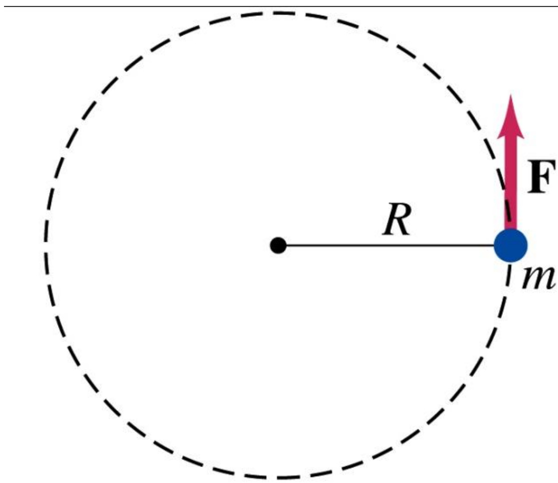
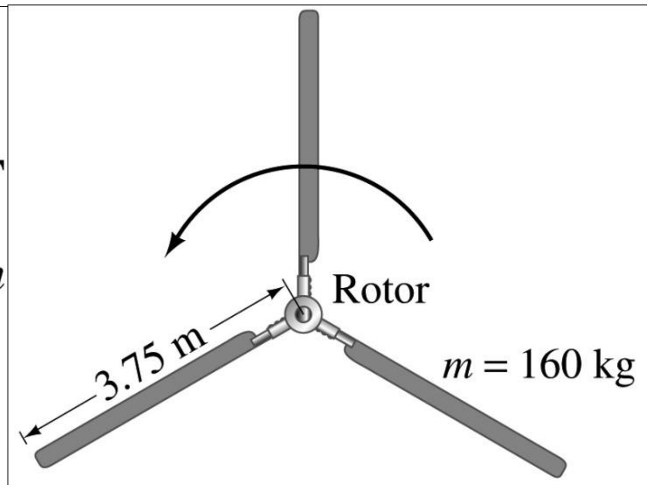
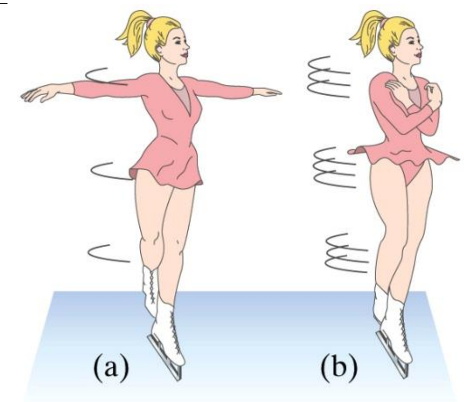
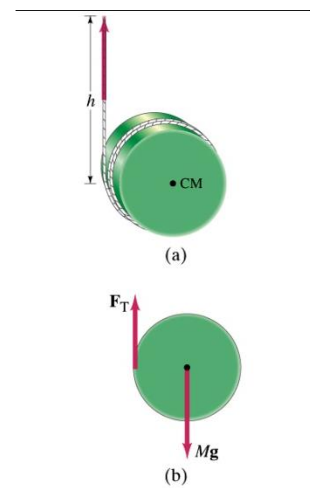
Chapter 9 Rotational Motion









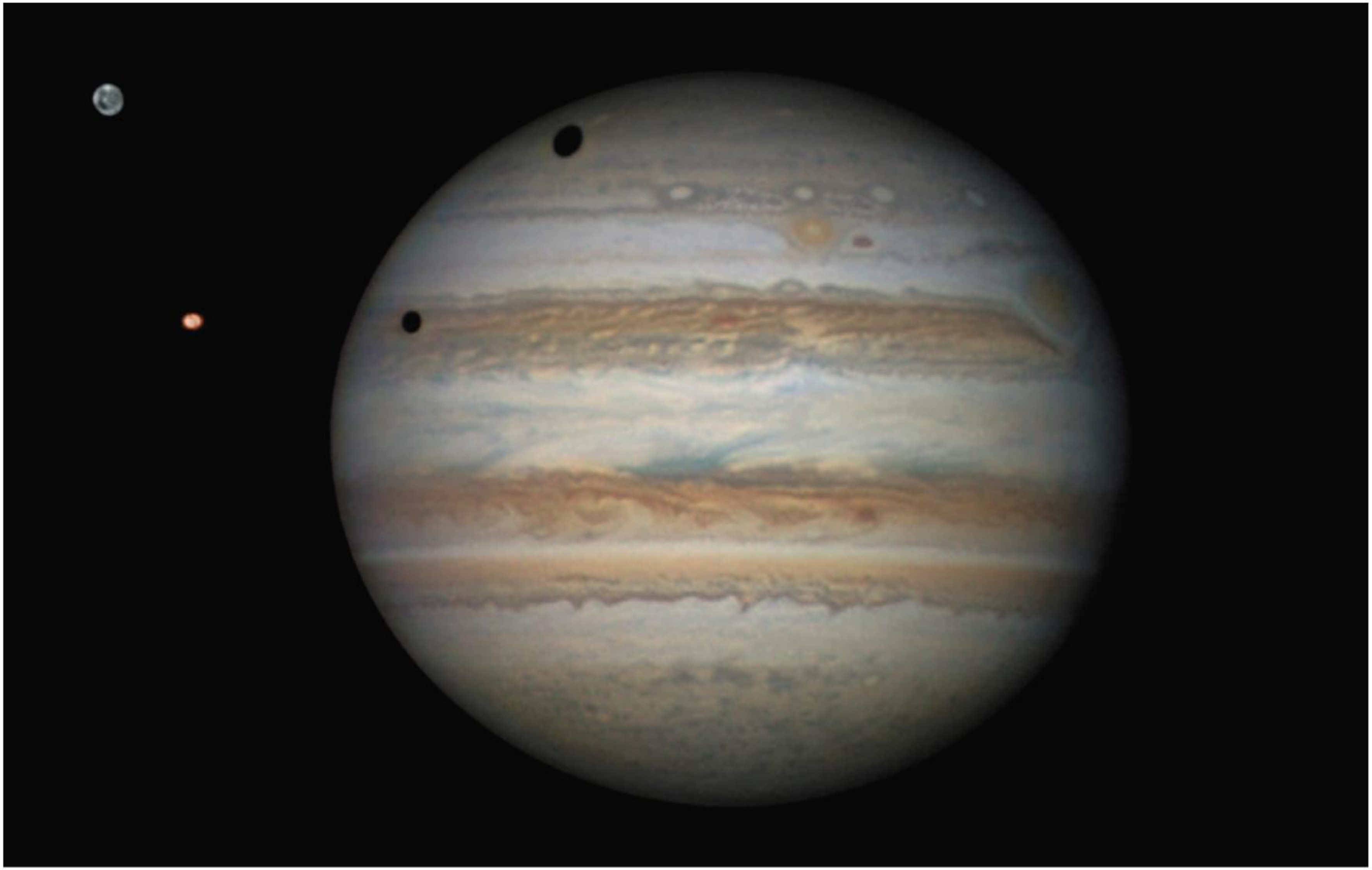
Rigid body instead of a particle
Rotational motion about a fixed axis
Rolling motion (without slipping)

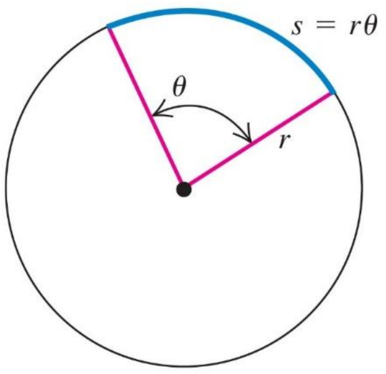
Chapter 9 Rotational Motion

- To study angular velocity and angular acceleration.
- To examine rotation with constant angular acceleration.
- To understand the relationship between linear and angular quantities.
- To determine the kinetic energy of rotation and the moment of inertia.
- To study rotation about a moving axis.

Courtesy of Wenhao Wu

Jupiter is the fastest rotating planet 1rotation takes about 10hours





9.1	Angular Position, Angular Velocity, and Angular Acceleration

Angular Position: θ in radians (rad.)

Average Angular Velocity: $\omega_{av} = \frac{\theta_2-\theta_1}{t_2-t_1} = \frac{\Delta\theta}{\Delta t}$

Instantaneous Angular Velocity : $\omega = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}$ (rad./s)

Units and Conversion:

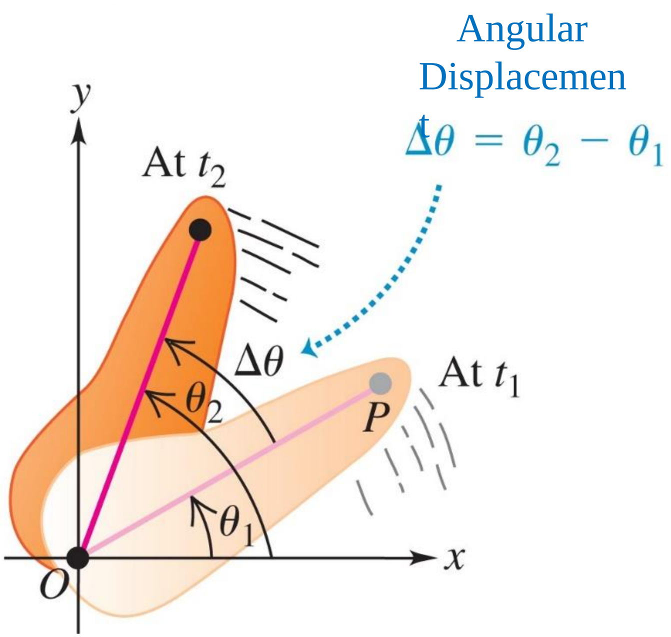
2π rad. = 360º; 1 rad. = 360º/2π = 57.3º

1 rev/s = 2π rad./s; 1 rpm = 2π/60 rad./s

Angular Acceleration (average): $\alpha_{av} = \frac{\omega_2-\omega_1}{t_2-t_1} = \frac{\Delta\omega}{\Delta t}$

Angular Acceleration (instantaneous): $\alpha = \lim_{\Delta t\to 0}\frac{\Delta\omega}{\Delta t}$

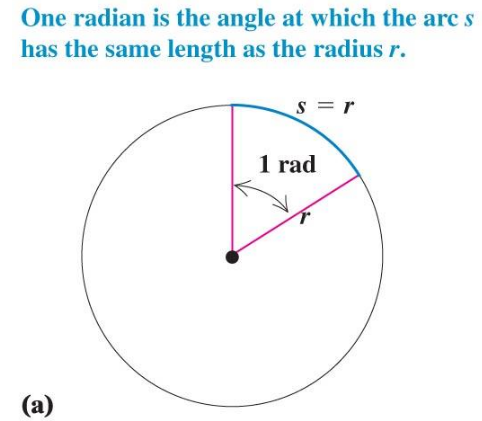
Units: rad./s^2



Courtesy of Wenhao Wu

3 word dictionary

Concepts of rotational motion



Distance d → angle θ
Velocity $\vec{v}$ → angular velocity $\vec{w}$
Acceleration $\vec{a}$ → angular acceleration $\vec{\propto}$

Angle θ

How many degrees are in one radian ? (rad is the unit if choice for rotational motion)

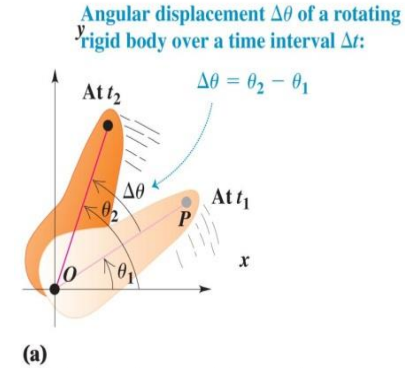
$$\theta = \frac{s}{r} \rightarrow \text{ratio of two lengths}$$

(dimensionless)

$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi\, rad \;\cong 360°$$

$$1\, rad \cong \frac{360°}{2\pi} = \frac{360°}{6.28} = 57° \;\; \therefore \text{Factors of unity } \frac{1\, rad}{57°} \text{ or } \frac{57°}{1\, rad}$$

1 radian is the angle subtended at the center of a circle by an arc with length
equal to the radius.



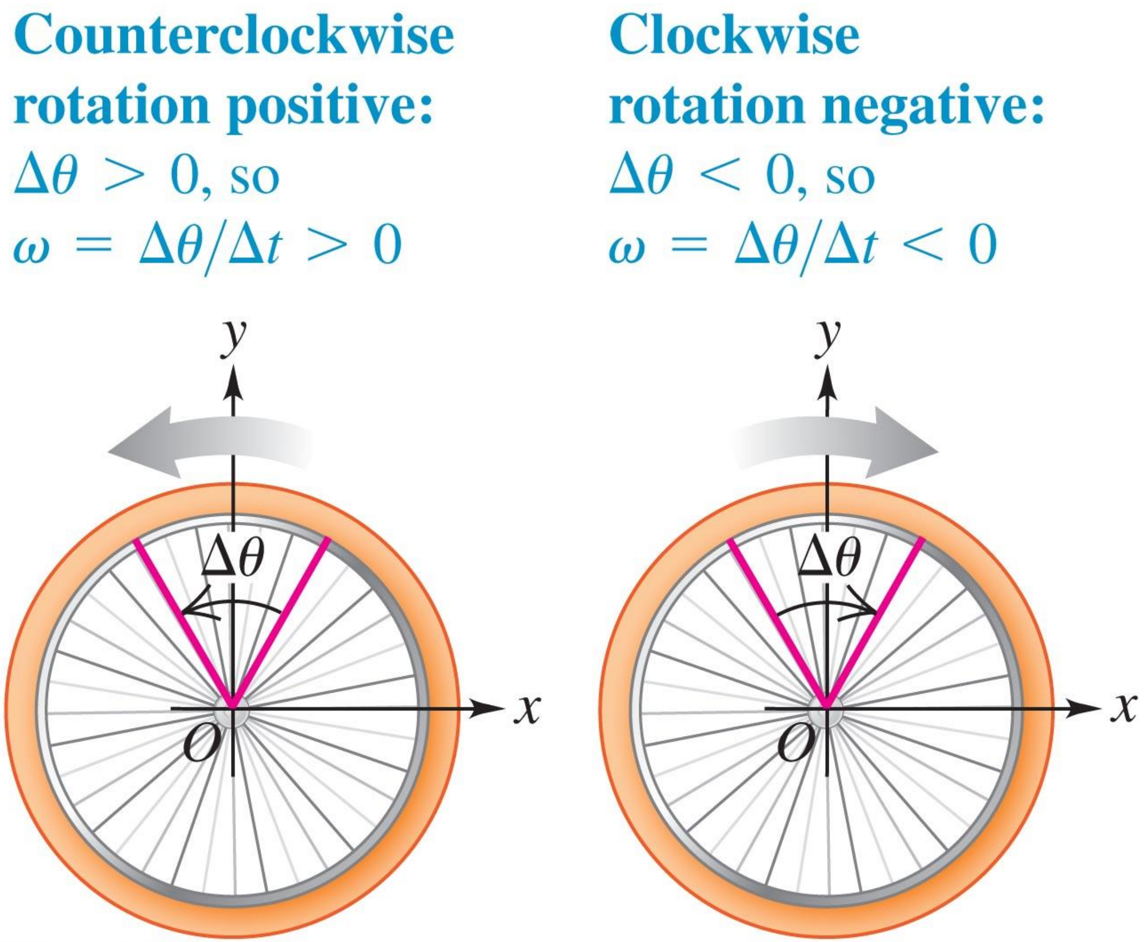
Angular velocity $\vec{w}$

$$w_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \;\; \left[\frac{rad}{s}\right] \;\; \rightarrow w = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \;\; \left[\frac{rad}{s}\right]$$

Other units are;

$$1\frac{rev}{s} = \frac{2\pi\, rad}{s} \qquad \therefore 1\frac{rev}{min} = 1\, rpm = \frac{2\pi}{60}\frac{rad}{s}$$

We Have a Sign Convention – Figure 9.7



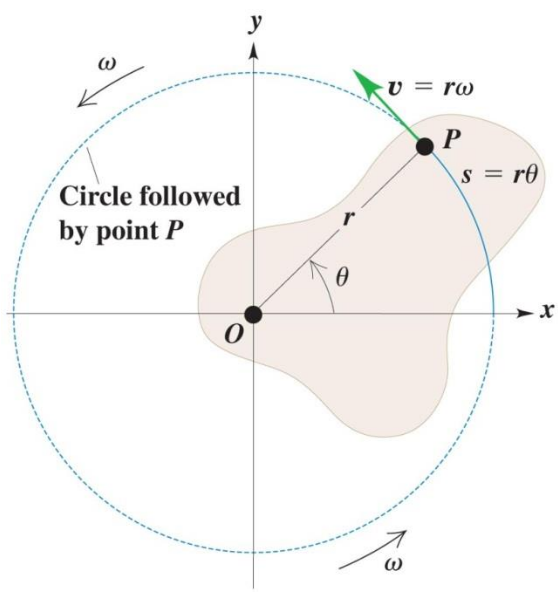
Angular acceleration $\vec{\propto}$

$$\propto_{av} = \frac{w_2 - w_1}{t_2 - t_1} = \frac{\Delta w}{\Delta t} \quad \left[\frac{rad}{s^2}\right] \quad \rightarrow \propto = \lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} \quad \left[\frac{rad}{s^2}\right]$$

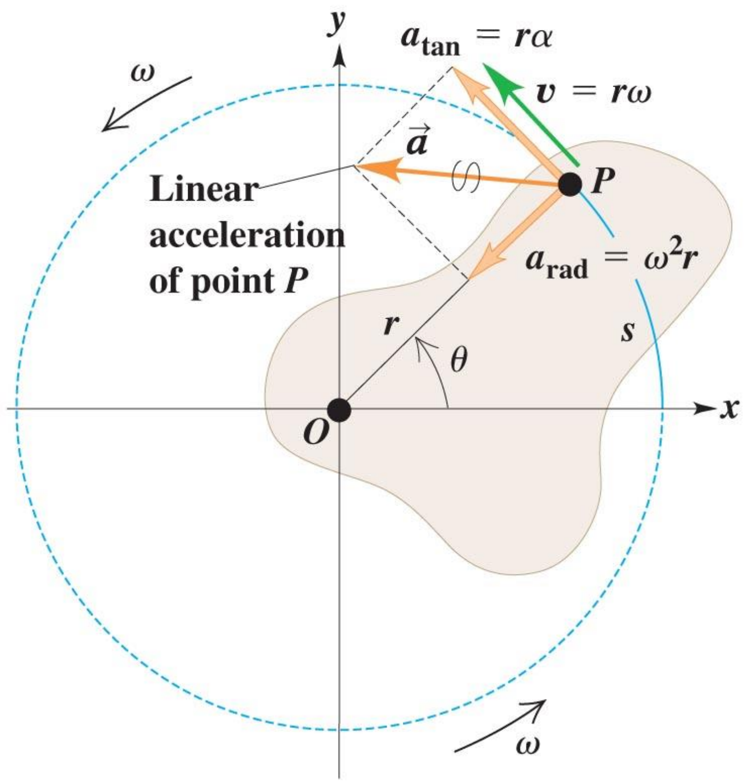
Relationship between linear and angular quantities

$$s = \theta r \rightarrow v_{av} = \frac{\Delta s}{\Delta r} = r\frac{\Delta \theta}{\Delta t} = r w_{av}$$

$$\therefore \Delta t \rightarrow 0 \quad gives \quad v = rw$$



Radial and tangential acceleration components:
- a_{rad} is point P's centripetal acceleration.
- a_{tan} means that P's rotation is speeding up (the body has angular acceleration).



Tangential component of acceleration;

$$\Delta v = r\Delta w$$

$$(a_{tangent})_{av} = \frac{\Delta v}{\Delta t} = r\frac{\Delta w}{\Delta t}$$

For; $\boldsymbol{\Delta t \to 0}$; $\boldsymbol{a_{tangent} = r \propto}$

Radial component

$$\boldsymbol{a_{rad} = \frac{v^2}{r} = w^2 r}$$

Magnitude of $\vec{a}$

$$\boldsymbol{|\vec{a}| = a = \sqrt{a_{rad}^2 + a_{tan}^2}}$$

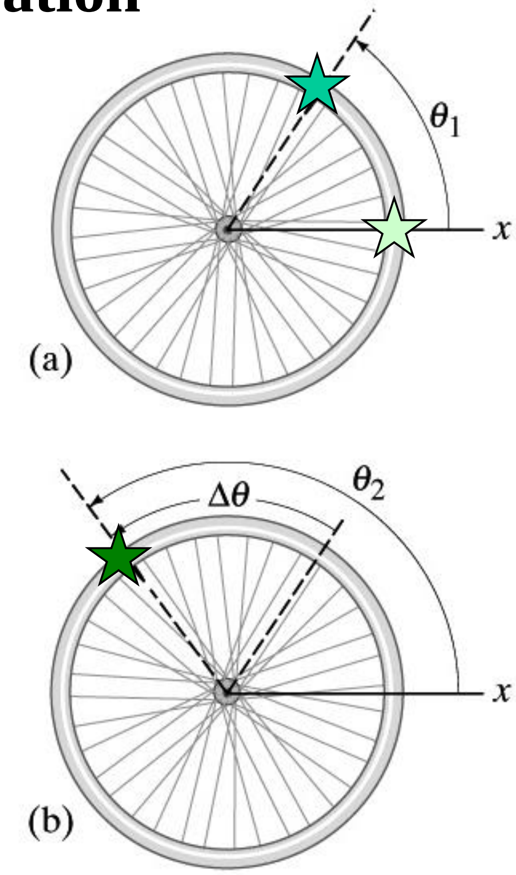
Angular Quantities

Kinematical variables to describe the rotational motion:
→Angular position, velocity and acceleration

$$\theta = \frac{l}{R} \quad (rad)$$

$$\omega = \lim_{\Delta t \to 0} \underbrace{\frac{\Delta\theta}{\Delta t}}_{\omega_{ave}} = \frac{d\theta}{dt} \quad (rad/s)$$

$$\alpha = \lim_{\Delta t \to 0} \underbrace{\frac{\Delta\omega}{\Delta t}}_{\alpha_{ave}} = \frac{d\omega}{dt} \quad (rad/s^2)$$



Angular Quantities: Vector

Kinematical variables to describe the rotational motion:

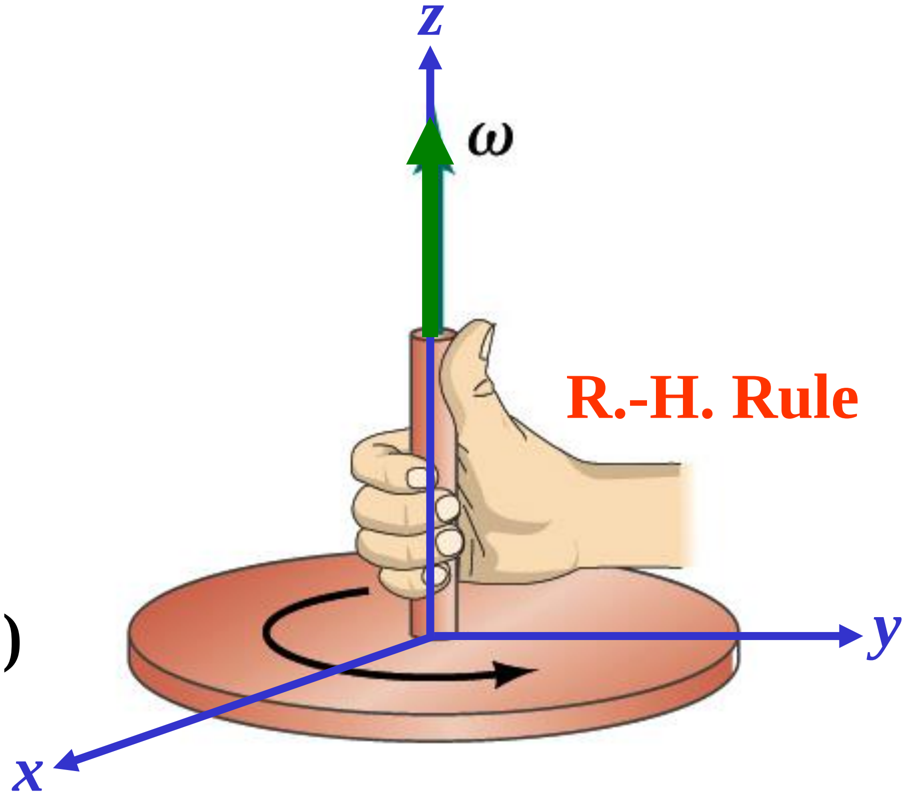
→Angular position, velocity and acceleration

→Vector natures

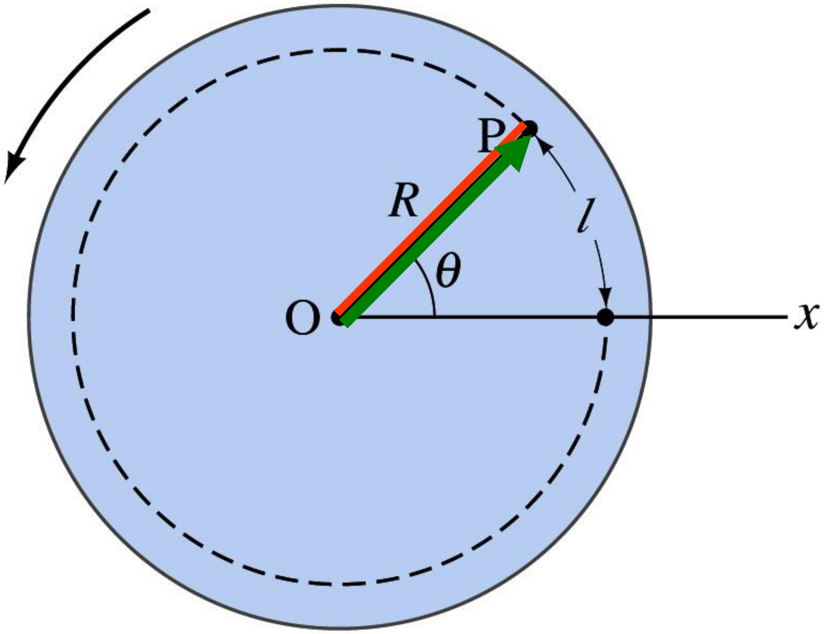
$$\theta = \frac{\boldsymbol{l}}{\boldsymbol{R}} \quad (\boldsymbol{rad})$$

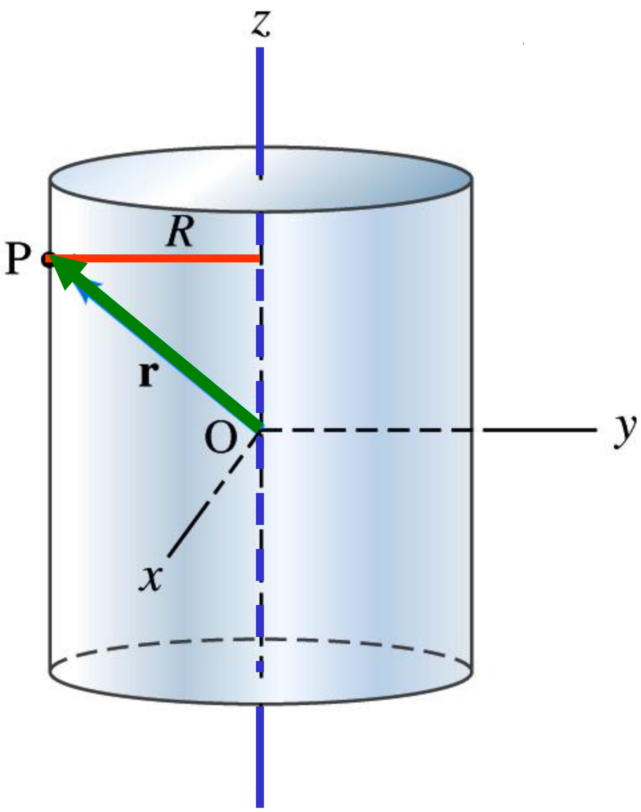
$$\omega \,\hat{\boldsymbol{k}} = \lim_{\Delta t \longrightarrow 0} \frac{\Delta\theta}{\Delta t}\hat{\boldsymbol{k}} = \frac{\boldsymbol{d}\theta}{\boldsymbol{dt}}\hat{\boldsymbol{k}} \quad (\boldsymbol{rad/s})$$

$$\alpha \,\hat{\boldsymbol{k}} = \lim_{\Delta t \longrightarrow 0} \frac{\Delta\omega}{\Delta t}\hat{\boldsymbol{k}} = \frac{\boldsymbol{d}\omega}{\boldsymbol{dt}}\hat{\boldsymbol{k}} \quad (\boldsymbol{rad/s^2})$$



"*R*" from the Axis (O)





Solid Disk **Solid Cylinder**

9.2 Rotation with Constant Angular Acceleration

Kinematic Equations for Rotational Motion ← compare → Kinematic Equations for Linear Motion

$$\omega(t) = \omega_0 + \alpha t \quad ....(9.7)$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 ...(2.11)$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \quad ...(9.12)$$

$$\omega_{av} = \frac{1}{2}[\omega(t) + \omega_0]......(9.8)$$

$$v_x(t) = v_{0x} + a_x t \quad ....(2.6)$$

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 ...(2.10)$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \quad ...(2.11)$$

$$v_{av,x} = \frac{1}{2}[v_x(t) + v_{0x}]......(2.7)$$

Courtesy of Wenhao Wu

Kinematical Equations for constant angular acceleration

$$\textbf{\textit{Conversion}} \; : \boldsymbol{x} \rightarrow \theta, \boldsymbol{v} \rightarrow \omega, \boldsymbol{a} \rightarrow \alpha$$

$$(1)\; \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$(2)\; \omega = \omega_0 + \alpha t$$

$$(3)\; \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\textbf{\textit{Note}} : \alpha = \textbf{\textit{constant}}$$

Q9.2

Clicker question

A DVD is initially at rest so that the line *PQ* on the disc's surface is along the +*x*-axis. The disc begins to turn with a constant $\alpha_z = 5.0 \text{ rad/s}^2$. At $t = 0.40$ s, what is the angle between the line *PQ* and the +*x*-axis?

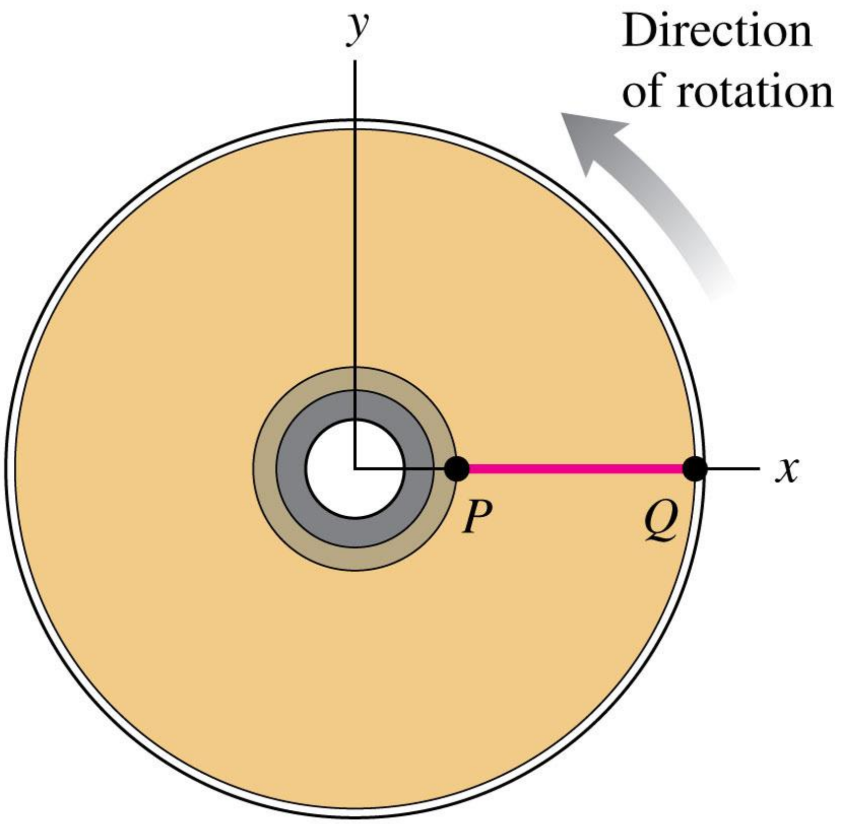
A. 0.40 rad

B. 0.80 rad

C. 1.0 rad

D. 1.6 rad

E. 2.0 rad

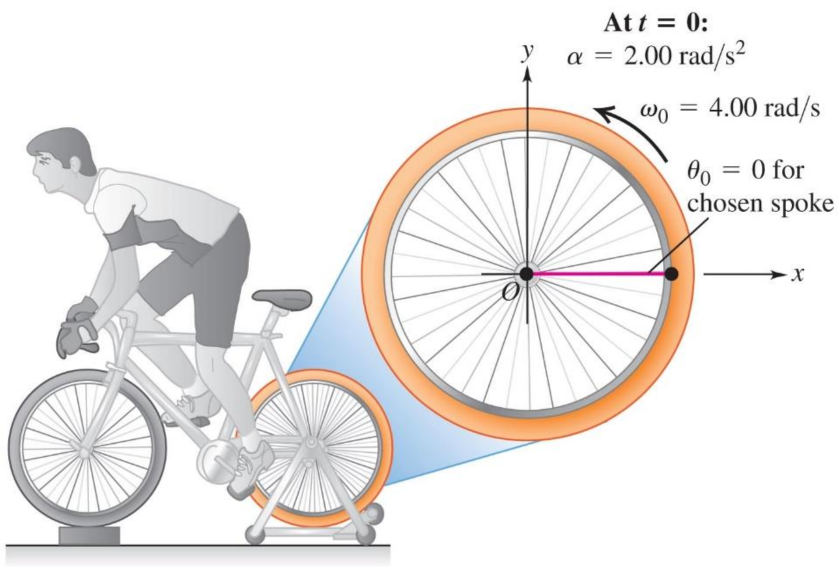


The rear wheel of the stationary bicycle is magnified for clarity

Example 9.2 on page 261
Rotation of a Bicycle Wheel

Given: (a) a = 2.00 rad/s^2
(b) At t = 0, $\theta_0 = 0$
(c) At t = 0, ω_0 = 4.00 rad/s

Find: (a) θ at t = 3.00 s
(b) ω at t = 3.00 s



Solution:

(a) $\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ = 0 + (4.00 rad/s^2)(3.00 s) + $\frac{1}{2}$ (2.00 rad/s^2)(3.00 s)^2
= 21.0 rad = (21.0 rad)/(2π rad/rev) = 3.34 rev

(b) $\omega(t) = \omega_0 + \alpha t$ = (4.00 rad/s) + (2.00 rad/s^2)(3.00 s) = 10.0 rad/s

or $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ = (4.00 rad/s)^2 + 2(2.00 rad/s^2)(21.0 rad) = 100 rad^2/s^2
ω = 10 rad/s

Rotation of a Bicycle Wheel – Figure 9.8

The rear wheel of the stationary bicycle is magnified for clarity

θ at t=3.00 s?

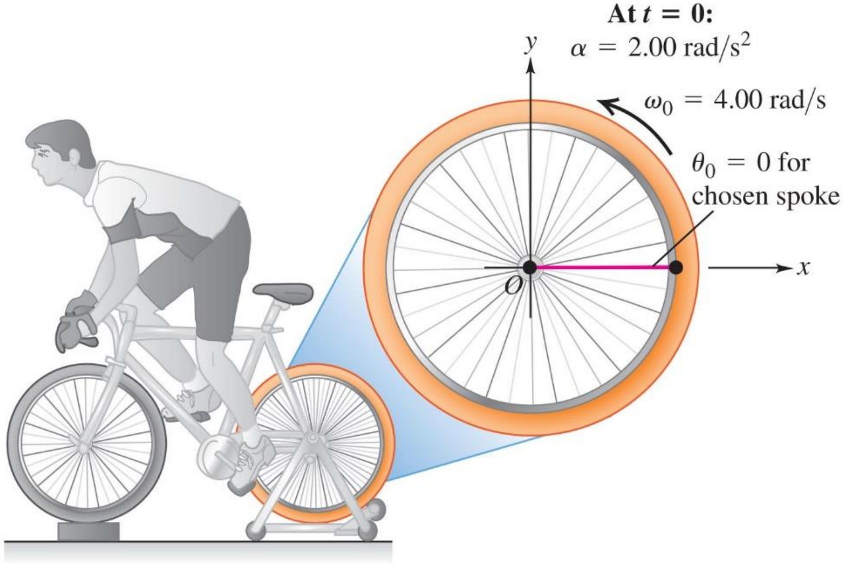
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 0 + \left(4.00\frac{\text{rad}}{\text{s}}\right)(3.00\ s) + \frac{1}{2}\left(2.00\frac{\text{rad}}{\text{s}^2}\right)(3.00\ \text{s})^2$$

$$= 21.0\ \text{rad} = 21.0\ \text{rad}\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 3.34\ \text{rev.}$$

3 complete revolutions with an additional 0.34 rev

$$(0.34\ \text{rev})\left(2\pi\frac{\text{rad}}{\text{rev}}\right) = 2.14\ \text{rad} = \mathbf{123^o}$$



See Example 9.2 in your text

ω at t=3.00 s?

$$\omega = \omega_0 + \alpha t$$

$$= 4.00\frac{\text{rad}}{\text{s}} + \left(2.00\frac{\text{rad}}{\text{s}^2}\right)(3.00\ \text{s}) = \mathbf{10.0\frac{rad}{s}}$$

OR using Δθ

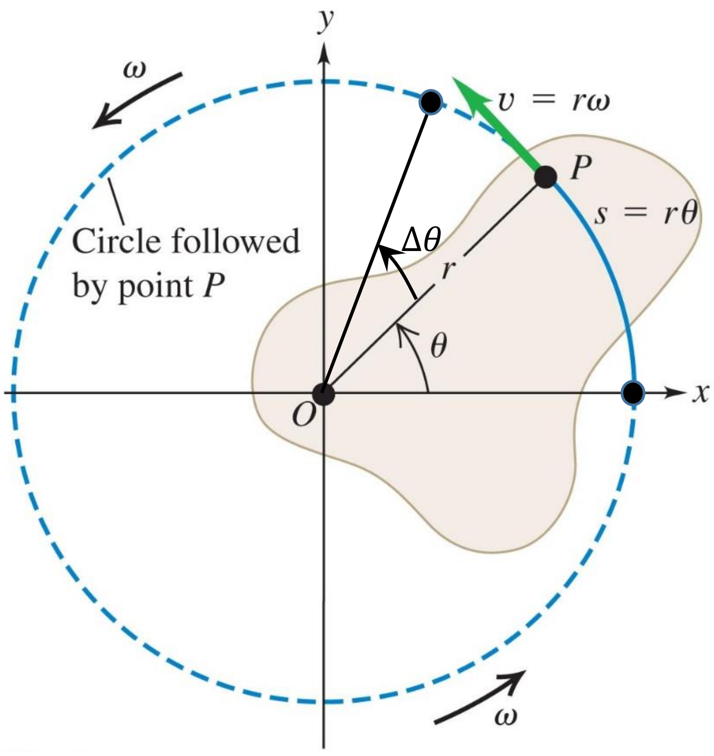
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$= \left(4.00\frac{\text{rad}}{\text{s}}\right)^2 + 2\left(2.00\frac{\text{rad}}{\text{s}^2}\right)(21.0\ \text{rad}) = 100\frac{\text{rad}^2}{\text{s}^2},$$

$$\omega = \sqrt{100} = \mathbf{10\frac{\mathit{rad}}{\mathit{s}}}$$

Remember counterclockwise rotation is positive

9.3 Relationship Between Linear and Angular Quantities



Length of Arc: $s = r\theta$

Average Speed: $v_{av} = \frac{\Delta s}{\Delta t} = \frac{\Delta(r\theta)}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega_{av}$

Instantaneous Tangential Velocity:

$$v = \lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t} = r\lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t} = r\omega$$

Direction: tangent to the circle.

Average Tangential Acceleration:

$$a_{tan,av} = \frac{\Delta v}{\Delta t} = \frac{\Delta(r\omega)}{\Delta t} = r\frac{\Delta\omega}{\Delta t} = r\alpha_{av}$$

Instantaneous Tangential Acceleration:

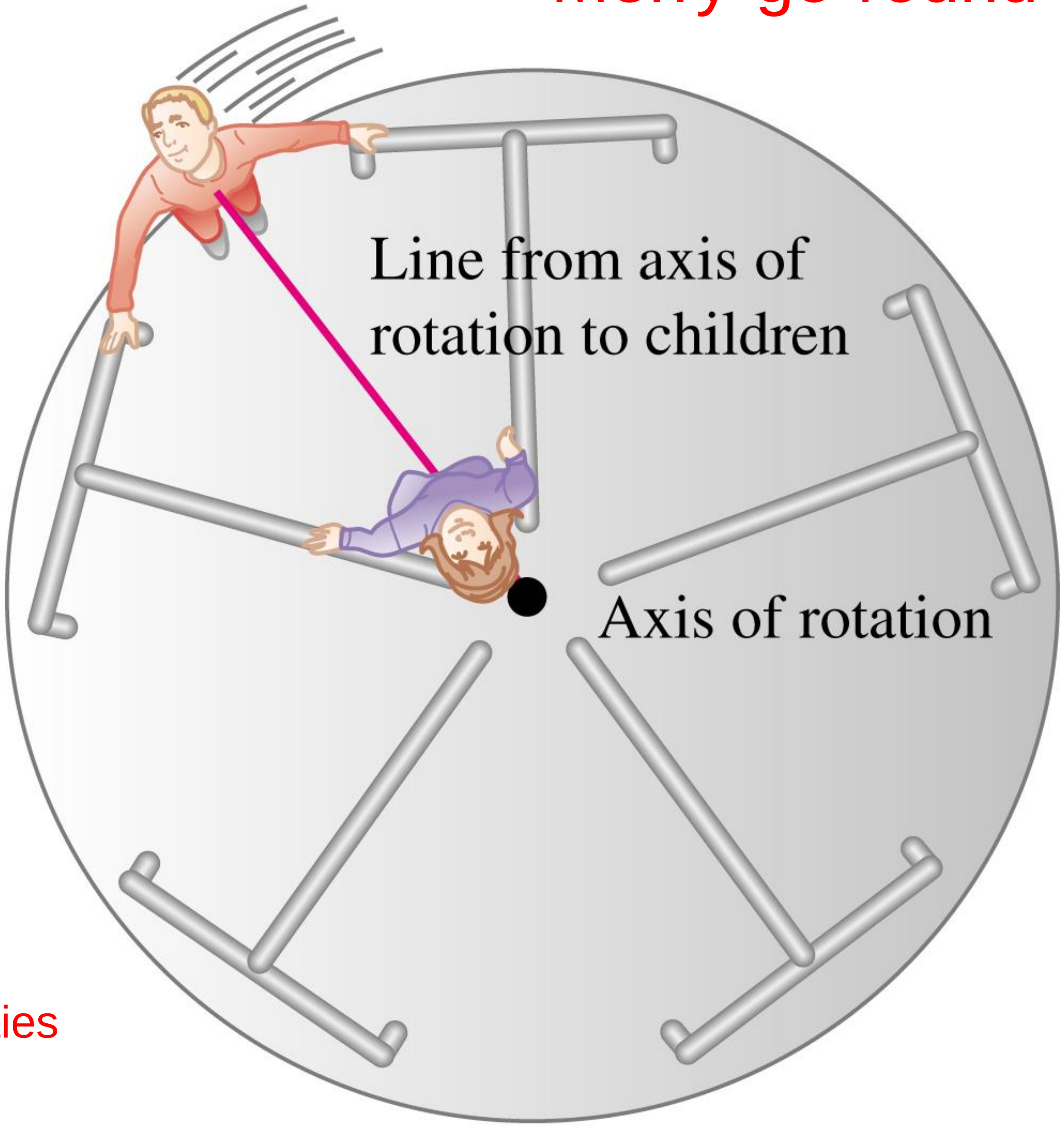
$$a_{tan} = \lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0}\frac{\Delta(r\omega)}{\Delta t} = r\lim_{\Delta t \to 0}\frac{\Delta\omega}{\Delta t} = r\alpha$$

Radial Acceleration: $a_{rad} = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r$

Magnitude of Acceleration: $a = \sqrt{a_{rad}^2 + a_{tan}^2}$

Figure 9.4

Merry-go-round

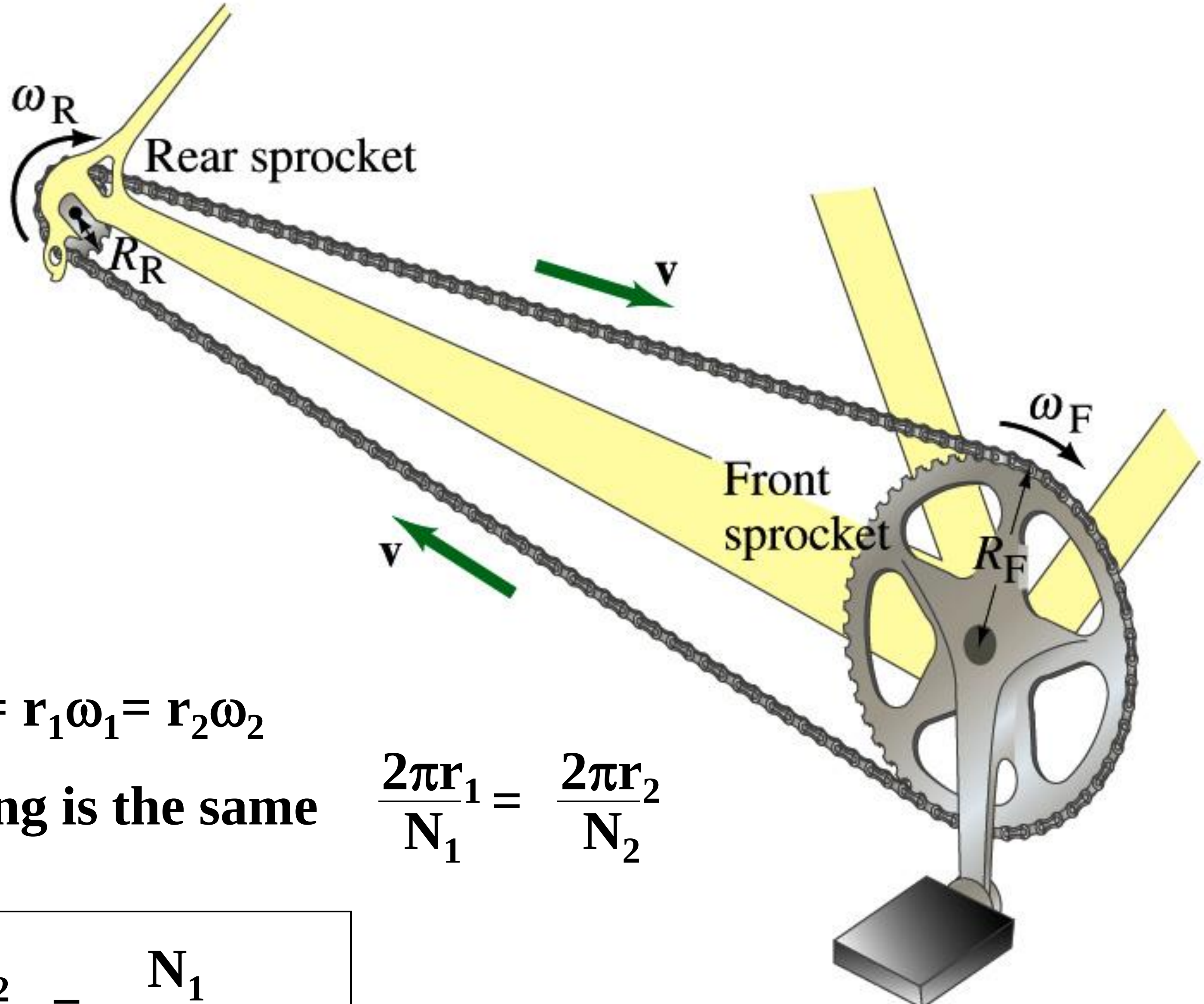


Compare the angular velocities

Clicker question

On a merry-go-round, you decide to put your toddler on an animal that will have a small angular velocity. Which animal do you pick?

a) Any animal; they all have the same angular velocity.
b) One close to the hub.
c) One close to the rim.



v_{tan}= same = $r_1\omega_1 = r_2\omega_2$

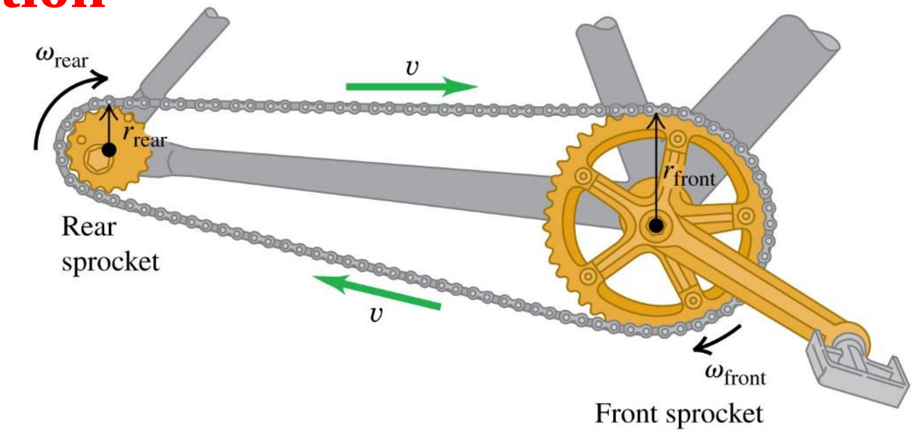
Tooth spacing is the same $$\frac{2\pi r_1}{N_1} = \frac{2\pi r_2}{N_2}$$

$$\frac{\omega_2}{\omega_1} = \frac{N_1}{N_2}$$

Q9.5

Clicker question

Compared to a gear tooth on the rear sprocket (on the left, of small radius) of a bicycle, a gear tooth on the *front* sprocket (on the right, of large radius) has



A. a faster linear speed and a faster angular speed.

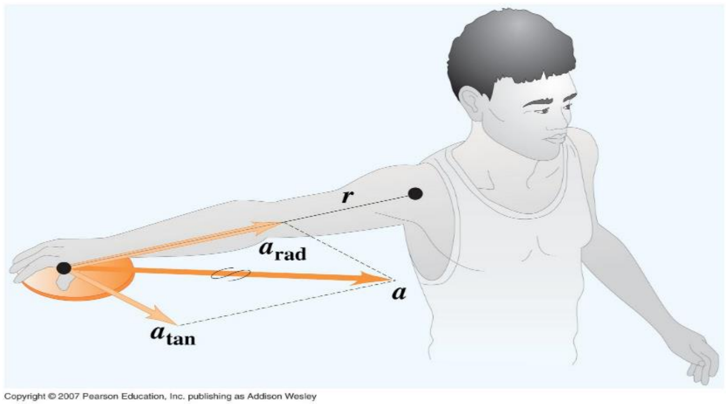
B. the same linear speed and a faster angular speed.

C. a slower linear speed and the same angular speed.

D. the same linear speed and a slower angular speed.

E. none of the above.

Example 9-3



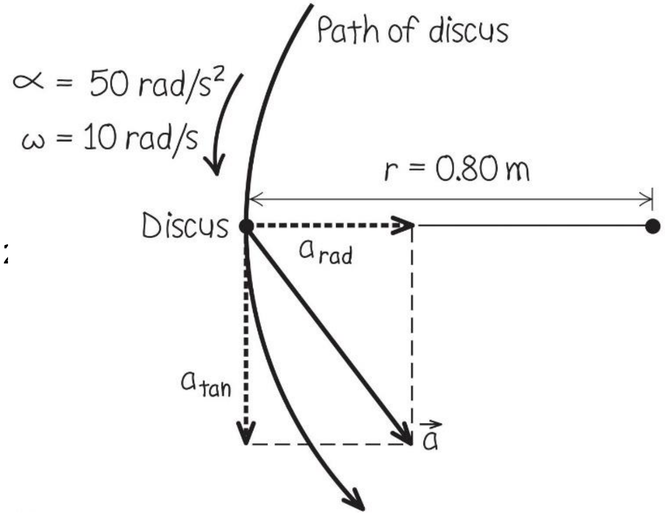
Given; $w = 10\frac{rad}{s}$; $\propto = 50\frac{rad}{s}$; $r = 0.8m$

$$a_{tan} = r\propto = 0.8 * 50 = 40\frac{m}{s^2}$$

$$a_{rad} = \frac{v^2}{r} = w^2 r = 10^2 * 0.8 = 80\frac{m}{s^2}$$

Now;

$$|\vec{a}| = \mathrm{a} = \sqrt{\mathrm{a}_{\mathrm{rad}}^2 + \mathrm{a}_{\mathrm{tan}}^2} = 89\frac{\mathrm{m}}{\mathrm{s}^2}$$

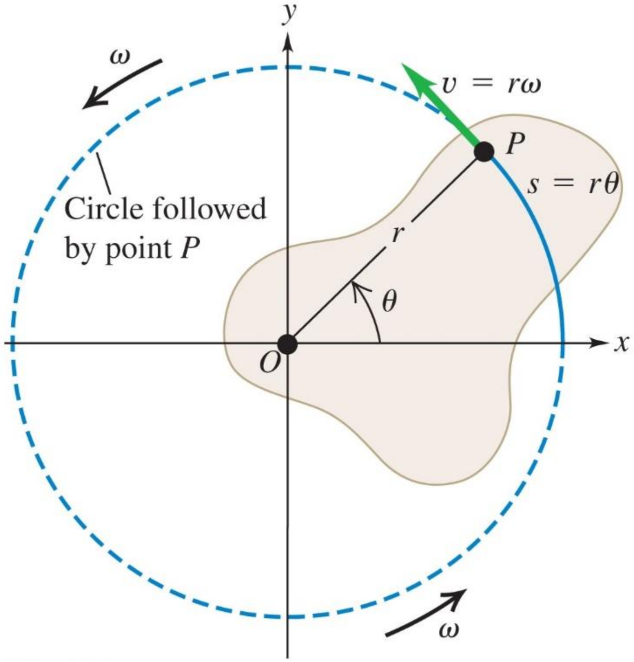


Energy in rotational motion and moment of inertia;

$$\sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i r_i^2 w^2 = \frac{1}{2}\underbrace{\left(\sum_i m_i r_i^2\right)}_{I} w^2 = \frac{1}{2} I w^2$$

$$K = \frac{1}{2} I w^2 \;\rightarrow\; I = \sum_i m_i r_i^2$$

$$I = cMR^2 \;\rightarrow\; I = MR^2$$



9.4 Kinetic Energy of Rotation and Moment of Inertia

How to calculate the kinetic energy of rotating rigid body?
Cut the rigid body into many small pieces, A, B, C, …

Kinetic Energy for piece A:

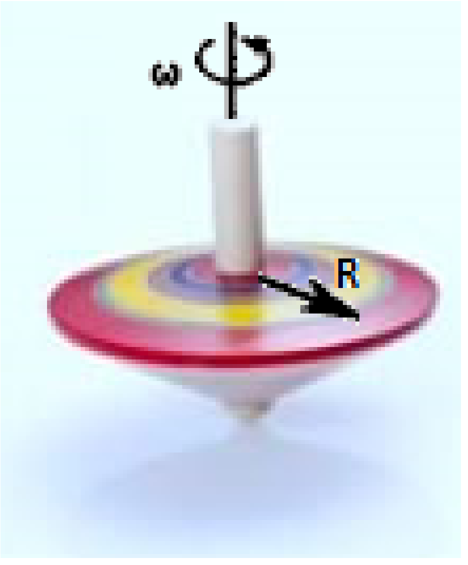
$$K_A = \frac{1}{2} m_A v_A^2 = \frac{1}{2} m_A (r\omega)_A^2 = \frac{1}{2} m_A r_A^2 \omega^2$$

Total Kinetic Energy:

$$K = \frac{1}{2} m_A r_A^2 \omega^2 + \frac{1}{2} m_B r_B^2 \omega^2 + \frac{1}{2} m_C r_C^2 \omega^2 + \cdots$$
$$= \frac{1}{2} (m_A r_A^2 + m_B r_B^2 + m_C r_C^2 + \cdots) \omega^2$$
$$= \frac{1}{2} I \omega^2$$

Define the Moment of Inertia:

$$I = m_A r_A^2 + m_B r_B^2 + m_C r_C^2 + \cdots$$

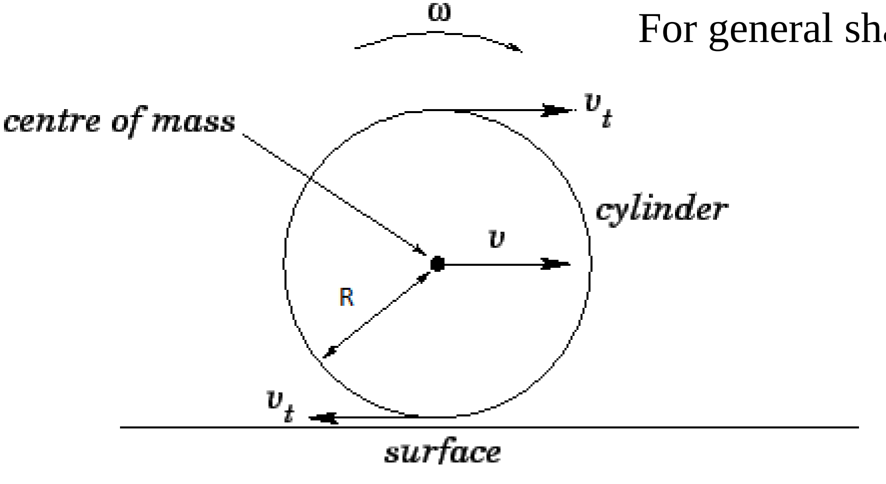


Energy in rotational motion and moment of inertia:

$$K_r = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i r_i^2 w^2 = \frac{1}{2} \underbrace{\left(\sum_i m_i r_i^2 \right)}_{I} w^2 = \frac{1}{2} I w^2$$

With moment of Inertia;
For large K_r it is better faster and not so much heavier

Example: fly wheel.
$K = \frac{1}{2} I w^2 \quad \rightarrow \qquad I = MR^2 = \sum_i m_i r_i^2$[kg m²]



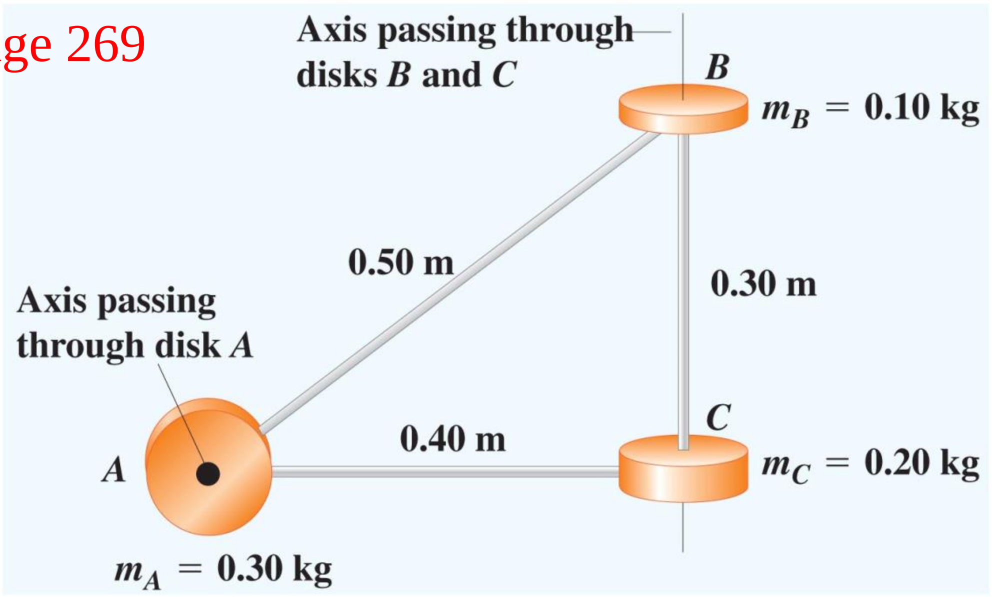
For general shapes; (**c** - factors)

$$I = cMR^2$$

Hula hoop = all point on the circumference $r_i = R$

Example 9.6, an abstract sculpture, on page 269

Note: In rotational motion the moment of inertia depends on the axis of rotation. It is not like a mass a constant parameter of an object



<u>Example 9.6</u>: An abstract sculpture

$$I = m_A r_A^2 + m_B r_B^2 + m_C r_C^2$$

a) For axis BC, disks B and C are on axis;

$$r_B = r_C = 0$$

$$I_{BC} = m_A r_A^2 = 0.3 * (0.4)^2 = 0.048\ kgm^2$$

b) For axis through A perpendicular to the plane;

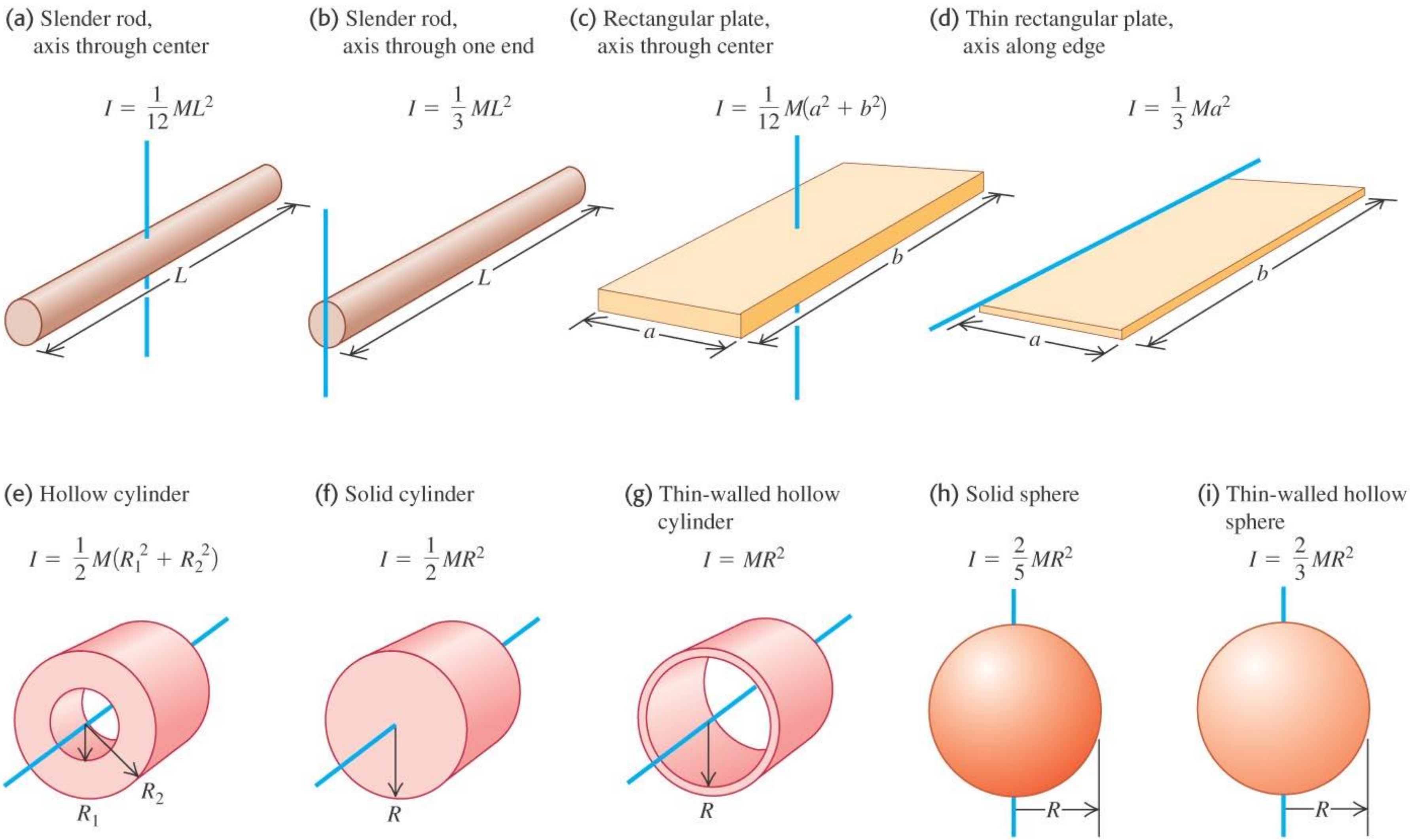
$$r_A = 0$$

$$I_A = m_B r_B^2 + m_C r_C^2 = 0.1 * (0.5)^2 + 0.2 * (0.4)^2 = 0.057\ kgm^2$$

c) If the object rotates with $w = 4\frac{rad}{s}$ around the axis through A perpendicular to the plane, what is K_{rot}?

$$K_{rot} = \frac{1}{2} I_A w^2 = \frac{1}{2} * 0.057 * (0.4)^2 = 0.46\ J$$

Table 9.2 Finding the moment of inertia for common shapes



Q9.6

Clicker question

You want to double the radius of a rotating solid sphere while keeping its kinetic energy constant. (The mass does not change.) To do this, the final angular velocity of the sphere must be

A. four times its initial value.

B. twice its initial value.

C. the same as its initial value.

D. half of its initial value.

E. one-quarter of its initial value.

Q9.7

Clicker question

The three **objects** shown here all have the same mass and the same outer radius. Each object is rotating about its axis of symmetry (shown in blue). All three objects have the *same* rotational kinetic energy. Which object is rotating *fastest*?

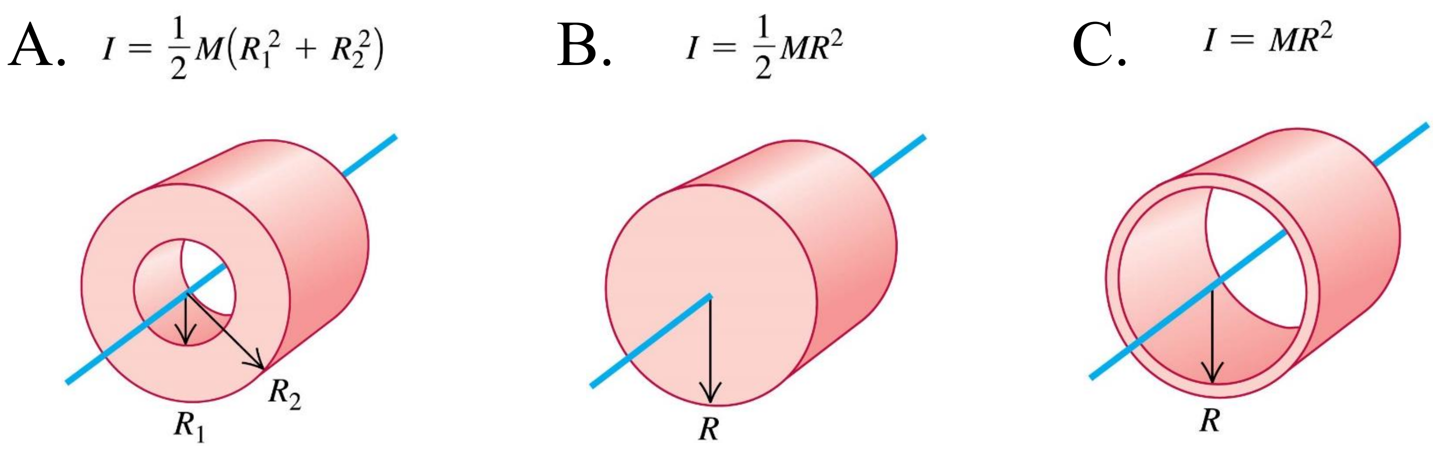
A. Object A is rotating fastest.

B. Object B is rotating fastest.

C. Object C is rotating fastest.

D. Two of these are tied for fastest.

E. All three rotate at the same speed.



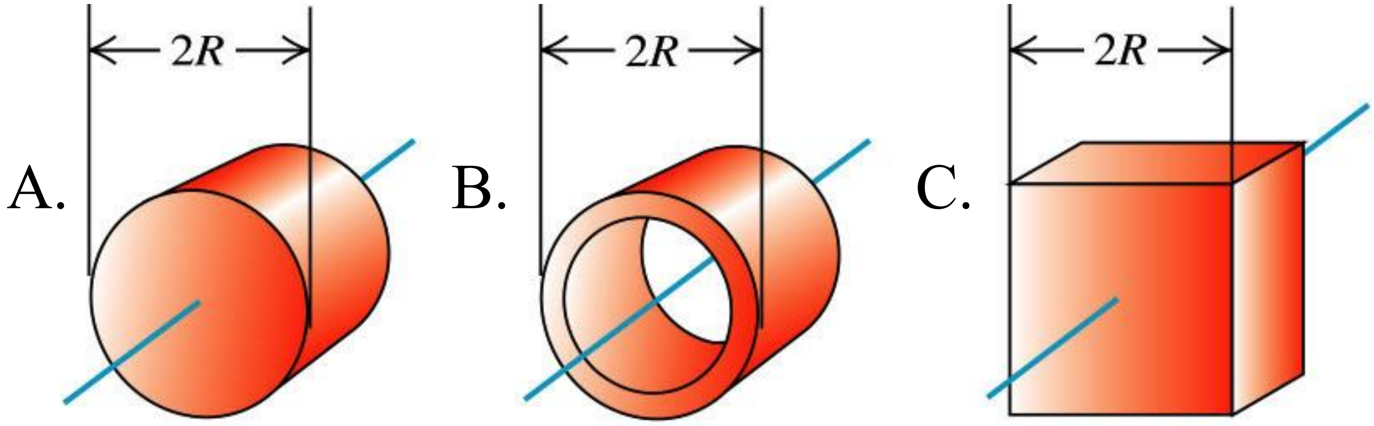
$$K = \frac{1}{2}Iw^2$$

B

Q-RT9.1

Clicker question

Objects A, B, and C all have the same mass, all have the same outer dimension, and are all uniform.



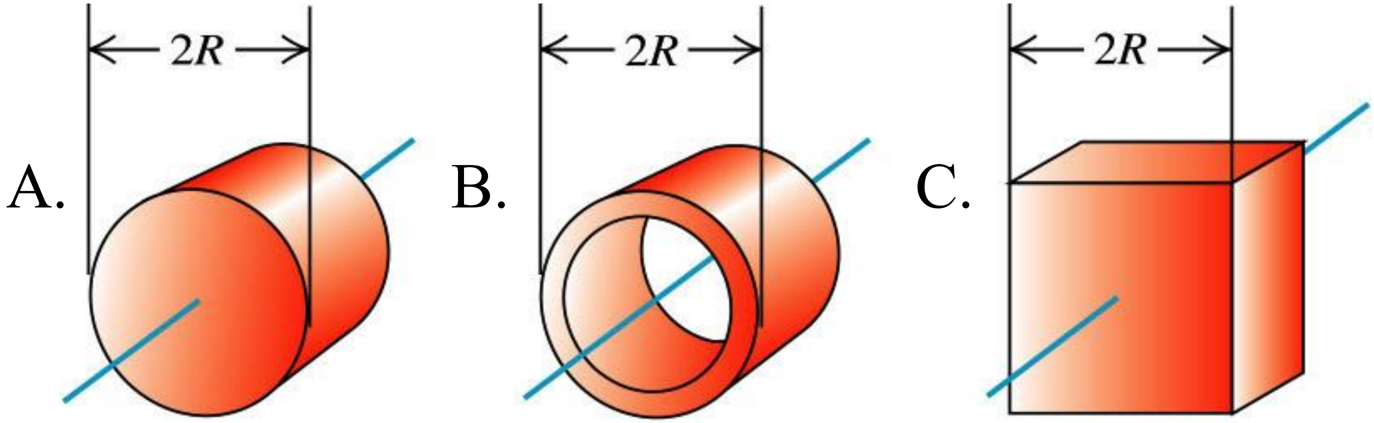
Rank these objects in order of their *moment of inertia* about an axis through its center (shown in blue), from largest to smallest.

BCA

Q-RT9.2

Clicker question

Objects A, B, and C all have the same mass, all have the same outer dimension, and are all uniform. Each object is rotating about an axis through its center (shown in blue). All three objects have the same rotational kinetic energy.

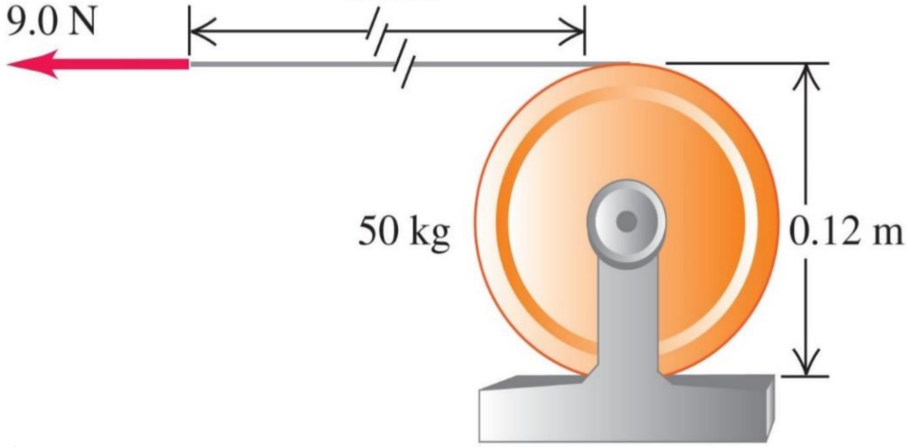


Rank these objects in order of their *angular speed* of rotation, from fastest to slowest.

ACB

Work-Energy Theorem:
$W_{total} = K_f - K_i$

Example 9.7 on page 271
A cable unwinding from a winch.



Find: (a) Final angular velocity .
(b) Final speed of cable

Solution:
Work done $W = Fd = (9.0\ \text{N})(2.0\ \text{m}) = 18\ \text{J}$

Apply the work-energy theorem

$$W = K_f - K_i = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2 = \frac{1}{2} I\omega_f^2$$

$$(18\ \text{J}) = \frac{1}{2} I_A\omega_f^2 = \frac{1}{2}\left[\frac{1}{2}(50\ \text{kg})(0.060\ \text{m})^2\right]\omega_f^2 = (0.045\ \text{kg})\omega_f^2$$

$$\omega_f = 20\ \text{rad/s}$$

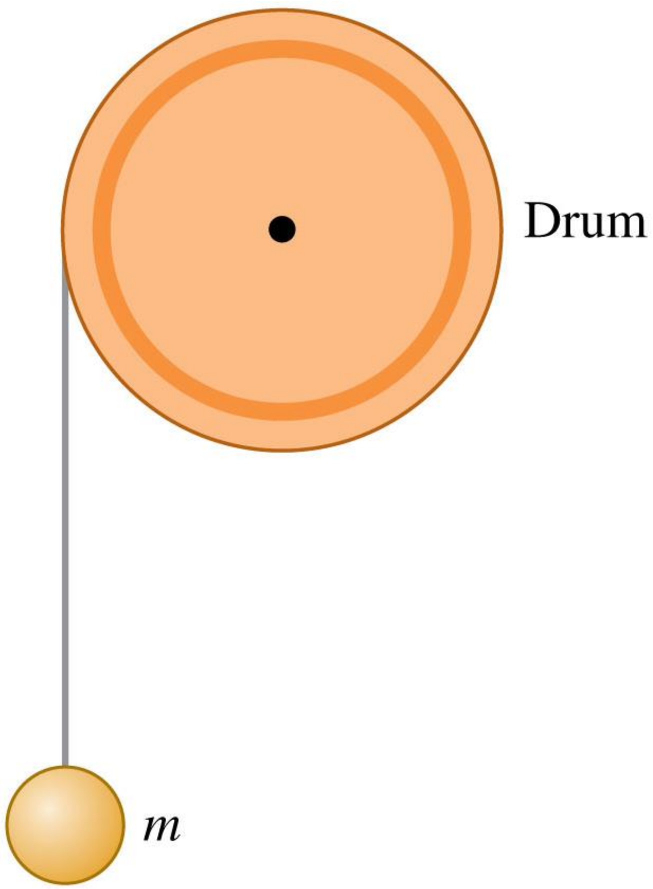
Speed of cable $v = r\omega_f = (0.06\ \text{m})(20\ \text{rad/s}) = 1.2\ \text{m/s}$

Courtesy of Wenhao Wu

Q9.8

Clicker question

A thin, very light wire is wrapped around a drum that is free to rotate. The free end of the wire is attached to a ball of mass m. The drum has the same mass m. Its radius is R and its moment of inertia is $I = (1/2)mR^2$. As the ball falls, the drum spins.



At an instant that the ball has translational kinetic energy K, what is the rotational kinetic energy of the drum?

A. K

B. $2K$

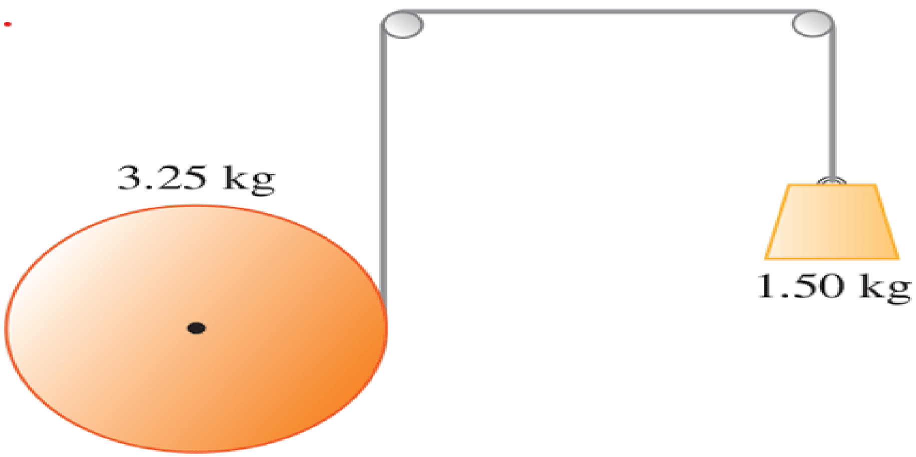
C. $K/2$

D. $K/4$

E. none of these

41. II A solid uniform 3.25 kg cylinder, 65.0 cm in diameter and 12.4 cm long, is connected to a 1.50 kg weight over two massless frictionless pulleys as shown in Figure 9.33. The cylinder is free to rotate about an axle through its center perpendicular to its circular faces, and the system is released from rest. (a) How far must the 1.50 kg weight fall before it reaches a speed of 2.50 m/s? (b) How fast is the cylinder turning at this instant?

Figure 9.33



Problem 41.

9.41. Set Up: The speed υ of the weight is related to ω of the cylinder by $\upsilon = R\omega$, where $R = 0.325$ m. Use coordinates where $+y$ is upward and $y_i = 0$ for the weight. $y_f = -h$, where h is the unknown distance the weight descends. Let $m = 1.50$ kg and $M = 3.25$ kg. For the cylinder $I = \frac{1}{2}MR^2$.

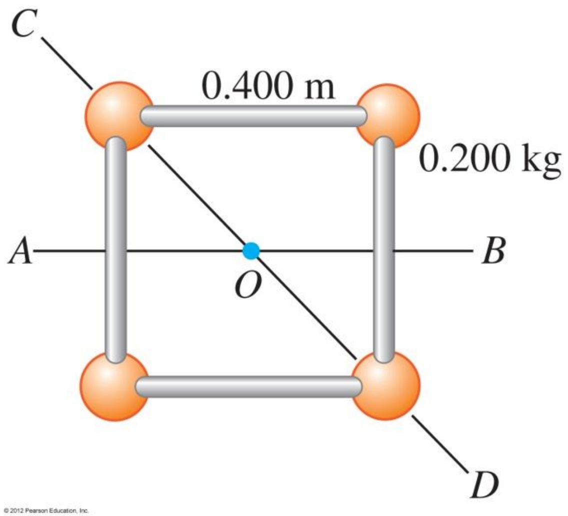
Solve: (a) Conservation of energy says $K_i + U_i = K_f + U_f$. $K_i = 0$ and $U_i = 0$. $U_f = mgy_f = -mgh$.

$$K_f = \frac{1}{2}m\upsilon^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}m\upsilon^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{\upsilon}{R}\right)^2 = \left(\frac{1}{2}m + \frac{1}{4}M\right)\upsilon^2$$

$$\left(\frac{1}{2}m + \frac{1}{4}M\right)\upsilon^2 - mgh = 0$$

$$h = \frac{\left(\frac{1}{2}m + \frac{1}{4}M\right)\upsilon^2}{mg} = \frac{\left[\frac{1}{2}(1.50\text{ kg}) + \frac{1}{4}(3.25\text{ kg})\right](2.50\text{ m/s})^2}{(1.50\text{ kg})(9.80\text{ m/s}^2)} = 0.664\text{ m}$$

(b) $\omega = \frac{\upsilon}{R} = \frac{2.50\text{ m/s}}{0.325\text{ m}} = 7.69\text{ rad/s}$



Problem 9.31:

a) Each mass is at a distance $r = \frac{\sqrt{2*(0.4)^2}}{2} = \frac{0.4}{\sqrt{2}}$ from the axis.

$I = \sum mr^2 = 4 * 0.2 * \left(\frac{0.4}{\sqrt{2}}\right)^2 = 0.064\ kgm^2$ **(axis is through center)**

b) Each mass is $r = 0.2$m from the axis.

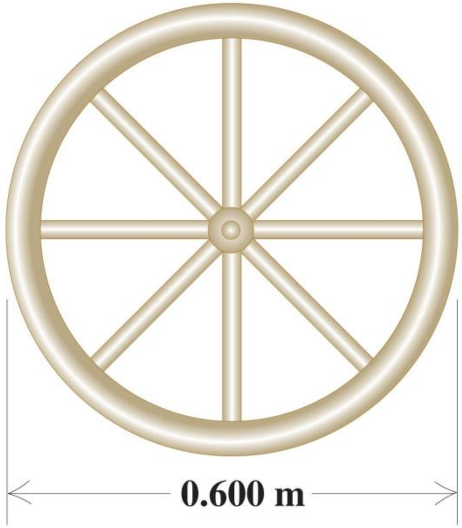
$I = 4 * 0.2 * (0.2)^2 = 0.032\ kgm^2$ (***axis is*** **along the line AB)**

c) Two masses are on the axis and two are $\frac{0.4}{\sqrt{2}}$ from the axis.

$$I = 2 * 0.2 * \left(\frac{0.4}{\sqrt{2}}\right)^2 = 0.032\ kgm^2$$

(axis is alongCD)

The value of ***I*** depends on the location of the axis.

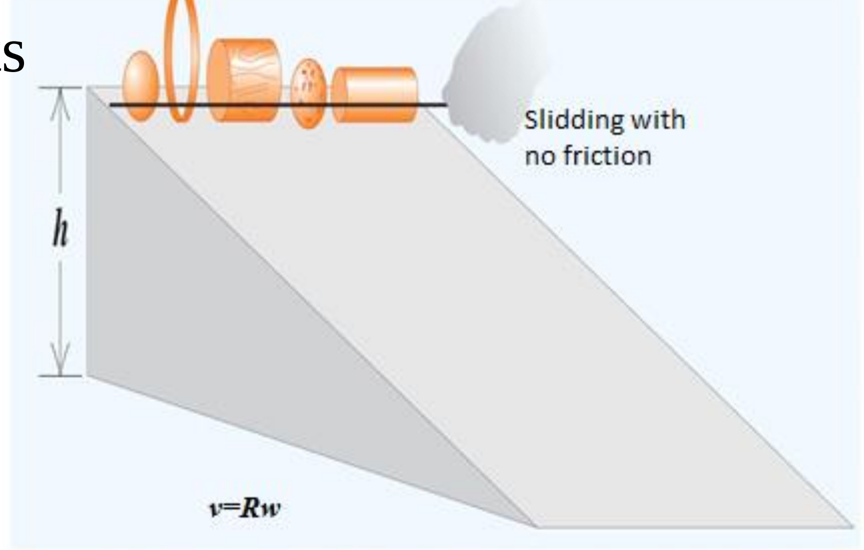


Problem 9.34:

$$I_o = m_{rim}R^2 + \frac{8}{3}m_{spoke}R^2 = 0.193\ kgm^2$$

Example 9.10 Race of rolling objects

Which object will win?



All moments of inertia in the previous table can be expressed as;

$$I_{cm} = \beta MR^2 = cMR^2 \text{ (c - number)}$$

Compare;

a) For a thin walled hollow cylinder $\beta = 1$

b) For a solid cylinder $\beta = \frac{1}{2}$ etc.

Conservation of energy;

$$0 + Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}Iw^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\beta MR^2(\frac{v}{R})^2 = \frac{1}{2}(1+\beta)Mv^2$$

$$\boldsymbol{v} = \sqrt{\frac{\mathbf{2}\boldsymbol{gh}}{\mathbf{1}+\boldsymbol{\beta}}}$$

Small β bodies wins over large β bodies.

On a horizontal surface , what fraction of the total kinetic energy is rotational?
a) a uniform solid cylinder . b) a uniform sphere c)a thin walled hollow sphere d)a hollow cylinder with outer radius R an inner radius R/2

9.49. Set Up: Apply Eq. (9.19). For an object that is rolling without slipping we have $v_{cm} = R\omega$.

Solve: The fraction of the total kinetic energy that is rotational is

$$\frac{(1/2)I_{cm}\omega^2}{(1/2)Mv_{cm}^2 + (1/2)I_{cm}\omega^2} = \frac{1}{1+(M/I_{cm})v_{cm}^2/\omega^2} = \frac{1}{1+(MR^2/I_{cm})}$$

(a) $I_{cm} = (1/2)MR^2$, so the above ratio is 1/3.

(b) $I_{cm} = (2/5)MR^2$ so the above ratio is 2/7.

(c) $I_{cm} = (2/3)MR^2$ so the ratio is 2/5.

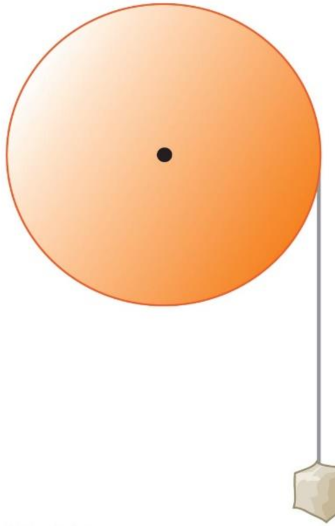
(d) $I_{cm} = (5/8)MR^2$ so the ratio is 5/13.

$$\mathbf{K}=\frac{1}{2}Mv^2+\frac{1}{2}Iw^2$$

Reflect: The moment of inertia of each object takes the form $I = \beta MR^2$. The ratio of rotational kinetic energy to total kinetic energy can be written as $\frac{1}{1+1/\beta} = \frac{\beta}{1+\beta}$. The ratio increases as β increases.

Small β bodies win over large β bodies

9.40 A light string is wrapped around the outer rim of a solid uniform cylinder of diameter 75 cm that can rotate about an axis through its center. A stone is tied to the free end of the string .When the string is released from rest the stone reaches a speed of 3.5m/sec after having fallen 2.5 m What is the mass of the cylinder??



Use coordinates where +*y* is upward. Take the origin at the final position of the stone, so for the stone $y_f = 0$ and $y_i = 2.50$ m. The cylinder has no change in gravitational potential energy. The cylinder has rotational kinetic energy and the stone has translational kinetic energy. Let *m* be the mass of the stone and let *M* be the mass of the cylinder. For the cylinder $I = \frac{1}{2}MR^2$. The speed of the stone and the angular speed ω of the cylinder are related by $\upsilon = R\omega$.

Solve: Conservation of energy says $U_i + K_i = U_f + K_f$. $K_i = 0$ and $U_f = 0$, so $U_i = K_f$. The conservation of energy expression becomes $mgy_i = \frac{1}{2}m\upsilon^2 + \frac{1}{2}I\omega^2$.

$$\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)(\upsilon/R)^2 = \frac{1}{4}M\upsilon^2, \text{ so } mgy_i = \frac{1}{2}m\upsilon^2 + \frac{1}{4}M\upsilon^2 \text{ and}$$

$$M = \frac{2m\left(2gy_i - \upsilon^2\right)}{\upsilon^2} = \frac{2(3.00\text{ kg})\left[2\left(9.80\text{ m/s}^2\right)(2.50\text{ m}) - (3.50\text{ m/s})^2\right]}{(3.50\text{ m/s})^2} = 18.0\text{ kg}$$

Compute the magnitude of the tangential and radial acceleration

a) at the start
b) b after it turned through 60and120 degrees

<u>9.24</u>

$a_{tan} = r\alpha$, $a_{rad} = r\omega^2$, $a = \sqrt{a_{tan}^2 + a_{rad}^2}$

α must be in rad/s² and ω must be in rad/s

α is constant and equal to 0.600 rad/s²

a) At the start $\omega = 0$ so $a_{rad} = 0$

$a_{tan} = r\alpha = (0.300 m)(0.600 rad/s^2) = 0.180 m/s^2$ and is constant

$a = 0.180 m/s^2$

b) Use a constant angular acceleration equation to find ω

$\theta - \theta_0 = 60.0° \left(\frac{2\pi rad}{360°}\right) = 1.05 rad$

$\omega_0 = 0$

$\alpha = 0.600 rad/s^2$

$\theta - \theta_0 = 1.05 rad$

$\omega = ?$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$\omega = \sqrt{2\alpha(\theta - \theta_0)} = \sqrt{2(0.600 rad/s^2)(1.05 rad)}$

$\omega = 1.12 rad/s$

$a_{rad} = r\omega^2 = (0.3m)(1.12 rad/s)^2 = 0.378 m/s^2$

$a = \sqrt{a_{tan}^2 + a_{rad}^2} = 0.419 m/s^2$

c) $\theta - \theta_0 = 120.0° \left(\frac{2\pi rad}{360°}\right) = 2.09 rad$

$\omega = \sqrt{2(0.600 rad/s^2)(2.09 rad)} = 1.58 rad/s$

$a_{rad} = (0.3m)(1.58 rad/s)^2 = 0.752 m/s^2$

$a = \sqrt{(0.180 m/s^2)^2 + (0.752 m/s^2)^2} = 0.774 m/s^2$

47. II A size-5 soccer ball of diameter 22.6 cm and mass 426 g rolls up a hill without slipping, reaching a maximum height of 5.00 m above the base of the hill. We can model this ball as a thin-walled, hollow sphere. (a) At what rate was it rotating at the base of the hill? (b) How much rotational kinetic energy did it then have?

***9.47. Set Up:** The ball has moment of inertia $I_{\text{cm}} = \frac{2}{3}mR^2$. Rolling without slipping means $\upsilon_{\text{cm}} = R\omega$. Use coordinates where +*y* is upward and $y = 0$ at the bottom of the hill, so $y_{\text{i}} = 0$ and $y_{\text{f}} = h = 5.00$ m.

Solve: (a) Conservation of energy gives $K_{\text{i}} + U_{\text{i}} = K_{\text{f}} + U_{\text{f}}$. $U_{\text{i}} = 0$, $K_{\text{f}} = 0$ (stops). Therefore $K_{\text{i}} = U_{\text{f}}$ and $\frac{1}{2}m\upsilon_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2 = mgh.$

$$\frac{1}{2}I_{\text{cm}}\omega^2 = \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{\upsilon_{\text{cm}}}{R}\right)^2 = \frac{1}{3}m\upsilon_{\text{cm}}^2 \text{ so } \frac{5}{6}m\upsilon_{\text{cm}}^2 = mgh$$

$$\upsilon_{\text{cm}} = \sqrt{\frac{6gh}{5}} = \sqrt{\frac{6(9.80\text{ m/s}^2)(5.00\text{ m})}{5}} = 7.67\text{ m/s}$$

and

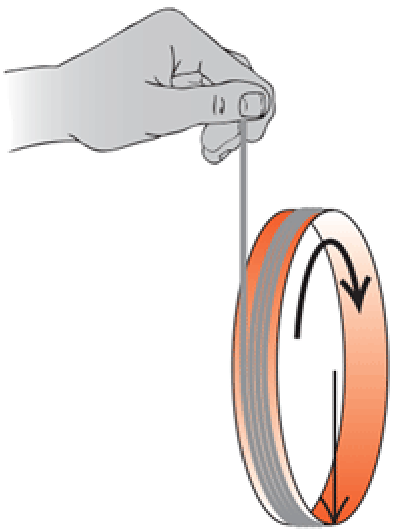
$$\omega = \frac{\upsilon_{\text{cm}}}{R} = \frac{7.67\text{ m/s}}{0.113\text{ m}} = 67.9\text{ rad/s}$$

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{3}m\upsilon_{\text{cm}}^2 = \frac{1}{3}(0.426\text{ kg})(7.67\text{ m/s})^2 = 8.35\text{ J}$

Reflect: Its translational kinetic energy at the base of the hill is $\frac{1}{2}m\upsilon_{\text{cm}}^2 = \frac{3}{2}K_{\text{rot}} = 12.52$ J. Its total kinetic energy is 20.9 J. This equals its final potential energy: $mgh = (0.426\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 20.9$ J.

50. II A string is wrapped several times around the rim of a small hoop with a radius of 0.0800 m and a mass of 0.180 kg. If the free end of the string is held in place and the hoop is released from rest (see Figure 9.34), calculate the angular speed of the rotating hoop after it has descended 0.750 m.

Figure 9.34



9.50. Set Up: Only gravity does work, so $W_{\text{other}} = 0$ and conservation of energy gives $K_i + U_i = K_f + U_f$. Let $y_f = 0$, so $U_f = 0$ and $y_i = 0.750$ m. The hoop is released from rest so $K_i = 0$. $K_f = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$. $v_{\text{cm}} = R\omega$. For a hoop with an axis at its center, $I_{\text{cm}} = MR^2$.

Solve: Conservation of energy gives $U_i = K_f$. $K_f = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2$, so $MR^2\omega^2 = Mgy_i$.

$$\omega = \frac{\sqrt{gy_i}}{R} = \frac{\sqrt{(9.80 \text{ m/s}^2)(0.750 \text{ m})}}{0.0800 \text{ m}} = 33.9 \text{ rad/s}$$

45. I A 2.20 kg hoop 1.20 m in diameter is rolling to the right without slipping on a horizontal floor at a steady **3.00 rad/s**. (a) How fast is its center moving? (b) What is the total kinetic energy of the hoop?

***9.45. Set Up:** Since there is rolling without slipping, $v_{cm} = R\omega$. The kinetic energy is given by Eq. (9.19). We have $\omega = 3.00$ rad/s and $R = 0.600$ m. For a hoop rotating about an axis at its center we have $I = MR^2$.

Solve: (a) $\upsilon_{cm} = R\omega = (0.600\text{ m})(3.00\text{ rad/s}) = 1.80\text{ m/s}$

(b) $K = \frac{1}{2}M\upsilon_{cm}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}M\upsilon_{cm}^2 + \frac{1}{2}(MR^2)(\upsilon_{cm}/R^2) = M\upsilon_{cm}^2 = (2.20\text{ kg})(1.80\text{ m/s})^2 = 7.13\text{ J}$

Reflect: For the special case of a hoop, the total kinetic energy is equally divided between the motion of the center of mass and the rotation about the axis through the center of mass.

51. II A 150.0 kg cart rides down a set of tracks on four solid steel wheels, each with radius 20.0 cm and mass 45.0 kg. The tracks slope downward at an angle of 20° to the horizontal. If the cart is released from rest a distance of 16.0 m from the bottom of the track (measured along the slope), how fast will it be moving when it reaches the bottom? Assume that the wheels roll without slipping, and that there is no energy loss due to friction.

9.51. Set Up: Solve this problem using energy conservation: $\Delta K = -\Delta U$. The change in potential energy of the cart is $\Delta U = Mg\Delta y$, where the total mass is $M = 150.0\ \text{kg} + 4(45.0\ \text{kg}) = 330.0\ \text{kg}$ and the vertical displacement of the cart is given by $\Delta y = -(16.0\ \text{m})\sin 20^\circ = -5.472\ \text{m}$. The kinetic energy of the cart consists of its translational kinetic energy and the rotational kinetic energy of its four identical wheels. The initial kinetic energy is zero, so $\Delta K = \frac{1}{2}M\upsilon^2 + 4\left(\frac{1}{2}I\omega^2\right)$. The moment of inertia of each wheel is that of a solid cylinder: $I = \frac{1}{2}mr^2$ with $m = 45.0$ kg.

Solve:

$$-Mg\Delta y = \frac{1}{2}M\upsilon^2 + 4\left(\frac{1}{2}I\omega^2\right)$$

$$= \frac{1}{2}M\upsilon^2 + 2\left(\frac{1}{2}mr^2\right)\left(\frac{\upsilon}{r}\right)^2$$

$$= \upsilon^2\left(\frac{M}{2} + m\right)$$

$$\upsilon = \pm\sqrt{\frac{-Mg\Delta y}{\frac{M}{2}+m}} = \sqrt{\frac{-g\Delta y}{\frac{1}{2}+\frac{m}{M}}}$$

$$= \sqrt{\frac{-\left(9.8\ \text{m/s}^2\right)(-5.472\ \text{m})}{\frac{1}{2}+\frac{45.0\ \text{kg}}{330.0\ \text{kg}}}} = 9.2\ \text{m/s}$$

Reflect: If we ignore the rotational energy of the wheels, we would get $\upsilon = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(5.472\ \text{m})} = 10\ \text{m/s}$ for the speed of the cart at the bottom of the slope. The actual speed of the cart is only slightly slower than this due to the relatively small moment of inertia of the wheels.

Clicker question

You are preparing your unpowered soapbox vehicle for a soapbox derby down a local hill. You're choosing between solid-rim wheels and wheels that have transparent hollow rims. Which kind will help you win the race?

a) Either kind; it doesn't matter.

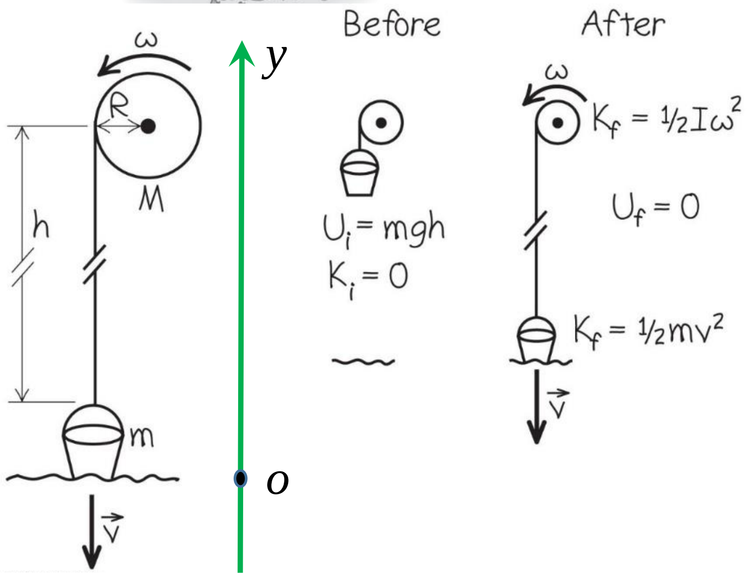
b) The solid-rim wheels.

c) The transparent hollow-rim wheels.

solid

hollow

Conservation of energy in a well



Conservation of Energy: $U_i + K_i = U_f + K_f$

Example 9.8 on page 269

Given: M, R, m, and h

Find v and ω just before hitting the water

Solution:

As the bucket descends, it potential energy decreases and is converted to kinetic energy.

Apply the conservation of energy:

$$mgh + 0 + 0 = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}\left(m + \frac{1}{2}M\right)v^2$$

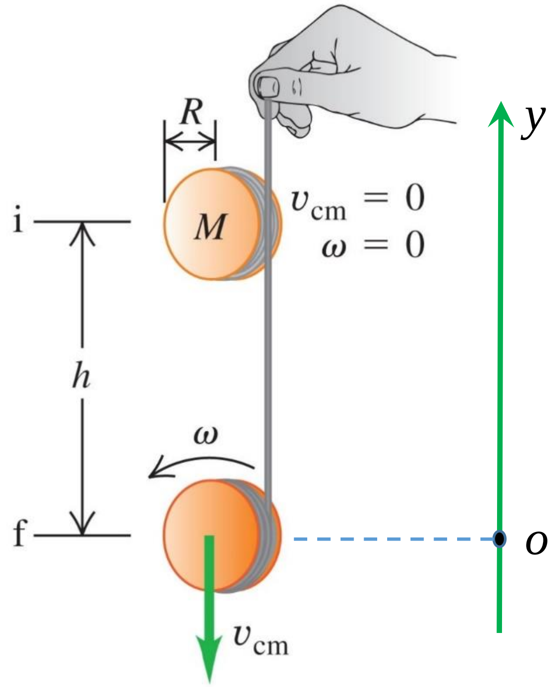
$$v = \sqrt{\frac{2gh}{1+M/2m}} \quad \text{and} \quad \omega = \frac{1}{R}\sqrt{\frac{2gh}{1+M/2m}}$$

Note: free fall velocity for **M<<<m**; all energy goes to kinetic and not rotation

9.5 Rotation about a Moving Axis

How to calculate the kinetic energy of a rigid body that is rotating and also having a linear motion?

$$K = K_{linear} + K_{rotational} = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$



Example 9.9 on page 272

Given: Given M, R, and h

Find: Velocity of the center of mass v_{cm}

Solution:

$U_i = Mgh$, $K_i = 0$, $U_f = 0$

$$K_f = K_{cm} + K_{rot} = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

$$= \frac{1}{2} M v_{cm}^2 + \frac{1}{2} \left(\frac{1}{2} M R^2\right) \left(\frac{v_{cm}}{R}\right)^2 = \frac{3}{4} M v_{cm}^2$$

Apply the conservation of energy: $Mgh + 0 = 0 + \frac{3}{4} M v_{cm}^2$

$$v_{cm} = \sqrt{4gh/3}$$

Courtesy of Wenhao Wu

Calculate the angular momentum and kinetic energy of a solid uniform sphere with a radius of 0.12 m and a mass of 14.0 kg if it is rotating at 6.00 rad/s about an axis through its center

$$I = \frac{2}{5}MR^2 = \frac{2}{5} * (14kg)(0.12m)^2 = 0.0806\, kgm^2$$

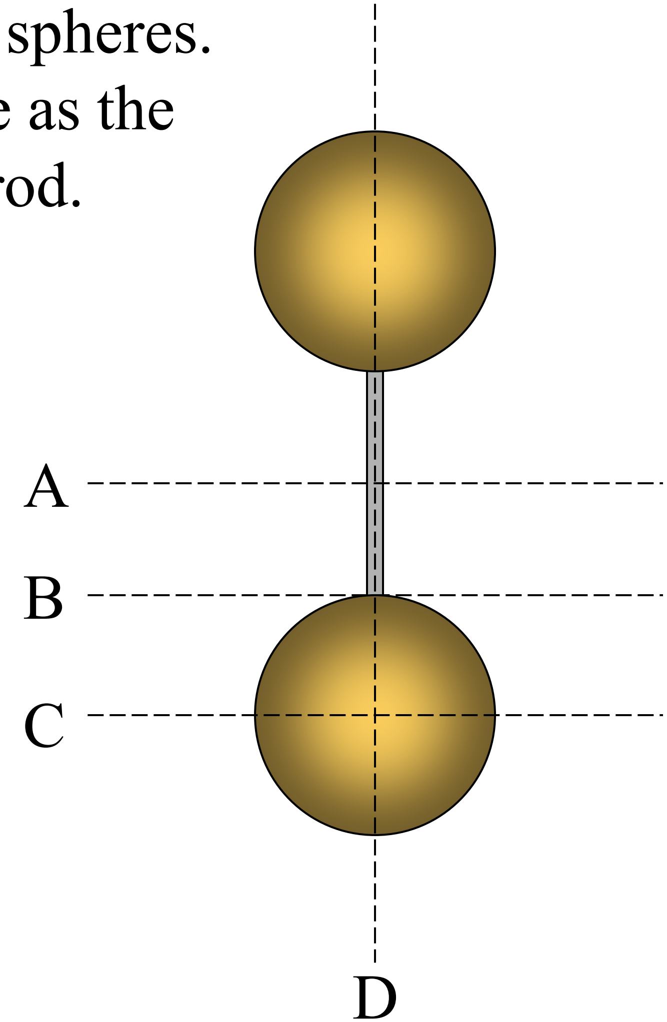
$$L = Iw = 0.0806\, kgm^2 \; * 6.0\text{ rad} = 0.484\text{ kg}\frac{m^2}{s}$$

$$K = \frac{1}{2}Iw^2 = \frac{1}{2}Lw = \frac{1}{2}\left(0.484\text{ kg}\frac{m^2}{s}\right) * 6\text{ rad} = 1.45\text{ J}$$

Q-RT9.3 **Clicker question**

Two identical uniform solid spheres are attached by a solid uniform thin rod. The rod lies on a line connecting the centers of mass of the two spheres. Axes A, B, C, and D are in the same plane as the centers of mass of the spheres and of the rod.

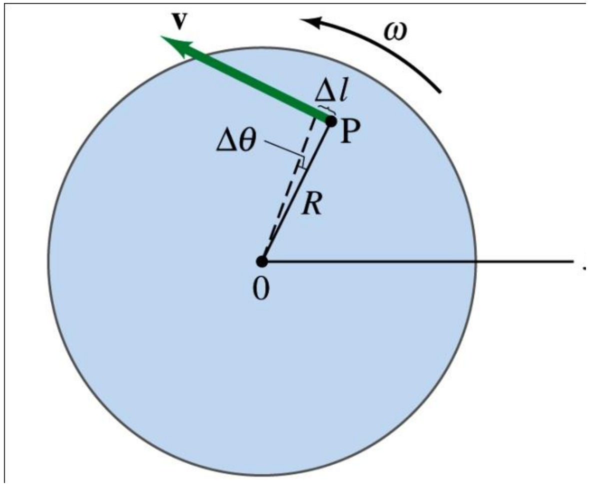
For the combined object of two spheres plus rod, **rank** the object's *moments of inertia* about the four axes, from largest to smallest.



CBAD

Relationship between Linear and Angular Quantities

$$(1)\ \theta\left(=\frac{l}{R}\right)=\omega t \quad (like\ \ l=v\,t)$$



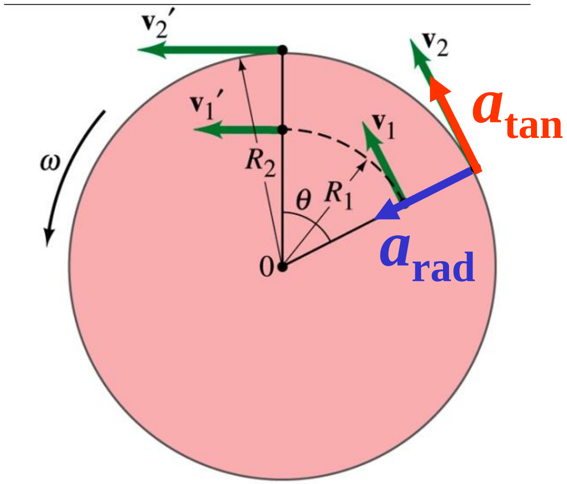
$$\therefore l=(R\omega)t$$

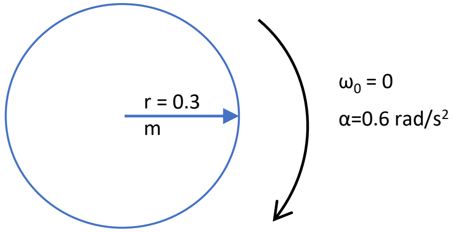
$$\therefore v = R\omega$$

$$(2)\ a_{\tan} = \frac{dv}{dt} = R\frac{d\omega}{dt}$$

$$\therefore a_{\tan} = R\alpha$$

$$(3)\ a_{\mathrm{rad}} = \frac{v^2}{R} = \frac{(\omega R)^2}{R} = \omega^2 R$$





Find the magnitude of the tangential component of acceleration (a_{tan}), the radial component of acceleration (a_{rad}), and the total acceleration (a) of a point on its rim..

a. At the start
b. After it has turned through 60°
c. After it has turned through 120°

$a_{tan} = r\alpha$, $a_{rad} = r\omega^2$, $a = \sqrt{a_{tan}^2 + a_{rad}^2}$

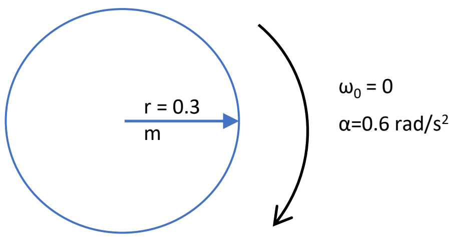
α must be in rad/s² and ω must be in rad/s

α is constant and equal to 0.600 rad/s²

a) At the start $\omega = 0$ so $a_{rad} = 0$

$a_{tan} = r\alpha = (0.300\,m)(0.600\,rad/s^2) = 0.180\,m/s^2$ and is constant

$a = 0.180\,m/s^2$



Find the magnitude of the tangential component of acceleration (a_{tan}), the radial component of acceleration (a_{rad}), and the total acceleration (a) of a point on its rim..

a. At the start
b. After it has turned through 60°
c. After it has turned through 120°

b) Use a constant angular acceleration equation to find ω

$$\theta - \theta_0 = 60.0° \left(\frac{2\pi \text{ rad}}{360°}\right) = 1.05 \text{ rad}$$

$\omega_0 = 0$

$\alpha = 0.600 \text{ rad/s}^2$

$\theta - \theta_0 = 1.05 \text{ rad}$

$\omega = ?$

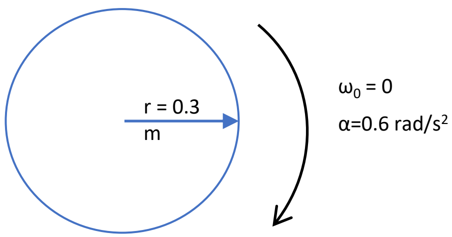
$$\omega^2 = \cancel{\omega_0^2} + 2\alpha(\theta - \theta_0)$$

$$\omega = \sqrt{2\alpha(\theta - \theta_0)} = \sqrt{2(0.600 \text{ rad/s}^2)(1.05 \text{ rad})}$$

$$\omega = 1.12 \text{ rad/s}$$

$$a_{rad} = r\omega^2 = (0.3 \text{ m})(1.12 \text{ rad/s})^2 = 0.378 \text{ m/s}^2$$

$$a = \sqrt{a_{tan}^2 + a_{rad}^2} = 0.419 \text{ m/s}^2$$



Find the magnitude of the tangential component of acceleration (a_{tan}), the radial component of acceleration (a_{rad}), and the total acceleration (a) of a point on its rim..

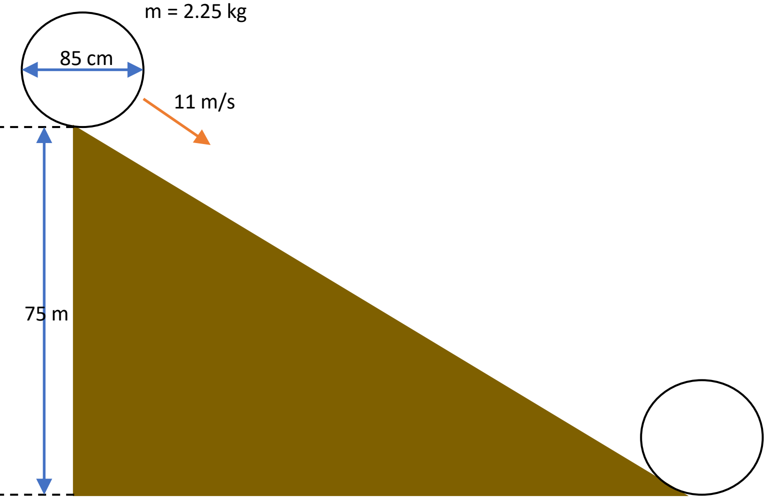
a. At the start
b. After it has turned through 60°
c. After it has turned through 120°

c) $\theta - \theta_0 = 120.0° \left(\frac{2\pi \text{ rad}}{360°}\right) = 2.09 \text{ rad}$

$\omega = \sqrt{2(0.600 \text{ rad/s}^2)(2.09 \text{ rad})} = 1.58 \text{ rad/s}$

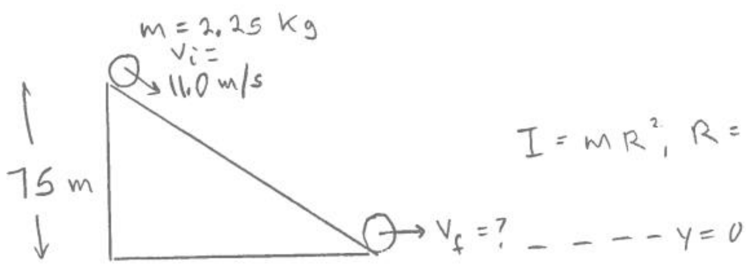
$a_{rad} = (0.3\text{m})(1.58 \text{ rad/s})^2 = 0.752 \text{ m/s}^2$

$a = \sqrt{(0.180 \text{ m/s}^2)^2 + (0.752 \text{ m/s}^2)^2} = 0.774 \text{ m/s}^2$



a) How fast is the wheel moving at the bottom of the slope (assume no slipping occurred)?
b) How much Kinetic Energy does the wheel have when it reaches the bottom?

a)



$I = mR^2$, $R = 0.425\ m$

$K_i + U_i + W_{other} = K_f + U_f$

$W_{other} = 0 \quad U_f = 0 \quad U_i = mgh$ with $h = 75\ m$

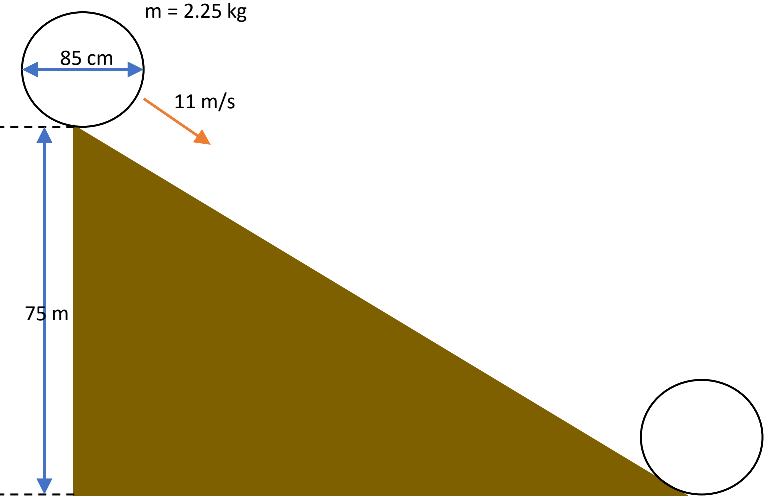
$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$I = mR^2$, no slipping means $V = R\omega$

$K = \frac{1}{2}mv^2 + \frac{1}{2}(mR^2)\left(\frac{V}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$

$mV_i^2 + mgh = mV_f^2$

$V_f = \sqrt{V_i^2 + gh} = \sqrt{(11.0\ m/s)^2 + (9.8\ m/s^2)(75\ m)} = 29.3\ m/s$



a) How fast is the wheel moving at the bottom of the slope (assume no slipping occurred)?
b) How much Kinetic Energy does the wheel have when it reaches the bottom?

b) $K_f = mv_f^2 = (2.25\,kg)(29.3\,m/s)^2 = 1930\,J$

Exam2 histogram