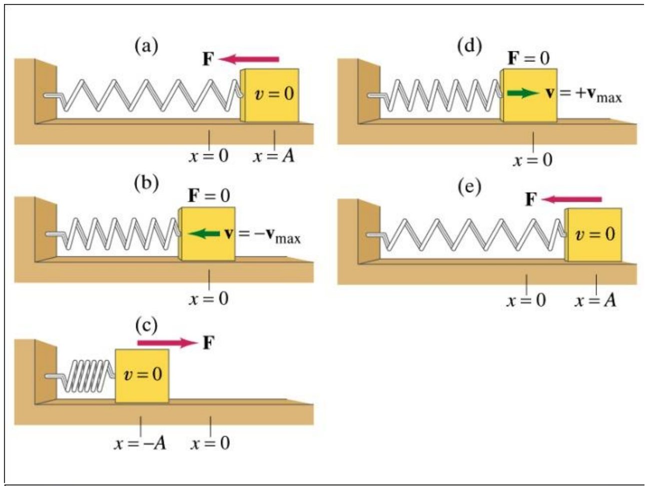
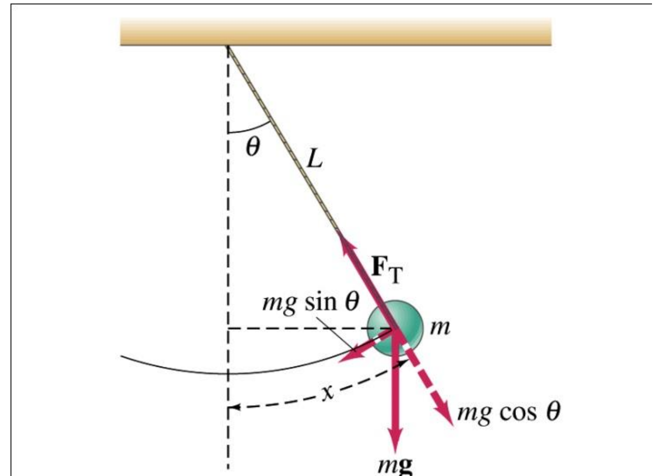
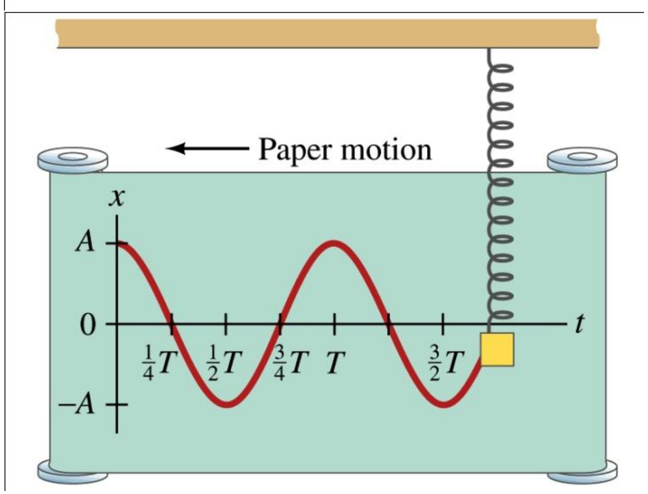
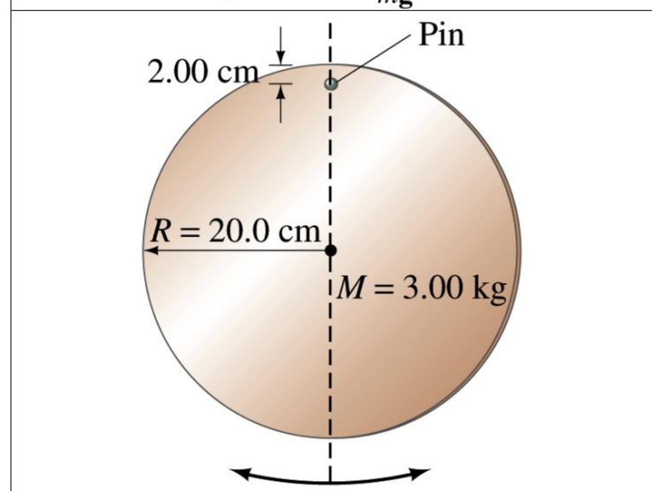
Chapter 11: Elasticity and Periodic Motion









Skiing with Harmonic motion in the German Alps

Chapter 11 Elasticity and Periodic Motion

- To understand stress, strain, and elastic deformation.
- To understand elasticity and plasticity.
- To understand simple harmonic motion (SHM).
- To solve equations of simple harmonic motion.
- To understand the pendulum as a an example of SHM.

Hooke's Law

5.4 Elastic Force
Spring, Spring Restoring Force, and Hooke's Law

Hooke's Law on spring restoring force:

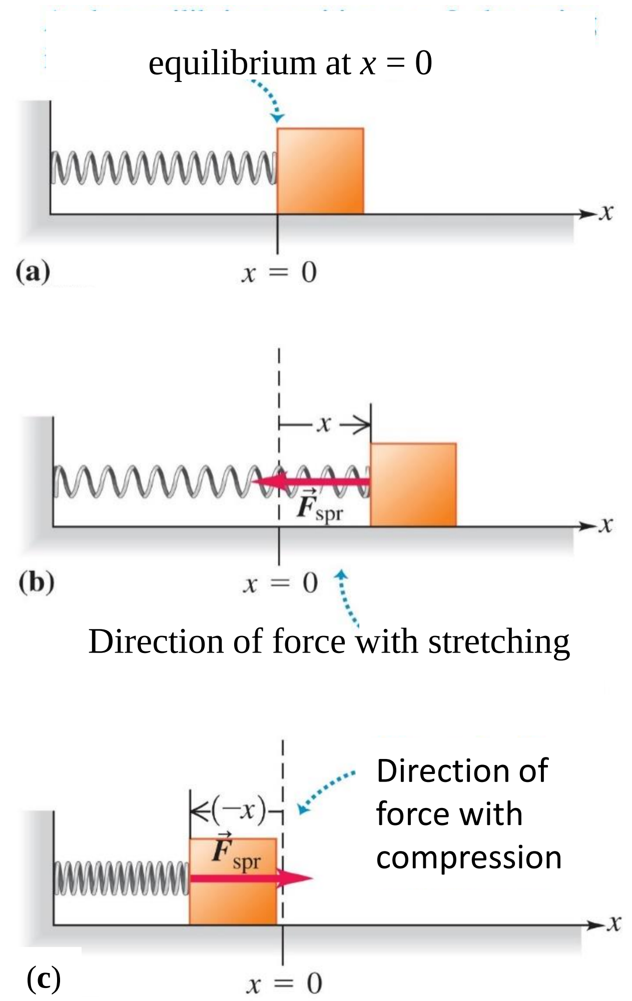
- In magnitude $F_{spr} = k\Delta L$
- Direction: Opposed to length change → restoring force
- A general expression taking care of the direction

$$F_{spr} = -kx$$

- x is measured with respect to the equilibrium position

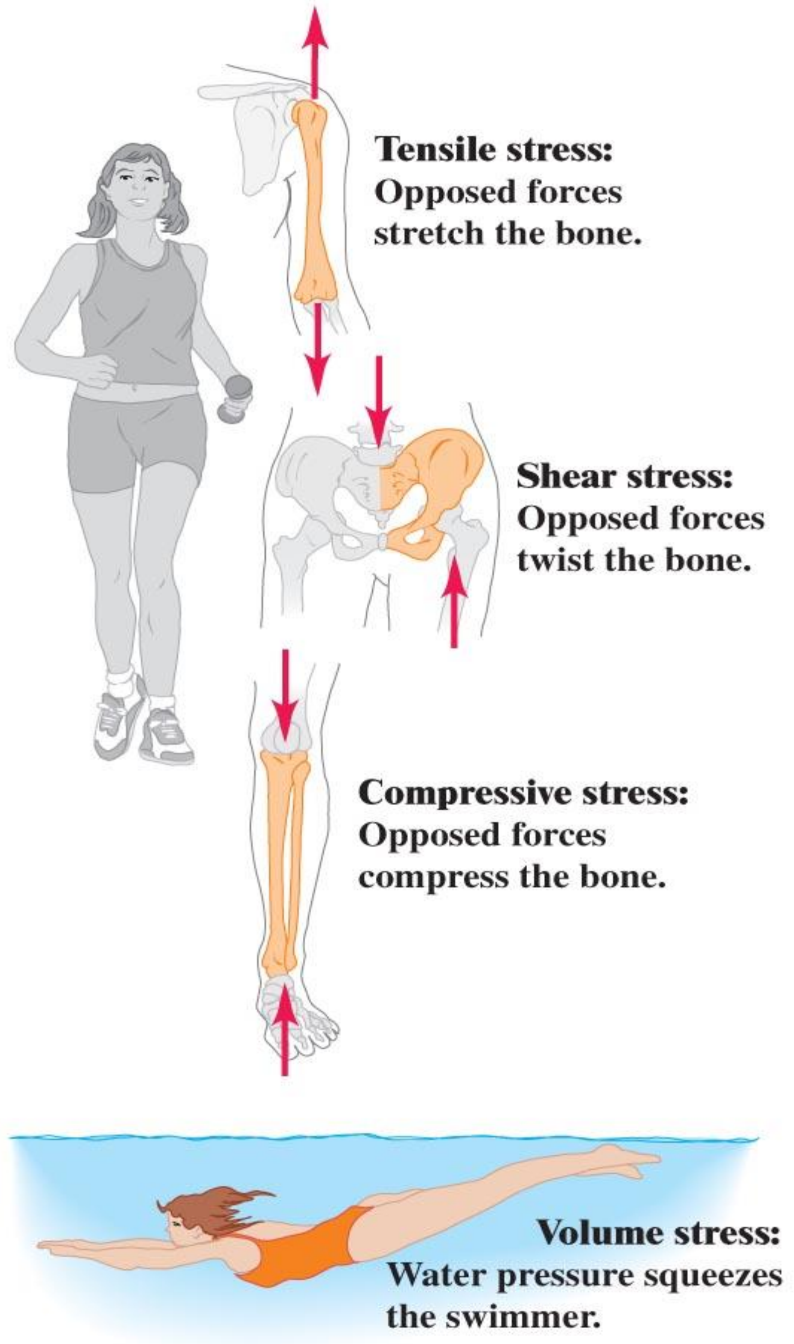
Example 5.14
A vertical spring balance scale stretches 1.00 cm when a 12.0 N weight is hung on it. If the 12.0 N weight is replaced by a 1.50 kg fish, by what amount is the spring stretched?

Answer:
Spring constant $k = F/\Delta L$ = (12.0 N)/(0.0100 m) = 1200 N/m
Stretching = $F/k = mg/k$ = 0.0123 m = 1.23 cm



We did this for springs in chapter 5, now for general objects

Stress and strain



11.1 Stress, Strain, and Elastic Deformation

What is stress?

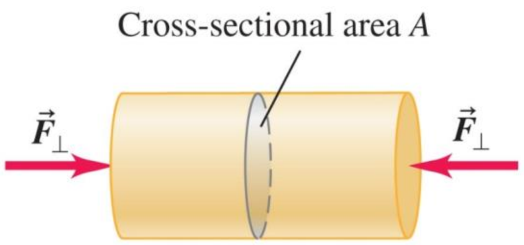
The "intensity" of the force exerting on an object quantified by the force per unit area.

What is strain?

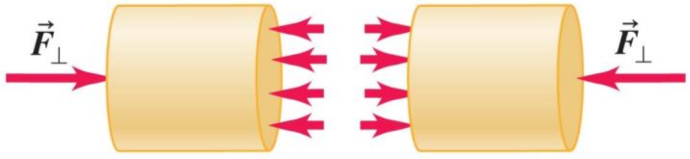
The amount of relative deformation appears to an object under the given stress.

The relationship between the two----If the stress is small, the resultant strain is proportional to the stress:

$$\frac{Stress}{Strain} = constant$$



(a) A bar in compression



(b) Force on a cross section through the bar

Tensile and Compressive Stress and Strain

Tensile and compressive stress

$$\text{Tensile stress} = \frac{F_{\perp}}{A}$$

Units: N/m² or pascal (Pa), in SI unit system
psi or pounds per square inches, in the British units

Tensile and compressive strain

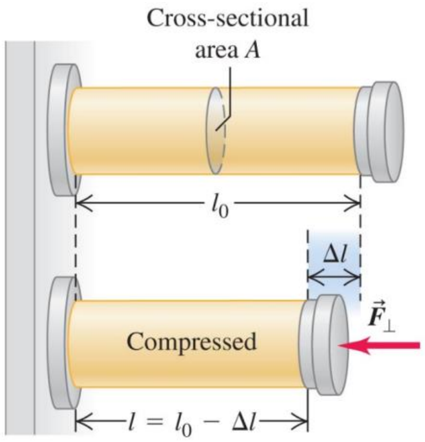
$$\text{Tensile strain} = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$

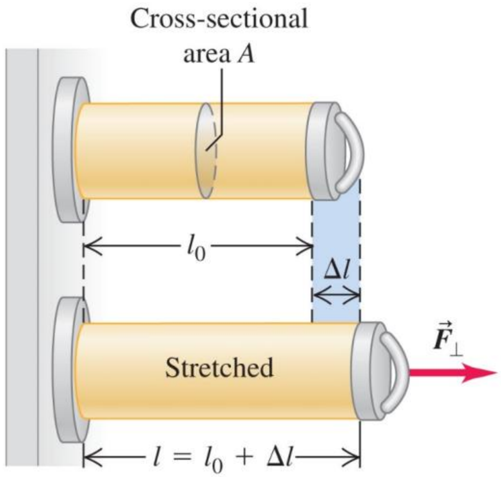
Units: none

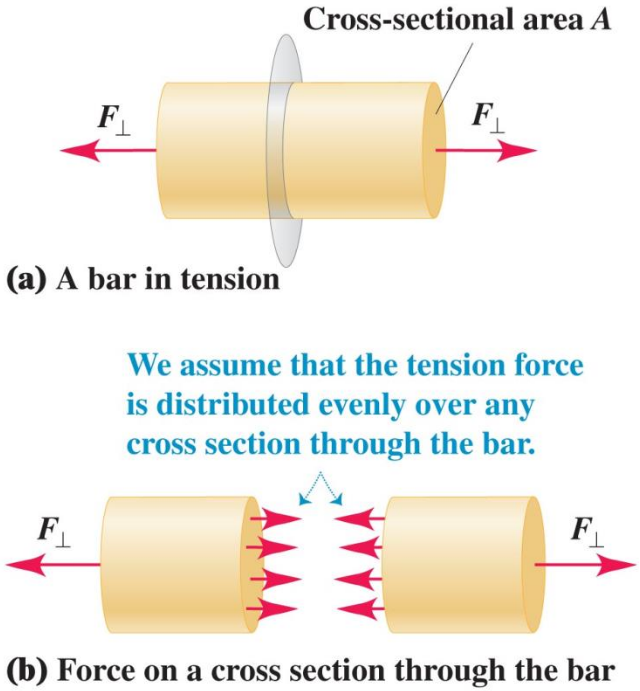
Young's modulus

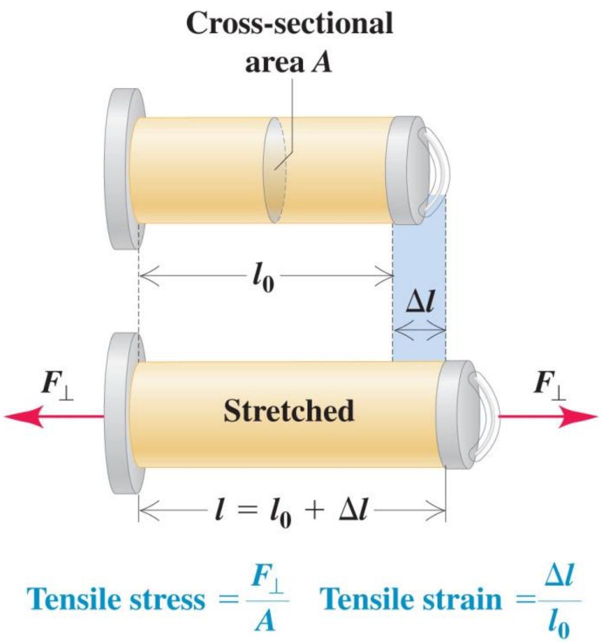
$$Y = \frac{Tensile\ Stress}{Tensile\ Strain} = \frac{F_{\perp}/A}{\Delta l/l_0} = \frac{l_0}{A}\frac{F_{\perp}}{\Delta l}$$

Units: N/m² or Pa







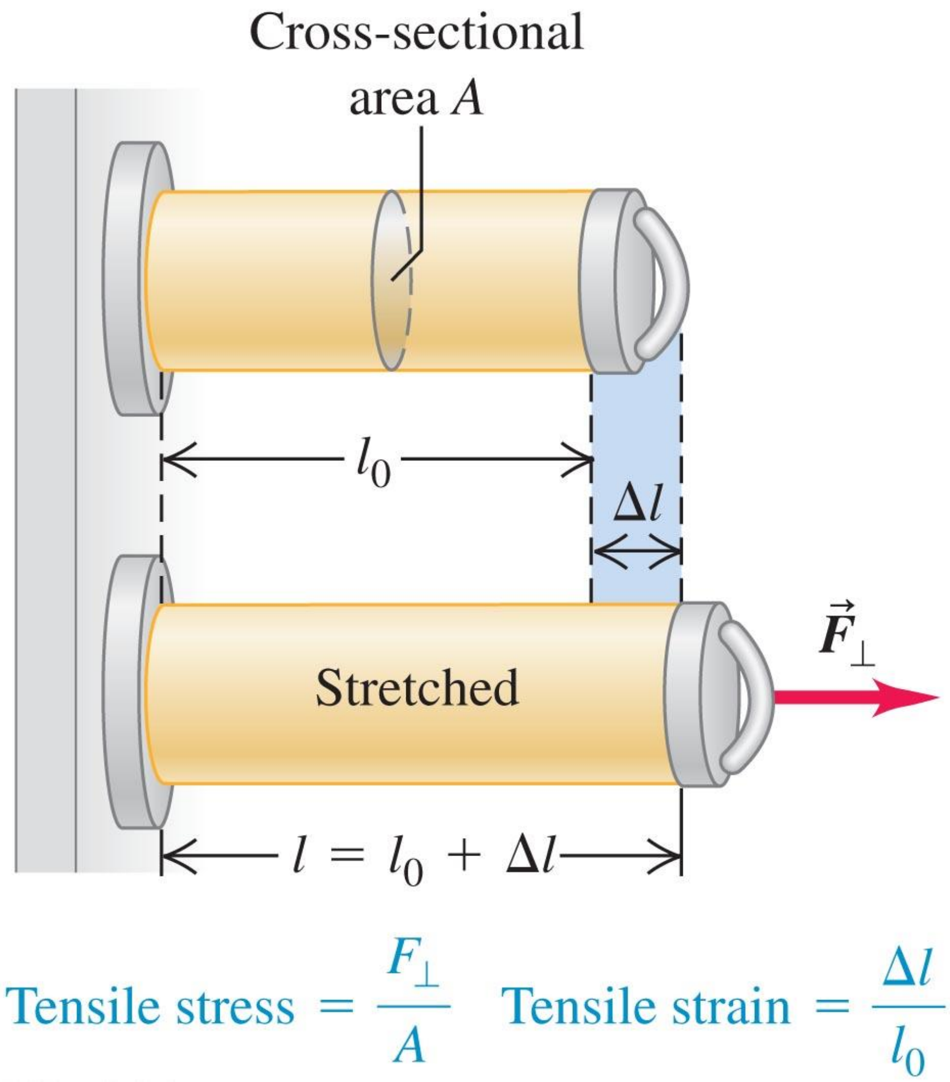


Compressive stress unit: pa=pascal=newton/m^2

Strain unit: dimensionless

The Reaction to Stress Is Strain – Figure 11.4

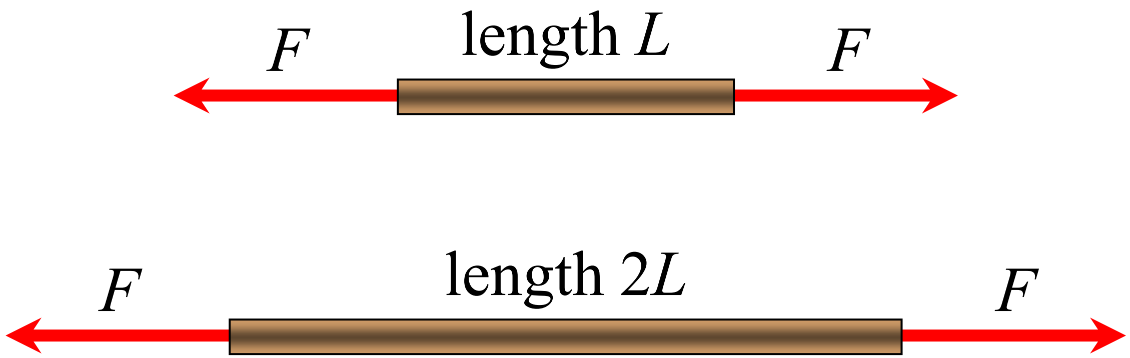
- If we return to the steel cable example, we could ask ourselves, "How much will the steel stretch under a load?"
- Strain, then, is $\Delta l/l_0$, the unitless change in length divided by the original length.



Q11.5

Clicker question

Two rods are made of the same kind of steel and have the same diameter. A force of magnitude F is applied to each end of each rod. Compared to the rod of length L, the rod of length $2L$ has



A. more stress and more strain.

B. the same stress and more strain.

C. the same stress and less strain.

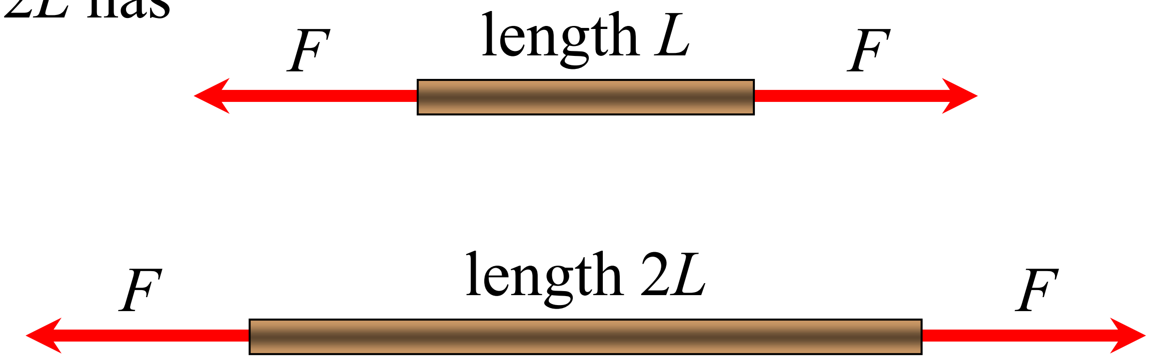
D. less stress and less strain.

E. the same stress and the same strain.

Q11.6

Clicker question

Two rods are made of the same kind of steel. The longer rod has a greater diameter. A force of magnitude F is applied to each end of each rod. Compared to the rod of length L, the rod of length $2L$ has



A. more stress and more strain.

B. the same stress and more strain.

C. the same stress and less strain.

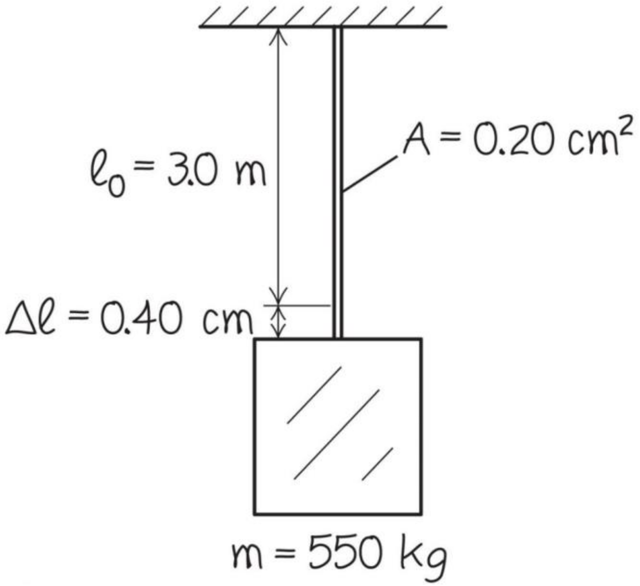
D. less stress and less strain.

E. the same stress and the same strain.

Example 11.1
A stretching elevator cable

Given: m, l_0, A, and Δl

Find: the cables stress, strain, and Young's modulus



Solution:

$$\text{Stress} = \frac{F_\perp}{A} = \frac{W}{A} = \frac{mg}{A} = \frac{(550\ kg)(9.8\ m/s^2)}{0.20\times10^{-4}\ m^2} = 2.7 \times 10^8\ \text{Pa}$$

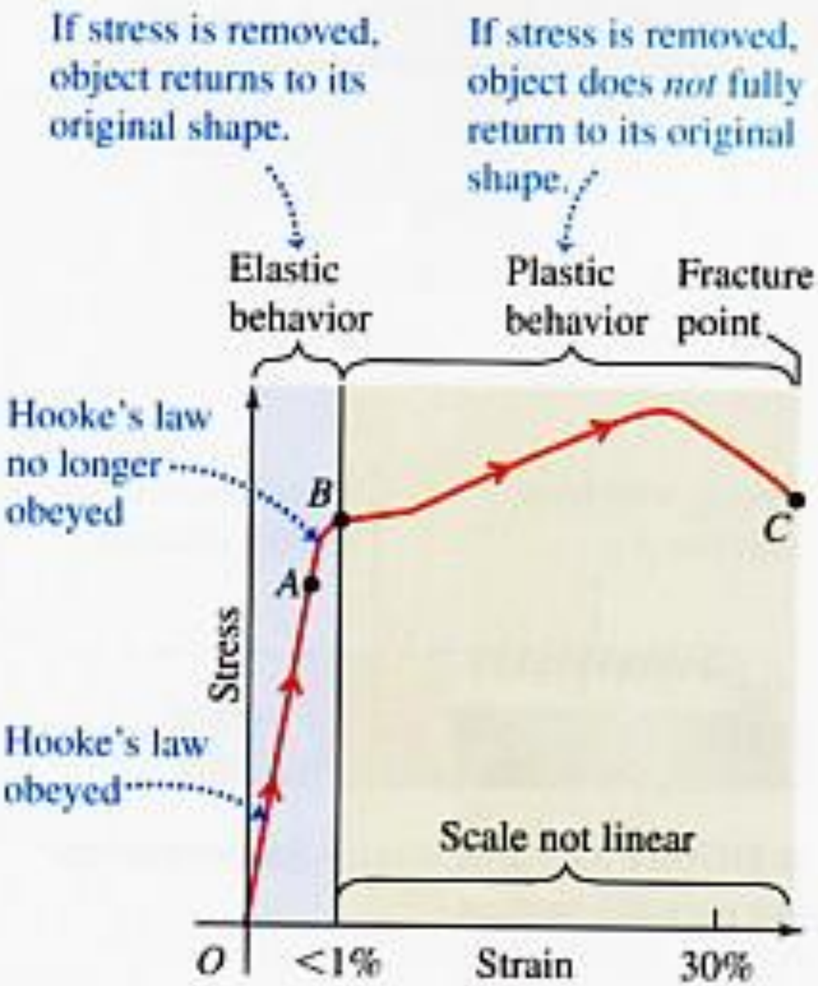
$$\text{Strain} = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0} = \frac{0.40\times10^{-2}\ m}{3.0\ m} = 0.00133$$

Young's Modulus

$$Y = \frac{Stress}{Strain} = \frac{2.7 \times 10^8\ \text{Pa}}{0.00133} = 2.0 \times 10^{11}\ Pa$$

Note: strain is dimensionless

Elasticity and Plasticity



▲ **FIGURE 11.12** Typical stress–strain diagram for a ductile metal under tension.

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} \quad \text{or} \quad \frac{\text{Compressive stress}}{\text{Compressive strain}},$$

$$Y = \frac{F_\perp / A}{\Delta l / l_0} = \frac{l_0}{A}\frac{F_\perp}{\Delta l}.$$

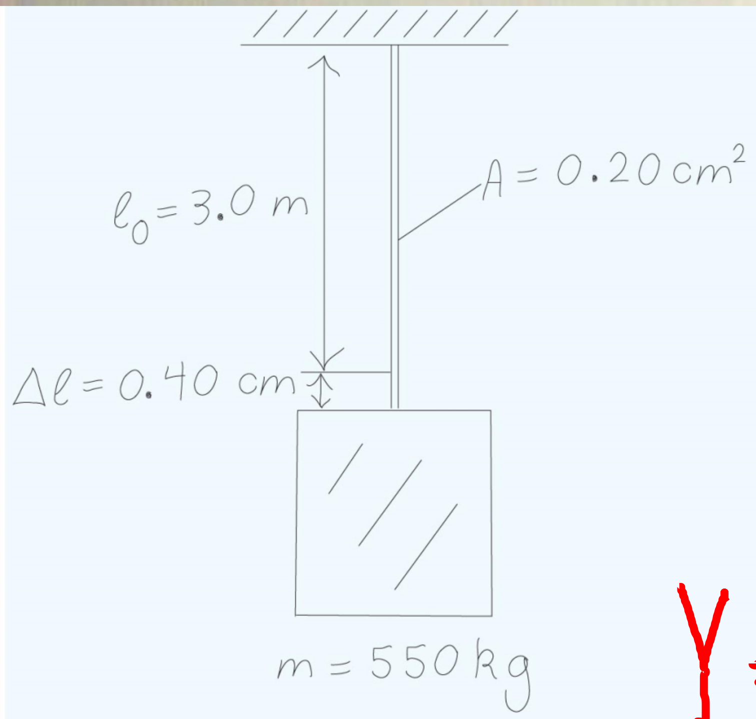
TABLE 11.1 Young's modulus

Material	Y (Pa)
Aluminum	0.70×10^{11}
Brass	0.91×10^{11}
Copper	1.1×10^{11}
Glass	0.55×10^{11}
Iron	1.9×10^{11}
Steel	2.0×10^{11}
Tungsten	3.6×10^{11}

$F_x = kx$ Hook's law

EXAMPLE 11.1 A stretching elevator cable

A small elevator with a mass of 550 kg hangs from a steel cable that is 3.0 m long when not loaded. The wires making up the cable have a total cross-sectional area of 0.20 cm², and with a 550 kg load, the cable stretches 0.40 cm beyond its unloaded length. Determine the cable's stress and strain. Assuming that the cable is equivalent to a rod with the same cross-sectional area, determine the value of Young's modulus for the cable's steel.



$$\text{Stress} = \frac{550\,kg \cdot 9.8\,\frac{m}{s^2}}{0.20 \cdot 10^{-4}\,m^2} = 2.7 \cdot 10^{8}\,Pa$$

$$\text{Strain} = \frac{\Delta\ell}{\ell_0} = \frac{0.40 \cdot 10^{-2}\,m}{3.0\,m} = 0.00133$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{2.7 \cdot 10^{8}\,Pa}{0.00133} = 2.0 \cdot 10^{11}\,Pa$$

1 Pa = 1N/m ^{2}

3. • A vertical solid steel post 25 cm in diameter and 2.50 m long is required to support a load of 8000 kg. You may ignore the weight of the post. What are (a) the stress in the post, (b) the strain in the post, and (c) the change in the post's length when the load is applied?

(a) $\text{Stress} = \dfrac{8000\,\text{kg} \cdot 9.8\,\frac{\text{m}}{\text{s}^2}}{\pi (25 \cdot 10^{-2}\,\text{m})^2 / 4} = 1.6 \cdot 10^{6}\,\text{Pa}$

(b) $\text{Strain} = -\dfrac{\text{Stress}}{Y} = -\dfrac{1.6 \cdot 10^{6}\,\text{Pa}}{2.0 \cdot 10^{11}\,\text{Pa}} = -8.0 \cdot 10^{-6}$

(c) $\Delta \ell = \text{Strain} \cdot \ell_0 = -2.0 \cdot 10^{-5}\,\text{m}$

Volume Stress and Strain

Volume stress, or, pressure

$$p = \frac{F_{\perp}}{A}$$

Units: N/m^2 or Pa, in SI units
psi, in the British units

Volume strain

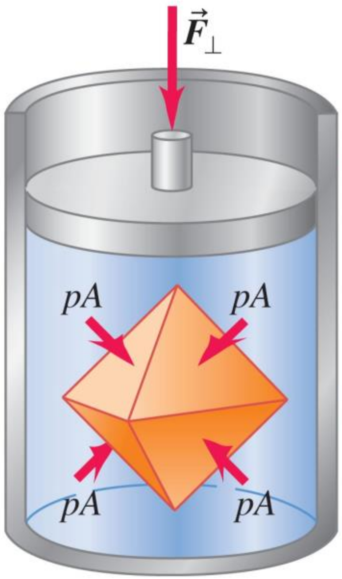
$$\text{Volume strain} = \frac{V - V_0}{V_0} = \frac{\Delta V}{V_0}$$

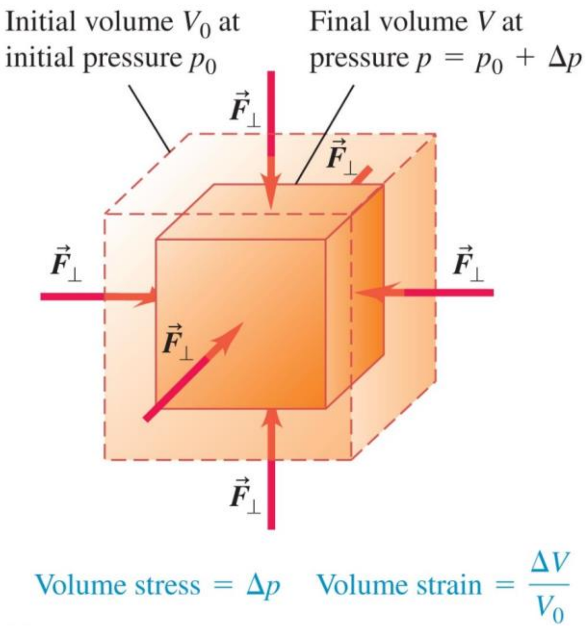
Units: none

Bulk modulus

$$B = \frac{\text{change in pressure}}{\text{resulting volume strain}} = \frac{\Delta p}{\Delta V / V_0}$$

Units: N/m^2 or Pa





Shear Stress and Strain

Shear stress

$$\text{Shear stress} = \frac{F_\parallel}{A}$$

Units: N/m^2 or Pa

Shear strain

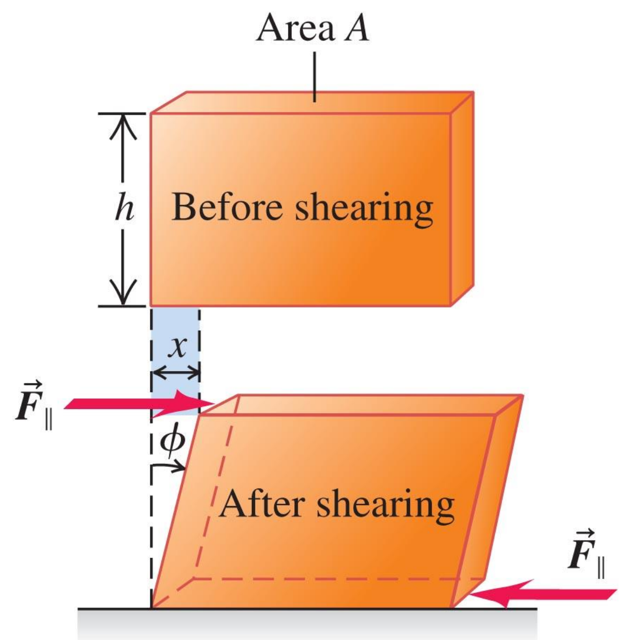
$$\text{Shear strain} = \frac{x}{h} = \tan\phi \approx \phi \qquad \text{for } x \ll h$$

Units: none

Shear modulus

$$S = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{F_\parallel/A}{x/h} = \frac{F_\parallel/A}{\phi}$$

Units: N/m^2 or Pa



13. • In the Challenger Deep of the Marianas Trench, the depth of seawater is 10.9 km and the pressure is 1.16×10^8 Pa (about 1150 atmospheres). (a) If a cubic meter of water is taken to this depth from the surface (where the normal atmospheric pressure is about 1.0×10^5 Pa), what is the change in its volume? Assume that the bulk modulus for seawater is the same as for freshwater (2.2×10^9 Pa). (b) At the surface, seawater has a density of 1.03×10^3 kg/m^3. What is the density of seawater at the depth of the Challenger Deep?

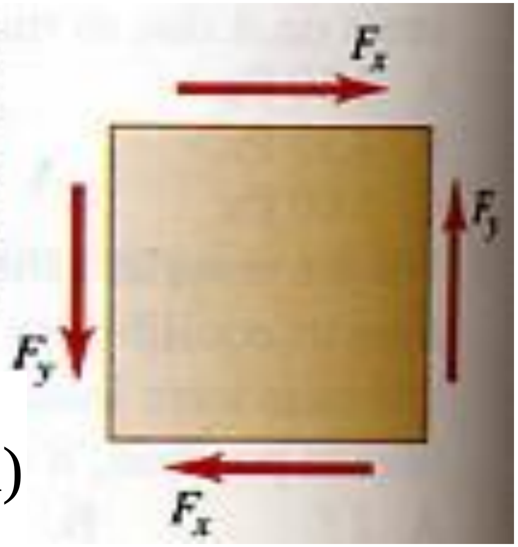
(a) $$\frac{\Delta V}{V_0} = -\frac{p}{B} = -\frac{1.16 \cdot 10^8 \,\text{Pa}}{2.2 \cdot 10^9 \,\text{Pa}} = -0.053$$

$$\Delta V = 1\,\text{m}^3 \cdot (-0.053) = -0.053\,\text{m}^3$$

(b) $$\rho = \frac{m}{V}; \quad \frac{\rho}{\rho_0} = \frac{V_0}{V_0 + \Delta V} = \frac{1\,\text{m}^3}{(1-0.053)\,\text{m}^3} = 1.056$$

$$\rho = 1.056 \cdot \rho_0 = 1.056 \cdot 1.03 \cdot 10^3 \frac{\text{kg}}{\text{m}^3} = 1.09 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$$

17. • In Figure 11.33, suppose the object is a square steel plate, 10.0 cm on a side and 1.00 cm thick. Find the magnitude of force required on each of the four sides to cause a shear strain of 0.0400.



(Shear modulus for steel 0.84 ·10^11Pa)

$$S = \frac{(F/A)}{\text{strain}}$$

$$F = A \cdot S \cdot \text{strain} = \underbrace{(0.1\,m \cdot 0.01\,m)}_{A} \cdot 0.84 \cdot 10^{11}\,Pa \cdot 0.0400 = 3.4 \cdot 10^{6}\ \text{N}$$

$$Stress = \frac{F}{A}(N/m^2) \text{ or (Pa)}$$

$$Strain = \frac{\Delta l}{l_0} \text{ (No dimension)}$$

Shear Stress and Strain

Shear stress

$$\text{Shear stress} = \frac{F_{\parallel}}{A}$$

Units: N/m^2 or Pa

Shear strain

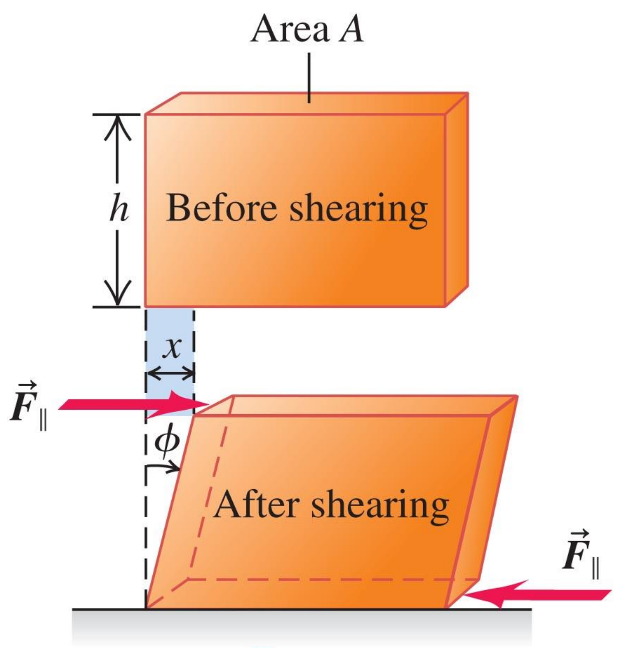
$$\text{Shear strain} = \frac{x}{h} = \tan\phi \approx \phi \qquad \text{for } x \ll h$$

Units: none

Shear modulus

$$S = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{F_{\parallel}/A}{x/h} = \frac{F_{\parallel}/A}{\phi}$$

Units: N/m^2 or Pa



Simple harmonic motion SMH

11.2 Periodic Motion

Simple Harmonic Motion (SHM) illustrated by the oscillations of a mass-loaded spring

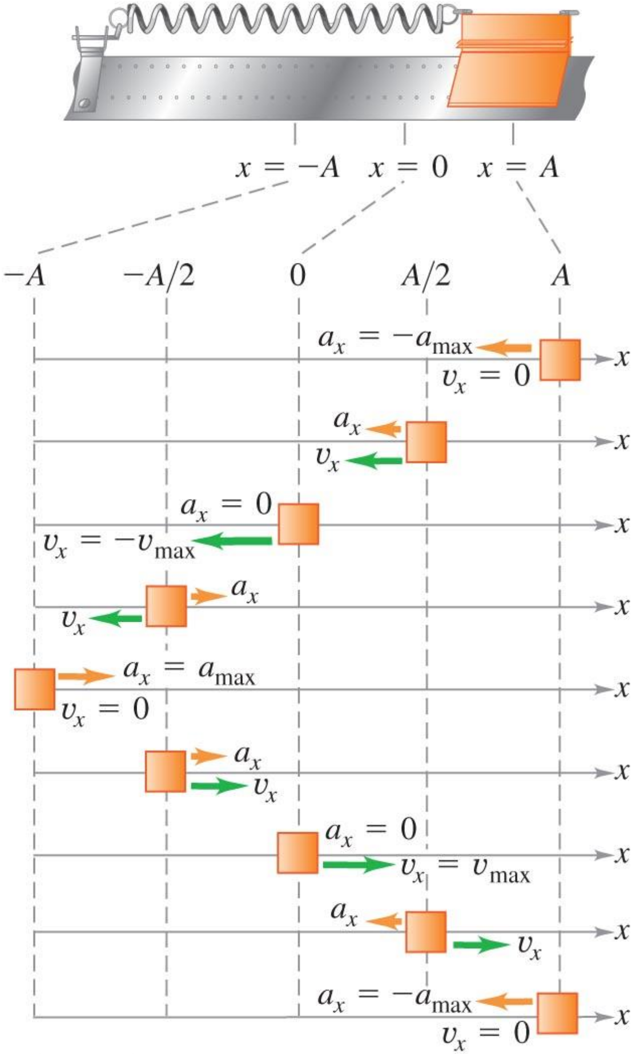
Spring restoring force: $F_x = -kx$

Acceleration of the mass: $a_x = \frac{F_x}{m} = -\frac{k}{m}x$

Defining simple harmonic motion: motion driven by a restoring force that is always opposite to the displacement and directly proportional to the displacement in magnitude.

Note:

(a) The restoring force F_x is opposite to displacement;
(b) F_x is not a constant;
(b) As a result, the acceleration a_x is not a constant;
(c) a_x varies between $(+ kA/m)$ and $(- kA/m)$;
(d) The magnitude of a_x has a maximum $a_{max} = kA/m$.

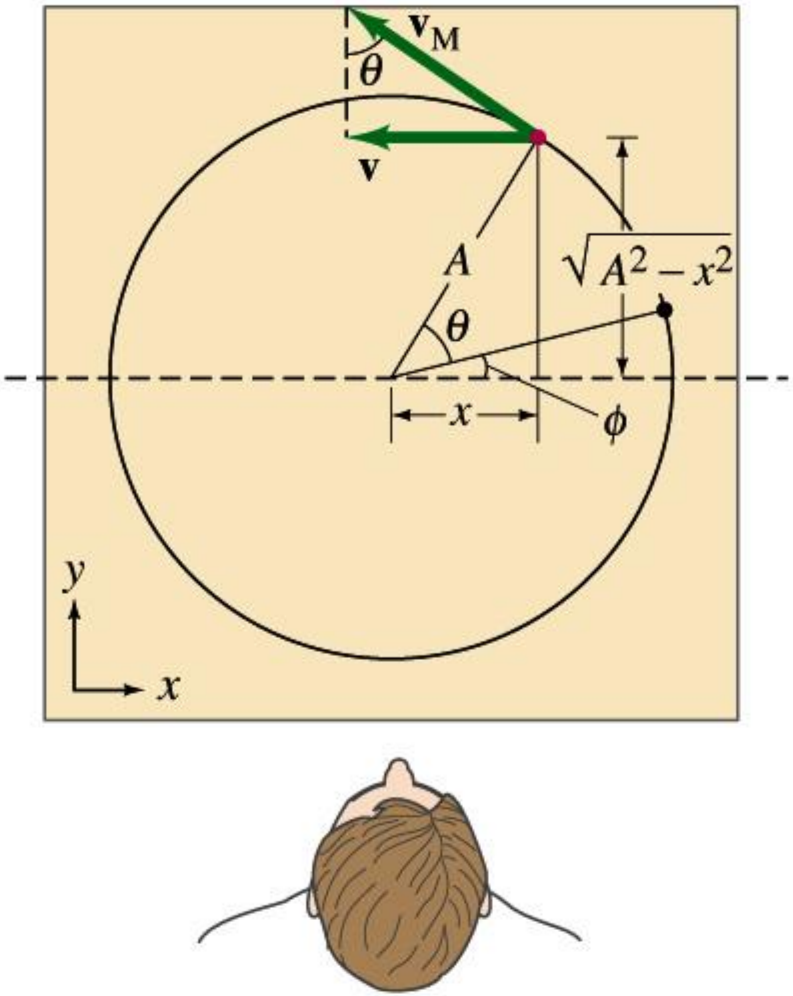


(5 pts) 15. A block with mass 4.0 kg is attached to a horizontal spring that has force constant k. The block moves in simple harmonic motion on a horizontal frictionless surface. The amplitude of the motion is 0.50 m and the maximum acceleration of the block has magnitude $20\ m/s^2$. What is the force constant k of the spring?

(a) 20 N/m
(b) 80 N/m
(c) 160 N/m
(d) 240 N/m
(e) none of the above answers

$$k = a\,m/A = (20\ m/s^2)\ 4\ kg/0.5\ m$$

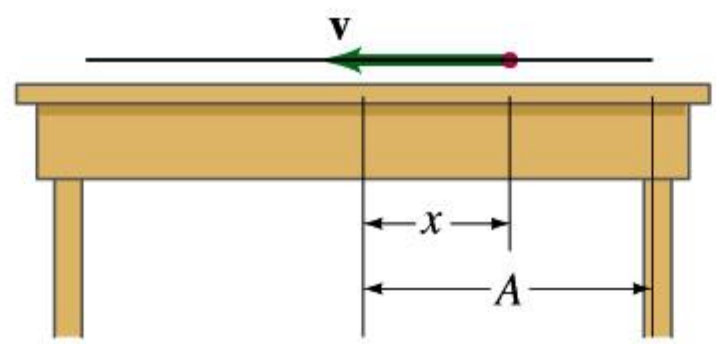
Circle of Reference



(a)

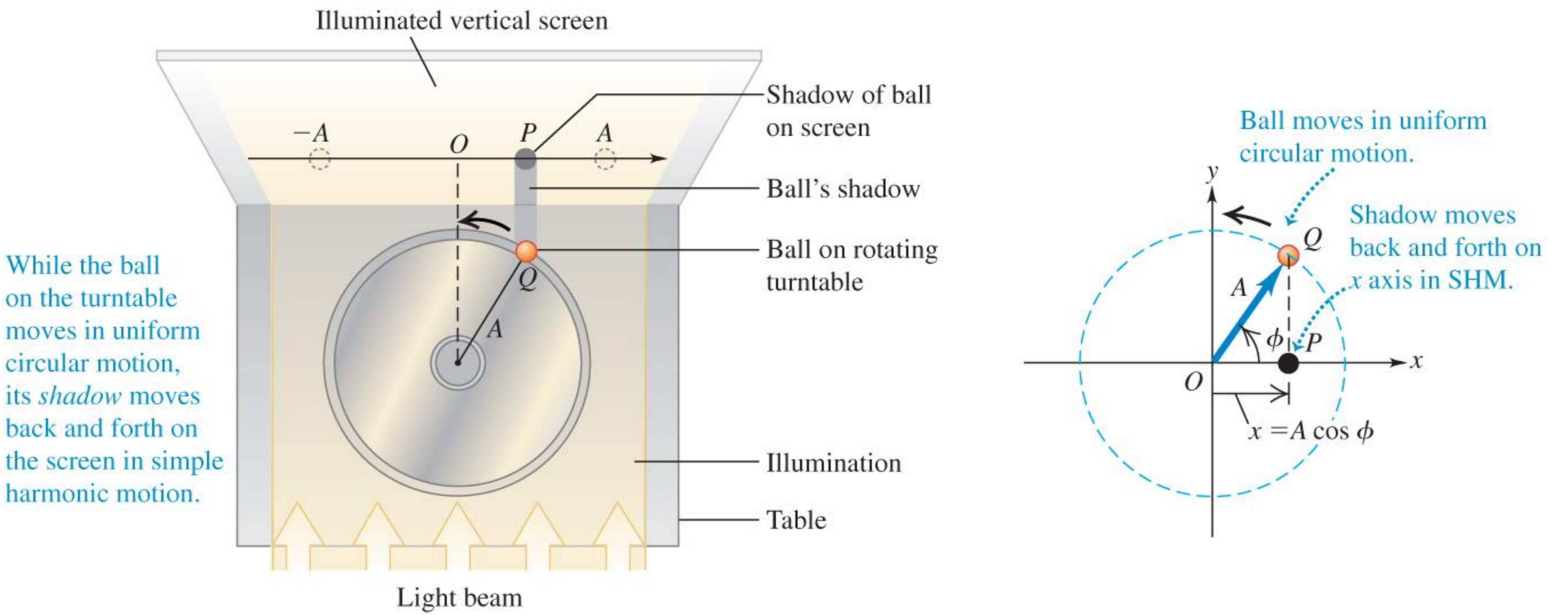
Simple harmonic motion is the projection of uniform circular motion on a diameter.

Consider a ball on a circular track on the table and looking at it from the side



(b)

Circle of Reference



(a) Apparatus for creating the reference circle

(b) An abstract representation of the motion in **(a)**.

Circle of Reference: Consider a small object undergoing a uniform circular motion as shown in the sketch. The x-component of the position of the object obeys SHM exactly. Therefore, the expressions describing the x-component of its positon can be used to describe SHM.

Let the angular velocity of the object be ω, and, its angular position $\phi_o = 0$ at $t = 0$. Let the radius of the circle be A.

The object's the angular position at time t is $\phi = \omega t$. We have the following x-component quantities for the object:

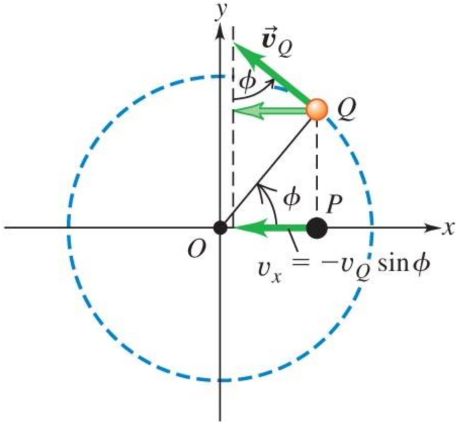
The position $x = A cos(\phi) = A cos(\omega t)$ .....Equation of SHM

The velocity $v_x = -\, v\, sin(\phi) = -(\omega A) sin(\omega t)$

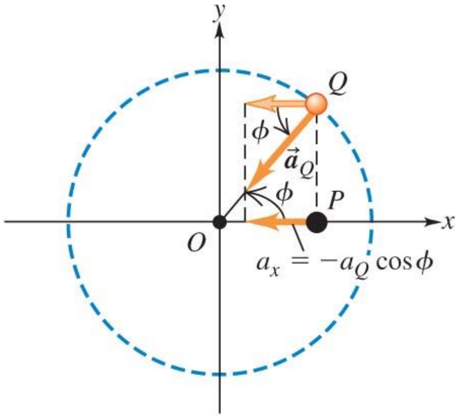
The acceleration $a_x = -\, a_{rad}\, cos(\phi) = -\omega^2 A\, cos(\omega t) = -\, \omega^2 x$

What is ω? Since for the spring along the x-axis $a_x = F_x/m = -kx/m$ we have $\omega^2 = k/m$ or $\omega = \sqrt{k/m}$,

and other quantities $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k/m}$, $T = \frac{1}{f} = 2\pi\sqrt{m/k}$



(a) Using the reference circle to determine the velocity of point P



(b) Using the reference circle to determine the acceleration of point P

$$x = A\cos wt = A\cos(2\pi f t)$$

$\boldsymbol{v} = \boldsymbol{rw}$ and (using the reference circle for the velocity of **Q**

Note: while **Q** rotates a full circle; **P** makes a complete back and forth vibration (1 cycle)

<u>"Vibrational velocity" of **P**:</u>

$$v_x = -v_Q \sin\phi = -wA\sin\phi \quad \rightarrow \quad \phi = wt$$

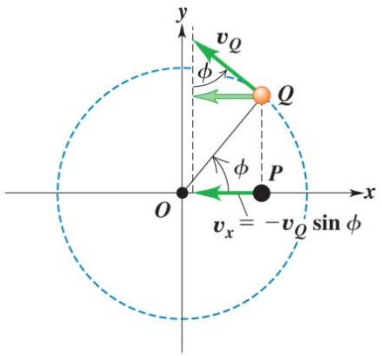
$\rightarrow v_x = -2\pi f A \sin(2\pi f t)$ ['-' sign at the instance shown v_x is to the left]

<u>"Vibrational acceleration" of **P**:</u>

$$\left.\begin{array}{r} a_{rad} = \dfrac{v^2}{A} \\ v = Aw \end{array}\right\} a_{rad} = Aw^2 = 4\pi^2 f^2 A$$

$$a_x = -a_{rad}\cos\phi = -w^2 A\cos\phi = 4\pi^2 f^2 \underbrace{A\cos(2\pi f t)}_{x}$$

$$\rightarrow a_x = -w^2 x$$



(a) Using the reference circle to determine the velocity of point *P*.

<u>Remember:</u>

Use Hooke's law

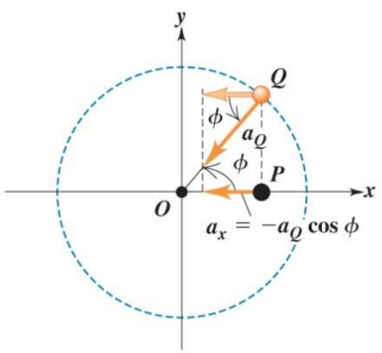
v=rw $\quad w^2 r = a_{rad}$

$$F_x = -kx = ma_x$$

$$\rightarrow a_x = \frac{k}{m}x$$

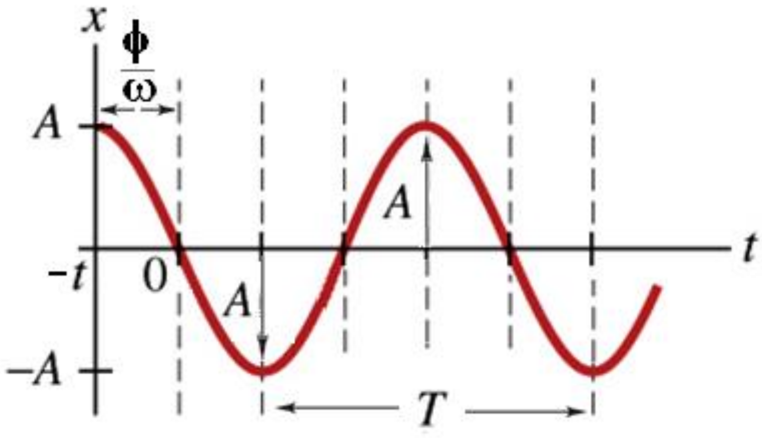
$$w^2 = \frac{k}{m}$$

$$w = \sqrt{\frac{k}{m}}$$



(b) Using the reference circle to determine the acceleration of point *P*.

Simple harmonic motion



$$x = A\cos(wt + \underbrace{\phi}_{2\pi\frac{cycles}{sec}\cdot sec})$$

$$wt + \phi = 0 \;\; \rightarrow \;\; \phi = -wt \quad \therefore -t = \frac{\phi}{w}$$

$$wT = 2\pi \;\; \rightarrow \;\; T = \frac{2\pi}{w}$$

$$\therefore f = \frac{1}{T} = \frac{w}{2\pi} = \frac{cycles}{sec} = Hz\ (hertz)$$

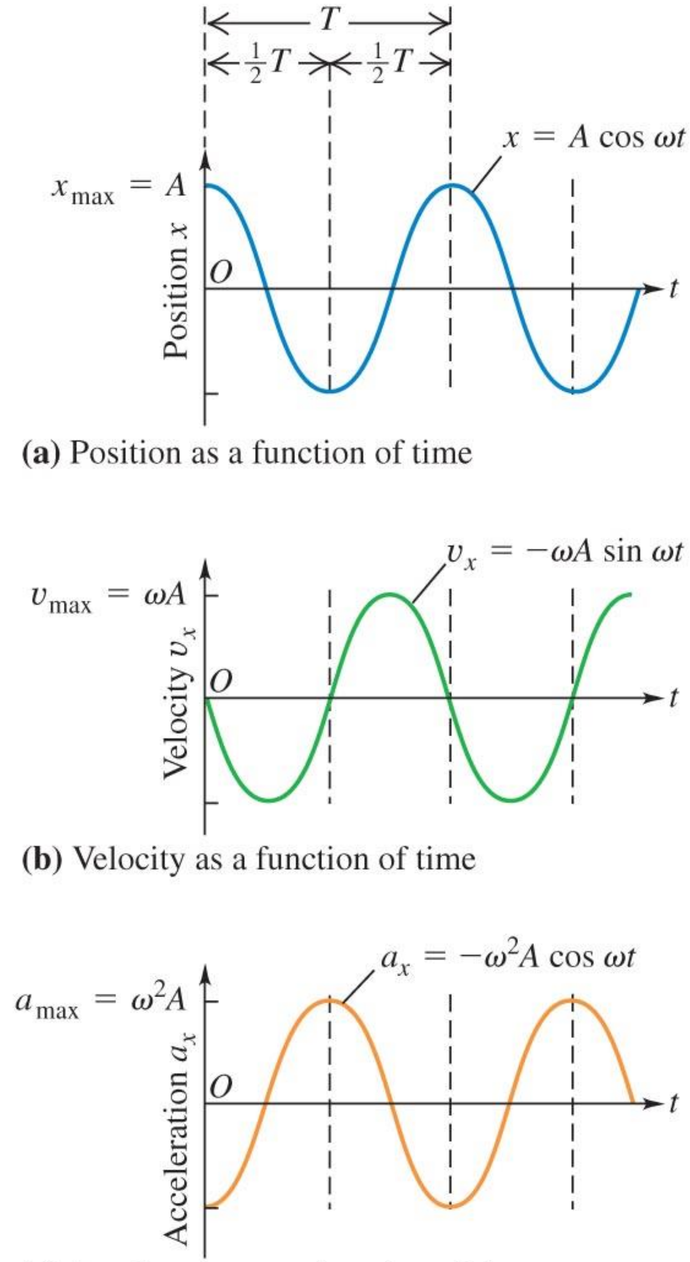
$$\underbrace{\boldsymbol{w}}_{\substack{\textbf{angular}\\ \textbf{frequency}}} = \boldsymbol{2\pi} \underbrace{\boldsymbol{f}}_{\substack{\textbf{linear}\\ \textbf{frequency}}}$$

Velocity; $v = -wA\sin(wt + \phi)$

Acceleration; $a = -w^2 A\cos(wt + \phi)$

Position, Velocity, and Acceleration

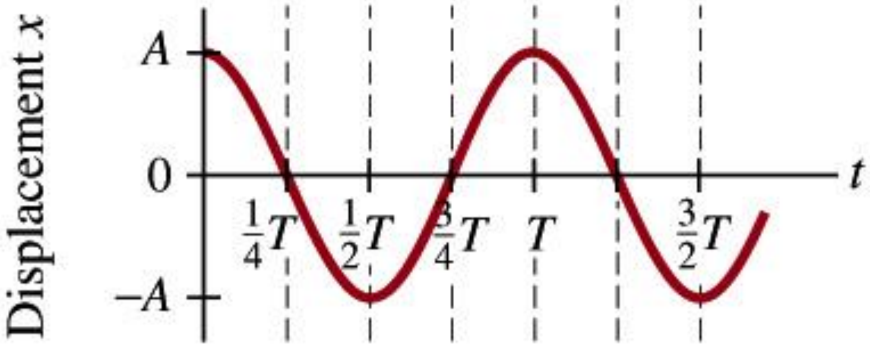
- Graphs from a particle undergoing simple harmonic motion.



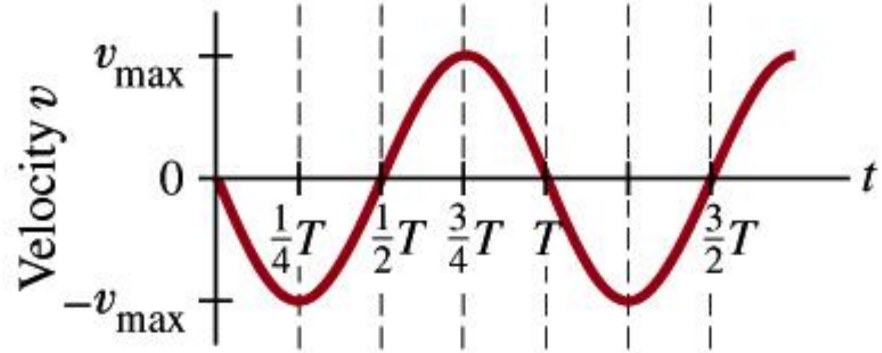
(a) Position as a function of time

(b) Velocity as a function of time

(c) Acceleration as a function of time



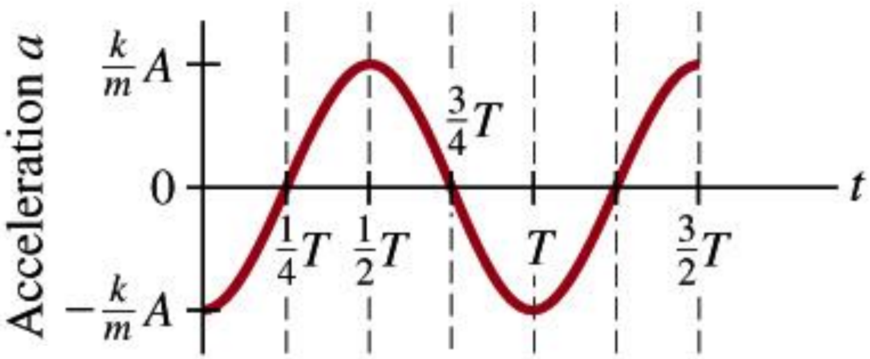
(a)



(b)

$$\mathbf{x = A\ Cos(\omega t + \phi\ )}$$

$$\mathbf{v = -\ \omega\ A\ Sin(\omega t + \phi\ )}$$



(c)

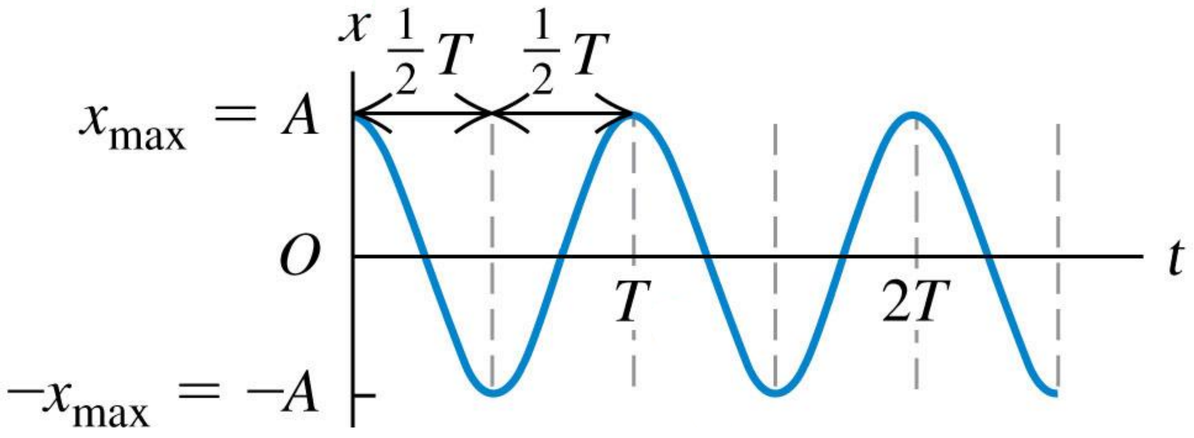
$$\mathbf{\omega = \frac{2\pi}{T} = 2\pi\ f}$$

$$\mathbf{-a = -\ \omega^2\ A\ Cos(\omega t + \phi\ )}$$

Q14.2

Clicker question

This is an *x*-*t* graph for an object in simple harmonic motion. At which of the following times does the object have the *most negative velocity* v_x?



A. $t = T/4$

B. $t = T/2$

C. $t = 3T/4$

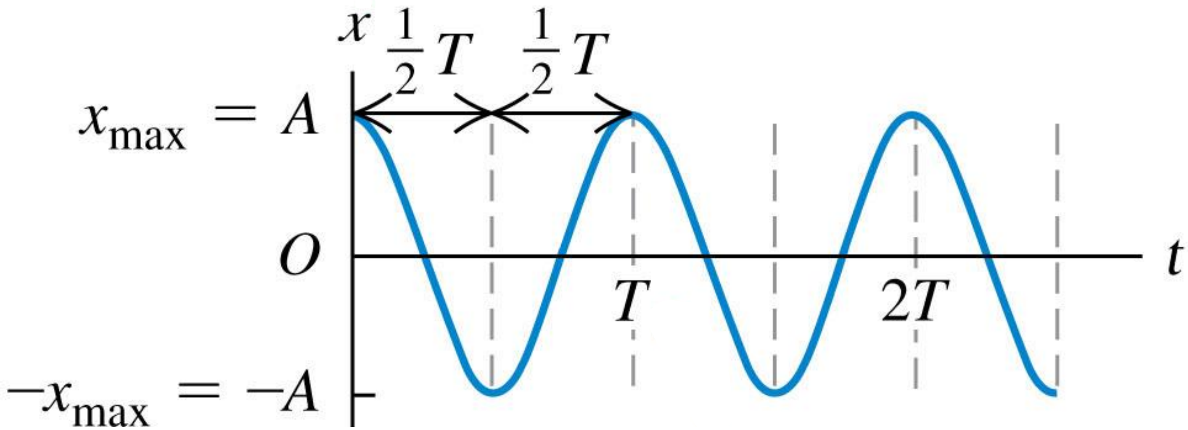
D. $t = T$

E. Two of the above are tied for most negative velocity.

Q14.3

Clicker question

This is an *x*-*t* graph for an object in simple harmonic motion. At which of the following times does the object have the *most negative acceleration* a_x?



A. $t = T/4$

B. $t = T/2$

C. $t = 3T/4$

D. $t = T$

E. Two of the above are tied for most negative acceleration.

Quantities that Describe Periodic Motion

- The amplitude of the motion is the maximum magnitude of the displacement,

$$A = |x|_{max}.$$

- One cycle of the motion: one complete round trip.
- The period T of the motion is the time it takes to complete one cycle, in units of s.
- The frequency f of the motion is the number of cycles the motion complete in 1 s.
- The angular frequency ω of the motion is 2π multiplies the frequency, $\omega = 2\pi f$.
- The relationships between T, f, and ω:

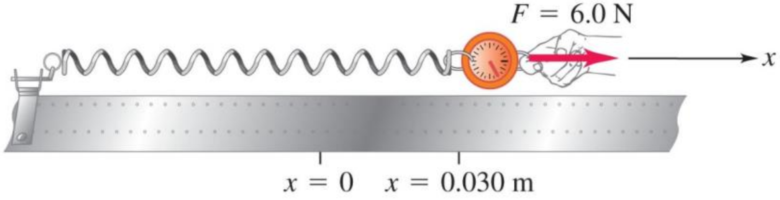
$$f = \frac{1}{T}$$ Units: hertz or Hz 1 Hz = 1 s^{-1}

$$\omega = 2\pi f = \frac{2\pi}{T}$$ Units: rad/s

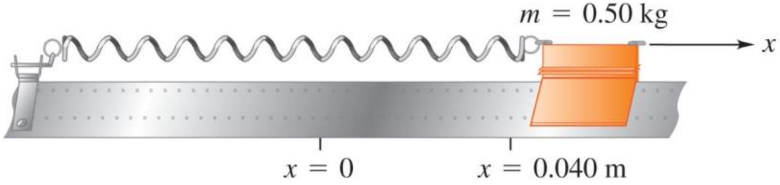
Example 11.5 on page 334, SHM on an air track

Given: m and A. Force constant k is give indirectly.

Find: (a) force constant k
(b) the maximum & minimum velocities
(c) the maximum & minimum accelerations
(d) v & a at half way to $x = 0$
(e) K, U, and E at half way to $x = 0$



(a)



(b)

(a) Force constant $k = F/x = (6.0\text{ N})/(0.030\text{ m}) = 200\text{ N/m}$

(b) $v_{max} = A\sqrt{k/m} = 0.80\text{ m/s}$, $v_{min} = -A\sqrt{k/m} = -0.80\text{ m/s}$, both at $x = 0$

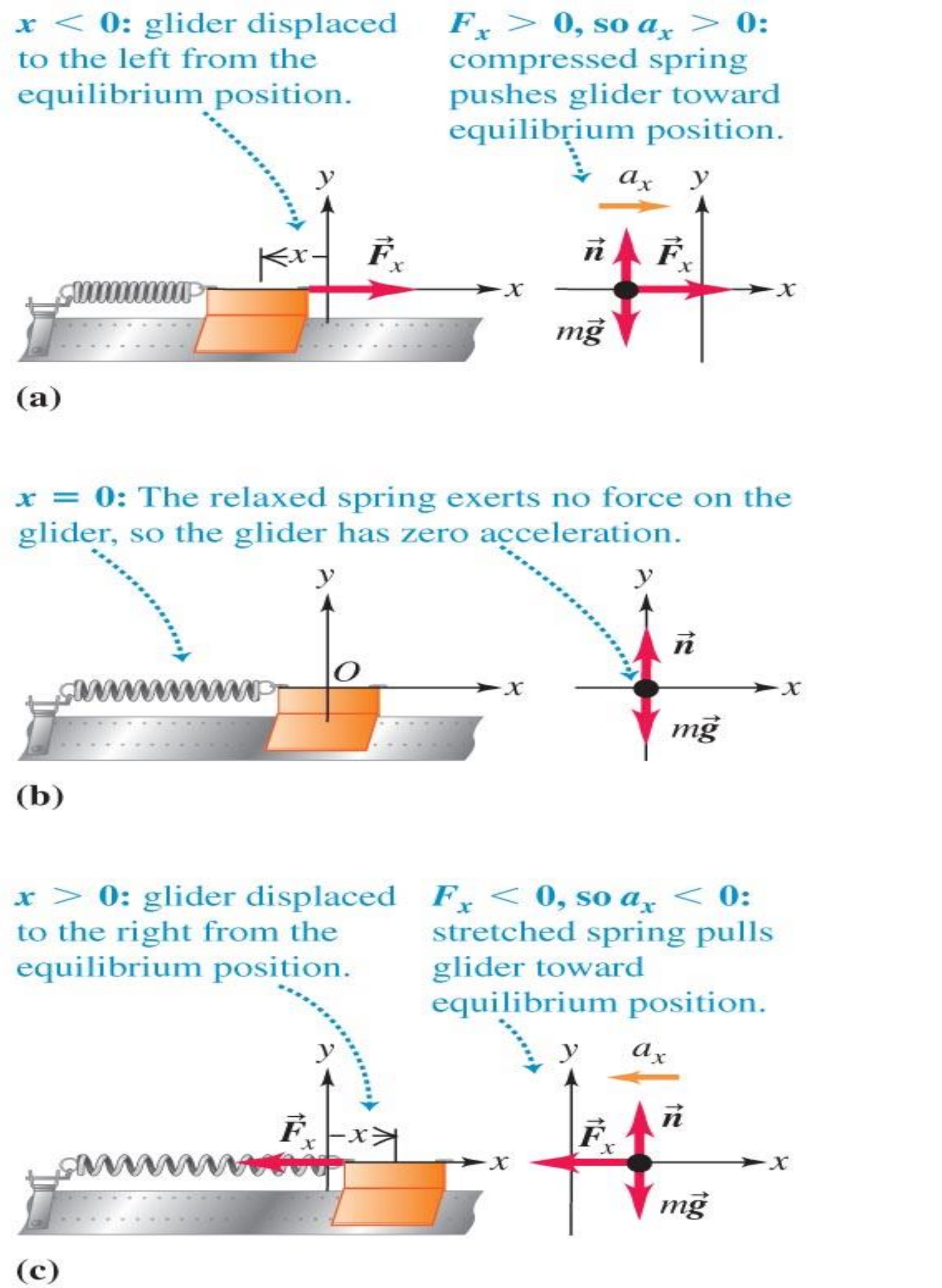
(c) $a_{\max} = +kA/m = 16.0\text{ m/s}^2$ at $x = -0.04$ m; $a_{\min} = -kA/m = -16.0\text{ m/s}^2$ at $x = +0.04$ m

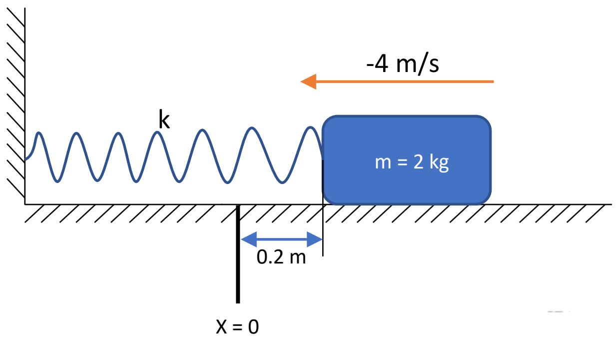
(d) $v_x = -\sqrt{\frac{k}{m}}\sqrt{(A^2 - x^2)} = -\sqrt{\frac{k}{m}}\sqrt{\left[A^2 - \left(\frac{1}{2}A\right)^2\right]} = -0.69\text{ m/s}$

$a_x = -\frac{k}{m}x = -\frac{k}{m}\left(\frac{1}{2}A\right) = -8.0\text{ m/s}^2$

(e) $K = \frac{1}{2}mv_x^2 = 0.12\text{ J}$ $U = \frac{1}{2}kx^2 = 0.040\text{ J}$ $E = K + U = 0.16\text{ J}$

- Elastic Situations Yield Simple Harmonic Motion





A block with mass m is attached to a spring with force constant k = 315 N/m. The spring is stretch in positive x direction by the amount shown, and the block has an initial velocity in the negative x direction.

a) Find the amplitude of the block.
b) Find the maximum acceleration of the block.
c) Find the maximum force the spring exerts on the block.

$m = 2.00\ kg \qquad k = 315\ N/m \qquad X = +0.200\ m \qquad V_x = -4.00\ m/s$

a) $-kx = ma_x$

$a_{max} = \frac{k}{m} A$

Use conservation of energy to solve for A

$\frac{1}{2} m v_x^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$

$A = \sqrt{x^2 + \frac{m}{k} v_x^2} = \sqrt{(0.2\ m)^2 + \frac{2\ kg}{315\ N/m}(-4.00\ m/s)^2} = 0.376\ m$

b) $a_{max} = \frac{315\ N/m}{2\ kg}(0.376\ m) = 59.3\ m/s^2$

c) $F = kx$ so $F_{max} = kA = (315\ N/m)(0.376\ m) = 118\ N$

11.3 Energy in Simple Harmonic Motion

Conservation of Energy in SHM

$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$$

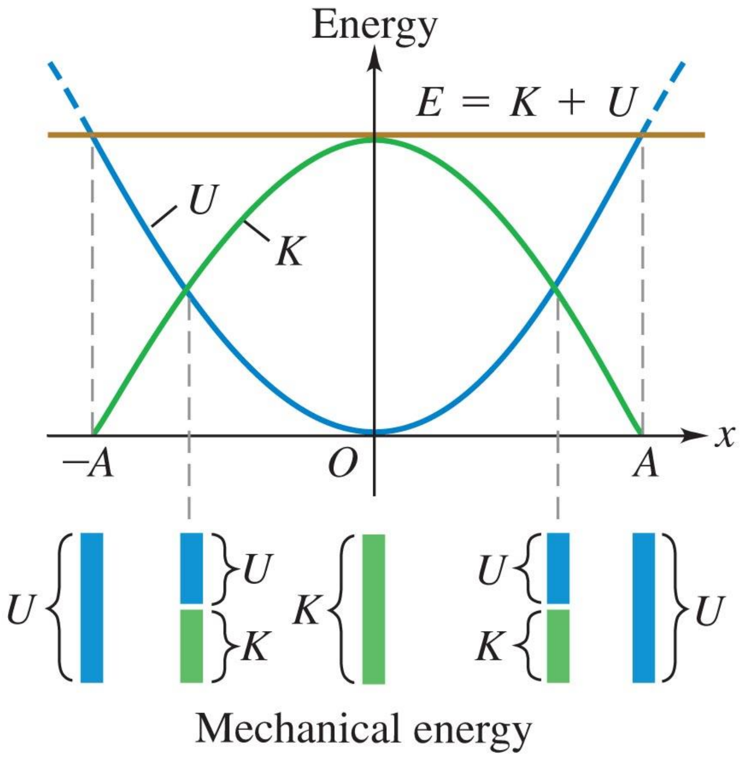
Velocity of an object in SHM as a function of position

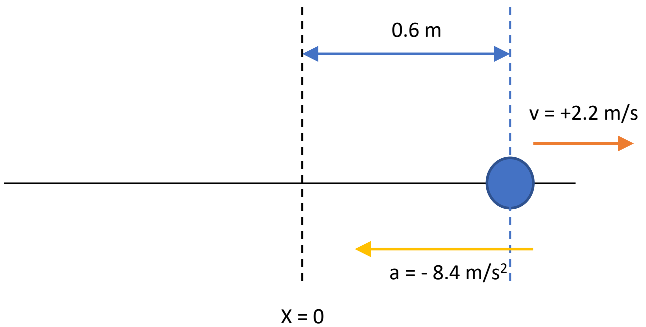
$$v_x = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\sqrt{\frac{k}{m}}\sqrt{(A^2 - x^2)}$$

Note:

(a) $v_x = 0$ when $x = \pm A$.

(b) Maximum speed $v_{x,max} = A\sqrt{k/m}$ when $x = 0$.





The object to the left is following Simple Harmonic Motion. It starts at the position shown with the velocity and acceleration as given. How much further from its current point will the object move before it stop momentarily and then starts to move back to the left?

Solve for the amplitude A. The additional distance the object will travel is $A - 0.600$ m.

The equation that relates velocity and position is

$\frac{1}{2} m v_x^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$ (conservation of energy)

The equation that relates acceleration and position is

$-kx = m a_x$ $(\Sigma \vec{F} = m\vec{a})$

$A = \sqrt{x^2 + \frac{m}{k} v_x^2}$

$\frac{m}{k} = -\frac{x}{a_x}$

Combine to get $A = \sqrt{x^2 - \frac{x}{a_x} v_x^2}$

$x = +0.600$ m, $v_x = +2.20$ m/s, $a_x = -8.40$ m/s²

$A = \sqrt{(0.600\text{m})^2 - \frac{0.600\text{m}}{-8.40\text{m/s}^2}(2.20\text{ m/s})^2}$

$A = \sqrt{(0.360\text{m}^2) + (0.3457\text{m}^2)} = 0.840$ m

additional distance is $0.840\text{ m} - 0.600\text{ m} = 0.240$ m

(5 pts) 16. A block with mass 0.200 kg is on a horizontal frictionless surface and is moving in SHM on the end of a spring. The amplitude of the motion is 0.150 m and the maximum speed of the block during its motion is 3.00 m/s. What is the maximum speed of the block if the amplitude is increased to 0.300 m?

(a) 0.750 m/s
(b) 1.50 m/s
(c) 3.00 m/s
(d) 6.00 m/s
(e) 9.00 m/s
(f) none of the above answers

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$

Clicker question

You construct a spring–glider system that oscillates with simple harmonic motion at a frequency *f*. If you replace the glider with one having one-fourth the mass, what is the system's new frequency?

a) *f*

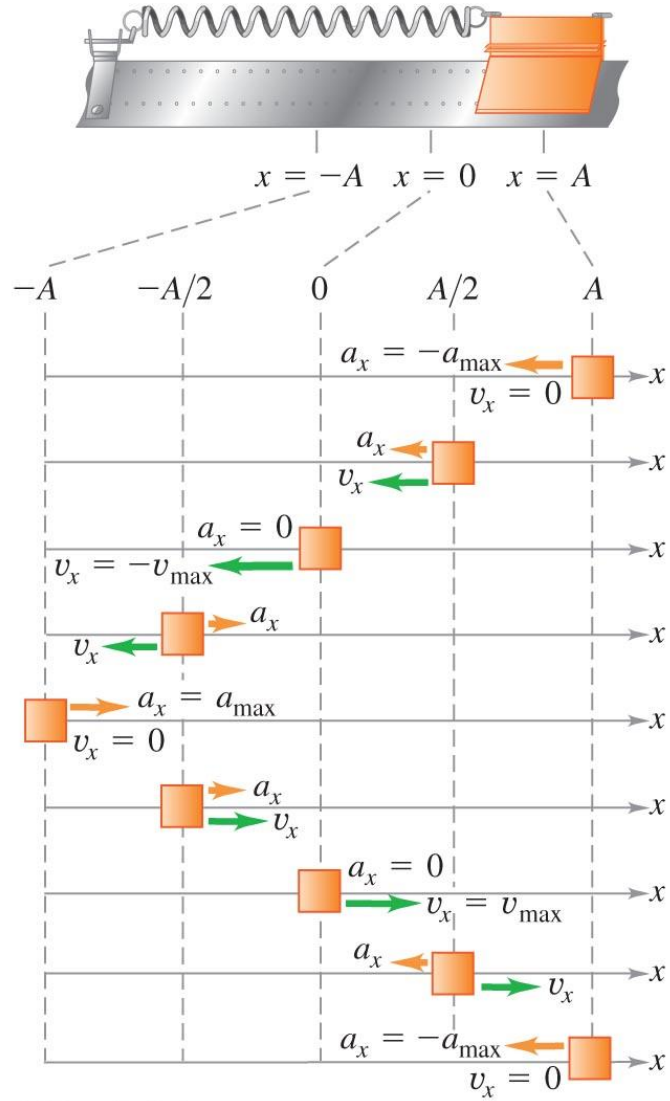
b) 2*f*

c) 1/2*f*

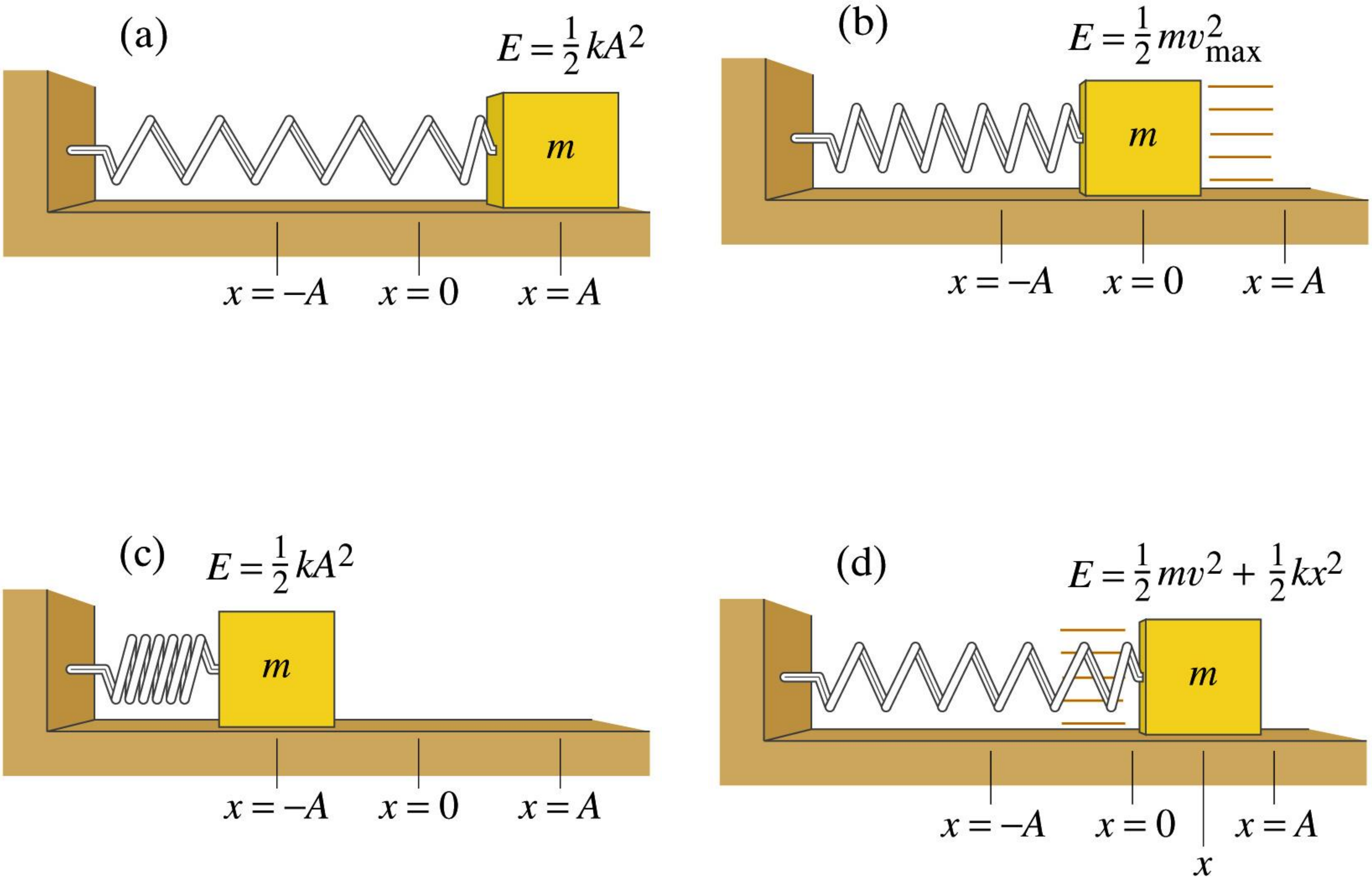
d) 1/4*f*

Pause and Consider Our Terminology – Figure 11.15

- Oscillation
- Restoring force
- SHM
- Hooke's Law
- Amplitude (*A*) … in meters
- Cycle
- Period (*T*) … in seconds
- Frequency (*f*) … in 1/s or Hz
- Angular frequency (ω) … in rad/s

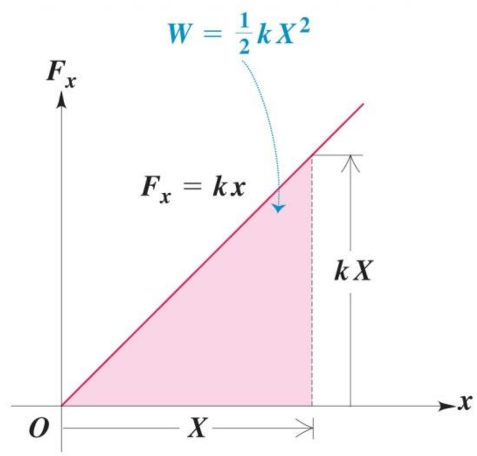


Energy in SHM



A force varying with distance is the basis of SHM

Hooke's law; $F = kx$ for Spring

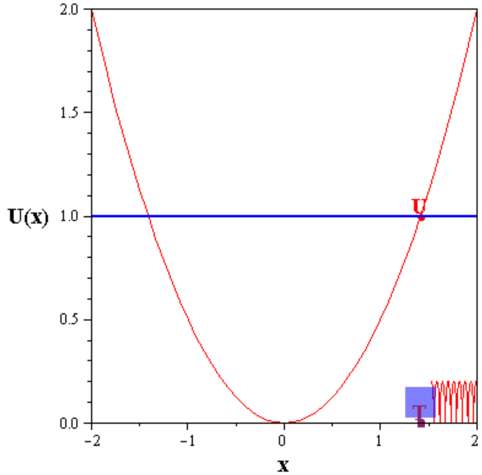


Force x distance = work = energy [J=N.m]

The total work done on spring by $\boldsymbol{F}$ is area under the above graph;

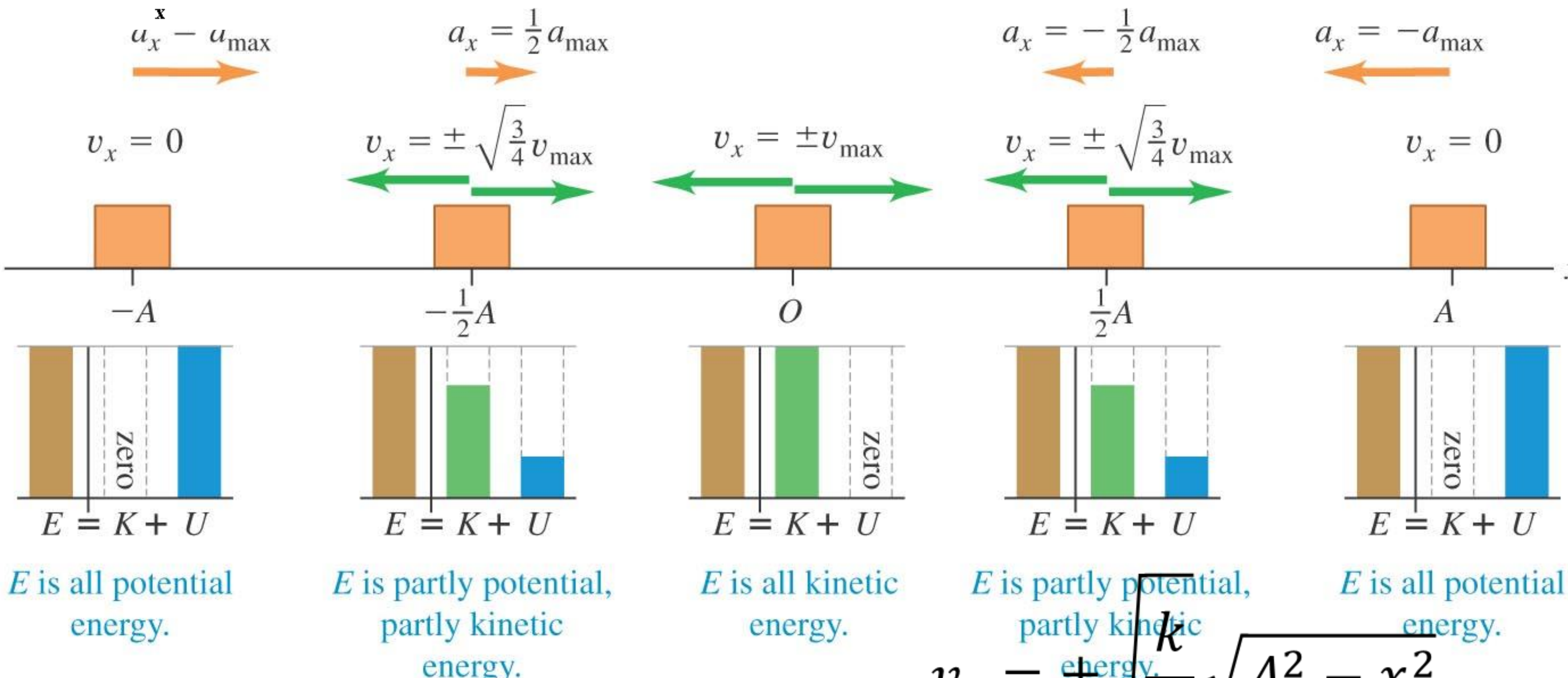
$$W = \frac{1}{2}xF = \frac{1}{2}x(kx) = \frac{1}{2}kx^2 = \text{potential energy}$$

Energy in SHM Energy Changes as the Oscillator Moves –Figure 11.17



- Energy is conserved during SHM and the forms (potential and kinetic) interconvert as the position of the object in motion changes.

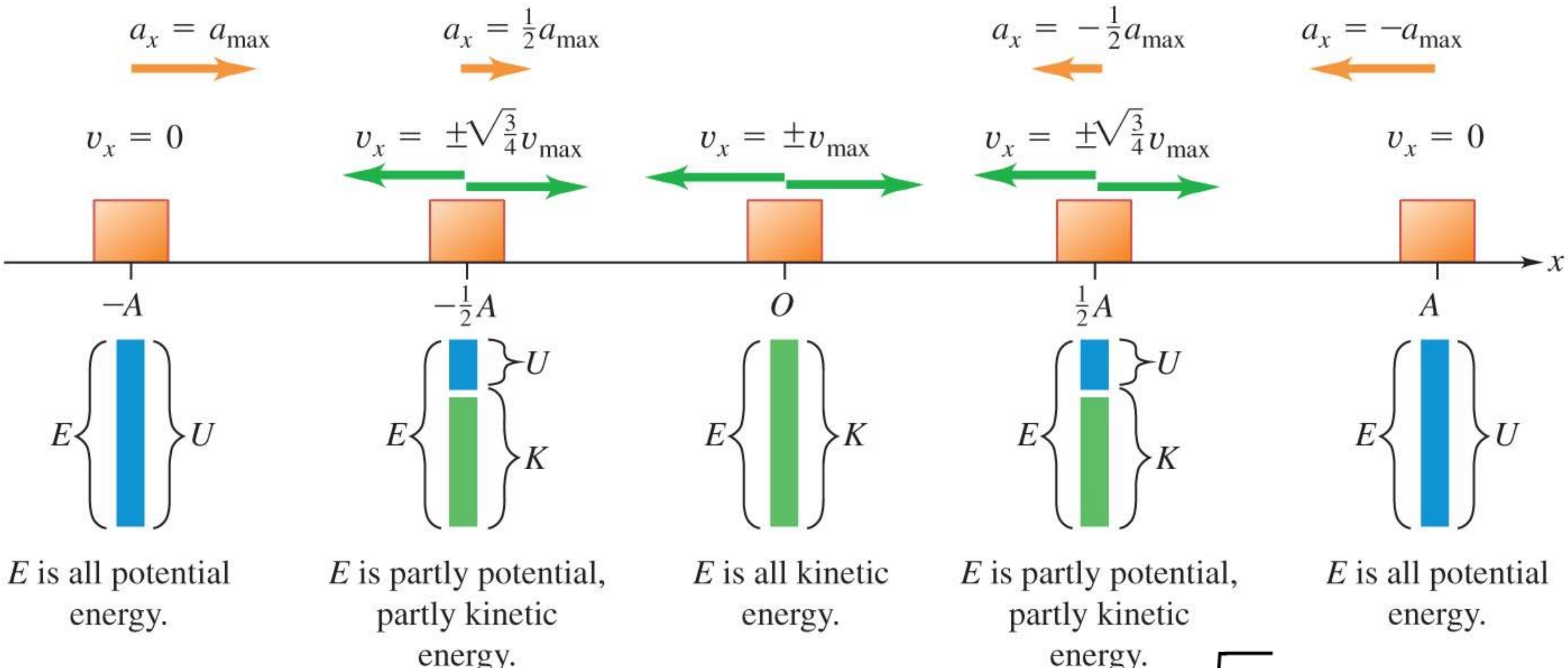
$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$



$$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$$

Energy Changes as the Oscillator Moves – Figure 11.17

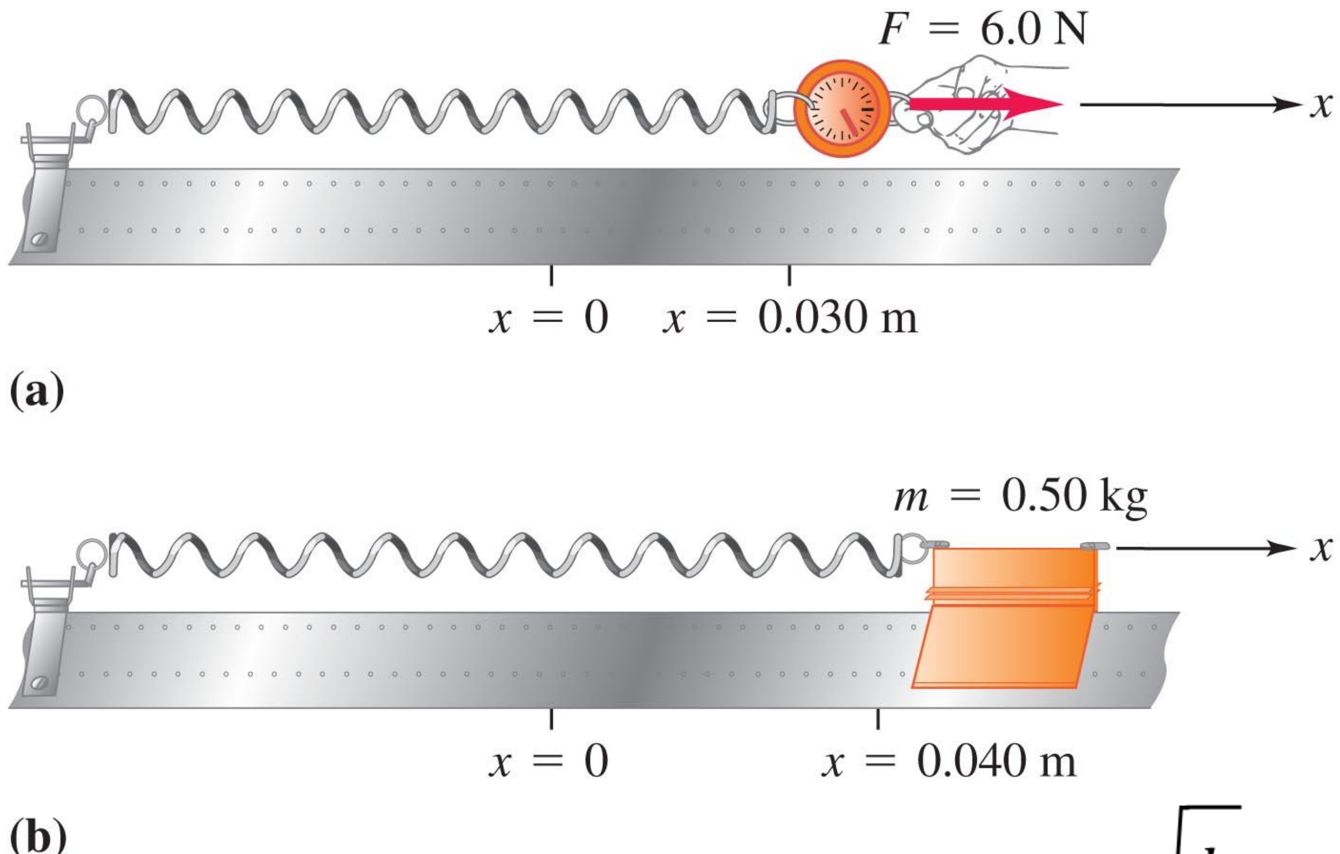
- Conserved in the absence of friction, energy converts between kinetic and potential.



$$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$$

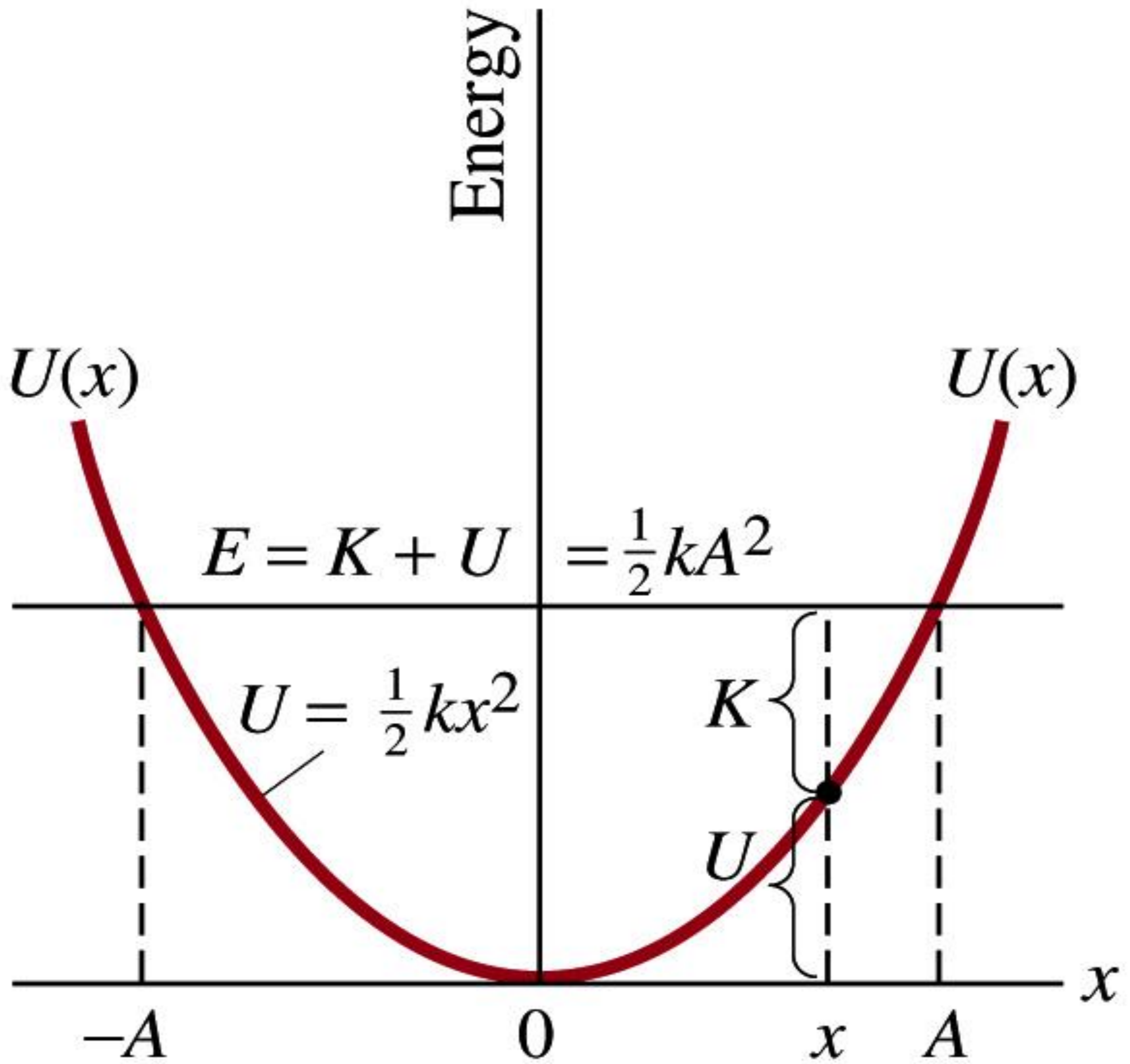
A Problem Using an Air Track – Example 11.5

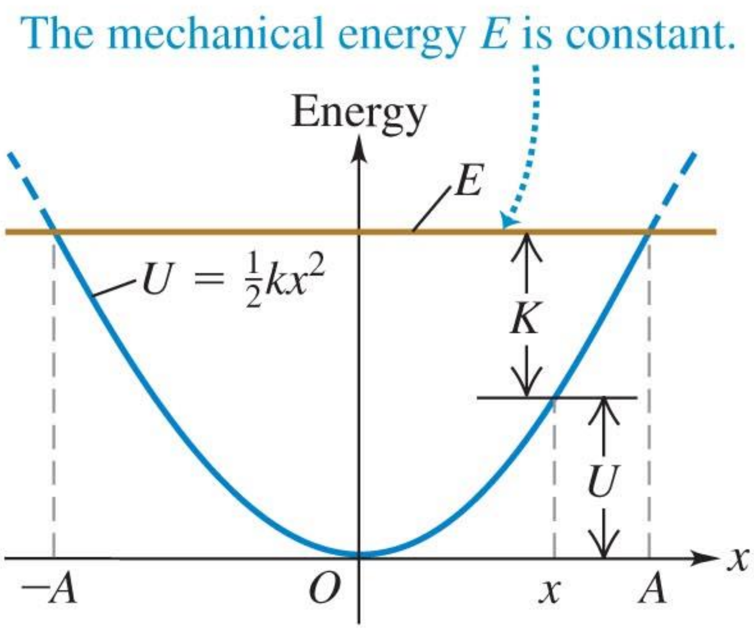
- Refer to the solved problem on page 334.



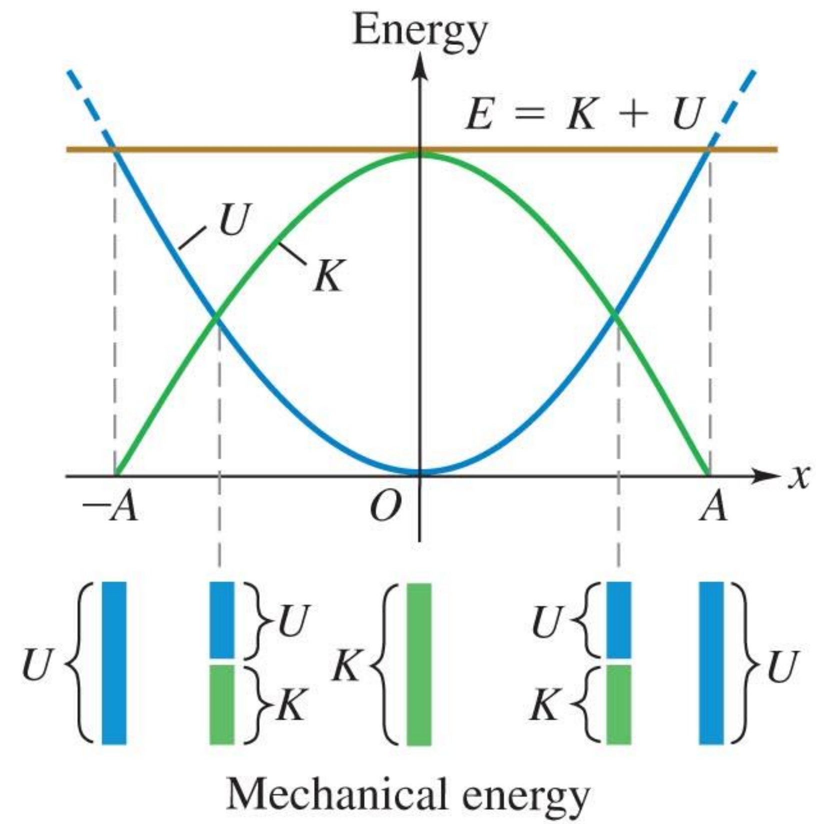
$$k = \frac{-F_x}{x} \quad E = \frac{1}{2}kA^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 \quad v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$$

Energy conservation in SHM





(a) The potential energy U and total energy E of an object in SHM as a function of x position

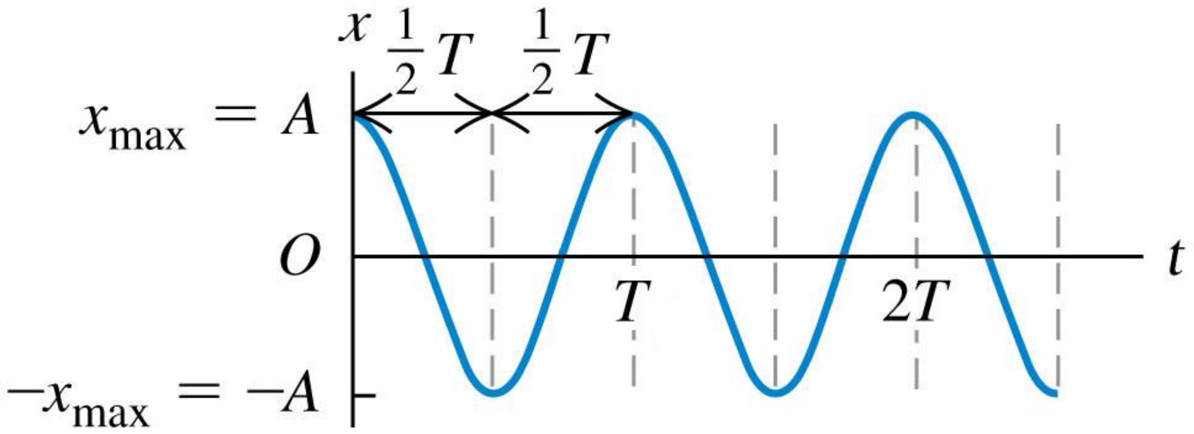


(b) The same graph as in **(a)**, showing kinetic energy K as well

Q14.6

Clicker question

This is an *x-t* graph for an object connected to a spring and moving in simple harmonic motion. At which of the following times is the *potential energy* of the spring the greatest?



A. $t = T/8$

B. $t = T/4$

C. $t = 3T/8$

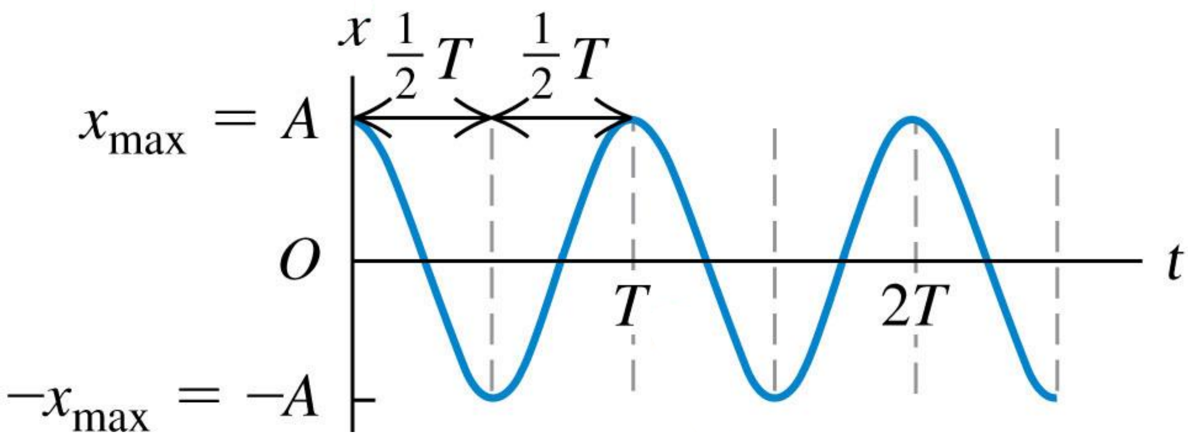
D. $t = T/2$

E. Two of the above are tied for greatest potential energy.

Q14.7

Clicker question

This is an *x*-*t* graph for an object connected to a spring and moving in simple harmonic motion. At which of the following times is the *kinetic energy* of the object the greatest?



A. $t = T/8$

B. $t = T/4$

C. $t = 3T/8$

D. $t = T/2$

E. Two of the above are tied for greatest kinetic energy.

Graphic Description of Position, Velocity, and Acceleration

The position: $x = A\cos(\phi) = A\cos(\omega t)$

The velocity: $v_x = -\,v\,\sin(\phi) = -(\omega A)\sin(\omega t)$

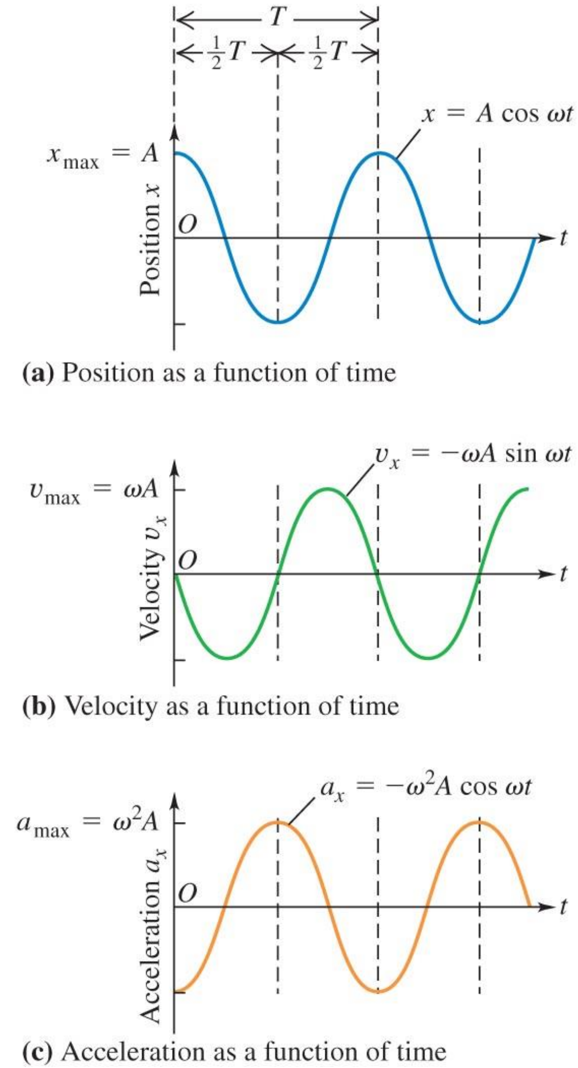
The acceleration: $a_x = -\,a_{rad}\,\cos(\phi) = -\omega^2 A\,\cos(\omega t)$
$= -\omega^2 x$

Parameters:

$$\omega^2 = k/m \quad \text{or} \quad \omega = \sqrt{k/m},$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k/m}$$

$$T = \frac{1}{f} = 2\pi\sqrt{m/k}$$



A few notes about SHM:

- The angular frequency, period, and frequency are all independent of the amplitude.

$$\omega = \sqrt{k/m}; \qquad T = \frac{1}{f} = 2\pi\sqrt{m/k}; \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k/m}$$

- If $\phi_o \neq 0$ at $t = 0$, the previously derived expressions remain correct if the angular
  position $\phi = \omega t$ is replaced by $\phi = \phi_o + \omega t$. For example, the position is

$$x = Acos(\phi) = Acos(\phi_o + \omega t)$$

- Since $x = Acos(\phi)$, we may re-write

$$v_x = -\omega Asin(\omega t) = \pm\omega A\sqrt{1 - [\cos(\omega t)]^2}$$

$$= \pm\omega\sqrt{A^2 - [A\cos(\omega t)]^2} = \pm\omega\sqrt{A^2 - x^2}$$

$$= \pm\sqrt{\frac{k}{m}}\sqrt{(A^2 - x^2)}$$

11.5 The Simple Pendulum

The mass moves along a circular path but the tangential speed is not a constant. Define $\theta = 0$ as the vertical and counter-clockwise as positive. Define x as the position of the mass along the arc in a similar way.

The tangential force is $F_x = -mgsin(\theta)$.

In the small angle limit $F_x \approx -mg\theta = -mg\frac{x}{L}$.

Newton's Second Law $-mg\frac{x}{L} = ma_x$ or $-\frac{g}{L}x = a_x$

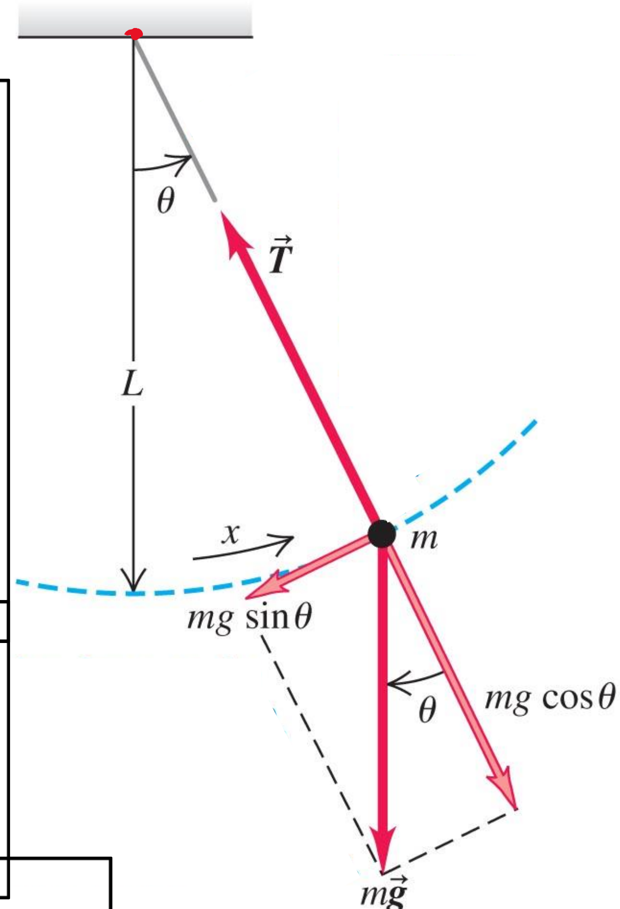
Compared with the SHM of a mass-loaded spring,

Spring restoring force $F_x = -kx$.

Newton's Second Law $-kx = ma_x$ or $-\frac{k}{m}x = a_x$

Conclusions for the simple pendulum:

$\omega = \sqrt{g/L}$; $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{g/L}$; $T = \frac{1}{f} = 2\pi\sqrt{L/g}$



(5 pts) 18. A small object with mass 0.20 kg swings as a pendulum on the end of a long light rope. For small amplitude of swing, the period of the motion is 3.0 s. If the object is replaced by one with mass 0.400 kg, what is the period for small amplitude of swing?

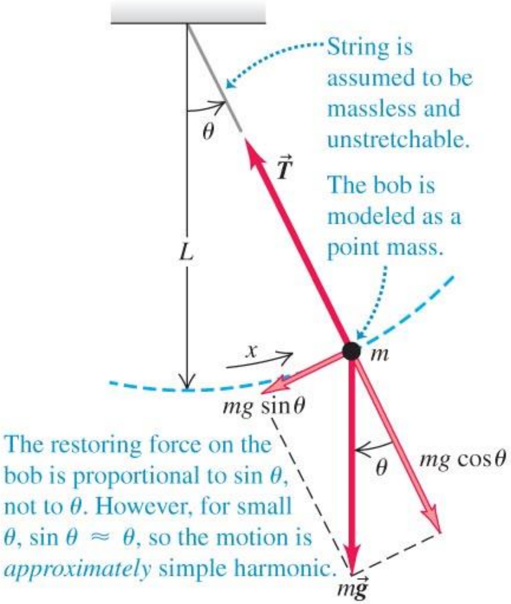
(a) 1.5 s
(b) 3.0 s
(c) 6.0 s
(d) 12.0 s
(e) none of the above answers

$$\omega = \sqrt{\frac{g}{L}} \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \quad T = 2\pi\sqrt{\frac{L}{g}}$$

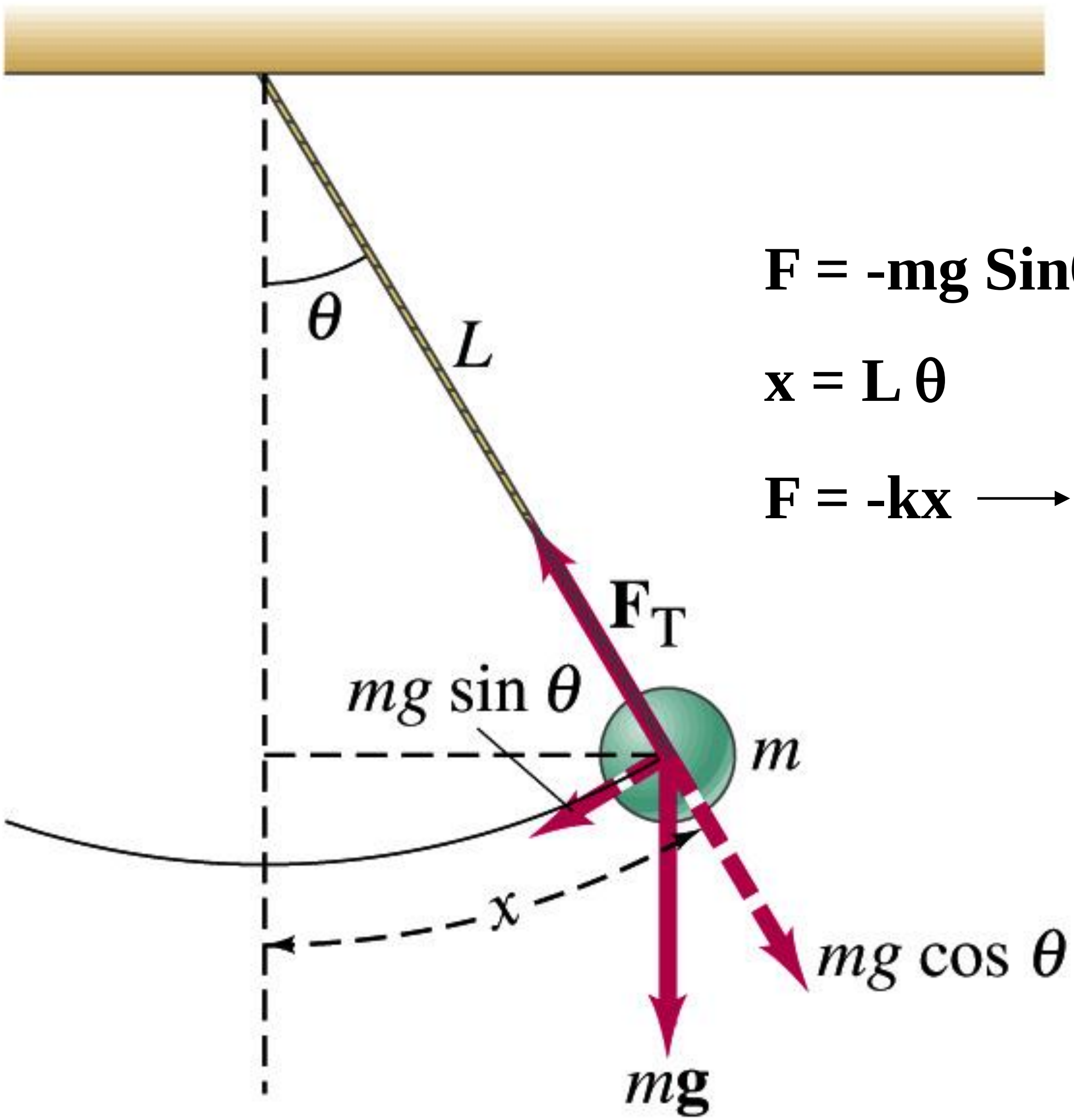
A Pendulum Undergoes Harmonic Motion –

- The pendulum is a good example of harmonic motion.
- Oscillations depend on the length of the pendulum and the gravitational restoring, force BUT not the mass.
- .

(a) A real pendulum



(b) An idealized simple pendulum



$$F = -mg\,\mathrm{Sin}\theta \;\approx -mg\,\theta$$

$$x = L\,\theta$$

$$F \approx -\frac{mg}{L}\,x$$

$$F = -kx \longrightarrow k = \frac{mg}{L}$$

$$\theta = \theta_{max}\,\mathrm{Cos}(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{L}}$$

Note: mass doesn't enter

amplitude doesn't enter

Clicker question

Foucault pendulum at TAMU

a) L=25 m
b) L=10 m
c) L=30 m
d) L=35 m

Clicker question

You install two rope swings from a tree in your yard. The rope for swing *A* is 1/4 as long as the rope for swing *B*. Assuming they behave like ideal pendulums, how do their periods compare?

a) $T_A = T_B$

b) $T_A = 1/2 T_B$

c) $T_A = 2T_B$

d) $T_A = \sqrt{2T_B}$

What is the period of a pendulum on mars (g(mass)=3.71 m/s^2), if the period of this pendulum on earth is 1.6 sec.

$$g'(mars) = 3.71\,\frac{m}{s^2}$$

$$\therefore T = 2\pi\sqrt{\frac{L}{g}}$$

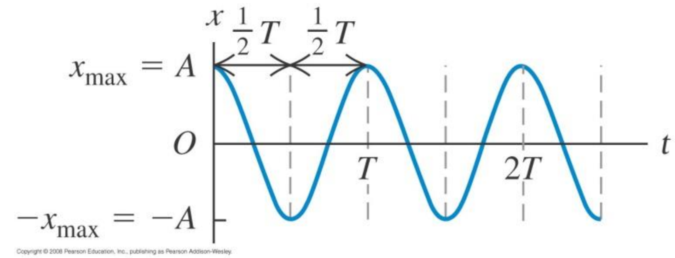
So; $$T'(mars) = 2\pi\sqrt{\frac{L}{g'}}$$

$$\frac{T'}{T} = \sqrt{\frac{g}{g'}} \rightarrow T' = T\sqrt{\frac{g}{g'}} = 1.6\sqrt{\frac{9.81}{3.71}} = 2.6\,sec$$

SUMMARY

Periodic motion: motion that repeats itself in a defined cycle. $f = \frac{1}{T}$ $T = \frac{1}{f}$ $\omega = 2\pi f = \frac{2\pi}{T}$

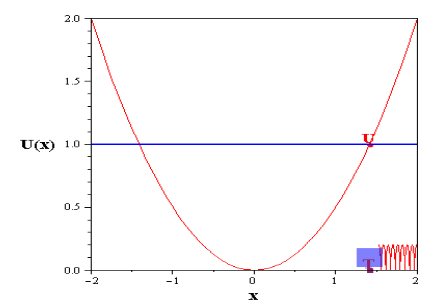
Simple harmonic motion: if the restoring force is proportional to the distance from equilibrium, the motion will be of the SHM type. The angular frequency and period do not depend on the amplitude of oscillation.



$$F_x = -kx \qquad a_x = \frac{F_x}{m} = -\frac{k}{m}x$$

$$\omega = \sqrt{\frac{k}{m}} \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

$$x = A\cos(\omega t + \phi)$$



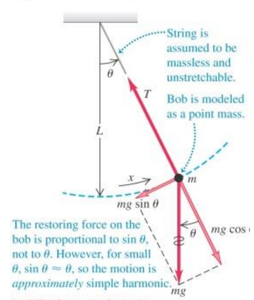
Energy in SHM:

$$E = \frac{1}{2}m\text{v}_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}m\text{v}_{max}^2$$

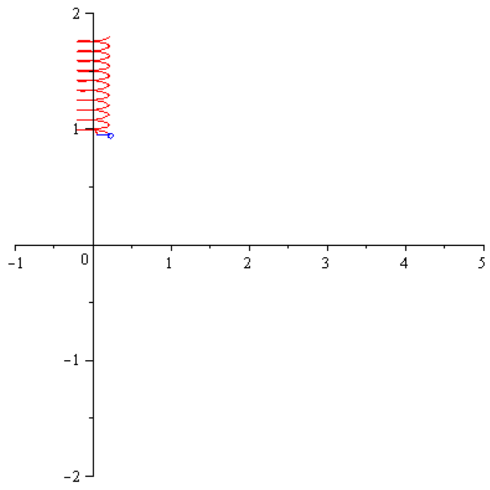
Simple pendulum:

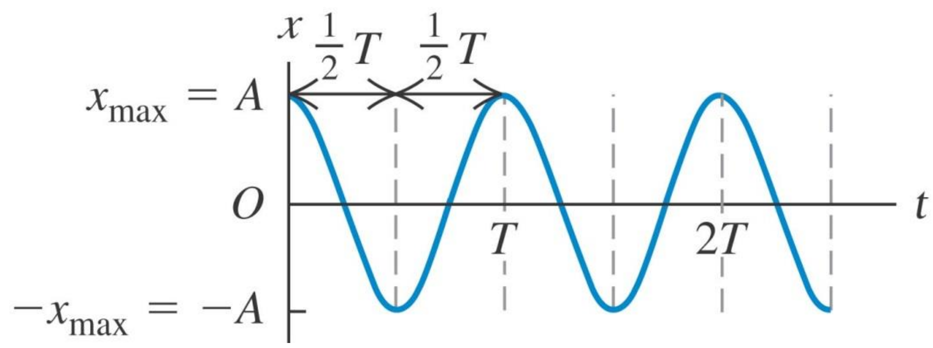
$$\omega = \sqrt{\frac{g}{L}} \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \quad T = 2\pi\sqrt{\frac{L}{g}}$$

(b) An idealized simple pendulum

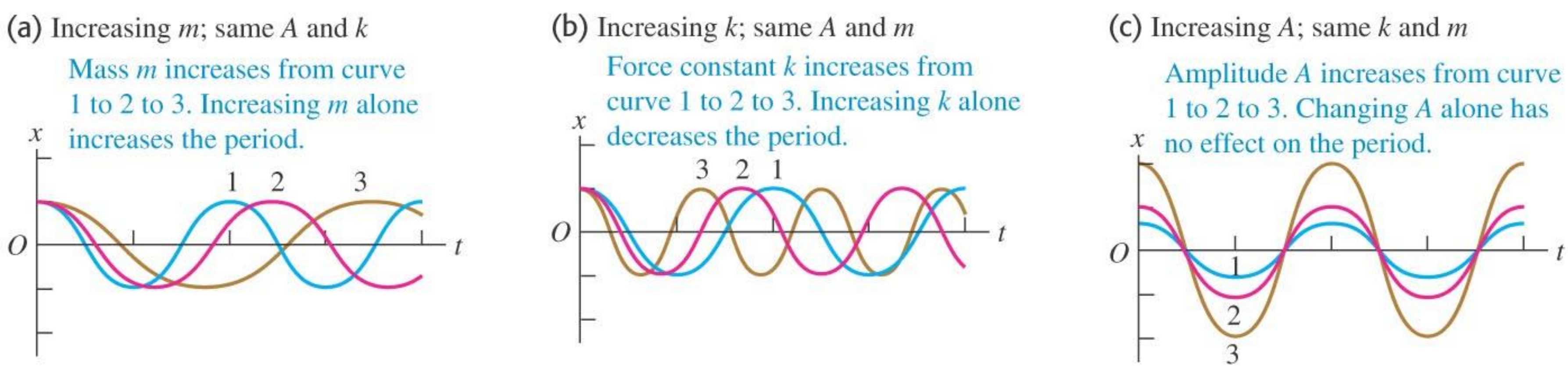


X versus *t* for SHM then simple variations on a theme





$$x(t) = A\cos(\omega t + \phi)$$



VERY IMPORTANT: frequency and period of oscillations DO NOT depend on the amplitude!!

Q14.9

Clicker question

A simple pendulum consists of a point mass suspended by a massless, unstrechable string. If the mass is doubled while the length of the string remains the same, the period of the pendulum

A. becomes four times greater.

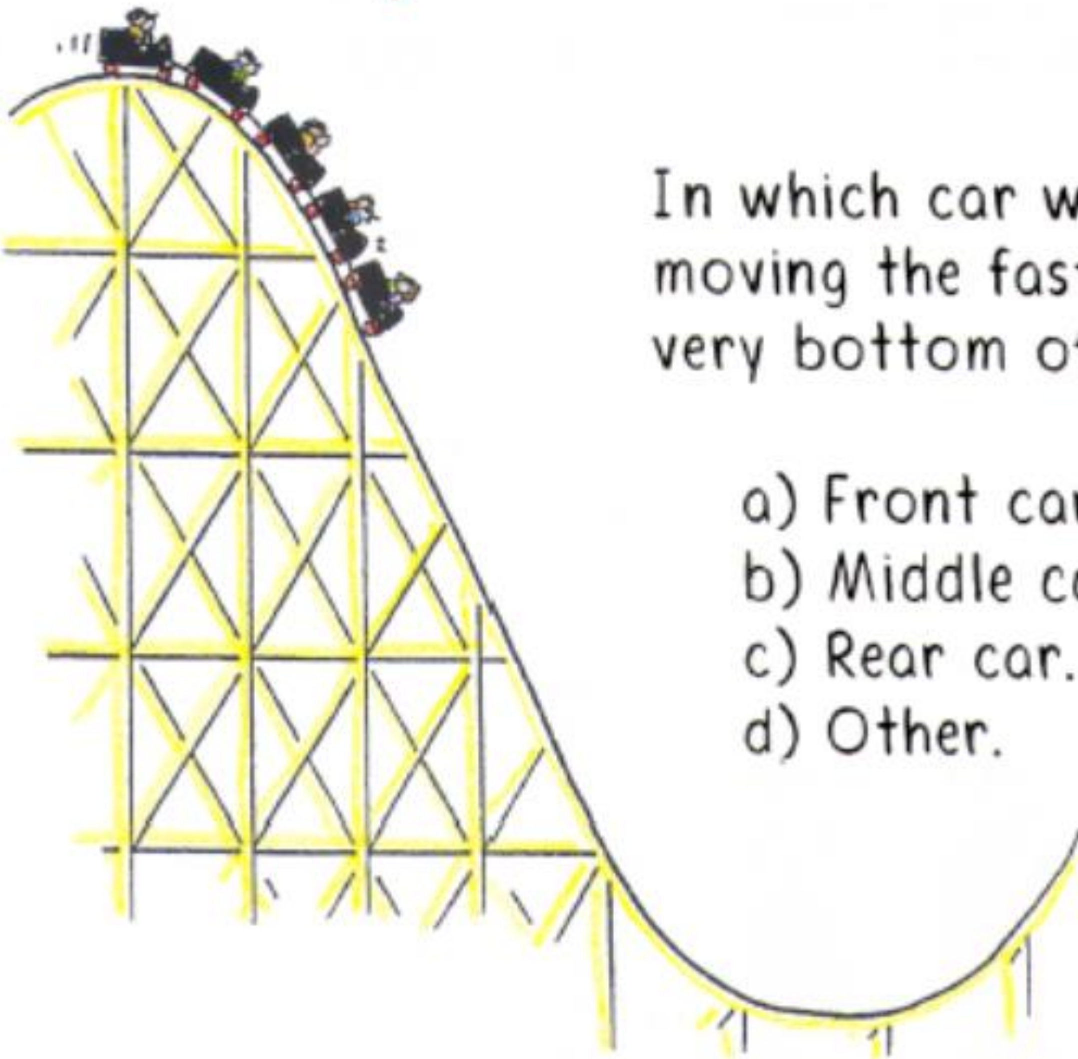
B. becomes twice as great.

C. becomes smaller by a factor of $\sqrt{2}$.

D. remains unchanged.

E. decreases.

Next-Time Question



In which car will you be moving the fastest at the very bottom of the incline?

a) Front car.
b) Middle car.
c) Rear car.
d) Other.

thanx to Joe Delaney

Hewitt Drew it!

Office hours:

Carlos Tuesday and Wednesday at1pm-2pm in MPHY 470

SI session on Sundays

- 'Jonah Dean' via 202331 PHYS 201 all <cs-phys201-202331@lists.tamu.edu>
- cs-phys201-202331@lists.tamu.edu
- SI session at 6:00 - 7:15 In ILCB 224, see you there!

Notification

- There will be no attendance quiz on Thursday
- I have updated the lecture for chapter 11
- Carlos updated study guide for exam3

Series & Parallel Spring

- Two identical springs are linked together first in series and then in parallel. A mass is hung from each configuration and the effective spring constant is measured and compared to the spring constant of a single spring.

a) k (effective) is the same for 2 springs in series
b) k (effective) is different 2 springs in parallel
c) k is constant for single spring