2}\right)$$

$$m(L - D) = MD \rightarrow m = \frac{M * D}{L - D} = \frac{90 * 1.5}{6 - 1.5} = 30kg$$

NEXT-TIME QUESTION



The broom balances at its center of gravity. If you saw the broom into two parts through the center of gravity, you'll have a "handle part" and a "broom part." If you then weigh each part on a scale, you'll find the part that weighs more will be the

a) handle part.

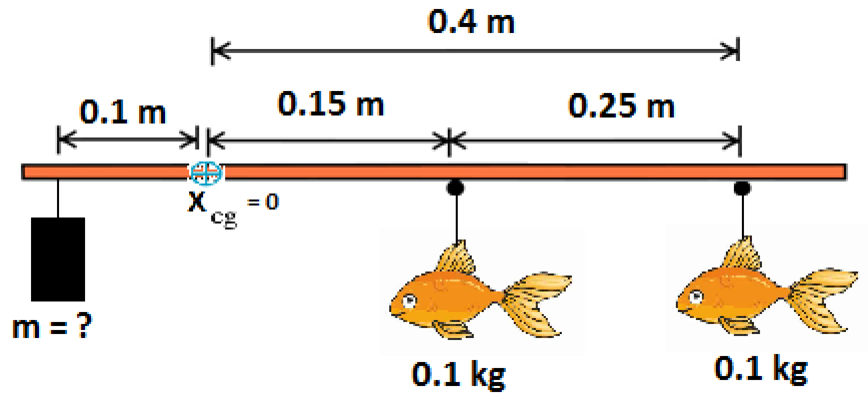
b) broom part.

c) neither, for both will weigh the same.

b)

A fishy wind chime

What is the counter weight of the mobile shown?



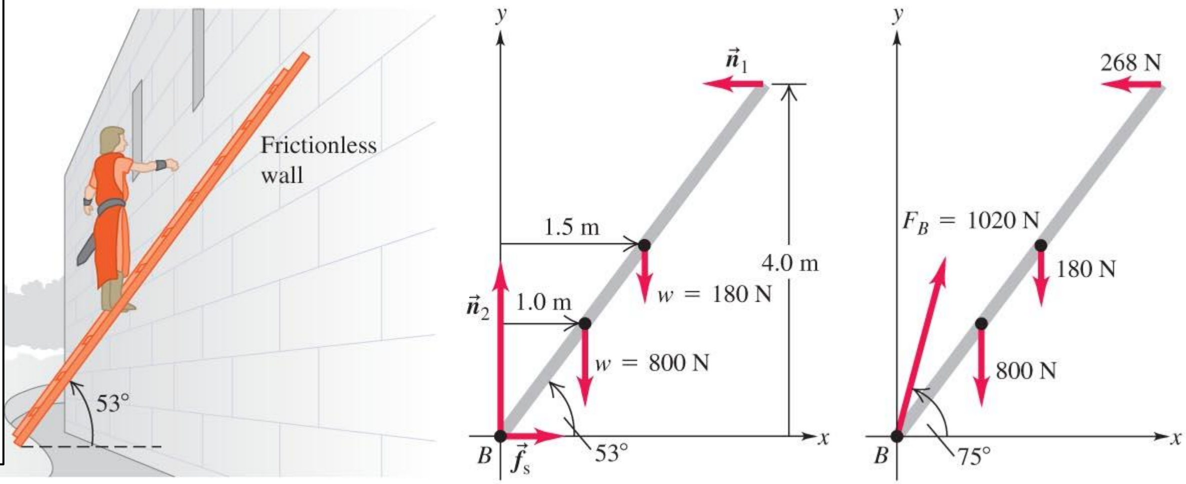
$$0 = X_{cg} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m(-0.1) + (0.1) \cdot (0.15) + (0.1) \cdot (0.4)}{m + 0.1 + 0.1}$$

$$m = \frac{(0.1) \cdot (0.15) + (0.1) \cdot (0.4)}{0.1} = 0.55\ kg$$

The center of gravity must be on the suspension point. So that the sum of torques is equal to zero.

Example 10.12
Climbing a ladder (length is 5 m)

Find:
(a) Normal and friction forces at the base of the ladder
(b) Minimum μ_s at the base
(c) Magnitude and direction of the contact force at the base



(a) (b) (c)

Solution: (a) $\sum F_x = 0$ $\quad f_s - n_1 = 0$

$\sum F_y = 0$ $\quad n_2 - (800\text{ N}) - (180\text{ N}) = 0$ $\quad n_2 = 980\text{ N}$

$\sum \tau = 0$ $\quad n_1(4.0\text{ m}) - (180\text{ N})(1.5\text{ m}) - (800\text{ N})(1.0\text{ m}) = 0$

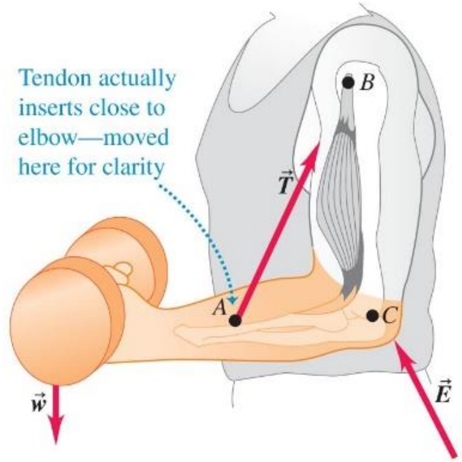
$n_1 = 268\text{ N}$

$f_s = 268\text{ N}$

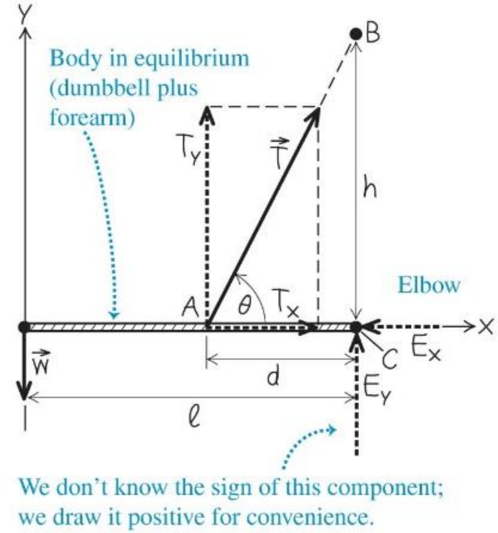
(b) Minimum $\quad \mu_s = (268\text{ N})/(980\text{ N}) = 0.27$

(c) $F_B = \sqrt{(268\ N)^2 + (980\ N)^2} = 1020\ N$ $\quad \theta = tan^{-1}\left(\frac{980\ N}{268\ N}\right) = 75°$

Balanced Forces During Exercise



(a)



(b)

Next we note that if we take torques about the elbow joint (point C), the resulting torque equation does not contain E_x, E_y, or T_x because the lines of action of all these forces pass through this point. The torque equation is then simply

$$\sum \tau_C = lw - dT_y = 0.$$

From this equation, we find that

$$T_y = \frac{lw}{d} \quad \text{and} \quad T = \frac{lw}{d \sin\theta}.$$

To find E_x and E_y, we could now use the first condition for equilibrium: $\sum F_x = 0$ and $\sum F_y = 0$. Instead, for added practice in using torques, we take torques about the point A where the tendon is attached:

$$(l - d)w + dE_y = 0 \quad \text{and} \quad E_y = -\frac{(l-d)w}{d}.$$

The negative sign shows that our initial guess for the direction of E_y was wrong; it is actually vertically *downward*.

Finally, we take torques about point B in the figure and remember to use the shortest perpendicular distance to the vector $\vec{w}$ to simplify our calculations:

$$\sum \tau_B = lw - hE_x = 0 \quad \text{and} \quad E_x = \frac{lw}{h}.$$

Evaluating our expressions for $w = 50\text{ N}$, $d = 0.10\text{ m}$, $l = 0.50\text{ m}$, and $\theta = 80°$, we get $\tan\theta = h/d$, and we find $h = d\tan\theta = (0.10\text{ m})(5.67) = 0.57\text{ m}$. We then have

$$T = \frac{(0.50\text{ m})(50\text{ N})}{(0.10\text{ m})(0.98)} = 250\text{ N},$$

$$E_y = -\frac{(0.50\text{ m} - 0.1\ldots}{0.10}$$

$$E_x = \frac{(0.50\text{ m})(50\text{ N})}{0.57\text{ m}} = 44\text{ N}.$$

The magnitude of the force at the elbow is

$$E = \sqrt{E_x{}^2 + E_y{}^2} = 200\text{ N}.$$

As we mentioned earlier, we have not explicitly used the first condition for equilibrium—that the vector sum of the forces is zero. To check our answer, we can compute $\sum F_x$ and $\sum F_y$ to verify that they really *are* zero. Such checks help verify internal consistency. Checking, we obtain

$$\sum F_x = E_x - T_x = 44\text{ N} - (250\text{ N})\cos 80° = 0,$$
$$\sum F_y = E_y + T_y - w$$
$$= -200\text{ N} + (250\text{ N})\sin 80° - 50\text{ N} = 0.$$

REFLECT Notice how much we have simplified these calculations by using a little ingenuity in choosing the point for calculating torques so as to eliminate one or more of the unknown quantities. In the last step, the force T has no torque about point B; thus, when the torques of T_x and T_y are computed separately, they must add to zero.

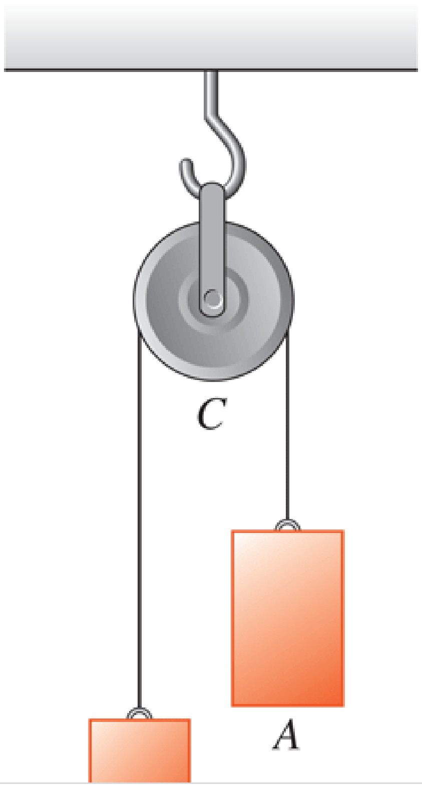
Practice Problem: Double the weight of the dumbbell. What is the new tension T in the tendon connected to the biceps muscle at point B? *Answer:* 510 N.

Refer to the worked example on page 308.

Atwood's machine problem 10.58

58. II Atwood's machine. Figure 10.77 illustrates an Atwood's machine. Find the linear accelerations of blocks A and B, the angular acceleration of the wheel C, and the tension in each side of the cord if there is no slipping between the cord and the surface of the wheel. Let the masses of blocks A and B be 4.00 kg and 2.00 kg, respectively, the moment of inertia of the wheel about its axis be $0.300\ \mathrm{kg \cdot m^2}$, and the radius of the wheel be 0.120 m.

Figure 10.77



Atwood's machine problem 10.58

10.58. Set Up: A accelerates downward, B accelerates upward and the wheel turns clockwise. Apply $\sum F_y = ma_y$ to blocks A and B. Let $+y$ be downward for A and y be upward for B. Apply $\sum \tau = I\alpha$ to the wheel, with the clockwise sense of rotation positive. Each block has the same magnitude of acceleration, a, and $a = R\alpha$. Call the tension in the cord between C and $A\,T_A$ and the tension between C and $B\,T_B$.

Solve: For A, $\sum F_y = ma_y$ gives $m_A g - T_A = m_A a$. For B, $\sum F_y = ma_y$ gives $T_B - m_B g = m_B a$. For the wheel, $T_A R - T_B R = I\alpha = I(a/R)$ and $T_A - T_B = \left(\frac{I}{R^2}\right)a$. Adding these three equations gives

$$(m_A - m_B)g = \left(m_A + m_B + \frac{I}{R^2}\right)a$$

$$a = \left(\frac{m_A - m_B}{m_A + m_B + I/R^2}\right)g = \left(\frac{4.00\text{ kg} - 2.00\text{ kg}}{4.00\text{ kg} + 2.00\text{ kg} + (0.300\text{ kg})/(0.120\text{ m})^2}\right)(9.80\text{ m/s}^2) = 0.730\text{ m/s}^2$$

$$\alpha = \frac{a}{R} = \frac{0.730\text{ m/s}^2}{0.120\text{ m}} = 6.08\text{ rad/s}^2$$

$$T_A = m_A(g - a) = (4.00\text{ kg})(9.80\text{ m/s}^2 - 0.730\text{ m/s}^2) = 36.3\text{ N}$$

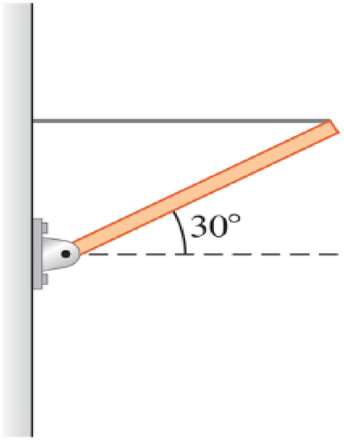
$$T_B = m_B(g + a) = (2.00\text{ kg})(9.80\text{ m/s}^2 + 0.730\text{ m/s}^2) = 21.1\text{ N}$$

Reflect: The tensions must be different in order to produce a torque that accelerates the wheel when the blocks accelerate.

A simple Hinge problem 10.56

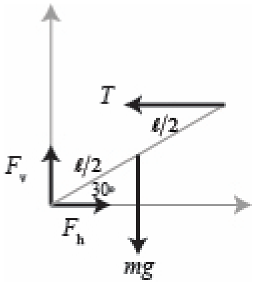
56. **I** One end of a 1.2-m-long beam is hinged to a vertical wall, and the other end is held up by a thin wire as shown in Figure 10.75. The wire will break if its tension exceeds 1000 N. What is the maximum mass that the beam can have and still be supported by the wire?

Figure 10.75



Problem 56.

10.56. Set Up: Apply $\sum \tau = 0$ to the beam. Denote the length of the beam as l. The free-body diagram of the beam is shown below. Consider the torques about the point where the beam joins the wall.

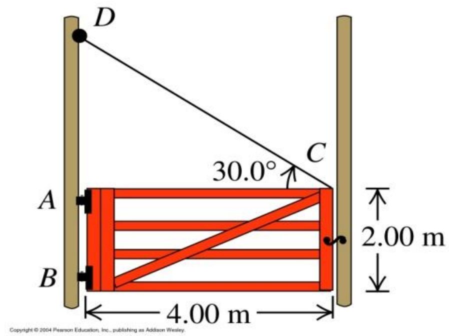


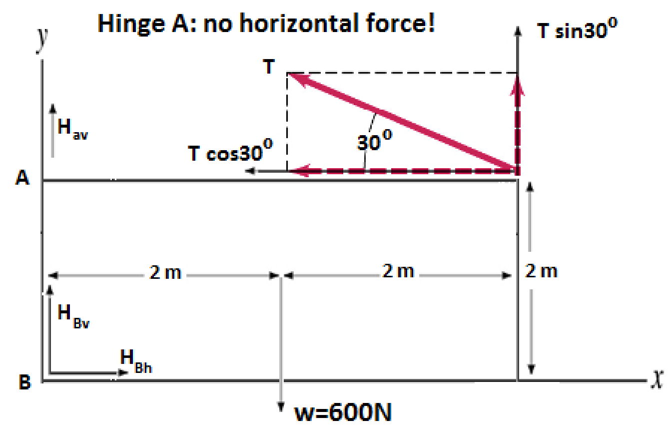
Solve: **(a)** $\sum \tau = 0$ gives $mg\frac{l}{2}\cos 30° - Tl\sin 30° = 0$. If the maximum permissible tension is T = 1000 N, the maximum mass of the beam is $m = \frac{2T}{g}\tan 30° = \frac{2(1000\text{ N})}{9.8\text{ m/s}^2}\tan 30° = 1.8\times 10^2$ kg

Reflect: Note that the length of the beam does not enter into the calculation.

Hanging a farm gate

Example 11-61





a)What is the tension in wire CD?
b)What is horizontal force on hinge B?
c) What is combined vertical force on hinges A and B?

Use coordinates with the origin at B. Let $\overrightarrow{H_A}$ and $\overrightarrow{H_B}$ exerted by the hinges A and B. The problem states that $\overrightarrow{H_A}$ has no horizontal component. Replace the tension $\vec{T}$ by its horizontal and vertical components.

a) $\sum \zeta_B = 0 \;\rightarrow +(T\,sin\,30)(4m) + (T\cos 30)(2m) - w(2m) = 0$

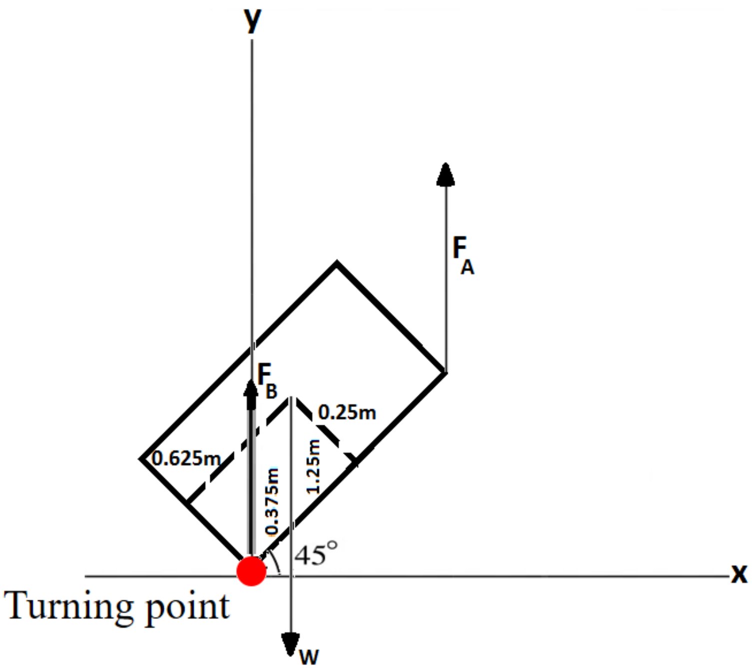
$$T(2\sin 30 + \cos 30) = w \;\rightarrow T = \frac{w}{(2\sin 30 + \cos 30)} = 322N$$

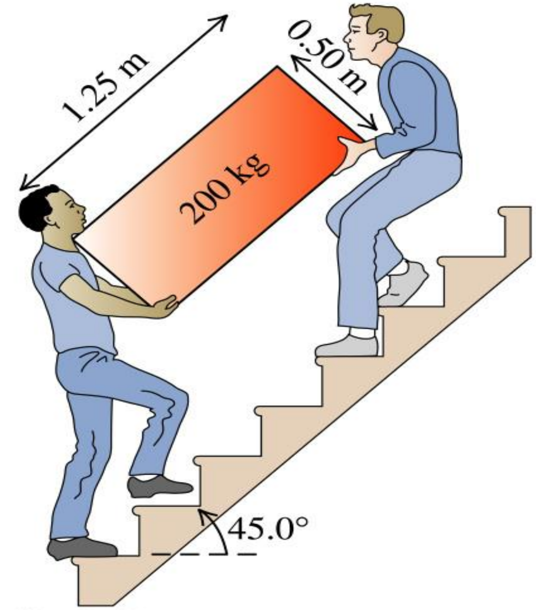
b) $\sum F_x = ma_x \;\rightarrow\; H_{Bh} - T\cos 30 = 0 \rightarrow\; H_{Bh} = T\cos 30 = (322N)\cos 30 = 279N$

c) $\sum F_y = ma_y \;\rightarrow\; H_{av} + H_{Bv} + T\sin 30 - w = 0$

$$\rightarrow H_{av} + H_{Bv} = w - T\sin 30 = 600N - (322N)sin30 = 439N$$

Carrying a box up the stairs





$$W = mg = 200 * 9.8 = 1960N$$

$$\sum \zeta = 0 \rightarrow F_A(1.25m)(cos45) - W(0.375m)(cos45) = 0$$

$$\rightarrow F_A = \frac{0.375}{1.25} W = 588N$$

$$\sum F_y = 0 \rightarrow F_A + F_B - W = 0 \rightarrow F_B = W - F_A = (1960 - 588) = 1372N$$

The person B below applies more than twice the force of A

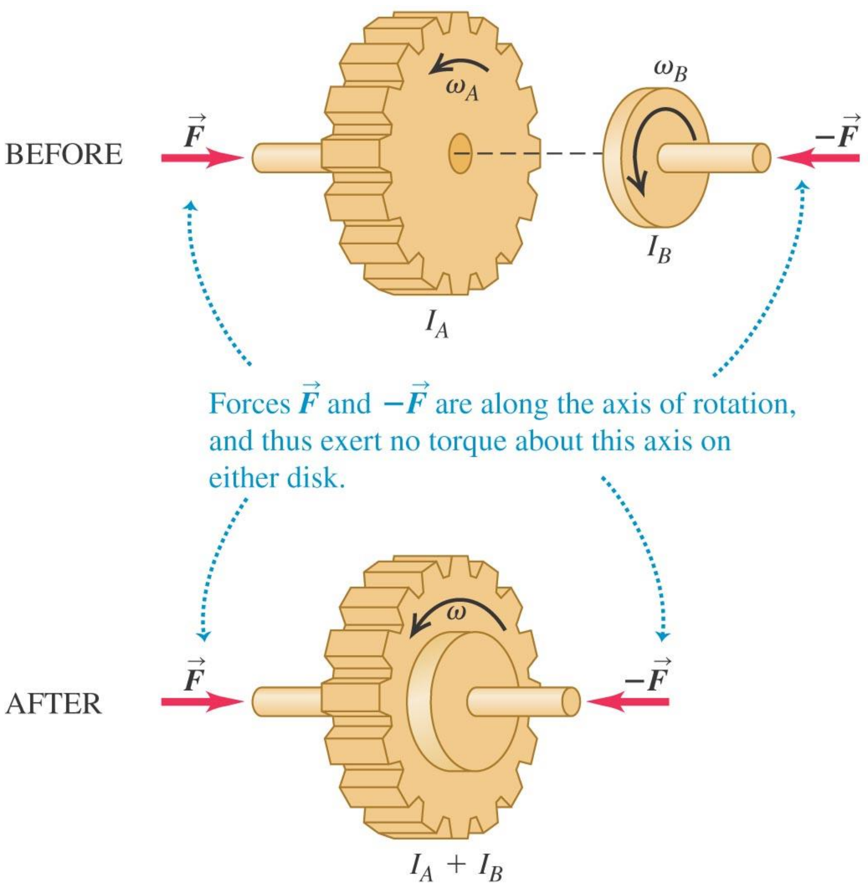
How a car’s clutch work

The clutch disk and the gear disk is pushed into each other by two forces that do not impart any torque, what is the final angular velocity when they come together?

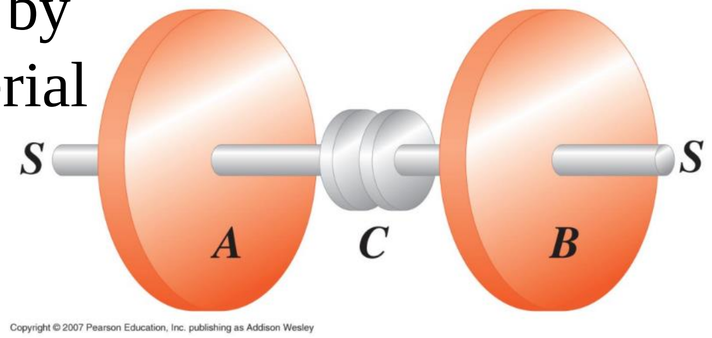
$$L_{z-before} = L_{z-after}$$

$$I_A \omega_A + I_B \omega_B = (I_A + I_B)\omega_{final}$$

$$\Rightarrow \quad \omega_{final} = \frac{I_A \omega_A + I_B \omega_B}{(I_A + I_B)}$$



Disk A and B are connected or disconnected by clutch C. Disk A is made from a lighter material



Problem 10. 66

. What is the original kinetic energy $\boldsymbol{K_i}$ of disk $\boldsymbol{A}$?

$$I_A = \frac{1}{3} I_B \rightarrow I_B = 3I_A$$

Initially;

$$L_i = w_o I_A \text{ and } K_i = \frac{1}{2} I_A w_o^2$$

Final;

$$I = I_A + 3I_A = 4I_A \text{ (after coupling together)}$$

Conservation of angular momentum;

$$L_i = w_o I_A = L_f = WI = W4I_A \rightarrow W = \frac{w_o}{4}$$

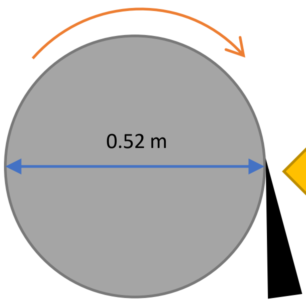
Final Kinetic energy;

$$K_f = \frac{1}{2} I W^2 = \frac{4}{2} I_A \frac{w_o^2}{4^2} = \frac{1}{4} K_i$$

$$\therefore \Delta K = K_f - K_i = -\frac{3}{4} K_i = -2400\, J \text{ (thermal energy developed during coupling)}$$

So, original energy;

$$K_i = -\frac{3}{4}(-2400) = 3200\, J$$



If the grindstone comes to a complete stop after 7.5s, what was the coefficient of kinetic friction between the axe and the stone? (Assume negligible friction in the rest of the system)

$R = 0.260\ m$, $m = 50.0\ kg$

$f_k = \mu_k n$, $n = 160\ N$

Apply $\sum \tau = I\alpha$

$\sum \tau = -f_k R$

$I = \frac{1}{2} mR^2$ (solid disk)

Use constant angular acceleration kinematic equations to find α

$\alpha = ?$

$\omega_0 = +850\ rev/min \left(\frac{2\pi\ rad}{1\ rev}\right)\left(\frac{1\ min}{60\ s}\right) = 89.0\ rad/s$

($\sum \tau = I\alpha$ requires that α be in rad/s^2)

$t = 7.50\ s$

$\omega = 0$

$\omega = \omega_0 + \alpha t$

$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 89.0\ rad/s}{7.50\ s} = -11.9\ rad/s^2$

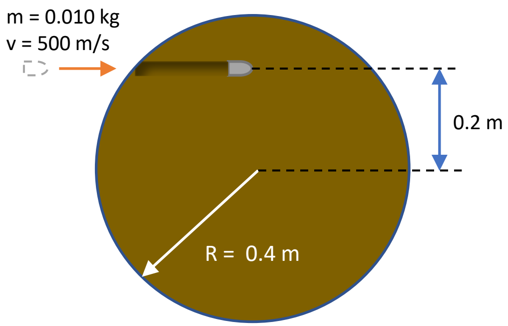
$\sum \tau = I\alpha$

$-f_k R = \frac{1}{2} mR^2 \alpha$

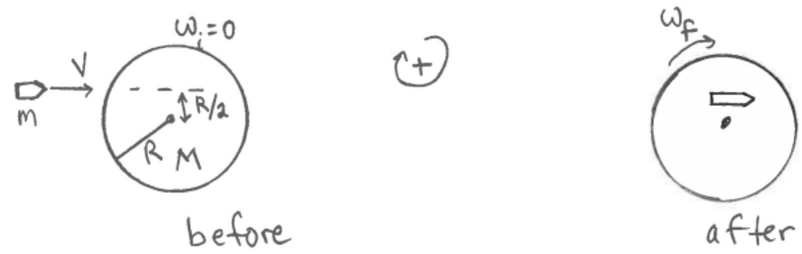
$f_k = -\frac{1}{2} mR\alpha = -\frac{1}{2}(50\ kg)(0.260\ m)(-11.9\ rad/s^2)$

$f_k = 77.4\ N$

$\mu_k = \frac{f_k}{n} = \frac{77.4N}{160\ N} = \underline{0.483}$



A bullet is shot into a solid disk of wood at rest as shown. It embeds itself in the disk, a distance of ½ R above the disk's center. What will the resulting angular velocity of the disk?



$v = 500\ m/s \quad m = 0.010\ kg \quad M = 2\ kg \quad R = 0.4\ m$

For the disk $I = \frac{1}{2}MR^2 = \frac{1}{2}(2kg)(0.4m)^2 = 0.16\ kg \cdot m^2$

No external torques so $L_i = L_f$

$L_i = mvl$ with $l = R/2 = 0.2\ m$

$L_f = I_f \omega_f$

$I_f = I_{disk} + I_{bullet} = 0.16\ kg \cdot m^2 + m(R/2)^2$

$I_f = 0.16\ kg \cdot m^2 + (0.010\ kg)(0.2m)^2 = 0.1604\ kg \cdot m^2$

$mvl = I_f \omega_f$

$$\omega_f = \frac{mvl}{I_f} = \frac{(0.010kg)(500\ m/s)(0.2m)}{0.1604\ kg \cdot m^2}$$

$\omega_f = 6.23\ rad/s$

Comments: In $L = I\omega$, ω must be in rad/s

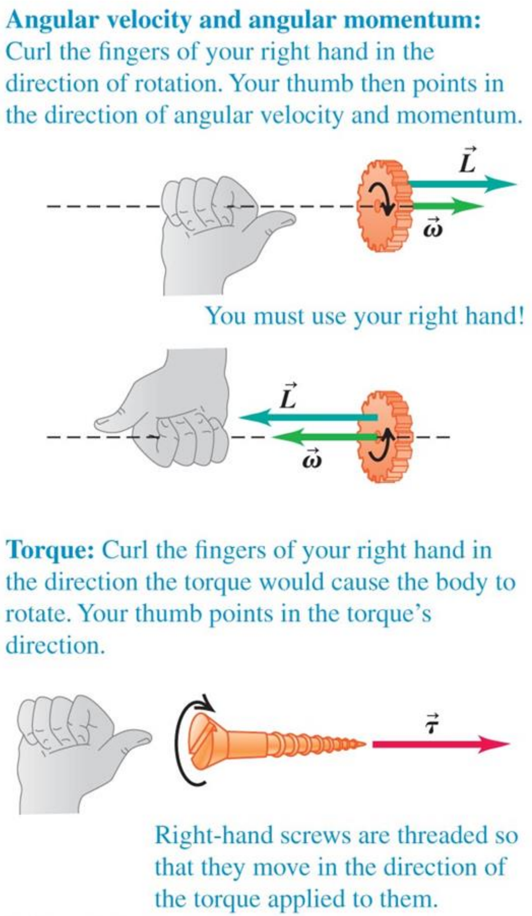
$K_i = \frac{1}{2}mv^2 = \frac{1}{2}(0.01\ kg)(500\ m/s)^2 = 1250\ J$

$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(0.1604\ kg \cdot m^2)(6.23\ rad/s)^2 = 0.194\ J$

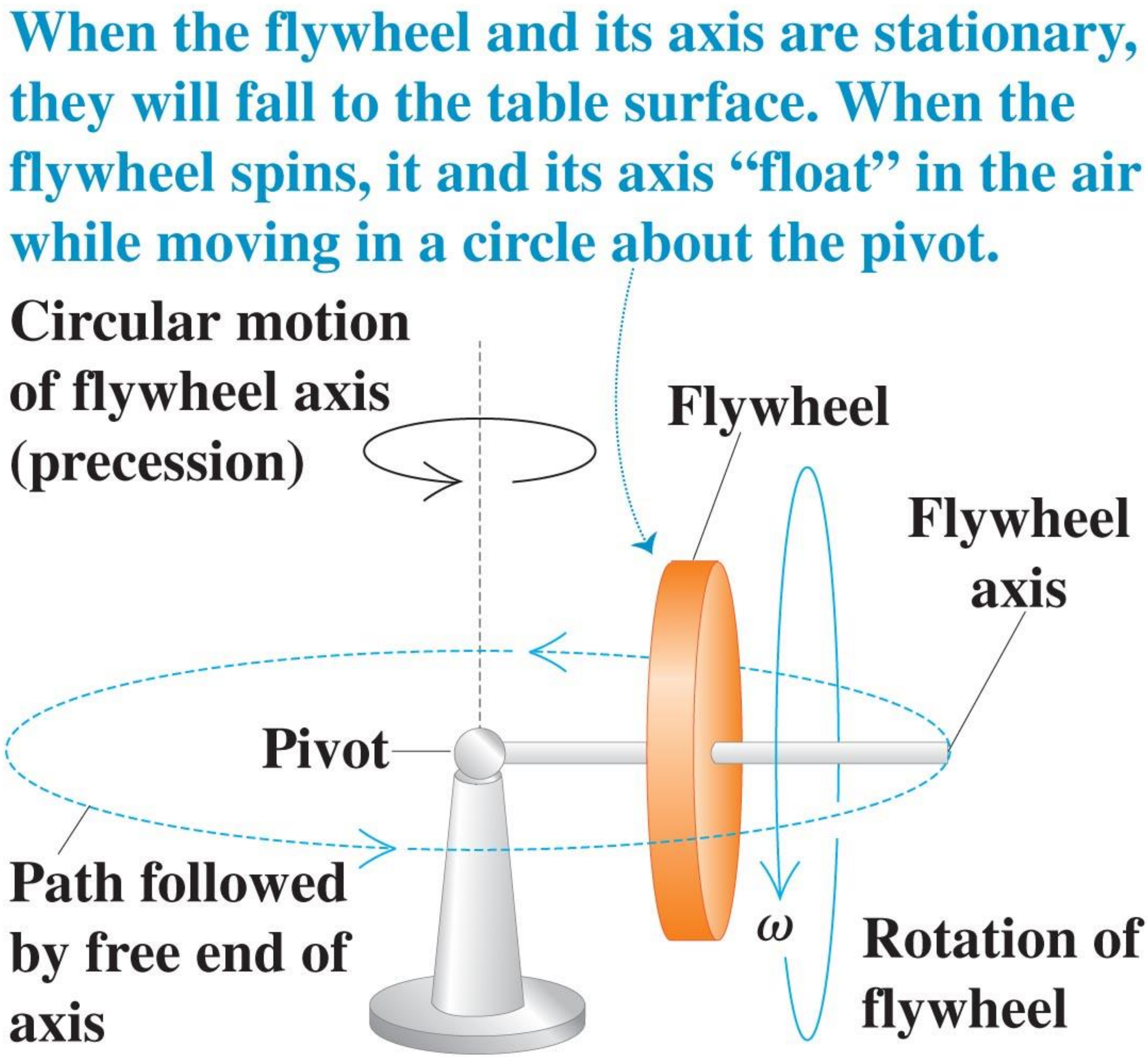
There is a large decrease in kinetic energy as a result of the collision

Angular Quantities Are Vectors – Figure 10.29

- The “right-hand rule” gives us a vector’s direction.

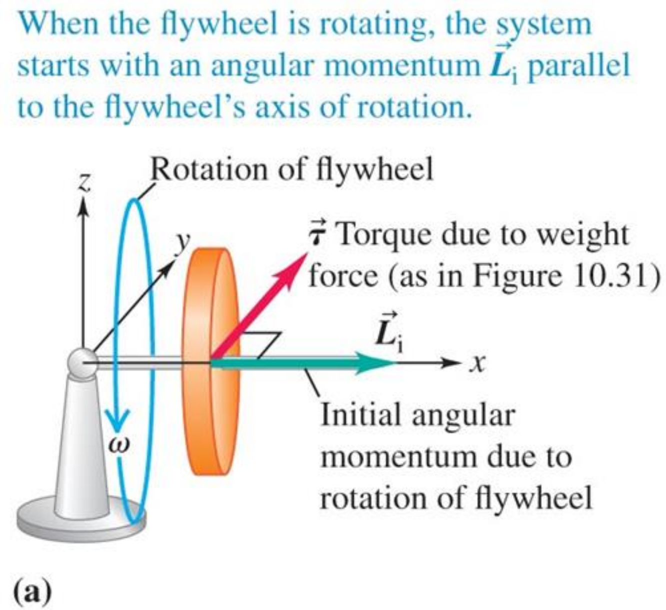


Vector nature of angular quantities

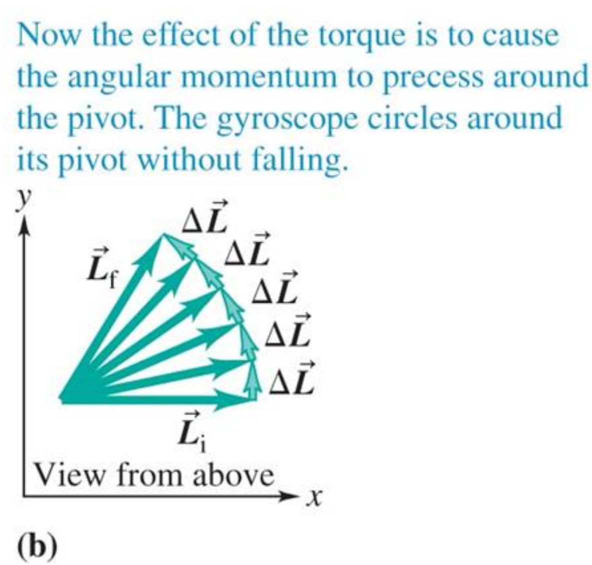


Gyroscopes Can Add Stability Figure 10.32

- The gyroscopic motion adds stability to bicycles, footballs, bullets and more.



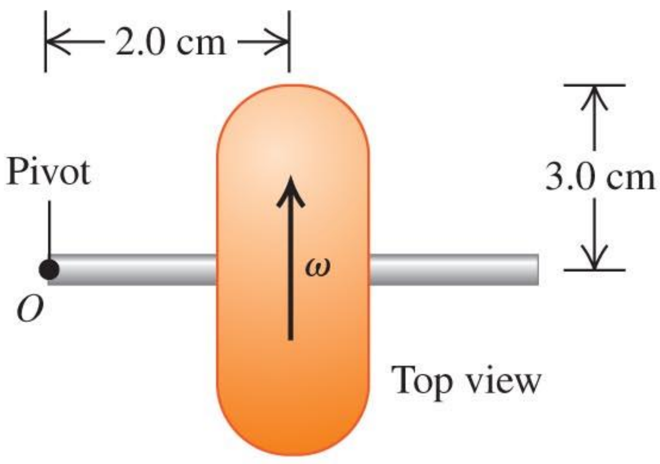
(a)



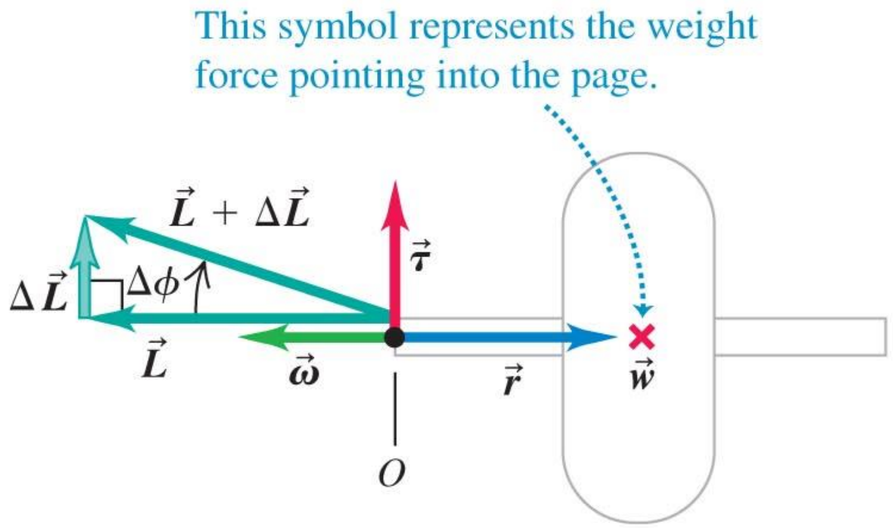
(b)

A Laboratory Gyroscope – Figure 10.33

- Refer to the worked example on page 309.



(a) Top view of spinning cylindrical gyroscope wheel



(b) Vector diagram

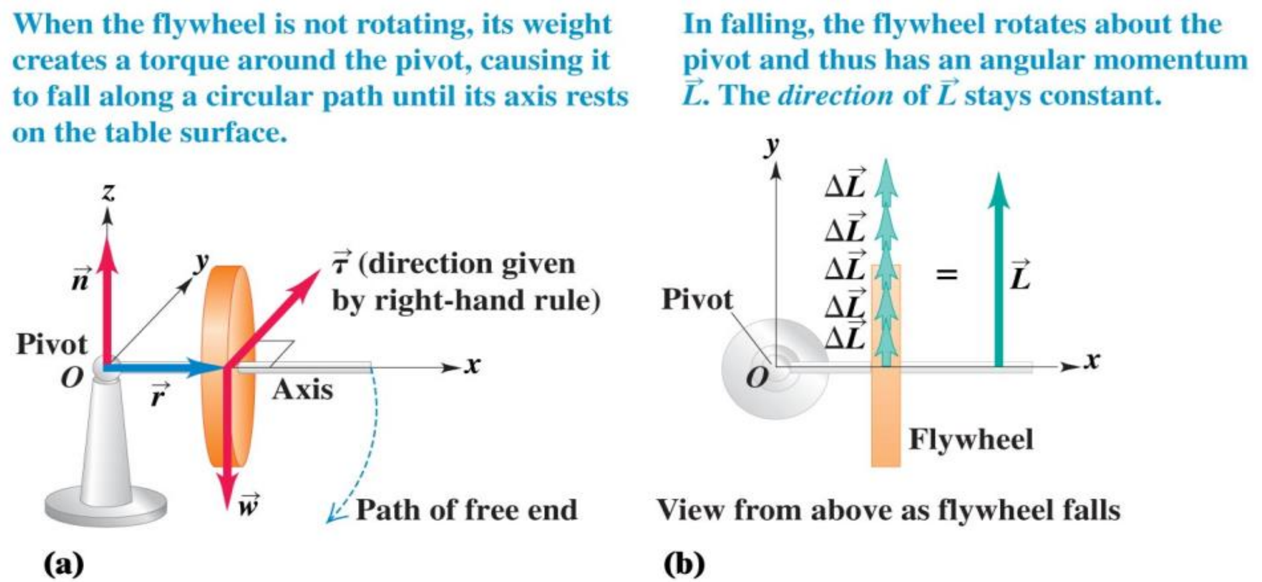
Honda 600RR

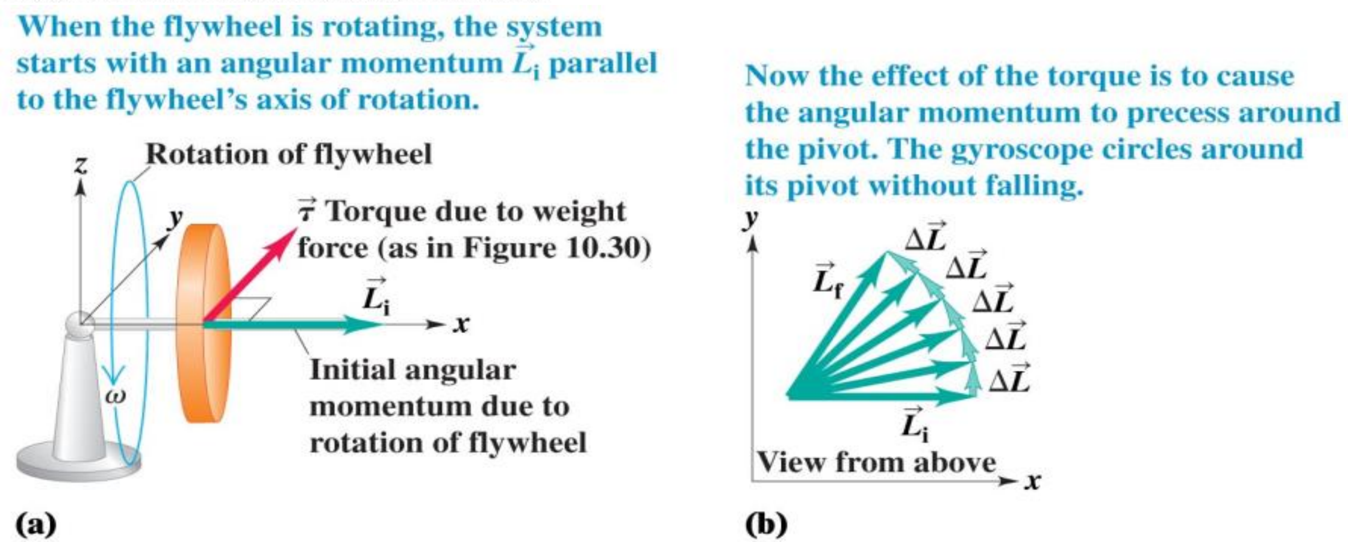
Who races this bike?

Why can anybody race it, if he just dares to go fast?

The oval track of a World Speedway allows speeds of 250 mph.

A non rotating and rotating gyroscope





Precesion Angular Velocity Ω

Ω is the rate $\frac{\Delta\phi}{\Delta t}$ at which the axis moves

$$\Delta\phi = \frac{|\Delta\vec{L}|}{|L|}$$

$$\vec{L} = I\,\vec{\omega}$$

$$\Delta\vec{L} = \sum \vec{\tau}\,\Delta t$$

$$\sum\vec{\tau} = \frac{\Delta\vec{L}}{\Delta t}$$ Analogous to $$\sum\vec{F} = \frac{\Delta\vec{P}}{\Delta t}$$

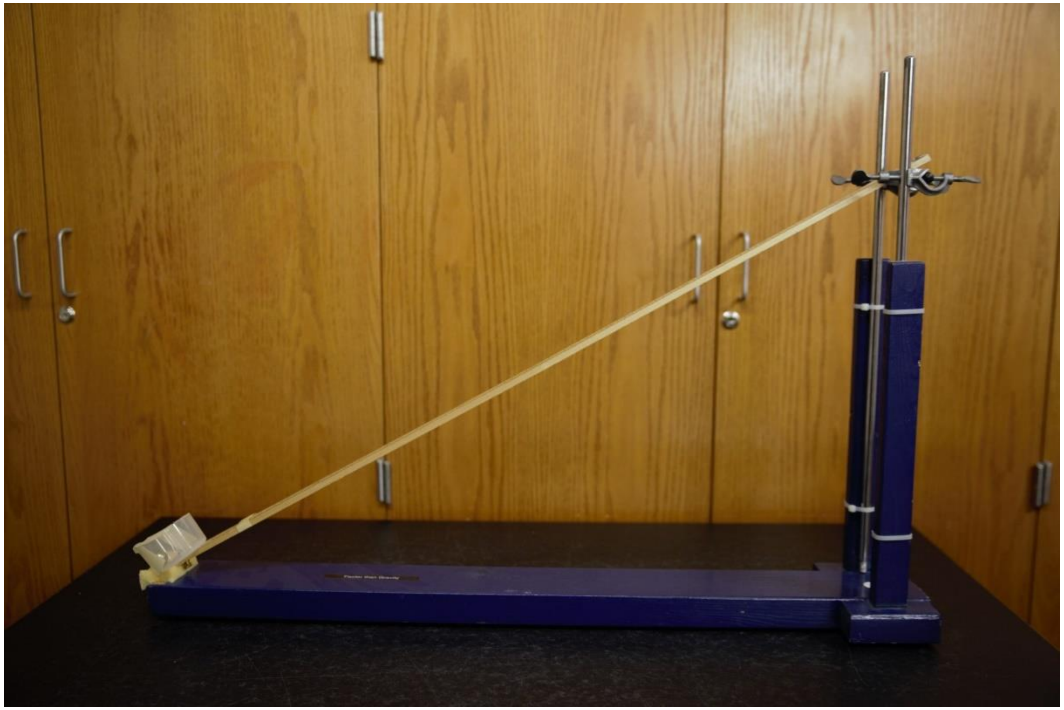
$$\Delta\phi = \frac{|\Delta\vec{L}|}{|L|}$$

$$\Omega = \frac{\Delta\phi}{\Delta t} = \frac{\frac{|\Delta\vec{L}|}{|L|}}{\Delta t} = \frac{\tau}{L} = \frac{Wr}{I\omega}\ \text{rad/s}$$

W is the weight of the flywheel, and r is the distance from the pivot point of the center of mass of the flywheel.


HONDA

Faster than Gravity



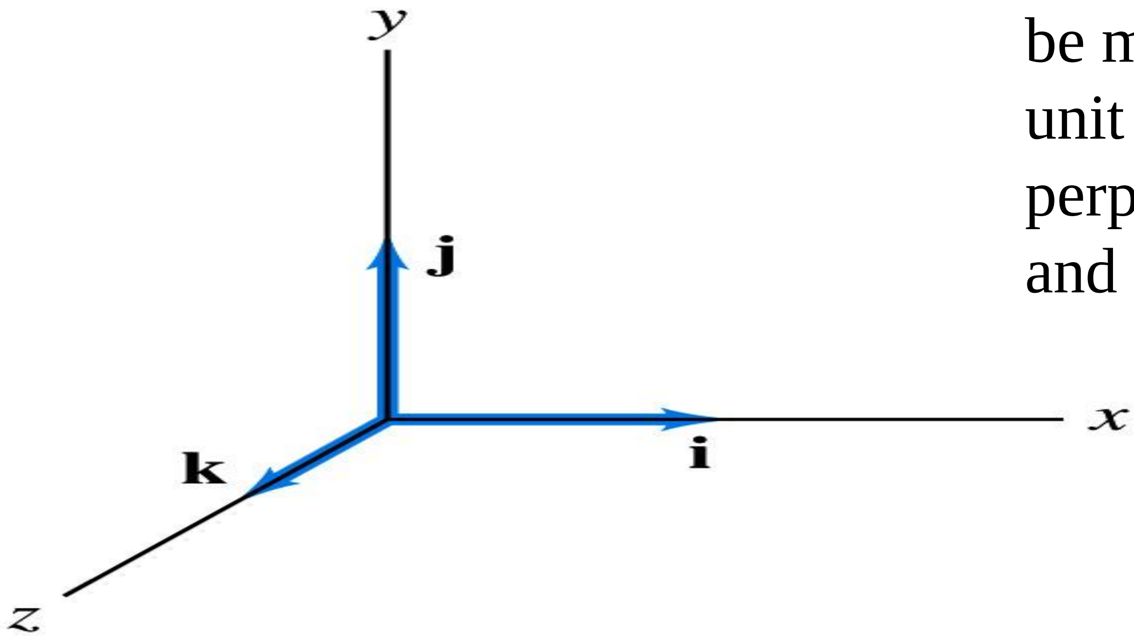
A small marble is placed on the tee at the very top of the raised end. A few inches below it is a plastic cup. The wooden board is held up by a wooden ruler. The ruler is quickly removed, allowing the board and the marble to fall due to the force of gravity. The center of percussion of the board is the point that has the acceleration of a free falling particle along the path that it follows, all points beyond the center of percussion descend with accelerations greater than they would have if they were particles moving freely on their respective paths. This is due to the board rotating about a hinged end. Thus, the cup falls at a faster rate than the marble, and the marble lands in the cup.

Vector Multiplication

Scalar product $\vec{A}\cdot\vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$

Vector product $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta = \left|\vec{C}\right|$

Magnitude C must be multiplied with a unit vector perpendicular to A and B



$$\vec{A}\times\vec{B} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{pmatrix} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

Next-Time Question

Suppose that the height of a rapidly-growing beanstalk on Earth doubles each day, and in 36 days reaches the Moon. The number of days required to reach halfway to the Moon would be

a) 18 days.

b) 27 days.

c) 35 days.

d) None of these.