Chapter 7 Work and Energy

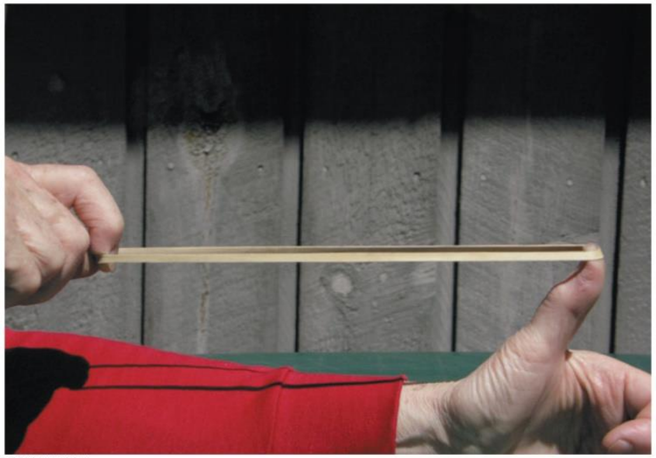
- Overview energy.
- Study work as defined in physics.
- Relate work to kinetic energy.
- Consider work done by a variable force.
- Study potential energy.
- Understand energy conservation.
- Include time and the relationship of work to power.

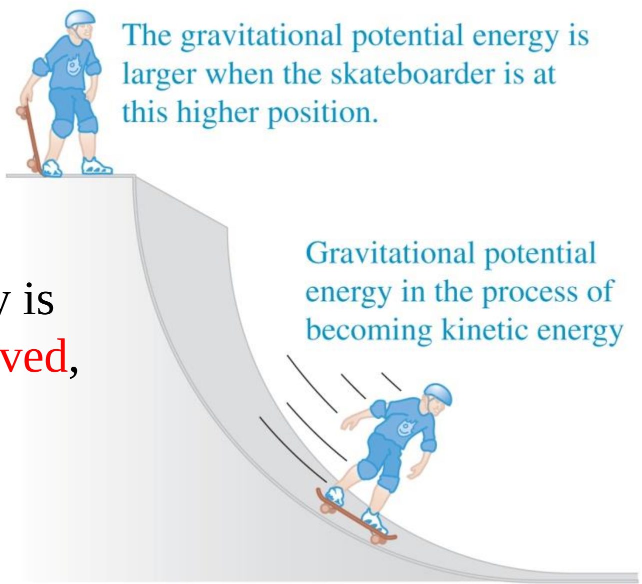
Note: In previous chapters, we studied motion. We used Newton's three laws to understand the motion of an object and the forces acting on it.

Sometimes this can be complicated and difficult. That was the reason we limited our studies to problems with a constant acceleration.

Now, we introduce *work and energy* as the next step.

Forms of Mechanical Energy





Energy is conserved,

TABLE 7.1 The analogy between energy and money

How money is like energy:

- It can take multiple forms (coins, bills, checks, bank accounts).
- You can transform it (e.g., by cashing a check) or transfer it to others.
- These transfers and transformations do not change its total amount.
- It can be stored or spent.

How money is not like energy:

- Money can be created or destroyed, whereas energy cannot be.

We introduce energy to solve problems with variable acceleration

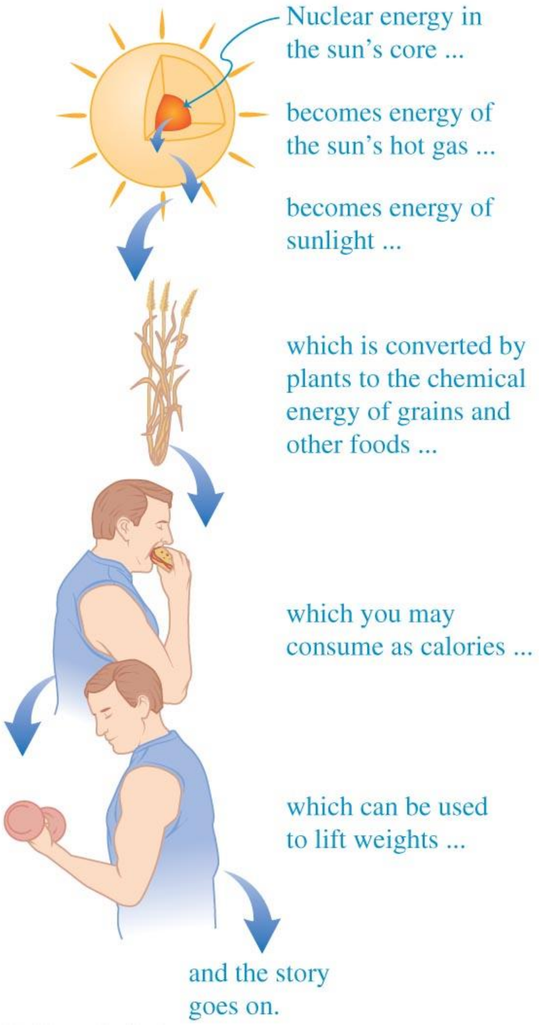
7.1 An Overview of Energy

Energy is the capability to do work.

- Energy is conserved, but can transfer from one form to another.
- Kinetic energy describes motion and relates to the mass of the object and its speed squared

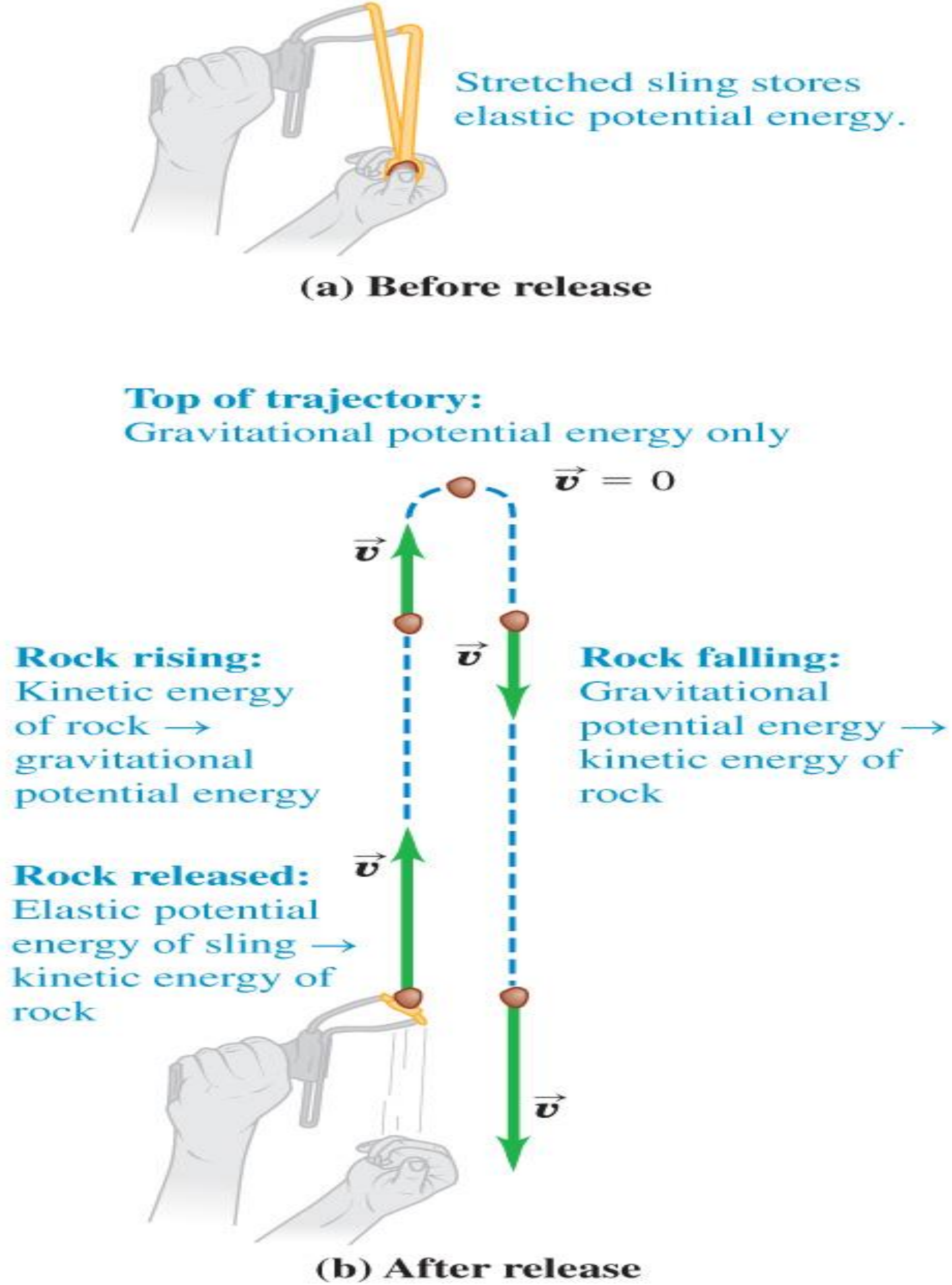
$$K = \frac{1}{2}mv^2$$

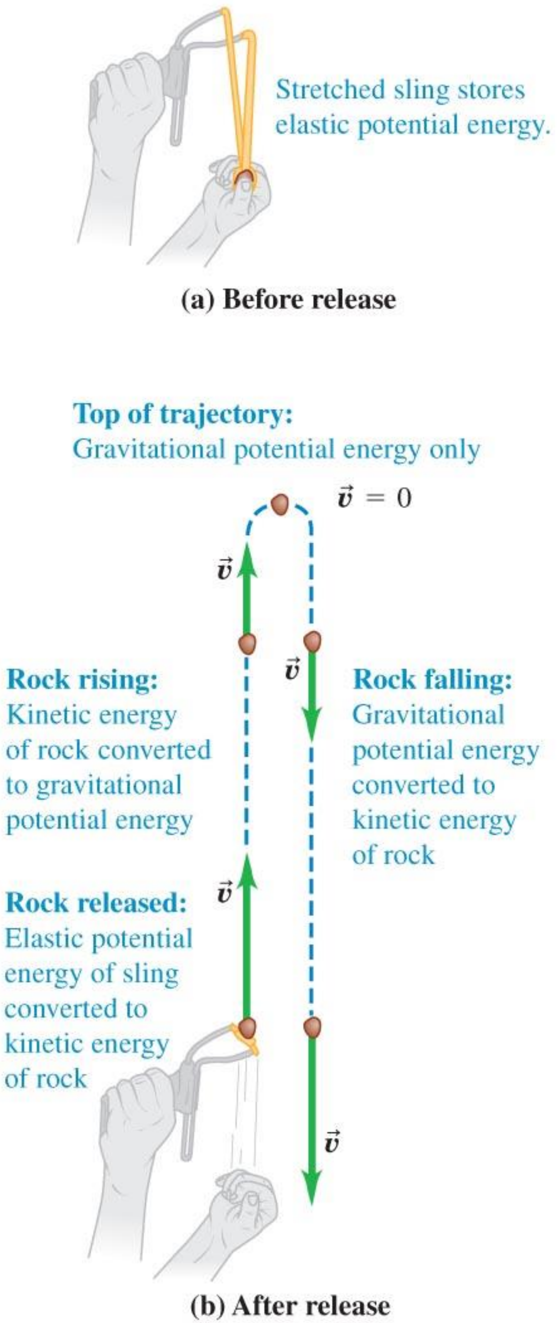
- Energy on earth originates from the sun.
- Energy on earth is stored thermally, chemically and physically.
- Chemical energy can be released by metabolism.
- Energy can be stored as potential energy in object height and mass and also through elastic deformation.



Courtesy of Wenhao Wu

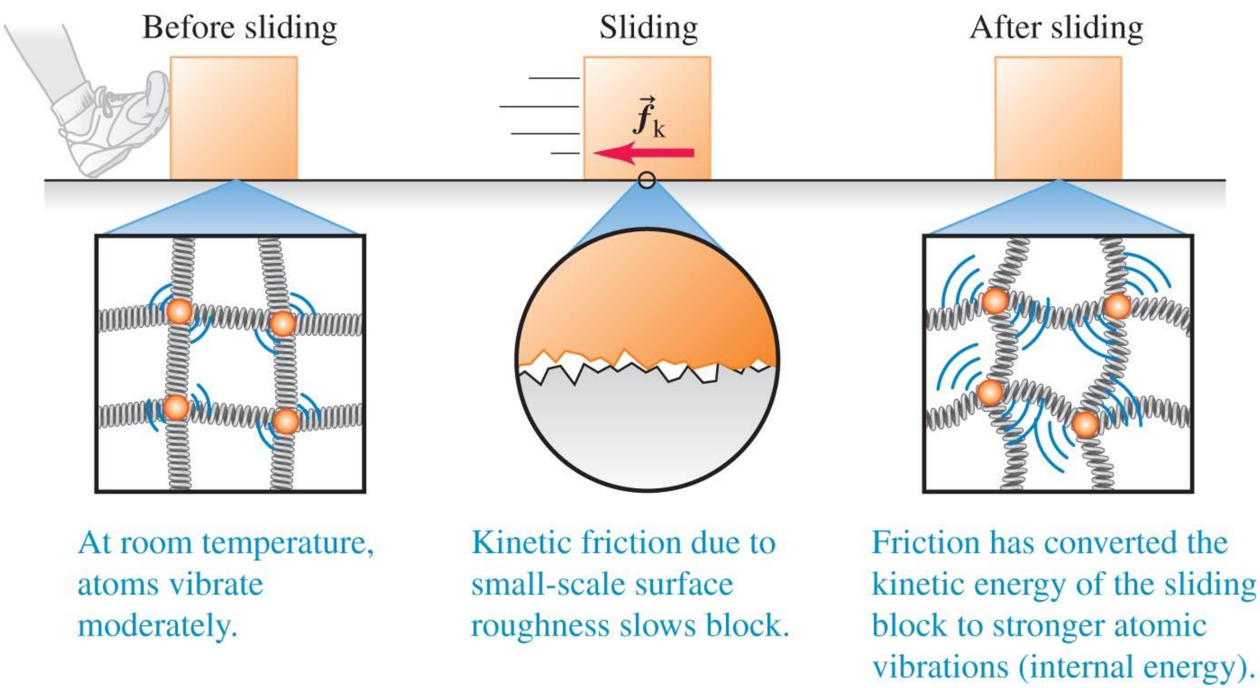
CONSERVATION OF ENERGY





Other Examples of Energy Transfer

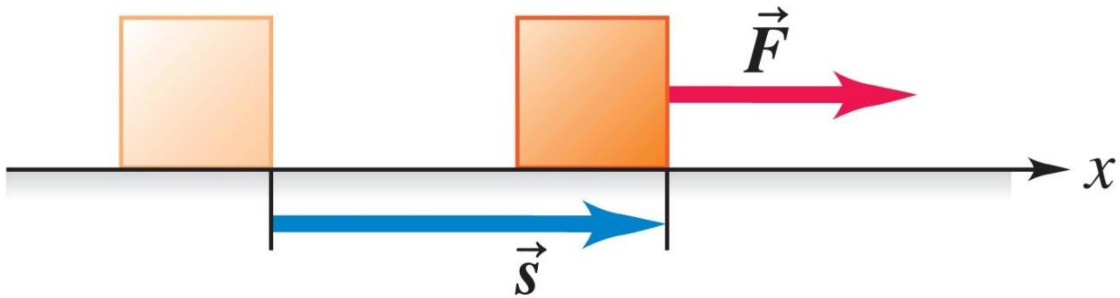
- Biological energy to elastic potential energy to kinetic energy to gravitational potential energy to kinetic energy
- Biological energy to kinetic energy to heat energy to kinetic energy



7.2 Work

How is "Work" Defined in Physics?

- In a simple definition, work is the product of a constant force ***F*** and a parallel displacement ***s*** made under the influence of the force.



Work done: $W = Fs$

Work is a scalar quantity.

Courtesy of Wenhao Wu

Units of Work - Energy

$$\boldsymbol{W} = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{d}} = |\boldsymbol{F}| \cdot |\boldsymbol{d}| \cos \propto$$

[1 Joule]=[1 N][1 m]

$$\text{Work} = \text{energy}$$

$$\boldsymbol{E} = \boldsymbol{mc^2}\textbf{(Einstein)}$$

$$\boldsymbol{E} = \frac{1}{2}\boldsymbol{mv^2}\textbf{(Kinetic energy)}$$

$$\text{Work – energy theorem}$$

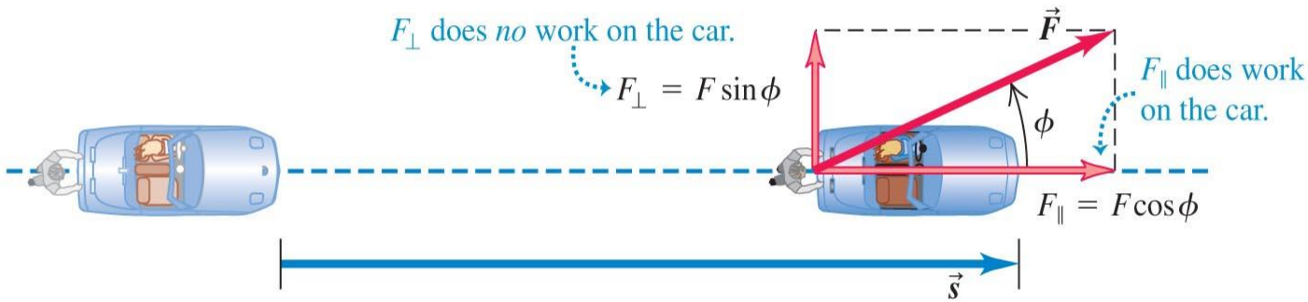
$$\boldsymbol{W_{net}} = \boldsymbol{k_2} - \boldsymbol{k_1} = \Delta \boldsymbol{k}$$

1Joule = 10^7 erg

$$\mathbf{1\ Joule} = \overbrace{\mathbf{kg}\frac{\mathbf{m}}{\mathbf{s^2}}}^{\textbf{Force}}\mathbf{m}\left(\frac{\mathbf{10^3 g}}{\mathbf{1\ kg}}\frac{\mathbf{10^2 cm}}{\mathbf{1\ m}}\frac{\mathbf{10^2 cm}}{\mathbf{1\ m}}\right) = \mathbf{10^7}\frac{\mathbf{g\ cm^2}}{\mathbf{s^2}}$$

Power → [Watt]=[Joule/second]

- More generally, work is the product of the component of the force in the direction of displacement and the magnitude *s* of the displacement.
- Forces applied at angles must be resolved into components.



$$W = F_{||}s$$

- W is a scalar quantity that can be positive, zero, or negative.
- If $W > 0$ ($W < 0$), energy is added to (taken from) the system.

Equivalent Representations: $W = F_{||}s = (F * cos\emptyset)s$

Clicker question

Work Done by a Constant Force

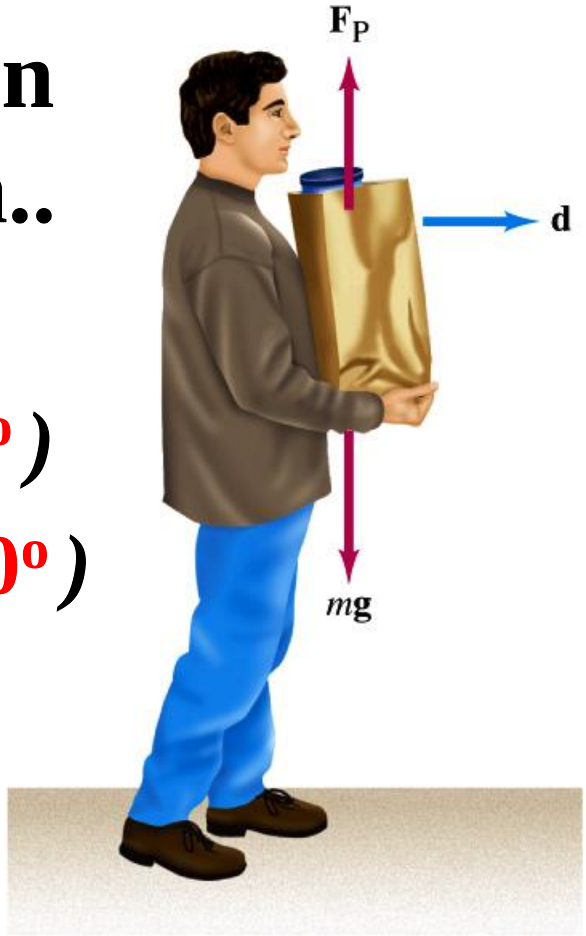
Example: Work done on the bag by the person..

→ ***Special case: $W = 0$ J***

a) $W_P = F_P\, d \cos(90^o)$

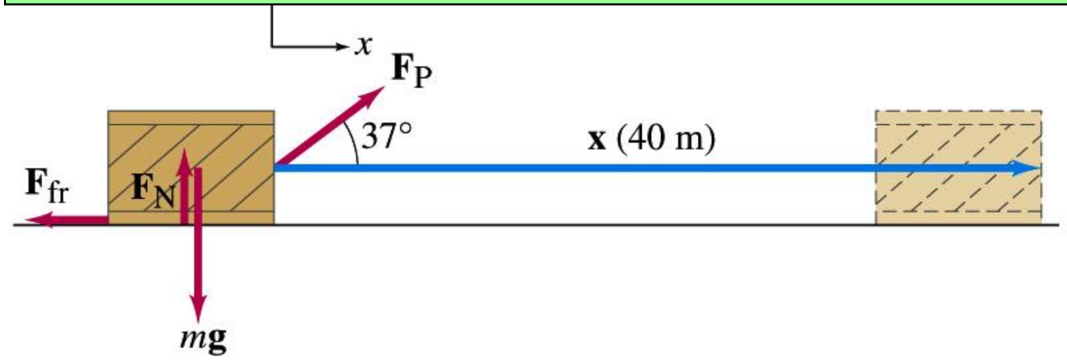
b) $W_g = m\, g\, d \cos(90^o)$

→ ***Nothing to do with the motion***



Example 1A

A 50.0-kg crate is pulled 40.0 m by a constant force exerted (F_P = 100 N and θ = 37.0°) by a person. A friction force F_f = 50.0 N is exerted to the crate. Determine the work done by each force acting on the crate.



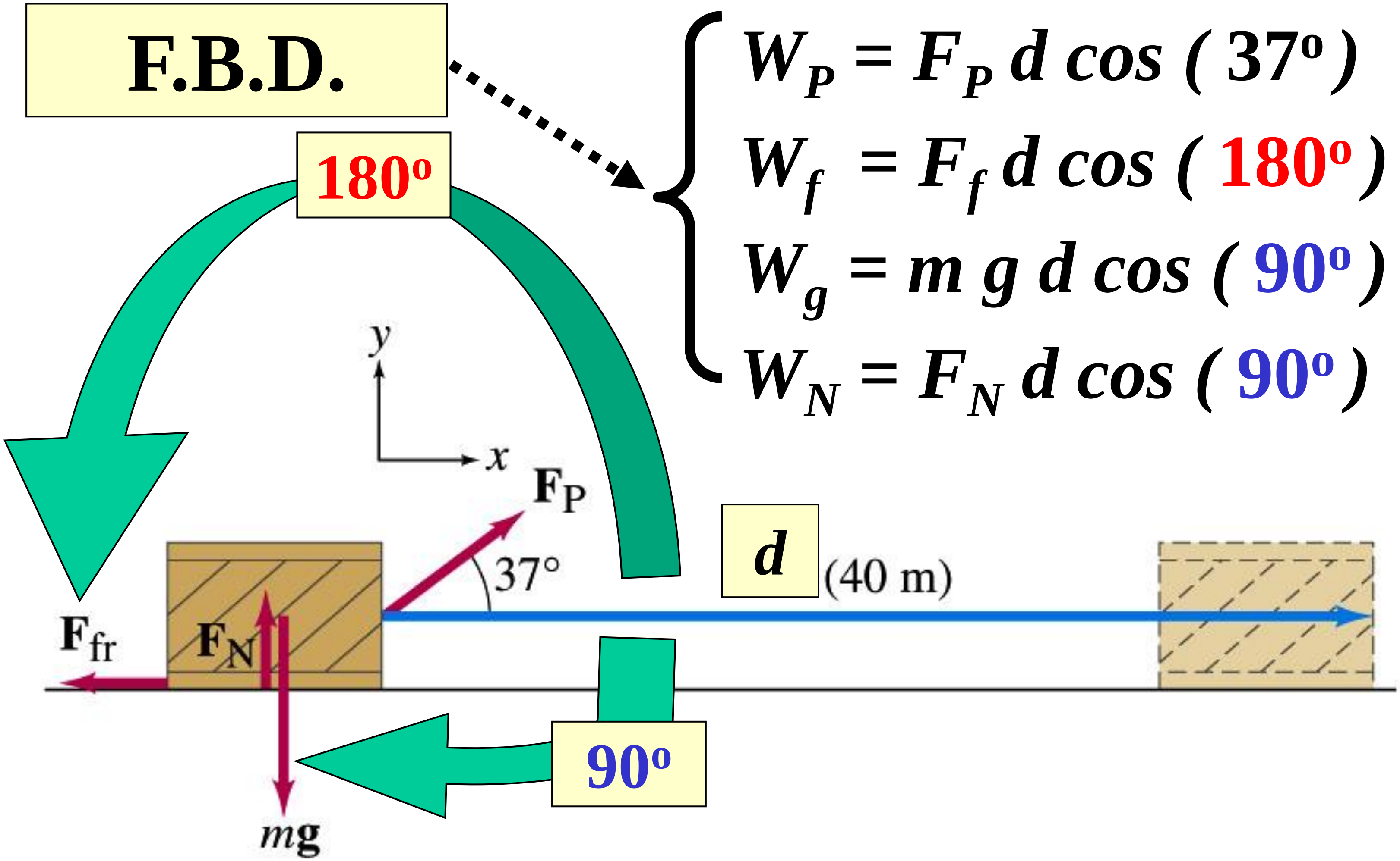
Example 1A (cont'd)

$$W_P = F_P \, d \cos(37^o)$$

$$W_f = F_f \, d \cos(180^o)$$

$$W_g = m \, g \, d \cos(90^o)$$

$$W_N = F_N \, d \cos(90^o)$$



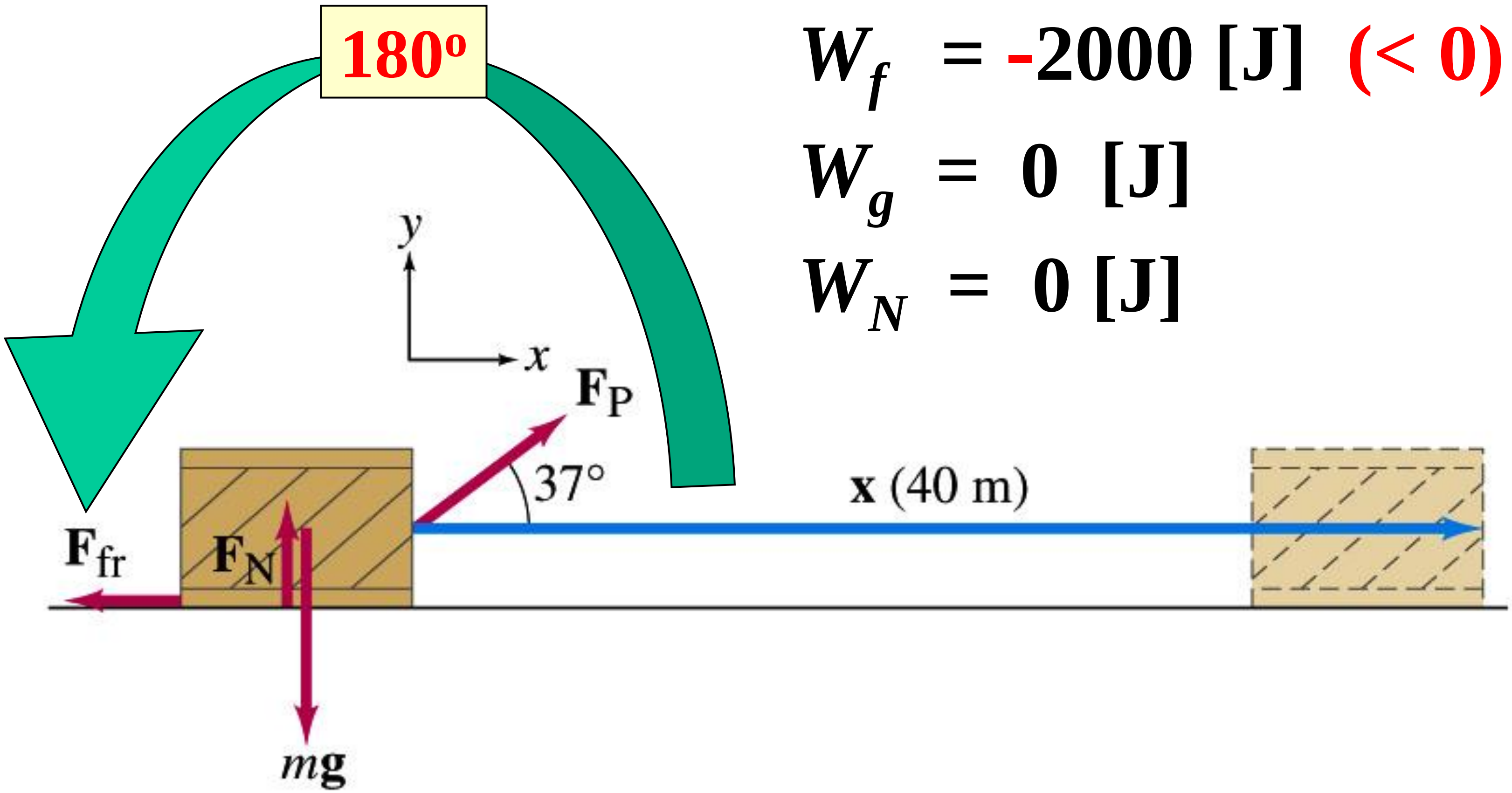
Example 1A (cont'd)

$W_P = 3195$ [J]

$W_f = -2000$ [J] (< 0)

$W_g = 0$ [J]

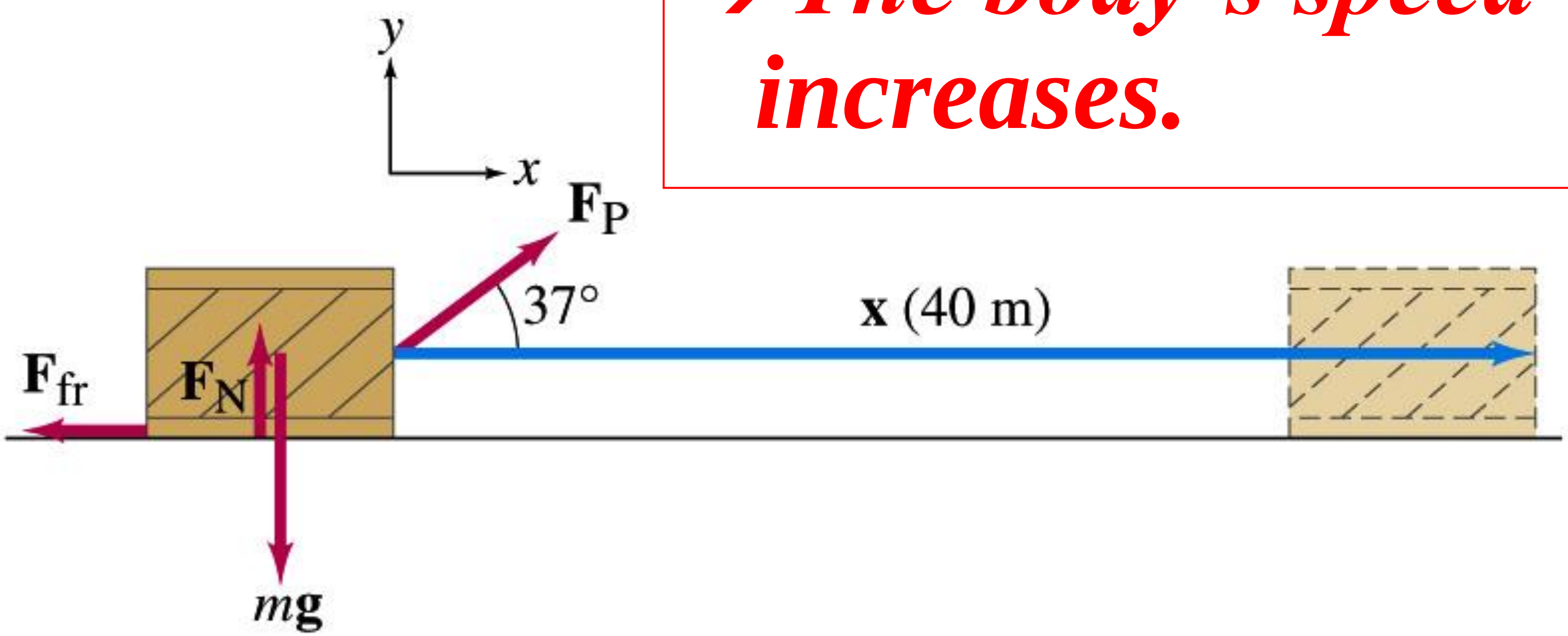
$W_N = 0$ [J]



Example 1A (cont'd)

$$W_{net} = \Sigma W_i$$
$$= 1195\ [J]\ \ (> 0)$$

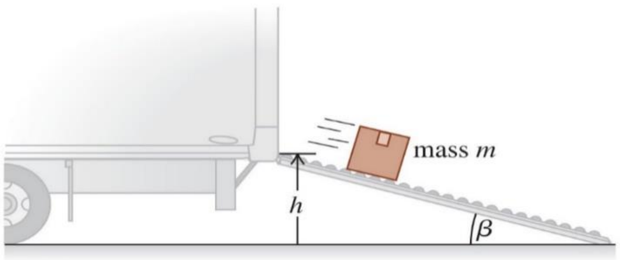
→ *The body's speed increases.*

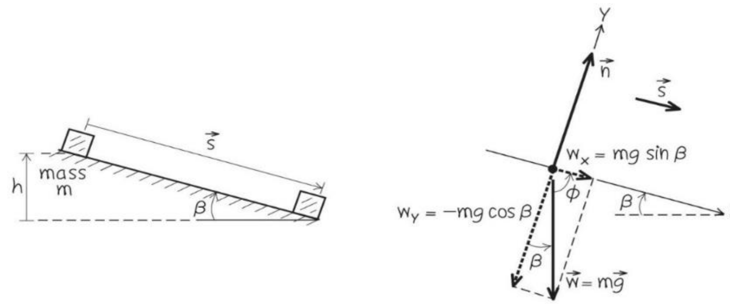


Example 7.2 on Page 187: Sliding on a Ramp

Package of mass m slides down a frictionless incline of height h and angle β.

Find: The total work done by all the relevant forces.





(a) Sketch of situation (b) Free-body diagram of package

Work done by the gravity force (the weight):

Component of the weight parallel to displacement = $mg\sin\beta$.

Work done: $(mg\sin\beta)s = mg[s(\sin\beta)] = mgh$

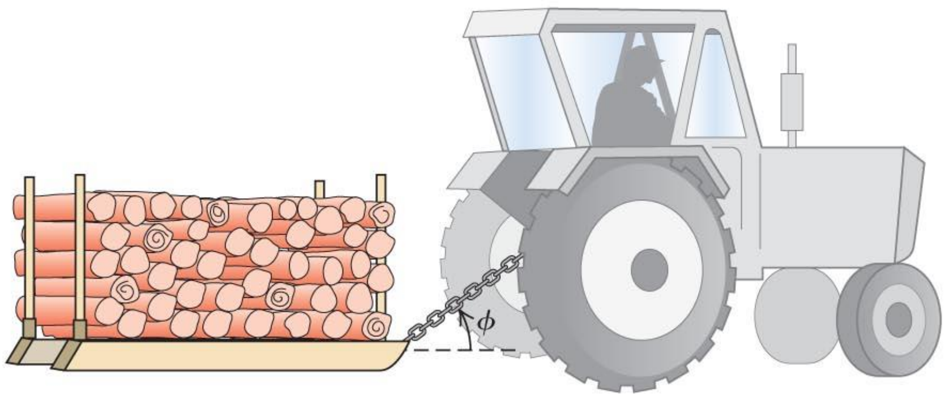
Work done by the normal force = 0

since the component of the normal force parallel to displacement = 0

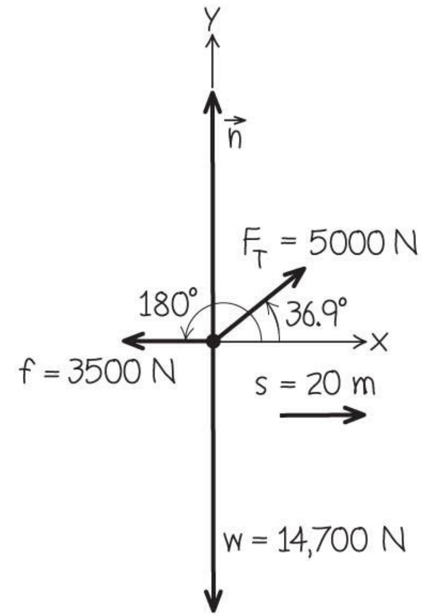
Total work done by all the relevant forces = mgh

Courtesy of Wenhao Wu

Work Done By Several Forces – Example 7.3



(a) A tractor pulls a sled.



(b) Free-body diagram of sled

$$W_T = (F_T \cos\phi)s = 80.0 \text{ kJ}$$

$$W_f = (f_k \cos 180°)s = -70.0 \text{ kJ}$$

$$W_w = W_n = 0$$ (weight and normal are perpendicular to displacement)

$$W_{total} = W_T + W_w + W_n + W_f$$

$$W_{total} = 80.0 \text{ kJ} + 0 + 0 - 70.0 \text{ kJ} = 10.0 \text{ kJ}$$

7.3 Work and Kinetic Energy

What if the net amount of work done on an object is not zero?

- The work-energy theorem:

$$W_{\text{total}} = K_{\text{f}} - K_{\text{i}} = \Delta K$$

- The kinetic energy K of a particle with mass m moving with speed v is $K = \frac{1}{2}mv^2$. It is the energy related to the motion of the particle.

- During any displacement of the particle, the net amount of work done by all the external force on the particle is equal to the change in its kinetic energy.

- Although the kinetic energy K is always positive, W_{total} may be positive, negative, or zero (energy added to, taken away, or left the same, respectively).

- If $W_{\text{total}} = 0$, then the kinetic energy does not change and the speed of the particle remains constant.

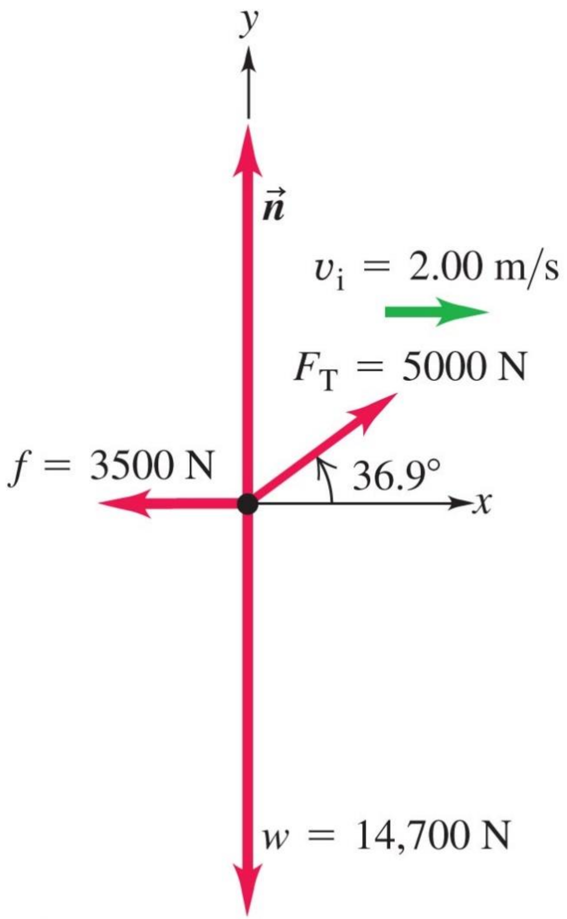
Work and Energy Related – Example 7.4

- Using work and energy to calculate speed.
- Returning to the tractor pulling a sled problem of Example 7.3:
  - If you know the initial speed, and the total work done, you can determine the final speed after displacement *s*.

$$K_f = K_i + W_{total}$$

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 + W_{total}$$

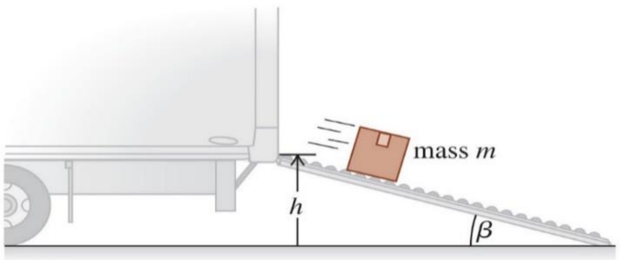
$$v_f = \sqrt{v_i^2 + 2W_{total}/m}$$

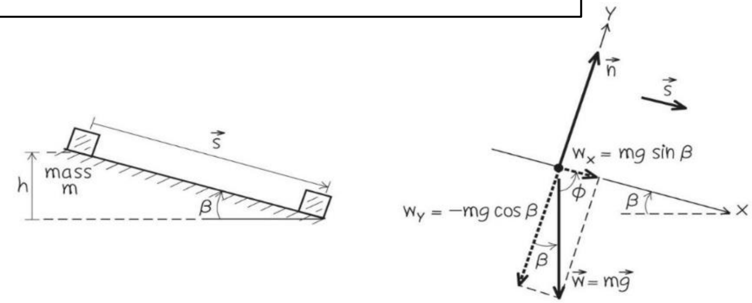


Revisit Example 7.2 on Page 186: Sliding on a Ramp

Package of mass *m*, initially at rest, slides down a frictionless incline of height *h* and angle *β*.

Find: Its speed the moment it reaches the bottom.





(a) Sketch of situation **(b)** Free-body diagram of package

As we calculated earlier, the total work done by all the relevant forces is

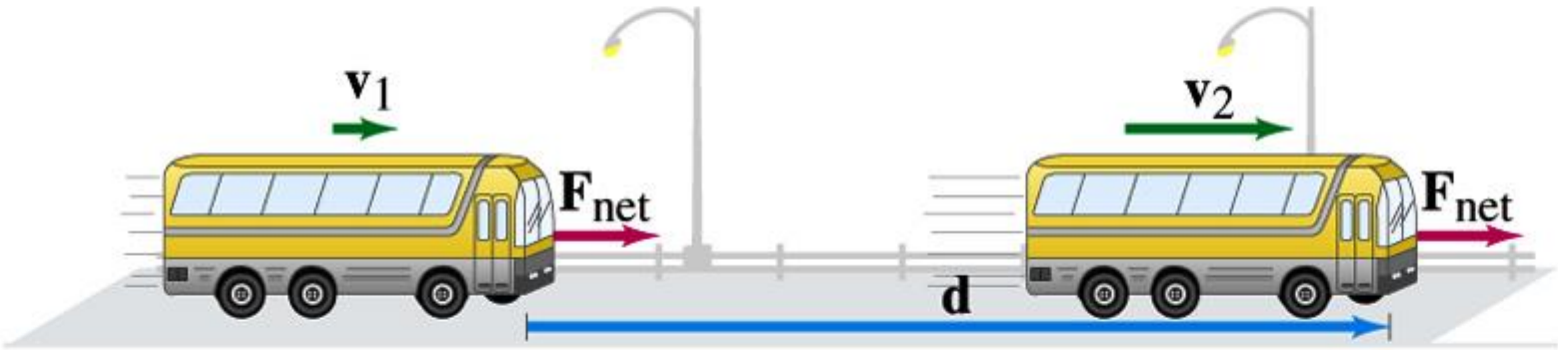
=(*mg*sin*β*)*s* =*W*mg[*s*(sin*β*)] = *mgh*.

Apply the Work-Energy Theorem:

$$W = mgh = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

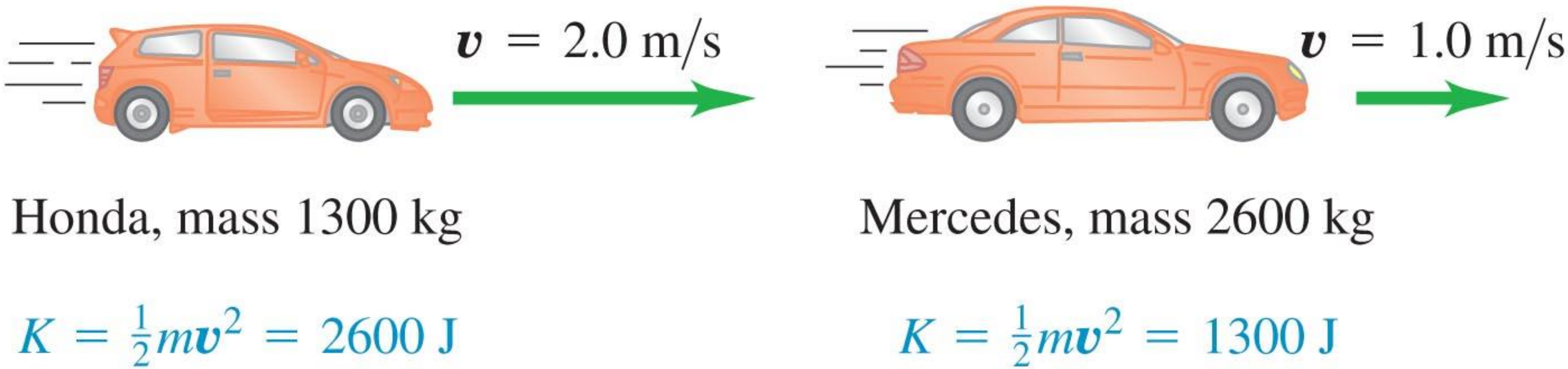
$$v_f = \sqrt{2gh}$$

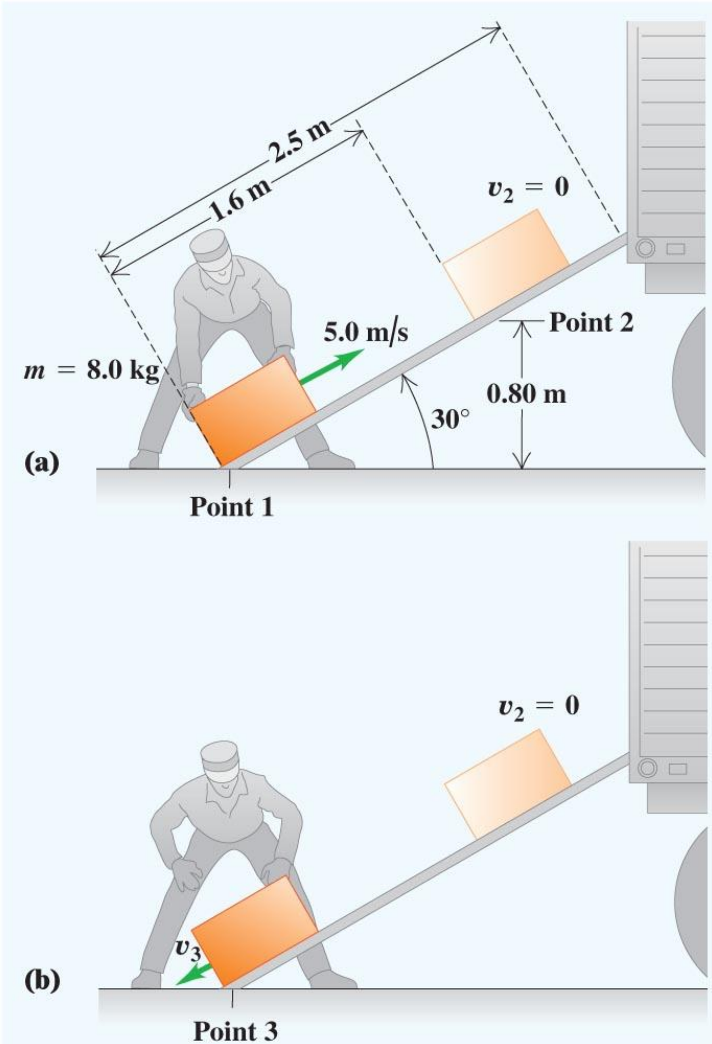
Work-Energy Theorem



$$
\begin{aligned}
W_{net} &= F_{net}\ d = (m\ a)\ d \\
&= m\ [\ (v_2{}^2 - v_1{}^2\ )\ /\ 2d\ ]\ d \\
&= (1/2)\ m\ v_2{}^2 - (1/2)\ m\ v_1{}^2 \\
&= K_2 - K_1
\end{aligned}
$$

Kinetic energy





Example 7.14

Loading a crate (with friction it does not go all the way up)

a) $K_1 = \frac{1}{2}mv^2 = \frac{1}{2}8(5)^2 = 100\text{ J}; K_2 = 0;\ U_1 = 0;\ \ U_2 = mgh = 8(9.8)(0.8) = 62.7\text{ J}$

$$W_{other} = -f_s = -f\,d = -f(1.6)$$

Here; f is the unknown friction force

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

$100\text{ J} + 0\ - f(1.6) = 0 + 62.7\text{ J}\ \rightarrow f = 23\text{ N}$

b) $W_{other} = W_f = (-2)(1.6)(23.3)$ =-74.6 J

$$K_1 = 100\text{ J};\ U_1 = U_3 = 0;\ K_3 = \frac{1}{2}mv_3^2 = \frac{1}{2}8v_3^2$$

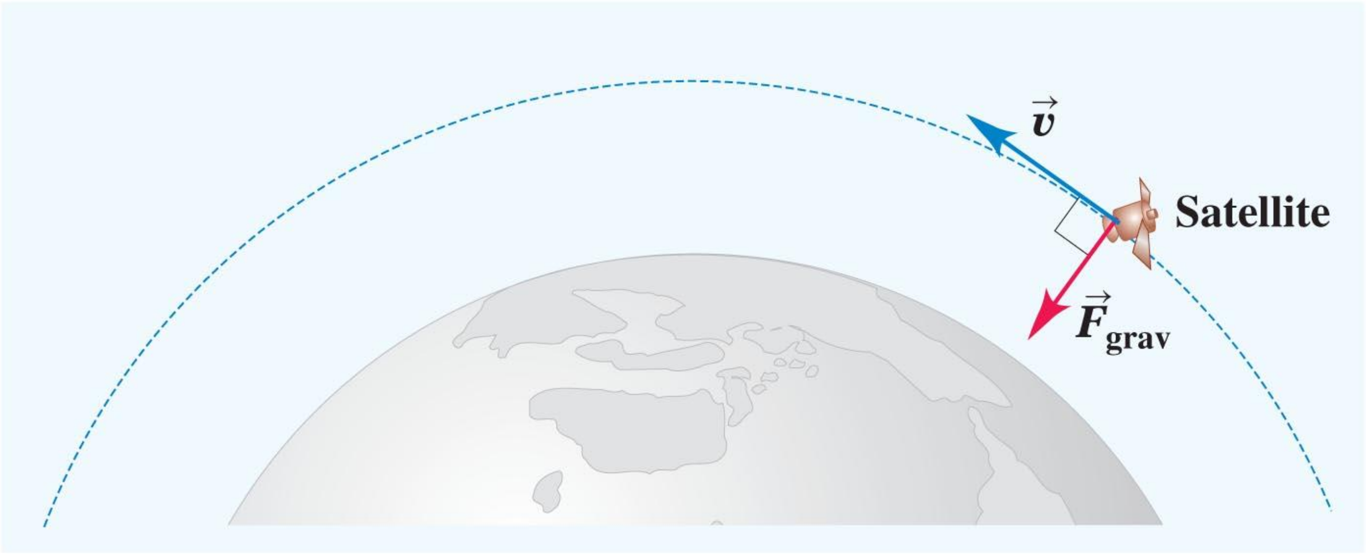
$$100\text{ J} + 0 - 74.6\text{J} = \frac{1}{2}8v_3^2 + 0$$

$$\rightarrow v_3 = 2.5\frac{\text{m}}{\text{s}}$$

The return velocity is smaller than the launch velocity due to friction

Satellite in a circular orbit

Does the Earth do work on the satellite?



Clicker question

What can you say about the work the sun does on the earth during the earth's orbit? (Assume the orbit is perfectly circular.)

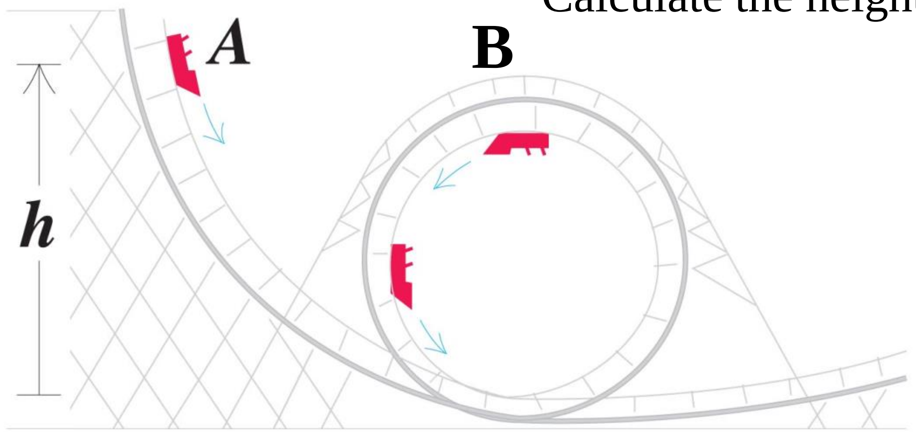
a) The sun does net negative work on the earth.
b) The sun does net positive work on the earth.
c) The work done on the earth is positive for half the orbit and negative for the other half, summing to zero.
d) The sun does no work on the earth at any point in the orbit.

Problem81. Riding a loop-the-loop starting at rest from a point **A**, find the minimum height ***h***, so that the car will not fall of the track at the top of the circular part of the loop, which has a radius of 20m.

At the top of the loop at point **B**

$F_{rad} = mg = m\frac{v^2}{R} \rightarrow v = \sqrt{gR} = v_B$ → minimum velocity needed to stay on the track.

Calculate the height h_A necessary to produce minimum speed v_B



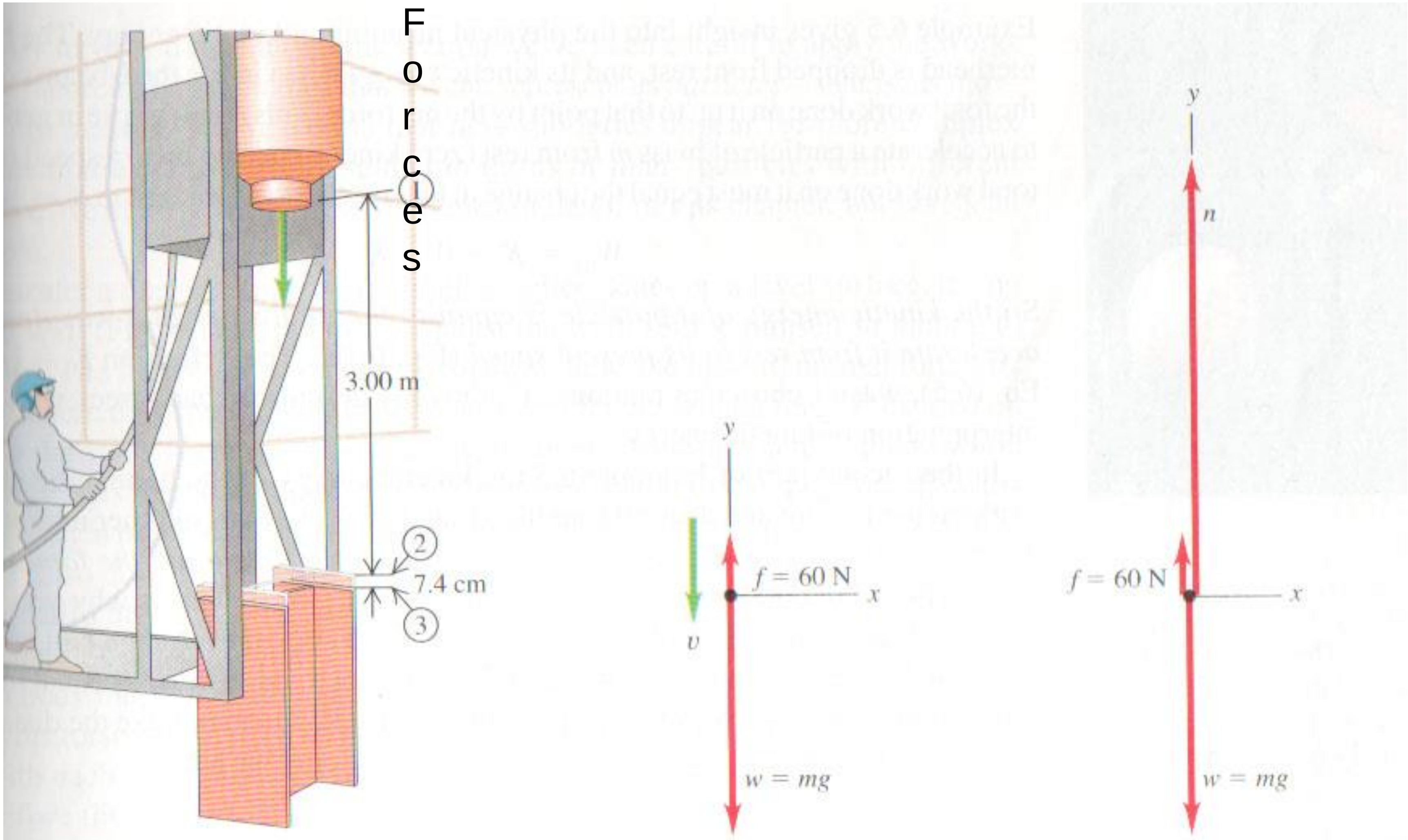
$$U_A + K_A = U_B + K_B$$
$$mgh_A + 0 = mgh_B + \frac{1}{2}mv_B^2$$
$$gh_A = g2R + \frac{1}{2}(\sqrt{gR})^2$$
$$h_A = \frac{5}{2}R$$

If **h** < 2.5**R** the car is moving too slow and falls off the track

If **h** > 2.5**R** than at point B more downward force than gravity is needed and this is provided by the normal force that the track exerts on the car.

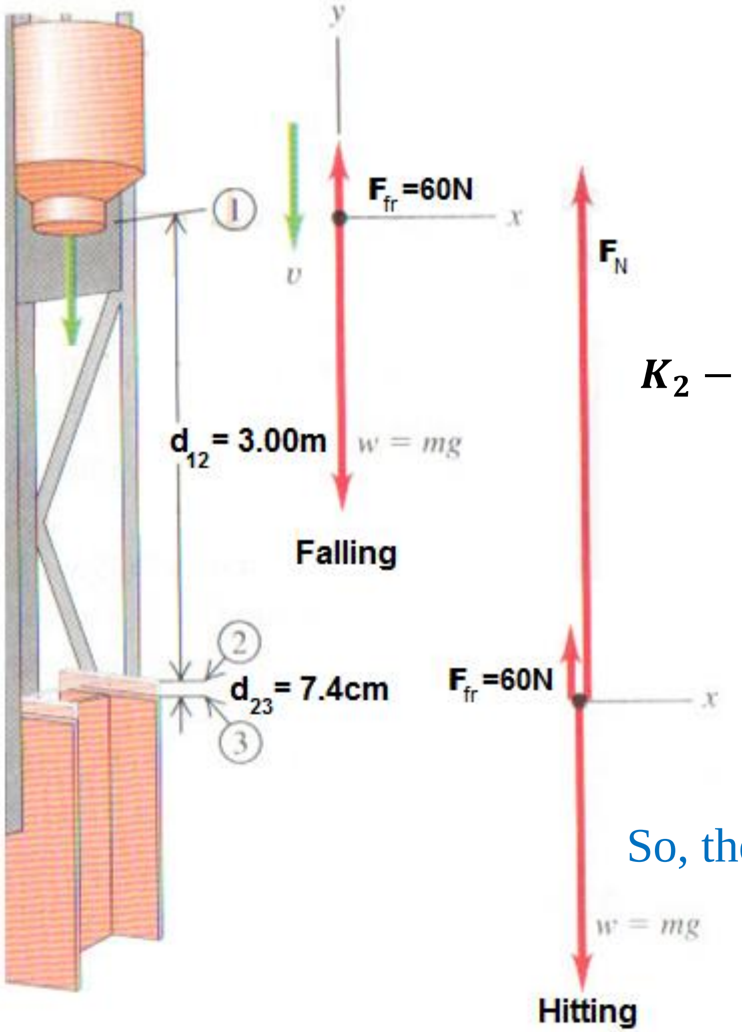
Forces on a hammerhead

F
o
r
c
e
s



(a) (b) (c)

Forces on a hammerhead



$$W = F \cdot d$$

$$W = 200kg \cdot 9.8\frac{m}{s^2} = 1960N$$

$$\text{Weight } w - F_{fr} = 1960 - 60 = 1900N$$

$$K_2 - K_1 = \frac{1}{2}mv_2^2 - 0 = W = (w - F_{fr}) \cdot d_{12} = 1900 \cdot 3 = 5700N$$

$$\rightarrow \frac{1}{2}mv_2^2 = 5700N \text{ (Falling) and } v_2 = \sqrt{\frac{2*5700}{200}} = 7.55\frac{m}{s}$$

$$K_3 - K_2 = (w - F_{fr} - F_N) \cdot d_{23} \text{ (Hitting)}$$

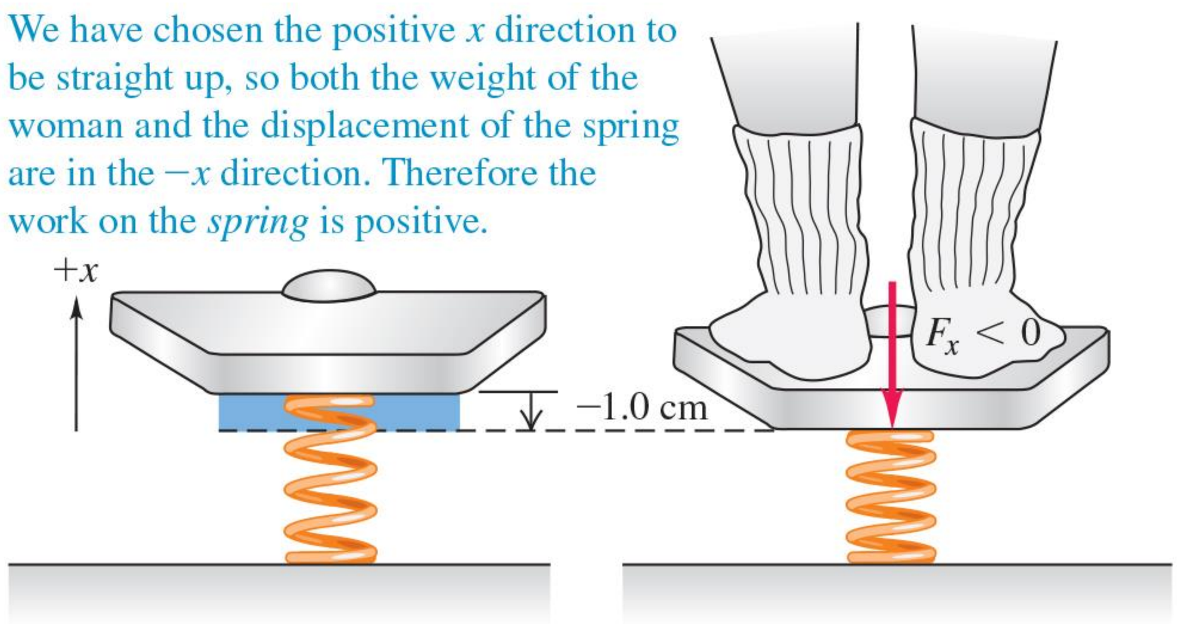
$$\rightarrow F_N = w - F_{fr} - \frac{K_3 - K_2}{d_{23}} = 1960 - 60 - \frac{0 - 5700}{0.074}$$

$$\therefore F_N = 79000N$$

So, the normal force is 40 times larger than weight of the hammer.

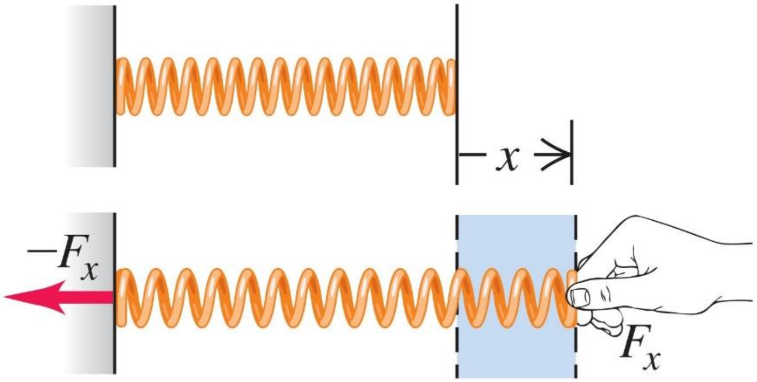
Work Done on a Spring Scale – Example 7.6

- Energy may be stored in compressed springs on a bathroom scale.
- Refer to the worked example on page 194.



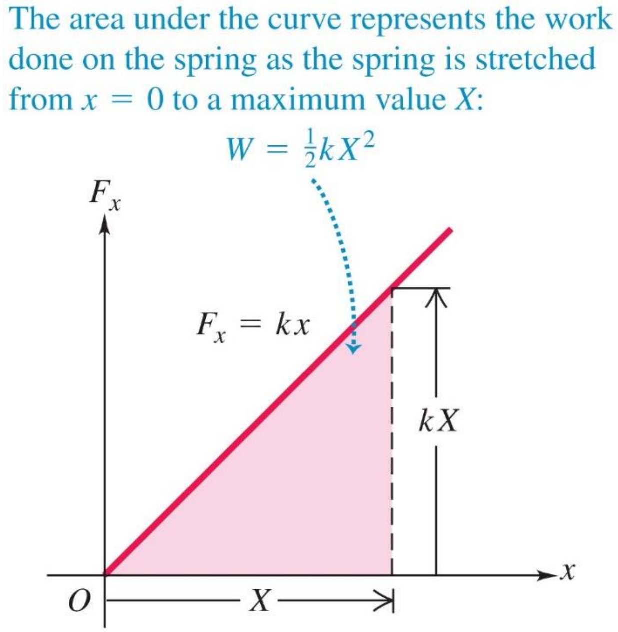
Work Done By a Varying Force

- Hooke's law: $$F_{spring} = kx$$



- As seen, this is a prime example of a varying force. The work done by a stretching/compressing a spring is equal to the area of the shaded triangle, or

$$W_{spring} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$



Example:
Work done on a spring by a Varying external force.

- The force needed to stretch or compress a spring

$$F = kx.$$

- The work done on the spring

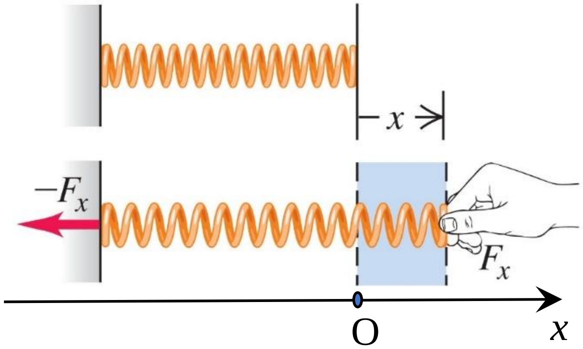
$$W = \left(\frac{1}{2}\right)(x)(kx) = \frac{1}{2}kx^2$$

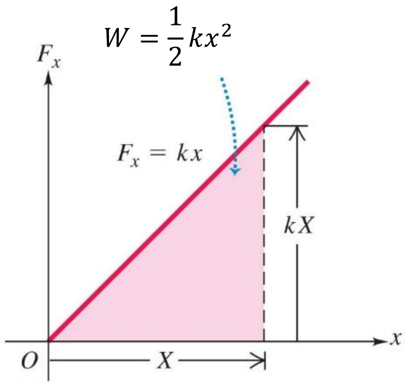
- The work done on the spring for stretching it from x_1 to x_2 is

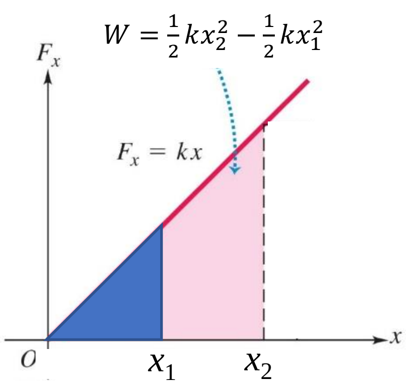
$$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

- The work done by the spring during this process

$$W_{spring} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$





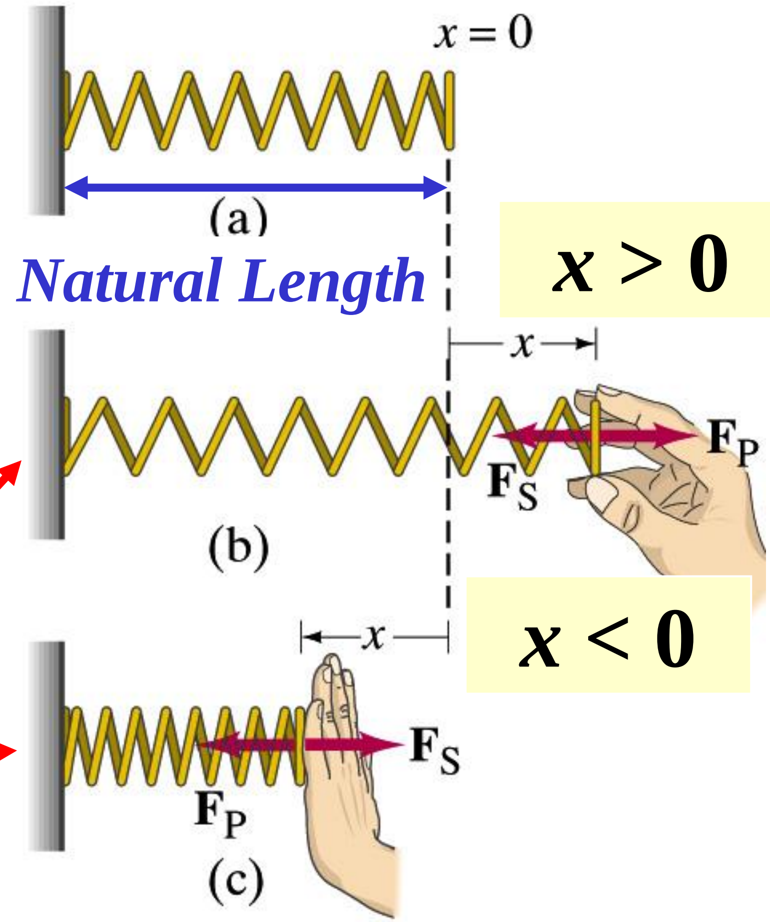


Spring Force (Hooke's Law)

Spring Force (Restoring Force):

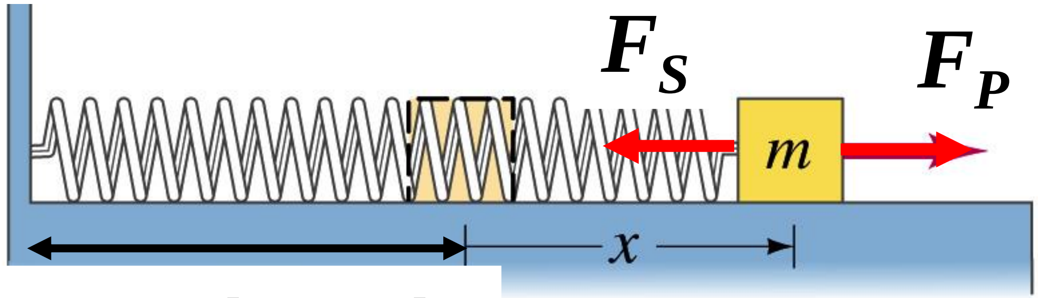
The spring exerts its force in the direction opposite the displacement.

$$F_S(x) = -k\,x$$



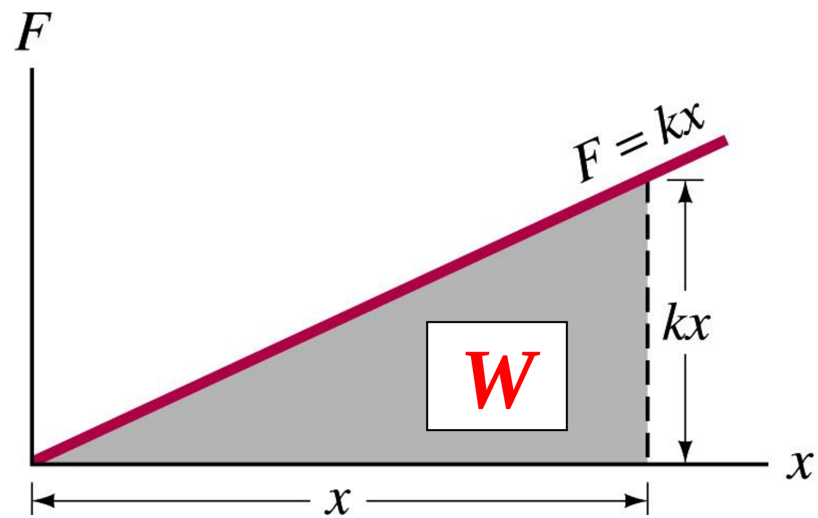
Work Done to Stretch a Spring

$$F_S(x) = -k\,x$$

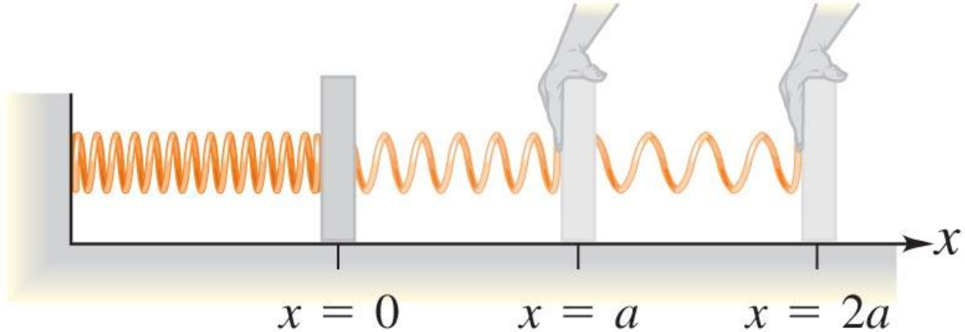


Natural Length

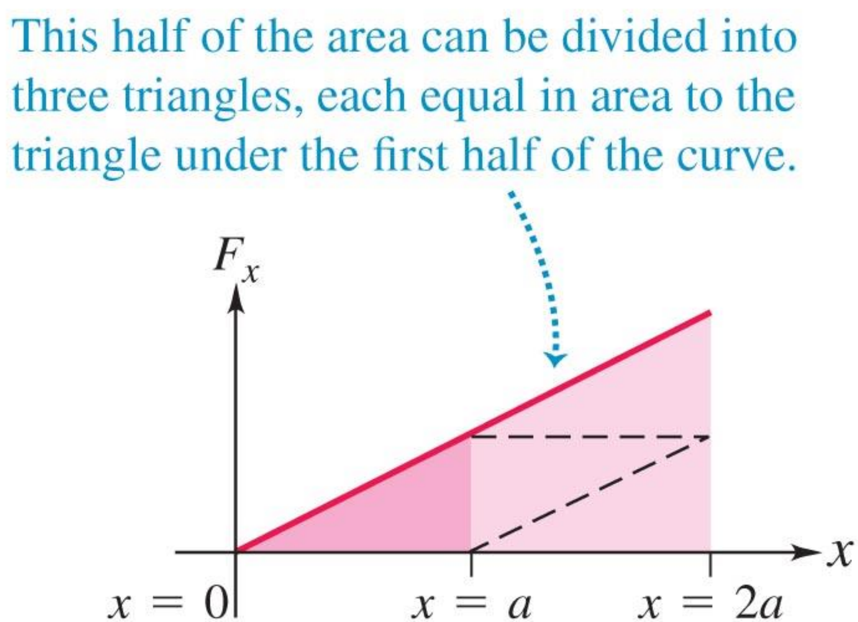
Area (triangle)
= baseline * height/ 2



$$k\,x \cdot x/2 = A$$



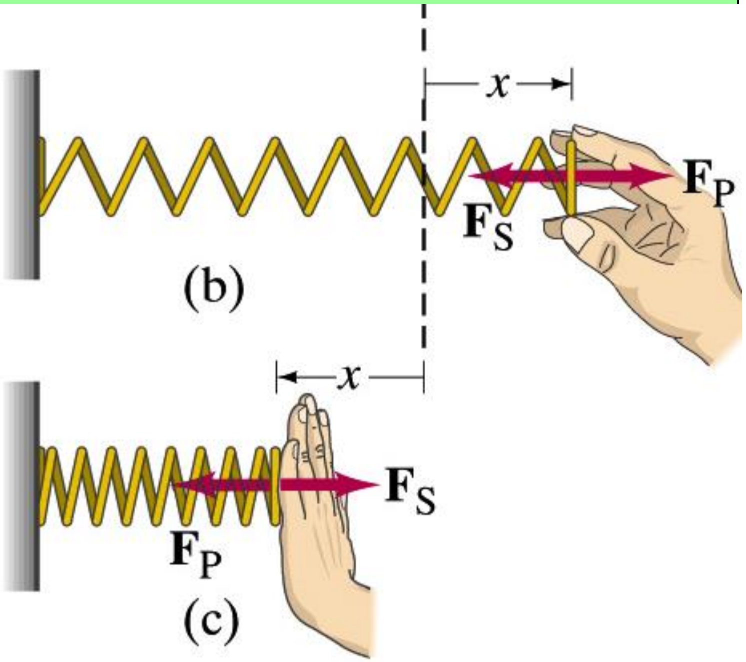
(a) Stretching a spring through two equal halves of a total displacement $2a$



(b) Force-versus-distance curve for the two halves of the displacement

Example 1A

A person pulls on the spring, stretching it 3.0 cm, which requires a maximum force of 75 N. How much work does the person do ? If, instead, the person compresses the spring 3.0 cm, how much work does the person do ?



Example 1A (cont'd)

(a) Find the spring constant *k*

$$k = F_{max} / x_{max}$$

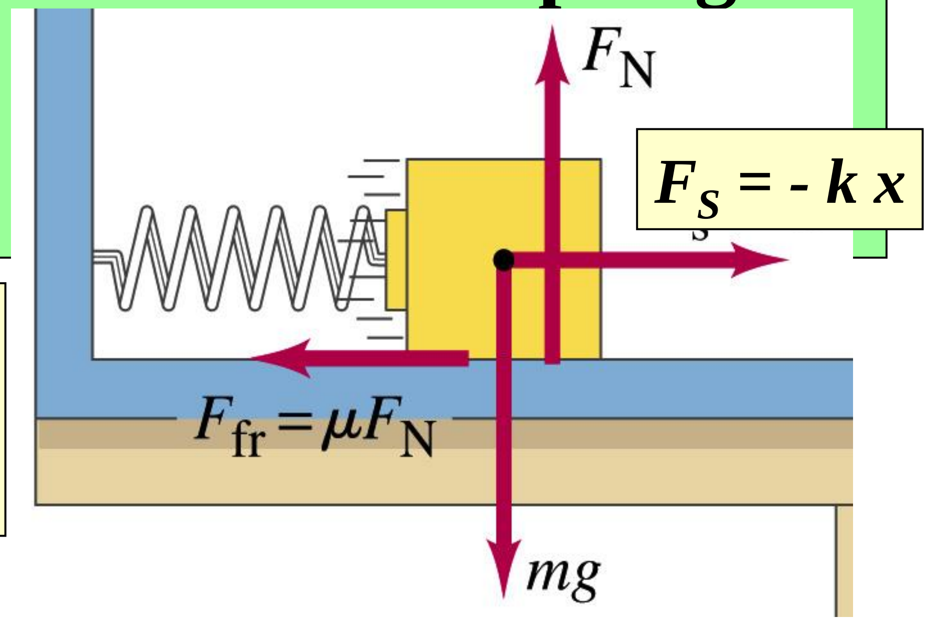
$$= (75\ \text{N}) / (0.030\ \text{m}) = 2.5 \times 10^3\ \text{N/m}$$

(b) Then, the work done by the person is

$$W_P = (1/2)\, k\, x_{max}^2 = 1.1\ \text{J}$$

Example 2

A 1.50-kg block is pushed against a spring (k = 250 N/m), compressing it 0.200 m, and released. What will be the speed of the block when it separates from the spring at x = 0? Assume μ_k = 0.300.



(i) **F.B.D. first !**

(ii) $x < 0$

Example 2 (cont'd)

(a) The work done by the spring is

$$W_S = 1/2\, k\, x^2 = ½\, (250 N/m)\, (0.2\, m^2\ ) = +5.00\ J$$

(b) $W_f = -\mu_k F_N (x_2 - x_1) = -4.41\,(0 + 0.200)$

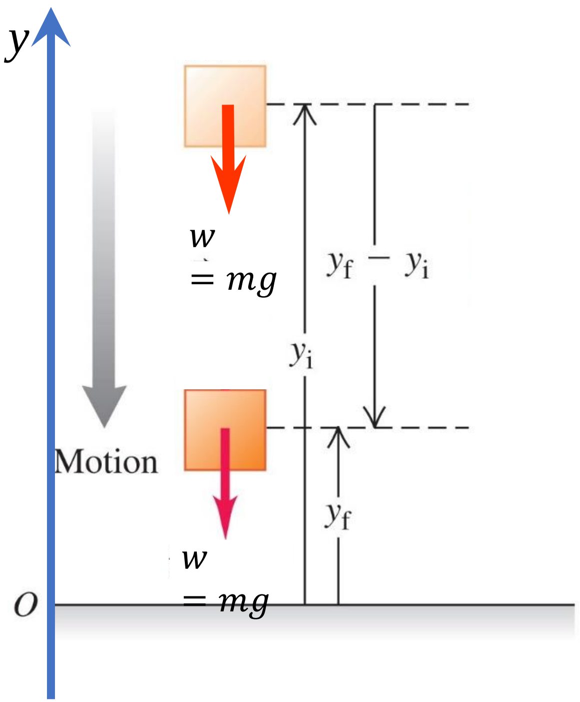
(c) $W_{net} = W_S + W_f = 5.00 - 4.41 \times 0.200$

(d) Work-Energy Theorem: $W_{net} = K_2 - K_1$

→ $4.12 = (1/2)\, m\, v^2 - 0$

→ $v = 2.34$ m/s

7.5 Potential Energy-------What is potential energy???
Using free-fall under the influence of gravity (weight) as an example.



How does the work-energy theorem describe this motion?

$$\text{Work} = (mg) * (y_i - y_f) = mgy_i - mgy_f$$

$$\text{Change in kinetic energy} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The work-energy theorem:

$$mgy_i - mgy_f = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Describing this motion using the concept of potential energy.

Define gravitational potential energy: $U_{grav} = mgy$

In the presence of a force, there is **energy** associated with the **position** of a particle----potential energy.

Unit = joule   can be positive ,negative, or zero

Courtesy of Wenhao Wu

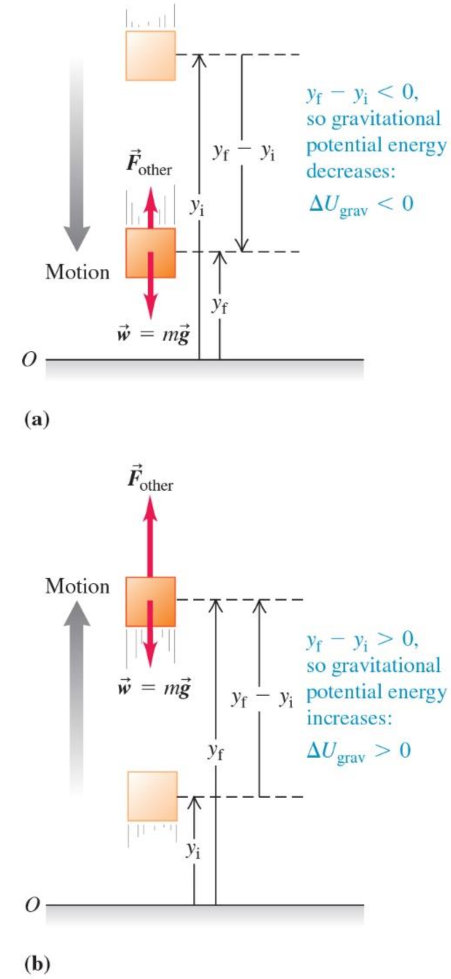
Examples of Force and Potential Energy

Force Type	Work Done by the Force $(i \to f)$	Potential Energy
Gravity $F_g = mg$	$W = mgy_i - mgy_f = U_i - U_f$	$U_{grav} = mgy$
Spring (elastic) $F_{spring} = -kx$	$W = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = U_i - U_f$	$U_{el} = \frac{1}{2}kx^2$

Potential Energy (1 of 2)

- In cases of conservative forces (gravity or elastic forces), there can be "stored" energy due to the spatial arrangement of a system, or **potential energy.**
- Gravitational potential energy (U_{grav}), near the surface of the earth can be written:

$$U_{grav} = mgy$$



Potential Energy (2 of 2)

- The change in the potential energy due to conservative forces is related to the work done by the net force:

$$W_{\text{grav}} = U_{\text{grav,i}} - U_{\text{grav,f}} = -\Delta U_{\text{grav}}$$

- If only conservative forces act, then by the work-energy theorem we can define the total mechanical energy:

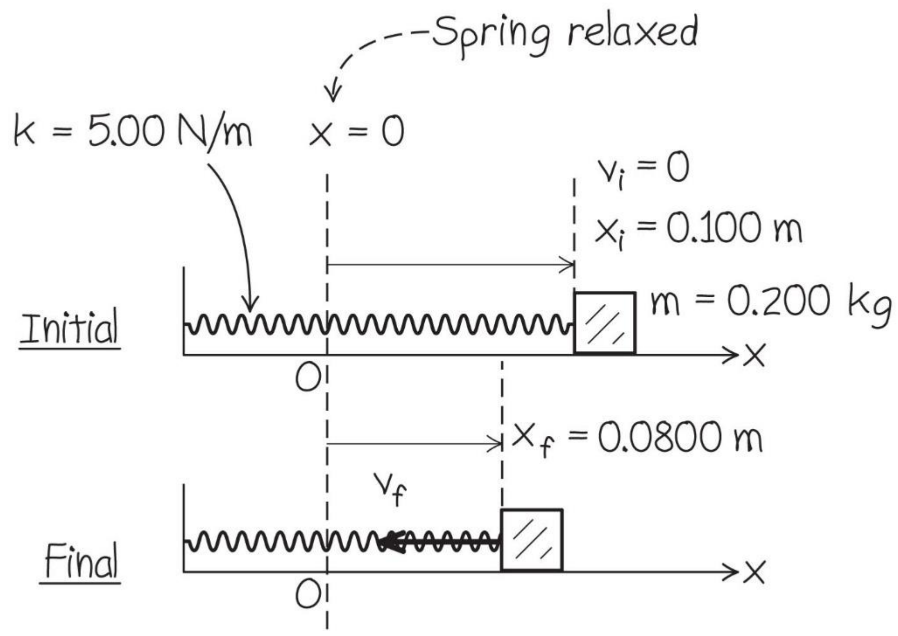
$$W_{\text{total}} = \left[W_{\textit{conservative}} = U_{\text{i}} - U_{\text{f}}\right] = K_{\text{f}} - K_{\text{i}}$$

$$K_{\text{i}} + U_{\text{i}} = K_{\text{f}} + U_{\text{f}}$$

$$\frac{1}{2}m\upsilon_{\text{i}}^2 + mgy_{\text{i}} = \frac{1}{2}m\upsilon_{\text{f}}^2 + mgy_{\text{f}}$$

Potential Energy on an Air Track with Mass and Spring

- Refer to Example 7.8.
- Knowing the initial state of our system and thus the total mechanical energy, we use this to find the final state at any position.
- Using conservation of total mechanical energy:



$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$$

7.6 Conservation of Energy

For a conservative force, the work done is related to the change in the potential energy:

$$W = U_i - U_f.$$

Apply the work-energy theorem, we obtain:

$$U_i - U_f = K_f - K_i \qquad \text{or} \qquad U_i + K_i = U_f + K_f.$$

The expression in red is the conservation of energy, when the force is a conservative force.

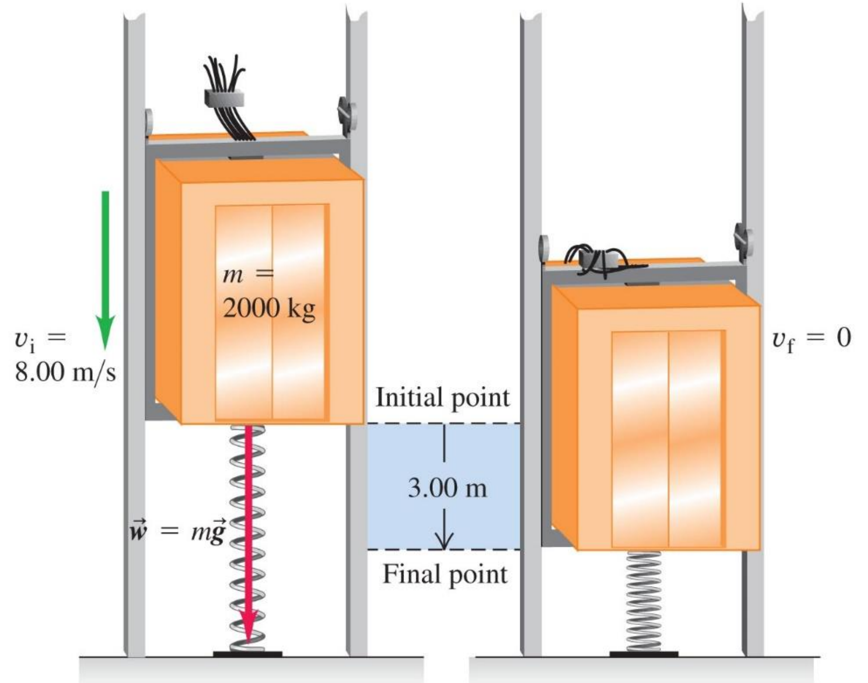
The total mechanical energy E is the sum of the potential energy and kinetic energy:

$$E = U + K$$

Conversion and Conservation – Example 7.11

- As kinetic and potential energy are interconverted, dynamics of the system may be solved.

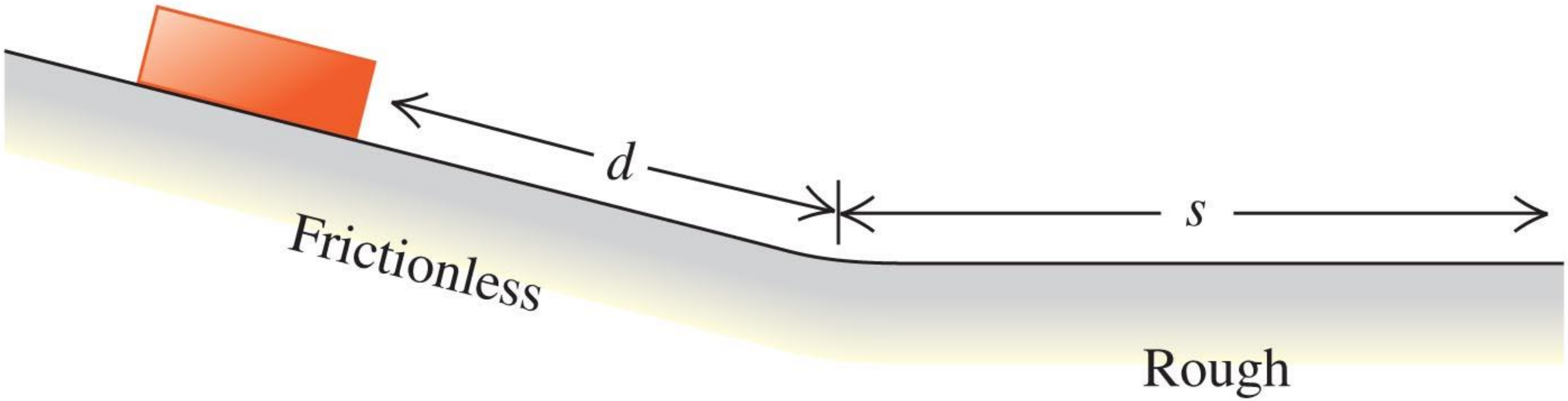
$$E_{\text{total mechanical}} = K + U_{\text{grav}} + U_{\text{el}} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$



$\mathbf{K_i = \frac{1}{2}mv^2 = 64\,000 J}$ $\mathbf{U_i = mgh = 58\,800\ J}$ $\mathbf{U_i + K_i = U_f + K_f}$

64 000 J+ 58 800 J = 0 + $\mathbf{\frac{1}{2}k(-3\ m)^2}$ $\mathbf{k = 2.73x10^2}$ **N/m**

Clicker question



A. K increases as it slides down the ramp and remains constant along the horizontal surface.
B. K remains constant as it slides down the ramp and decreases along the horizontal surface.
C. K increases as it slides down the ramp and deceases along the horizontal surface

How about the potential energy with the same questions??

Useful Expressions for Applications

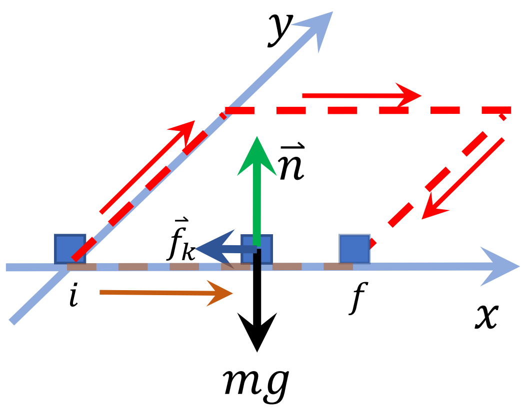
With gravity force:	$E = U_{grav} + K = mgy + \frac{1}{2}mv^2$
Conservation of energy:	$mgy_i + \frac{1}{2}mv_i^2 = mgy_f + \frac{1}{2}mv_f^2$
With spring force:	$E = U_{el} + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$
Conservation of energy:	$\frac{1}{2}kx_i^2 + \frac{1}{2}mv_i^2 = \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2$
With both g&s forces:	$E = U_{grav} + U_{el} + K = mgy + \frac{1}{2}ky^2 + \frac{1}{2}mv^2$
Conservation of energy:	$mgy_i + \frac{1}{2}ky_i^2 + \frac{1}{2}mv_i^2 = mgy_f + \frac{1}{2}ky_f^2 + \frac{1}{2}mv_f^2$

7.7 Conservative and Nonconservative Forces

Nonconservative Forces

Friction Force

- Work done depends on the path.
- Cannot define a potential energy.



Conservative Forces

Work done by a conservative force depends on the initial and the final positions only. It is independent of the details of the path along which the object moves from the initial to the final positions.

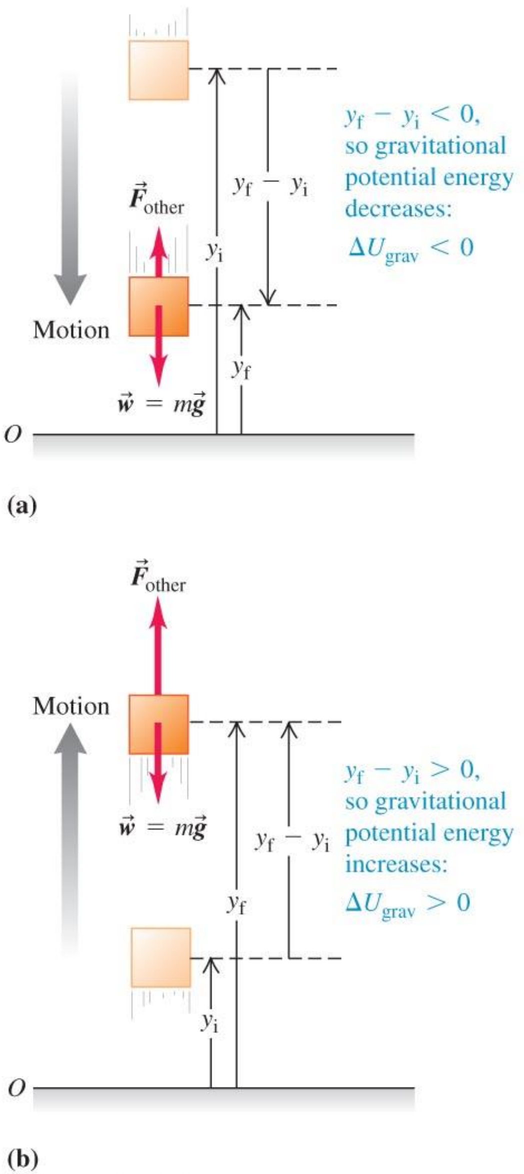
When both conservative and nonconservative forces act on an object

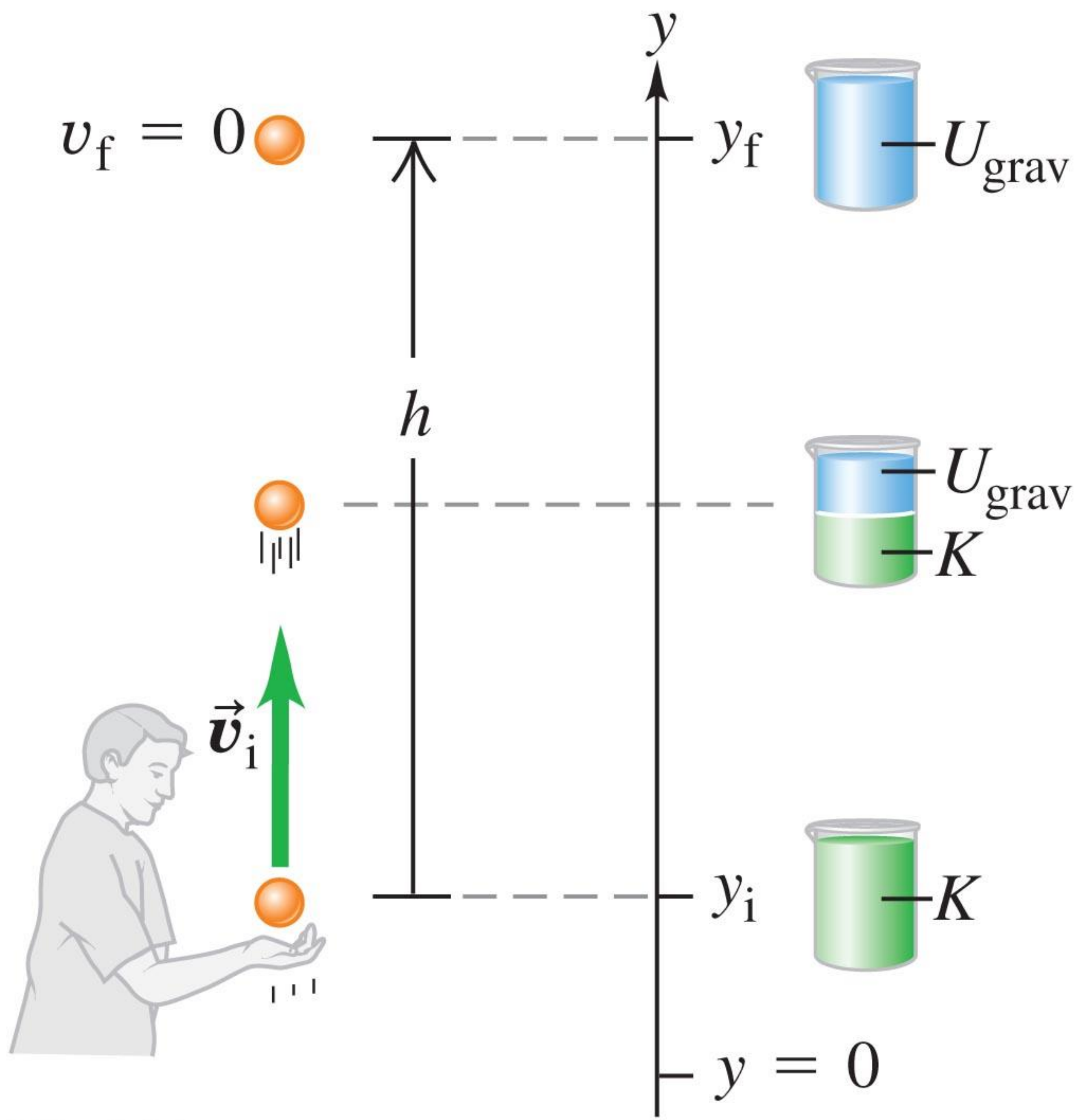
$$\left(U_{grav,i} + U_{el,i} + \frac{1}{2}mv_i^2\right) + W_{other} = \left(U_{grav,f} + U_{el,f} + \frac{1}{2}mv_f^2\right)$$

Gravitational potential energy

- In cases of conservative forces (gravity or elastic forces), there can be "stored" energy due to the spatial arrangement of a system, or **potential energy.**
- Gravitational potential energy (U_{grav}), near the surface of the earth can be written:

$$U_{grav} = mgy$$





$v_f = 0$
h
$\vec{v}_i$
y
y_f
y_i
$y = 0$
U_{grav}
U_{grav}
K
K

Potential Energy

- The change in the potential energy due to conservative forces is related to the work done by the net force:

$$W_{\text{grav}} = U_{\text{grav,i}} - U_{\text{grav,f}} = -\Delta U_{\text{grav}}$$

- If only conservative forces act, then by the work-energy theorem we can define the total mechanical energy:

$$W_{\text{total}} = \left[W_{conservative} = U_{\text{i}} - U_{\text{f}}\right] = K_{\text{f}} - K_{\text{i}}$$

$$K_{\text{i}} + U_{\text{i}} = K_{\text{f}} + U_{\text{f}}$$

$$\tfrac{1}{2}mv_{\text{i}}^2 + mgy_{\text{i}} = \tfrac{1}{2}mv_{\text{f}}^2 + mgy_{\text{f}}$$

Work Done by the Gravitational Force (III)

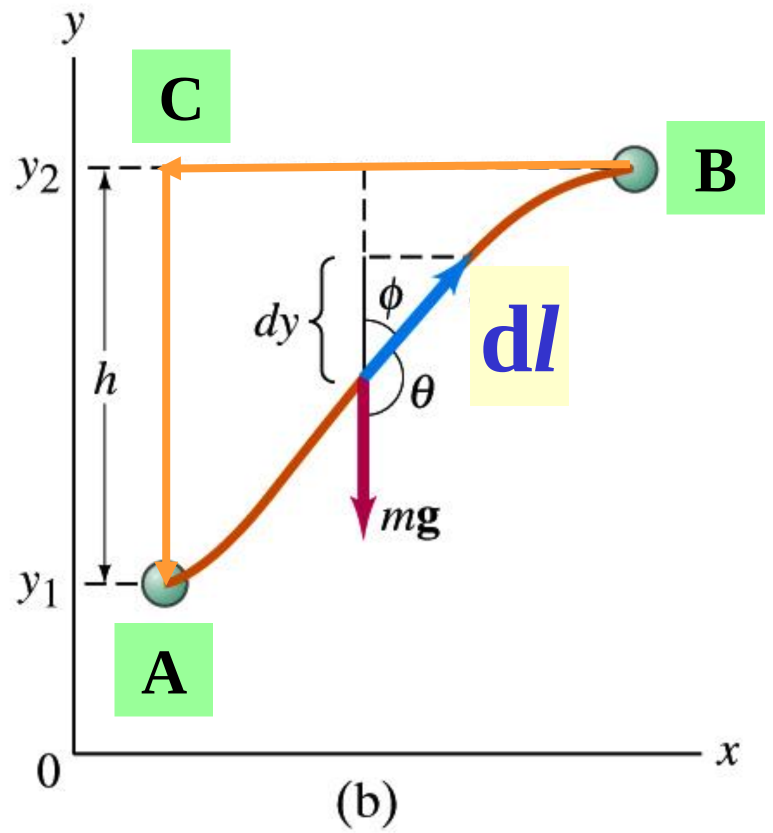
$$W_g < 0 \quad \text{if} \quad y_2 > y_1$$

$$W_g > 0 \quad \text{if} \quad y_2 < y_1$$

The work done by the gravitational force depends only on the initial and final positions.

Work Done by the Gravitational Force (IV)

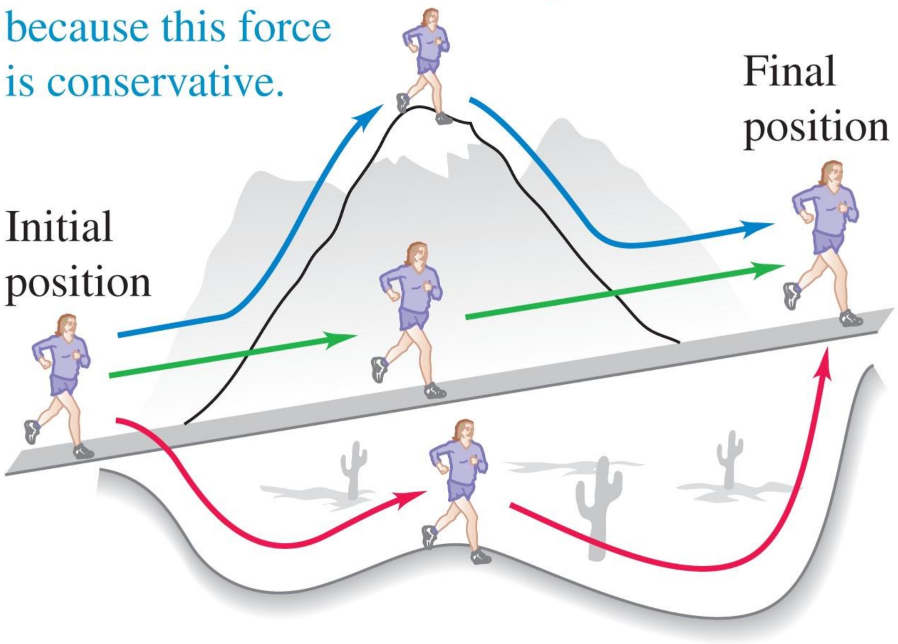
$$W_{g(A\rightarrow B\rightarrow C\rightarrow A)}$$
$$= W_{g(A\rightarrow B)} + W_{g(B\rightarrow C)} + W_{g(C\rightarrow A)}$$
$$= mg(y_1 - y_2) + 0 + mg(y_2 - y_1)$$
$$= 0$$



$W_g = 0$ for a closed path

Conservative Forces II – Figure 7.34

- The work done by a conservative force is independent of the path taken.



- When the starting and ending points are the same, the total work is zero.

Conservative and Nonconservative Forces

- In the previous section, we discussed that if we had *only conservative forces* acting, then we could used conservation of total mechanical energy.
- If we have nonconservative forces which do work, we have to add this to the total energy:

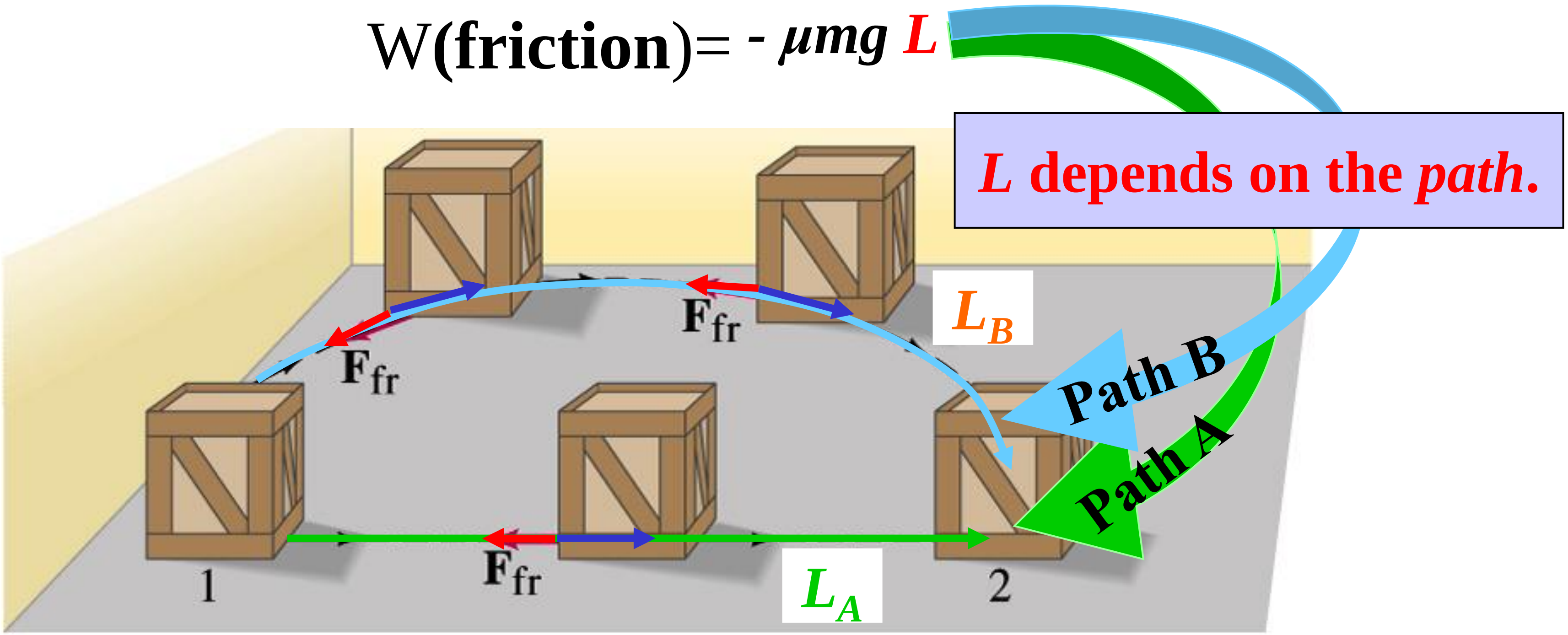
$$W_{\text{other}} = \left(K_{\text{f}} + U_{\text{grav,f}} + U_{\text{el,f}}\right) - \left(K_{\text{i}} + U_{\text{grav,i}} + U_{\text{el,i}}\right)$$

- W_{other} is the work done by nonconservative forces (e.g. friction).

Work Done by F_f (I)

$$W_f = \int_{l_1(=0)}^{l_2(=L)} \vec{F}_f \bullet d\vec{l}$$
$$= (-\mu mgl)\Big|_0^L =$$

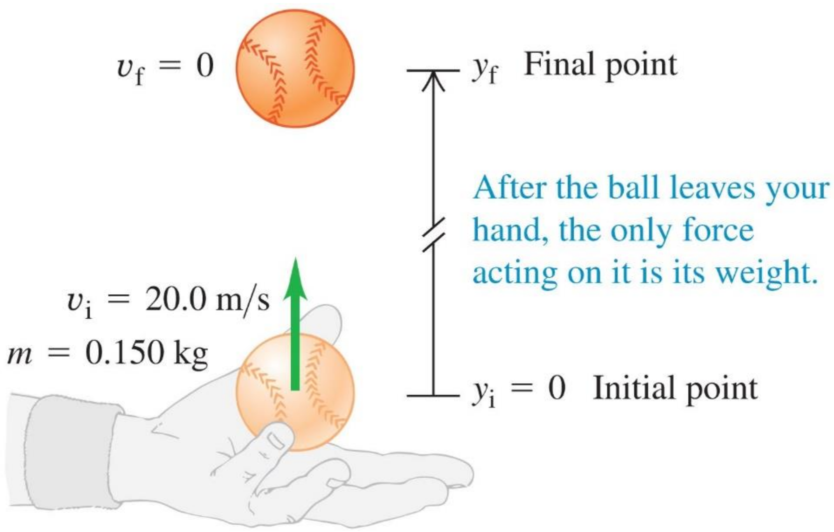
W(**friction**)= - μmg L

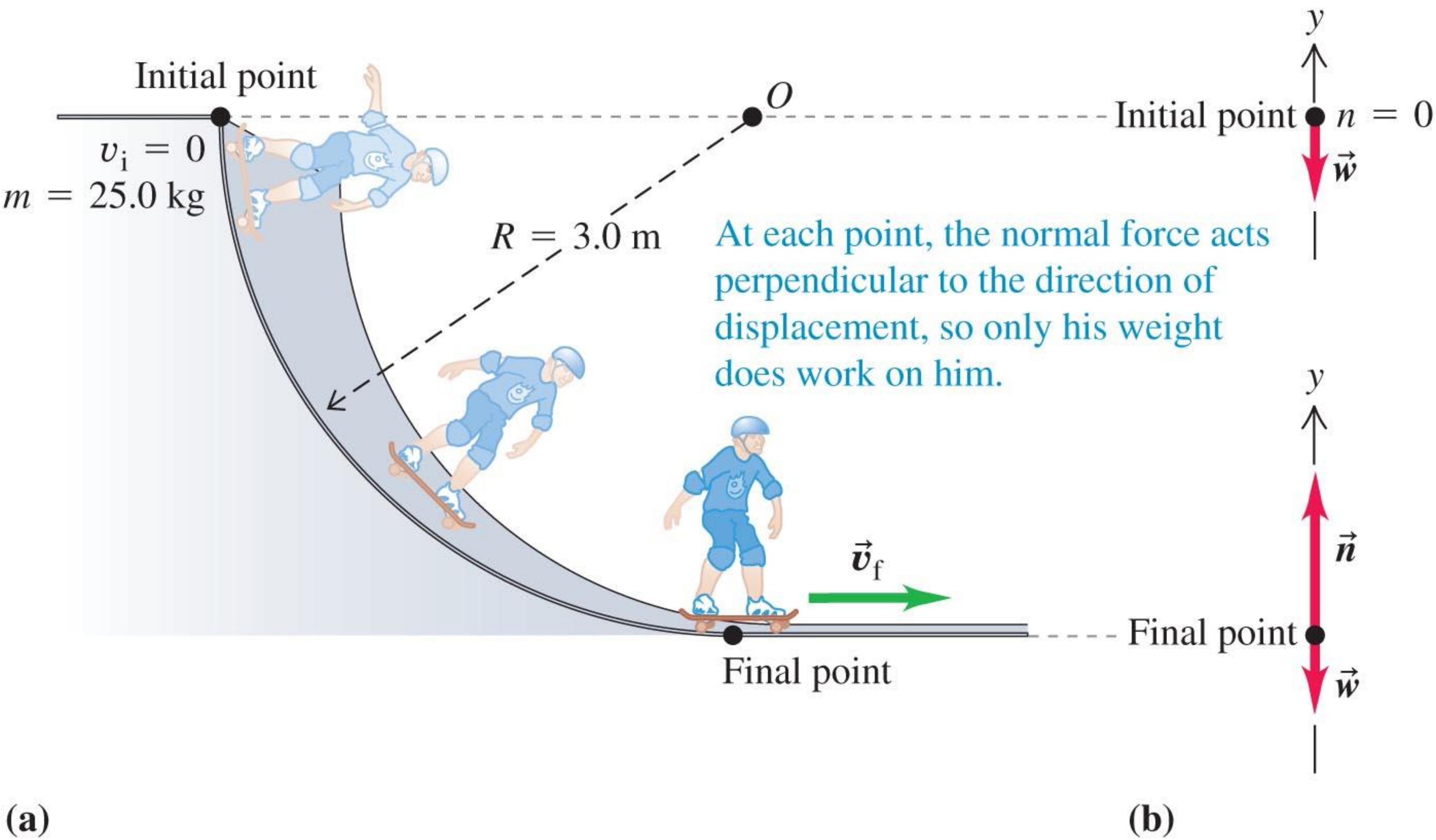


A Solved Baseball Problem – Example 7.7

- When the ball, with initial speed v_i is thrown straight upward, it slows down on the way up as the kinetic energy is converted to potential energy (*mgy*>0).
- At the top, the kinetic energy is zero and potential energy is maximum.
- On the way back down, the potential energy is converted back to kinetic energy, and the ball speeds up.

Conservation of total mechanical energy

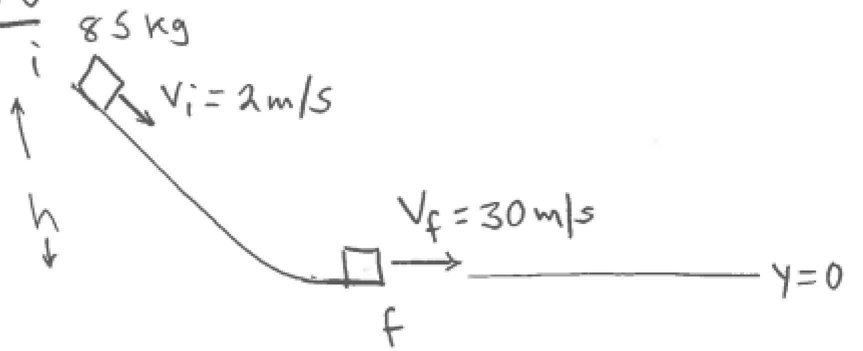




Initial point
O
$v_i = 0$
$m = 25.0$ kg
$R = 3.0$ m
At each point, the normal force acts perpendicular to the direction of displacement, so only his weight does work on him.
$\vec{v}_f$
Final point
y
Initial point
$n = 0$
$\vec{w}$
y
$\vec{n}$
Final point
$\vec{w}$
(a)
(b)

Ski jump ramp

7.70



Friction does negative work so $W_f = -4000\,J$

$$K_i + U_i + W_{other} = K_f + U_f$$

$$W_{other} = W_f = -4000\,J$$

take $y = 0$ at bottom of ramp so $U_i = mgh$ and $U_f = 0$

$$K_i = \frac{1}{2} m V_i^2 = \frac{1}{2}(85\,kg)(2.0\,m/s)^2 = 170\,J$$

$$K_f = \frac{1}{2} m V_f^2 = \frac{1}{2}(85\,kg)(30\,m/s)^2 = 38250\,J$$

$$170\,J + mgh - 4000\,J = 38{,}250\,J + 0$$

$$mgh = 38{,}250\,J + 4000\,J - 170\,J = 42{,}080\,J$$

$$h = \frac{42{,}080\,J}{(9.8\,m/s^2)(85\,kg)} = 50.5\,m$$

Clicker question

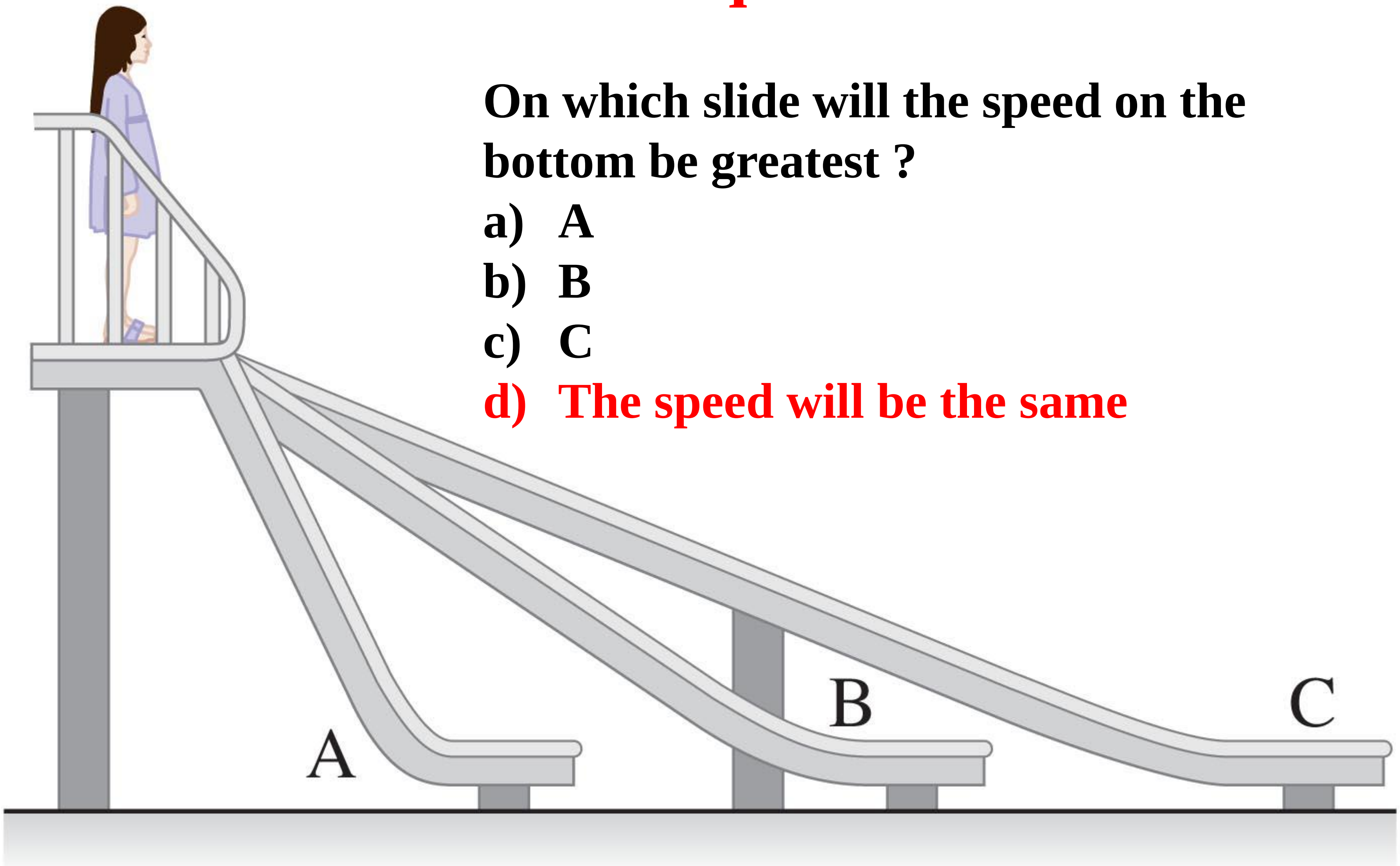
On which slide will the speed on the bottom be greatest ?

a) A

b) B

c) C

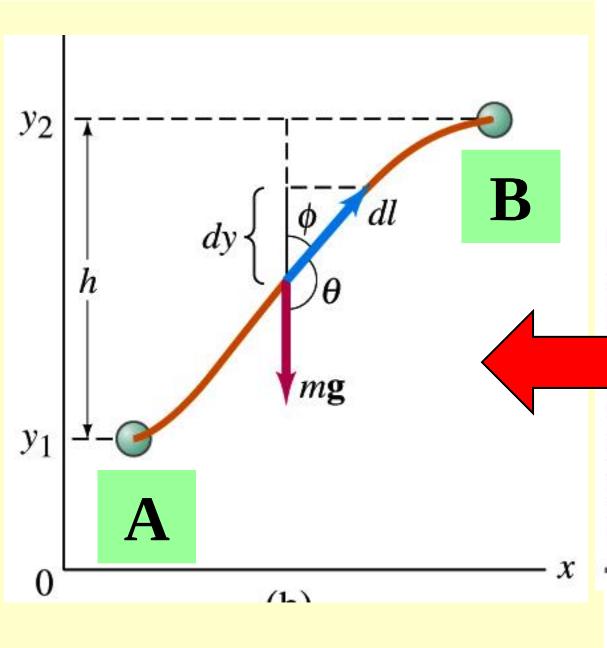
d) The speed will be the same

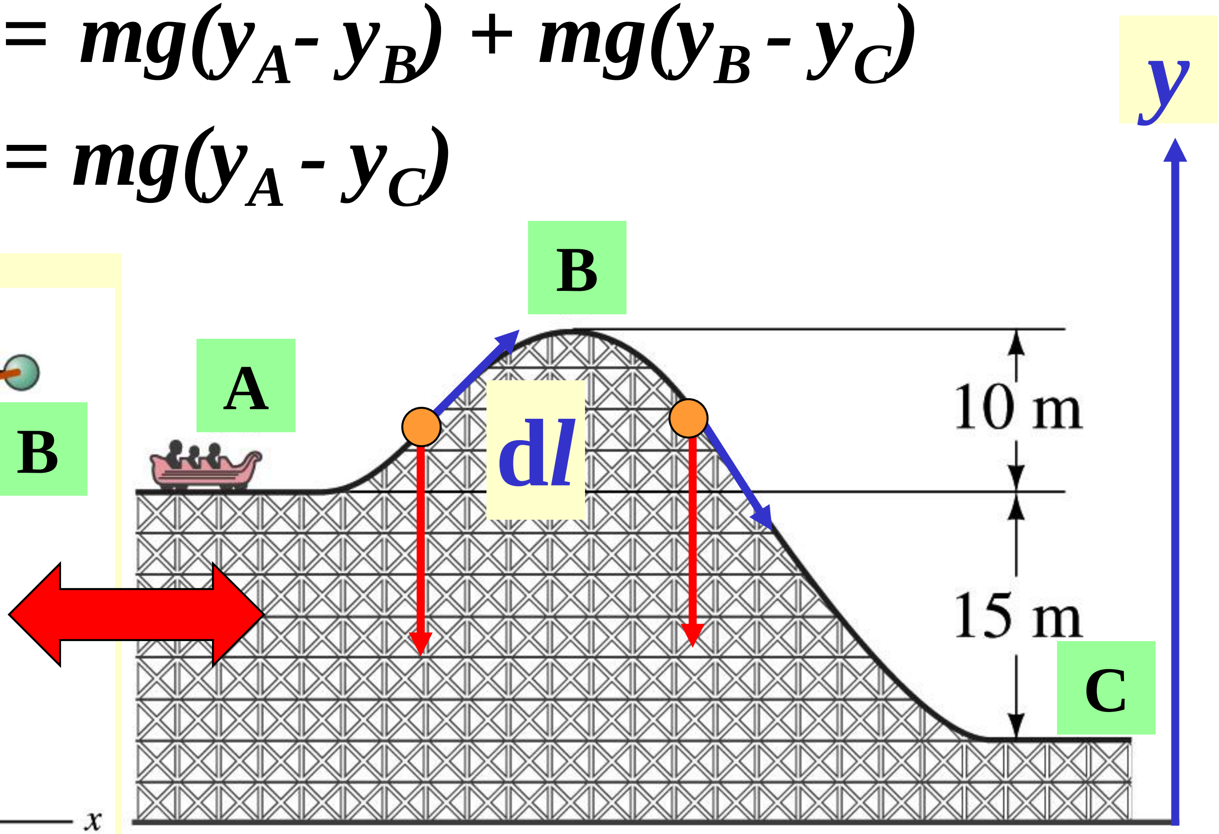


In each case the final energy of the child is the same $K_f = U_f - U_I = mgy$

$W_{g(A \to C)} = U_g(y_A) - U_g(y_C)$

$$W_{g(A \to B \to C)} = W_{g(A \to B)} + W_{g(B \to C)}$$
$$= mg(y_A - y_B) + mg(y_B - y_C)$$
$$= mg(y_A - y_C)$$





Power – How fast work is done

- When a quantity of work ΔW is done during a time interval Δt, the average power P_{av} or work per unit time is:

$$P_{av} = \frac{\Delta W}{\Delta t}$$

- Units of watt [W], or 1 watt = 1 joule per second [J/s]
- The rate at which work is done is not always constant. When it varies, we define the instantaneous power P as:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = F_{||} v$$

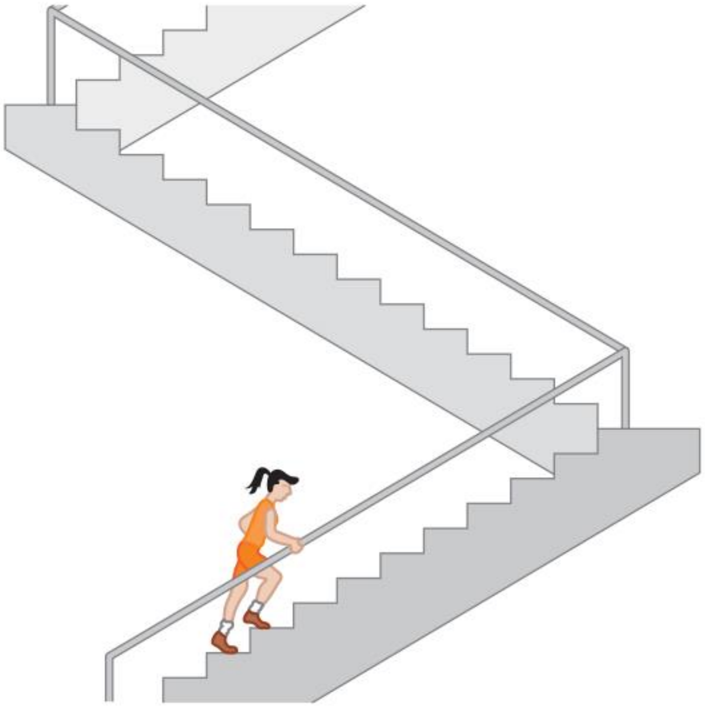
Power – Considers Work and Time to Do It (2 of 2)

- Example 7.16: A marathon stair climb
- If the runner is initially at rest and ends at rest, the work done by the runner is equal to the work done by gravity on the runner.

$$W_{runner} = mgh$$

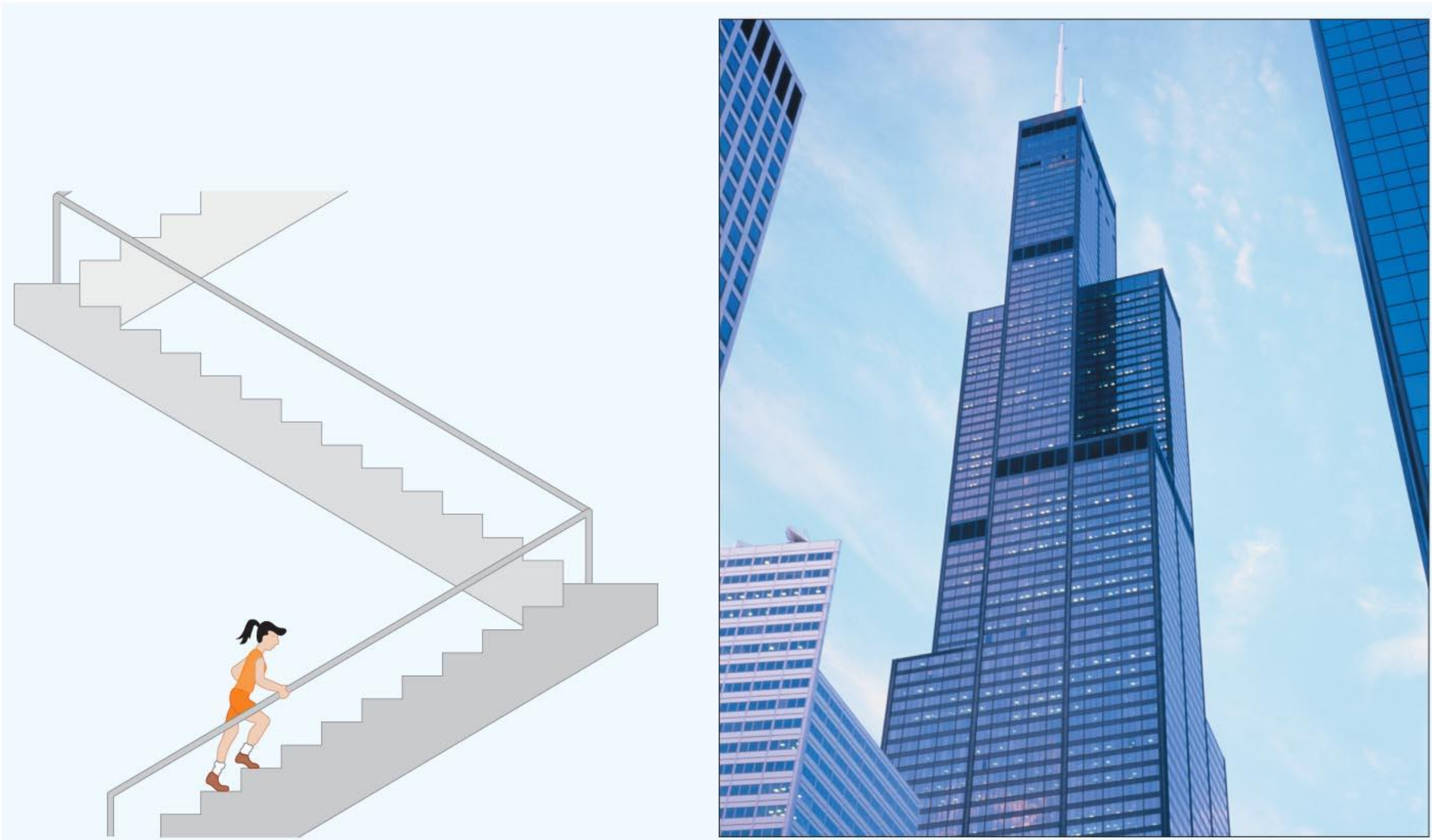
- The average power output by the runner is then:

$$P_{\text{av}} = \frac{W_{runner}}{\Delta t} = \frac{mgh}{\Delta t}$$
$$= F\upsilon_{av} = mg\upsilon_{av}$$



Climbing the Sears tower

The **Willis Tower** (formerly the **Sears Tower**) is a 108-story, 1,450-foot (42.
41 m) skyscraper in Chicago.



Power

Power = rate of energy production and consumption = $[\frac{J}{s}]$=**watt**

$$1\ hp \cong 746\ Watt$$

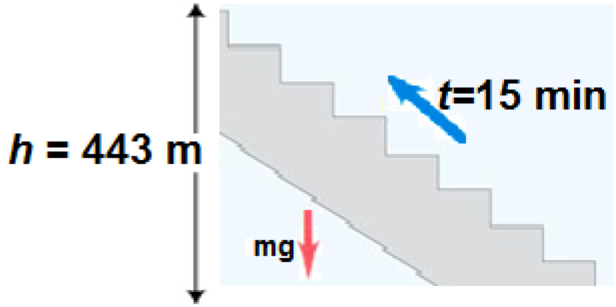
What do we pay to the electricity company for 100 watt bulb? (11 cents / 1 kwatt.h)

$$1\ kwatt \cdot h = 10^3 watt \cdot h \frac{3.6x10^3 s}{1h} = 3.6x10^6 J \cdot \frac{1kcal}{4.18x10^3 J} = 860kcal$$

Is electricity cheap? 1 kWatt-hour costs 11 cents. The exercise bike shows 100watt. How long must you pedal to produce energy of 1 kWatt-hour?

$$100\ watt \cdot 10\ hour = 0.1\ kwatt \cdot 10\ hour = 1\ kwatt \cdot h \rightarrow 11\ cents$$

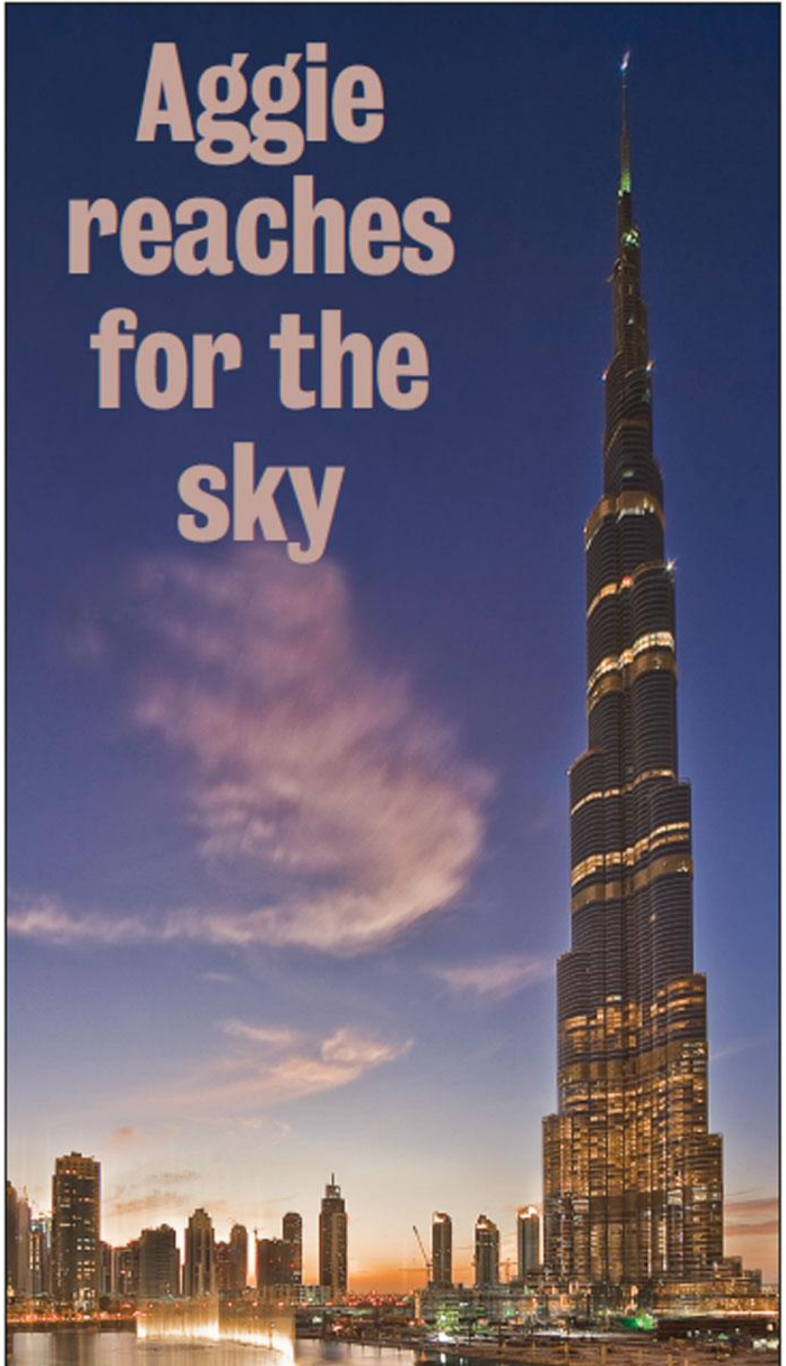
So; need to pedal for 10 hours.

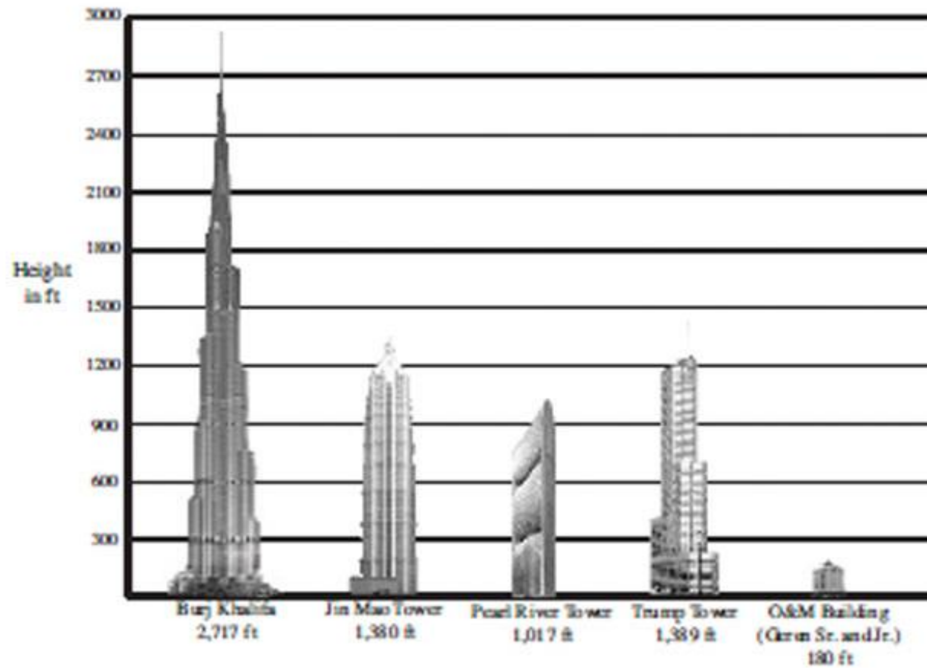


A power climb: **P=?** and **m=50kg**

$$mg \cdot h = 50(9.8)443 = 2.17x10^5 J \text{ and } \mathbf{t} = 15\ min\ \frac{60\ s}{1\ min} = 900\ s$$

$$\therefore P = \frac{2.17x10^5}{900} = 241\ Watt$$





The Burj Khalifa is the largest man made structure in the world and was designed by Adrian Smith class of 1966

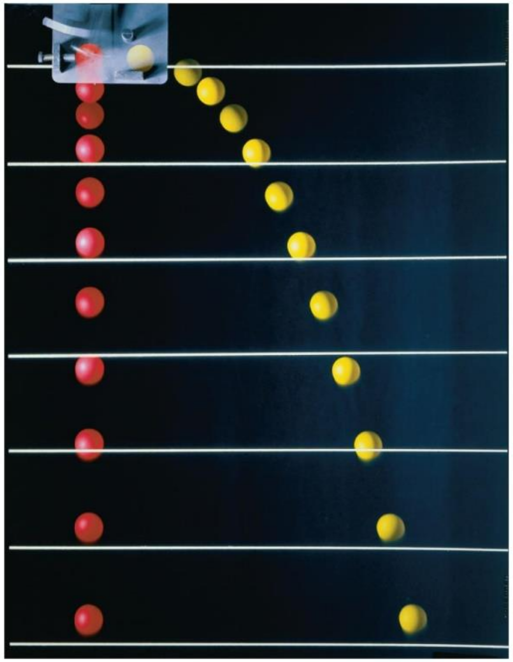
thebatt.com Febuary 25th

How many pillars of steel and concrete must be in the foundation? How long and wide it should be, so this building can be constructed on desert sand? What are the friction forces, which must be present, so it does not sink in the dessert sand?

Comments on Force and Potential Energy

- Potential energy is defined for conservative forces only.

- What kind of forces are conservative forces? As an example, let's consider the free-fall motions of a particle of mass *m* under the influence of gravity, with different horizontal initial velocities.



v_{ix} varies; $v_{iy} = 0$

$v_{fx} = v_{ix}$; $v_{fy} = \sqrt{2g(y_i - y_f)}$ independent of the paths

Apply the work-energy theorem and remember that $v^2 = v_x^2 + v_y^2$:

$$W_g = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = mgh = mg(y_i - y_f)$$

Conclusion: The work done by the gravity force depends on the initial and the final positions only.
It is independent of the exact path.

- If the work done by a force depends only on the initial and the final positions and is independent of the path of motion, this force is a conservative force.

7.6

I A 128.0 N carton is pulled up a frictionless baggage ramp inclined at above the horizontal by a rope exerting a 72.0 N pull parallel to the ramp's surface. If the carton travels 5.20 m along the surface of the ramp, calculate the work done on it by (a) the rope, (b) gravity, and (c) the normal force of the ramp. (d) What is the net work done on the carton? (e) Suppose that the rope is angled at above the horizontal, instead of being parallel to the ramp's surface. How much work does the rope do on the carton in this case?

7.6. Set Up: Use $W = F_p s = (F\cos\phi)s$. Calculate the work done by each force. In each case, identify the angle ϕ. In part (d), the net work is the algebraic sum of the work done by each force.

Solve: **(a)** Since the force exerted by the rope and the displacement are in the same direction,

$$\phi = 0° \text{ and } W_{\text{rope}} = (72.0\text{ N})(\cos 0°)(5.20\text{ m}) = +374\text{ J}$$

(b) Gravity is downward and the displacement is at $30.0°$ above the horizontal, so $\phi = 90.0° + 30.0° = 120.0°$.

$$W_{\text{grav}} = (128.0\text{ N})(\cos 120°)(5.20\text{ m}) = -333\text{ J}$$

(c) The normal force n is perpendicular to the surface of the ramp while the displacement is parallel to the surface of the ramp, so $\phi = 90°$ and $W_n = 0$.

(d) $W_{\text{net}} = W_{\text{rope}} + W_{\text{grav}} + W_n = +374\text{ J} - 333\text{ J} + 0 = +41\text{ J}$

(e) Now $\phi = 50.0° - 30.0° = 20.0°$ and $W_{\text{rope}} = (72.0\text{ N})(\cos 20.0°)(5.20\text{ m}) = +352\text{ J}$

Reflect: In part (b), gravity does negative work since the gravity force acts downward and the carton moves upward. Less work is done by the rope in part (e), but the net work is still positive.

7. 73

III Pendulum. A small 0.12 kg metal ball is tied to a very light (essentially massless) string that is 0.8 m long. The string is attached to the ceiling so as to form a pendulum. The pendulum is set into motion by releasing it from rest at an angle of with the vertical. (a) What is the speed of the ball when it reaches the bottom of the arc? (b) What is the centripetal acceleration of the ball at this point? (c) What is the tension in the string at this point?

7.73. Set Up: At any point, the potential energy is given by $mgl(1-\cos\theta)$, where l is the string length and θ is the angle the string makes with the vertical. At the top of the swing, the kinetic energy is zero while the potential energy is a maximum since $\theta = \theta_{max} = 45°$; whereas, at the bottom of travel, the kinetic energy becomes a maximum equal to the U_{grav} of the top of the swing. In terms of conservation of energy, $K_f = U_{grav,i}$ or $\frac{1}{2}mv_f^2 = mgl(1-\cos 45°)$, where "f" refers to the vertical position of the pendulum and "i" refers to top of travel.

Solve: (a) Solving for the velocity,

$$v_f = \sqrt{2gl(1-\cos 45°)} = \sqrt{2(9.80\ \text{m/s}^2)(0.80\ \text{m})(1-\cos 45°)} = 2.1\ \text{m/s}$$

(b) The ball is undergoing circular motion, so its centripetal acceleration is $a = \frac{v^2}{r} = \frac{(2.1\ \text{m/s})^2}{0.8\ \text{m}} = 6\ \text{m/s}^2$.

(c) The tension in the string at this instant is found using a force balance of the weight and radial acceleration,

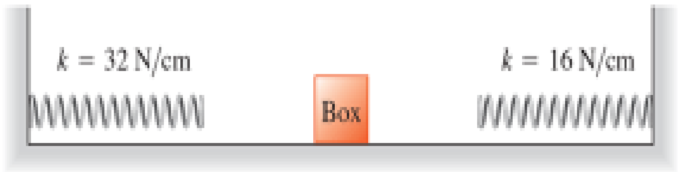
$$T = mg + \frac{mv_f^2}{l} = mg + \frac{2mgl(1-\cos 45°)}{l} = mg[1+2(1-\cos 45°)]$$

$$T = (0.12\ \text{kg})(9.80\ \text{m/s}^2)(1.5858) = 1.9\ \text{N}$$

Reflect: As the string passes through the vertical the ball has an upward acceleration, due to its circular motion. Therefore, there is a net upward force so the tension is greater than the weight of the ball

7. 44

I A 1.5 kg box moves back and forth on a horizontal frictionless surface between two different springs, as shown in Figure 7.46. The box is initially pressed against the stronger spring, compressing it 4.0 cm, and then is released from rest. (a) By how much will the box compress the weaker spring? (b) What is the maximum speed the box will reach?



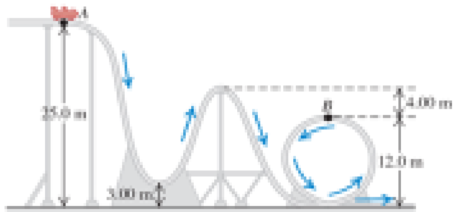
7.44. Set Up: In part (a), substitute $K_i = K_f = 0$ into $K_f + U_f = K_i + U_i$ to obtain $U_{el,f} = U_{el,i}$. Also, let subscript 1 refer to the stronger spring and 2 to the weaker spring. Then $\frac{1}{2}k_1x_1^2 = \frac{1}{2}k_2x_2^2$ and $x_2 = x_1\sqrt{k_1/k_2}$. In part (b), apply $U_{el,i} = K_f$ which results in $\frac{1}{2}mv_{max}^2 = \frac{1}{2}k_1x_1^2$.

Solve: (a) $x_2 = x_1\sqrt{k_1/k_2} = (4.0 \text{ cm})\sqrt{\dfrac{32 \text{ N/cm}}{16 \text{ N/cm}}} = (4.0 \text{ cm})(\sqrt{2}) = 5.7 \text{ cm}$

(b) $v_{max} = x_1\sqrt{\dfrac{k_1}{m}} = (4.0\times10^{-2} \text{ m})\sqrt{\dfrac{3200 \text{ N/m}}{1.5 \text{ kg}}} = 1.8 \text{ m/s}$

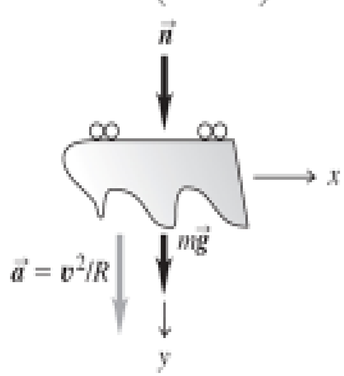
7. 75

II A 350 kg roller coaster starts from rest at point and slides down the frictionless loop-the-loop (a) How fast is this roller coaster moving at point ? (b) How hard does it press against the track at point B?



***7.75. Set Up:** For part (a), apply conservation of energy to the motion from point A to point B: $K_B + U_{grav,B} = K_A + U_{grav,A}$ with $K_A = 0$. Defining $y_B = 0$ and $y_A = 13.0$ m, conservation of energy becomes $\frac{1}{2}mv_B^2 = mgy_A$ or $v_B = \sqrt{2gy_A}$. In part (b), the free-body diagram for the roller coaster car at point B is shown in the figure below. $\sum F_y = ma_y$ gives $mg + n = ma_{rad}$, where $a_{rad} = v^2/r$. Solving for the normal force results in

$$n = m\left(\frac{v^2}{r} - g\right)$$



Solve: **(a)** $v_B = \sqrt{2(9.80\ \text{m/s}^2)(13.0\ \text{m})} = 16.0\ \text{m/s}$

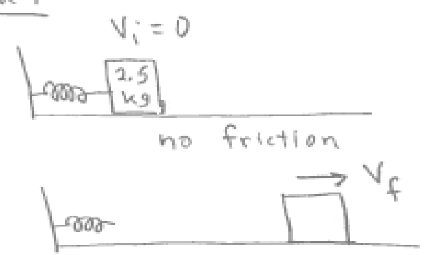
(b) $n = (350\ \text{kg})\left[\frac{(16.0\ \text{m/s})^2}{6.0\ \text{m}} - 9.80\ \text{m/s}^2\right] = 1.15 \times 10^4\ \text{N}$

Reflect: The normal force n is the force that the tracks exert on the roller coaster car. The car exerts a force of equal magnitude and opposite direction on the tracks.

Mass pushed on horizontal spring

Recitation Oct. 3 - 6

7.27



i spring compressed, block at rest

f block has left spring

a) Since no friction block moves at constant speed after leaving spring

$$K_i + U_i + W_{other} = K_f + U_f$$

No change in height so no work done by gravity and no change in U_{grav}.

So, $U = U_{el}$ (spring)

$U_i = 11.5\ J$ $\quad$ $U_f = 0$

$K_i = 0$ $\quad$ $K_f = \frac{1}{2} m V_f^2$

Only spring force does work so $W_{other} = 0$

$U_i = K_f$ $\quad$ energy stored in spring converted to kinetic energy of block

$$11.5\ J = \frac{1}{2}(2.5\ kg)\ V_f^2$$

$$V_f = 3.03\ m/s$$

Block has maximum speed after leaves spring.

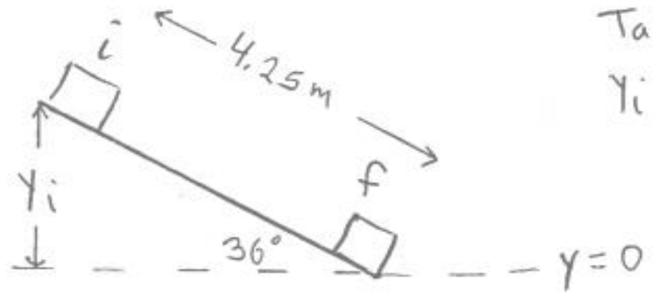
b) $F = ma$ so $a = \frac{F}{m}$. a is max when F is max. $|F_{spr}| = kx$ so a max when x is max and this is just after the mass is released, when the spring is compressed the most.

Find max compression: $U_i = \frac{1}{2} k x_{max}^2$ so $x_{max} = \sqrt{\frac{2 U_i}{k}} = \sqrt{\frac{2(11.5\ J)}{2500\ N/m}} = 0.0959\ m$

Note unit conversion for k

Roofer and toolbox

7.51



Take $y_f = 0$

$y_i = (4.25\,m)(\sin 36°) = 2.50\,m$

$m = \frac{w}{g} = \frac{85.0\,N}{9.8\,m/s^2} = 8.67\,kg$

Work done by the kinetic friction is

$W_f = -(22.0\,N)(4.25\,m) = -93.5\,J$ (friction work is negative)

$K_i + U_i + W_{other} = K_f + U_f$

No springs so $U = U_{grav}$

$K_i = 0 \qquad K_f = \frac{1}{2} m v_f^2 \qquad U_i = mgy_i = (85.0\,N)(2.50\,m) = 212.5\,J$

$U_f = 0$

$W_{other} = W_f = -93.5\,J$

$U_i + W_f = K_f$

$K_f = 212.5\,J + (-93.5\,J) = 119\,J$

$\frac{1}{2} m v_f^2 = K_f$ so $v_f = \sqrt{\frac{2(K_f)}{m}} = \sqrt{\frac{2(119\,J)}{8.67\,kg}} = 5.24\,m/s$