Chapter 13 Fluid Mechanics

Fluid statics = equilibrium situations

$$\boxed{\text{Density} = \rho = \frac{m}{V} = \frac{mass}{volume}\ [\text{kg/m}^3]}$$

<u>The density ρ has a wide range</u>

Osmium = $22.5\text{x}10^3$ kg/m^3
Gold = $19.3\text{x}10^3$ kg/m^3
Lead = $11.3\text{x}10^3$ kg/m^3
Air (gases) = 1.2 kg/m^3
Water = $1.0\text{x}10^3$ kg/m^3
Ice = $0.92\text{x}10^3$ kg/m^3

Chapter 13 Fluid Mechanics

- 13.1 To understand density.
- 13.2 To understand pressure in a fluid.
- 13.3 To apply Archimedes principle of buoyancy.

Sections not covered:

- 13.4 To understand surface tension and capillary action.
- 13.5 To understand fluid flow
- 13.6 To Bernoulli's equation.
- 13.7 The application of Bernoulli's equation.
- 13.8 To understand turbulence and viscosity in real fluids.

Goals for Chapter 13

- To study density and pressure in a fluid withPascal’s Law
- To apply Archimedes principle of buoyancy.

13.1 Density

- Mass is an extensive quantity.
- Density: mass per unit volume
  - Density $\rho = \dfrac{m}{V}$ (kg/m^3)
  - Density is an intensive quantity

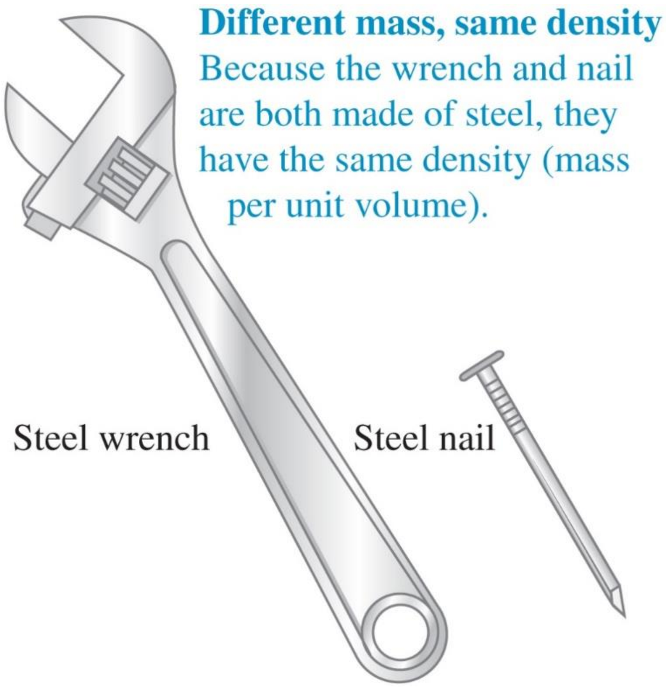
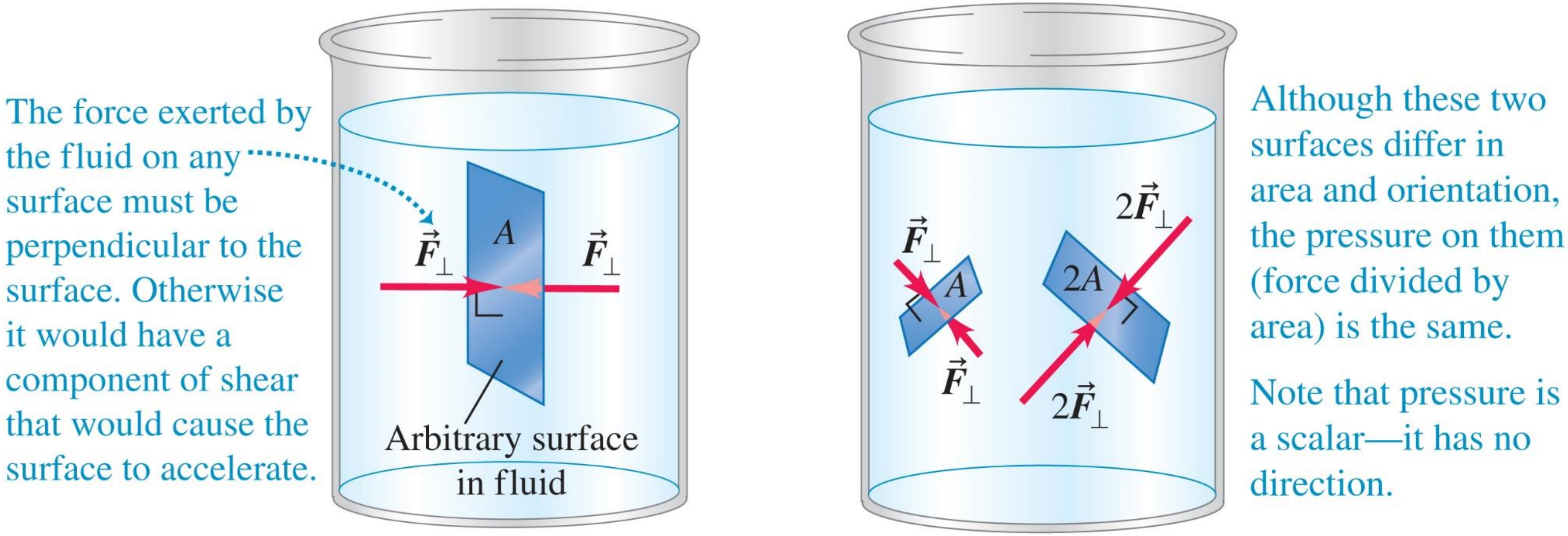


TABLE 13.1 Densities of some common substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Gas		Concrete	2.0×10^3
Air (1 atm, 20°C)	1.20	Aluminum	2.7×10^3
		Iron, steel	7.8×10^3
Liquids		Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ethanol	0.81×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Gold	19.3×10^3
Blood	1.06×10^3	Platinum	21.4×10^3
Mercury	13.6×10^3		
		Astrophysical	
Solids		White-dwarf star	10^{10}
Glycerin	1.26×10^3	Neutron star	10^{18}
Ice	0.92×10^3		

*To obtain the densities in grams per centimeter, simply divide by 10^3.

13.2 Pressure in a Fluid



(a) Why forces due to fluid pressure must act perpendicular to a surface

(b) Pressure equals force divided by area.

13.2 Pressure in a Fluid

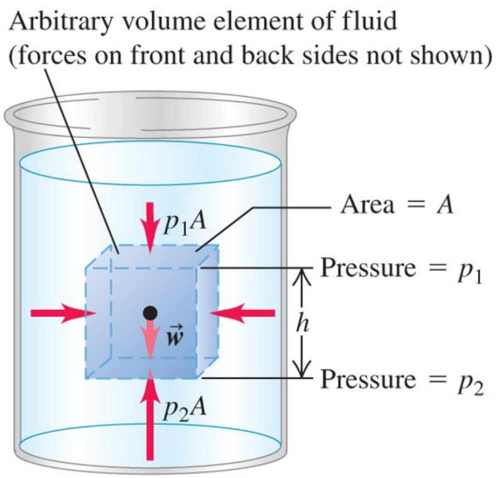
- Pressure: The perpendicular component of the force acting on a surface divided of the area. Or, the perpendicular force per unit area.

$$p = \frac{F_\perp}{A} \qquad (\mathrm{N/m^2} \text{ or Pa})$$

- Force is a vector quantity.
- Pressure is a scalar quantity.
- Atmosphere pressure

p_{atm} = 1.013×10^5 Pa = 14.7 psi = 1.013 bars = 1013 millibars (mbar) = 760 mm Hg = 76 cm Hg

Question: How does the pressure vary in a fluid?

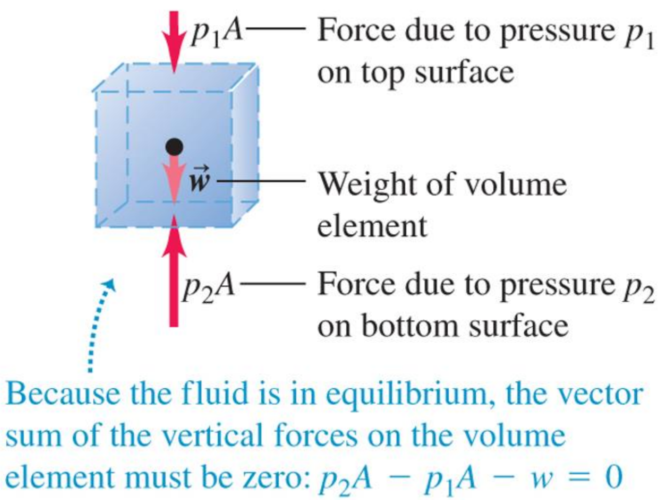


In equilibrium:

$$F_2 - F_1 - mg = 0$$

$$p_2A - p_1A - \rho ghA = 0$$

$$p_2 - p_1 - \rho gh = 0$$



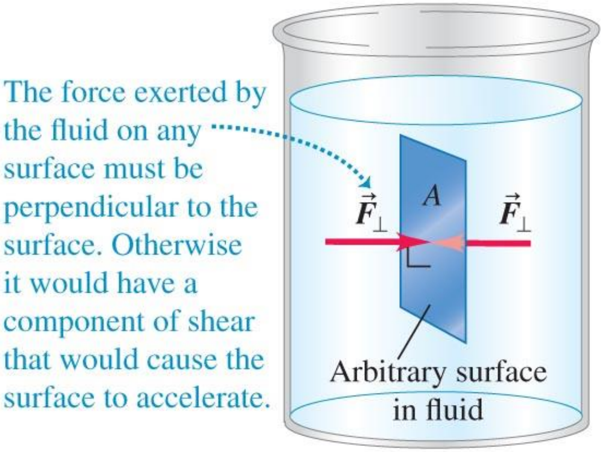
Variation of pressure with depth in a fluid

$$p_2 = p_1 + \rho gh$$

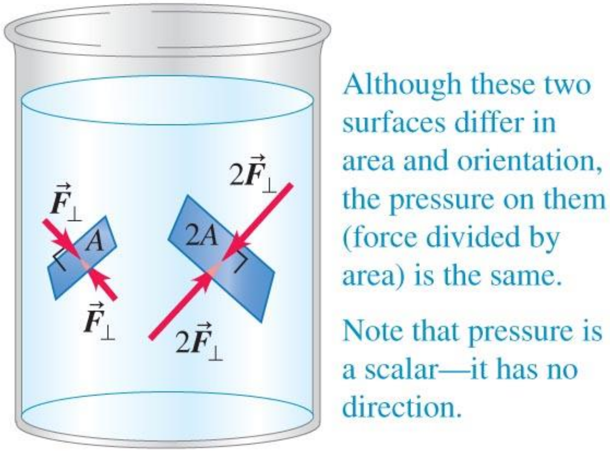
ρ is the density of the fluid;
h is the depth below the fluid level p_1 is measured;
p is the pressure which is always positive.

Pressure In a Fluid

- $P = F/A$
- The pressure is equal to force (in N) per unit area (in m^2).
- A new derived unit N/m^2 = 1 Pascal = 1 Pa
- Atmospheric pressure is 1 atm = 760 mm Hg = 14.7lb/in^2 = 101325 Pa = 1.013 bars
- 1mm Hg=1 torr=133.3Pa



(a) Why forces due to fluid pressure must act perpendicular to a surface



(b) Pressure equals force divided by area.

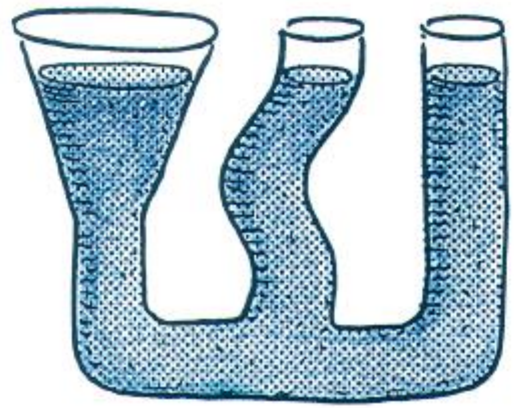
Pressure in a fluid open to air

Variation of pressure with depth in a fluid open to the air at the top

$$p = p_{atm} + \rho g h$$

p_{atm} is the atmospheric pressure at the surface of the fluid;
ρ is the density of the fluid;
h is the depth below the surface of the fluid;
p is the pressure at depth h below the surface. p is always positive.

Clicker - Questions



Everybody knows that "water seeks its own level," but very few people know **why** water seeks its own level. The reason has most to do with

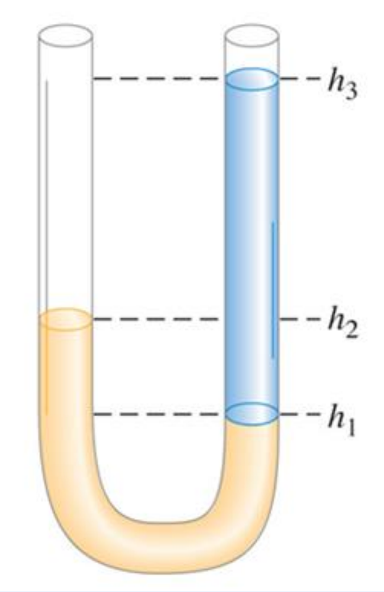
a) atmospheric pressure.

b) water pressure depending on depth.

c) water's density.

Clicker - Questions

The U tube in the figure contains two liquids in equilibrium. At which level(s) must the pressure be the same in both sides of the tube?



A) h_1

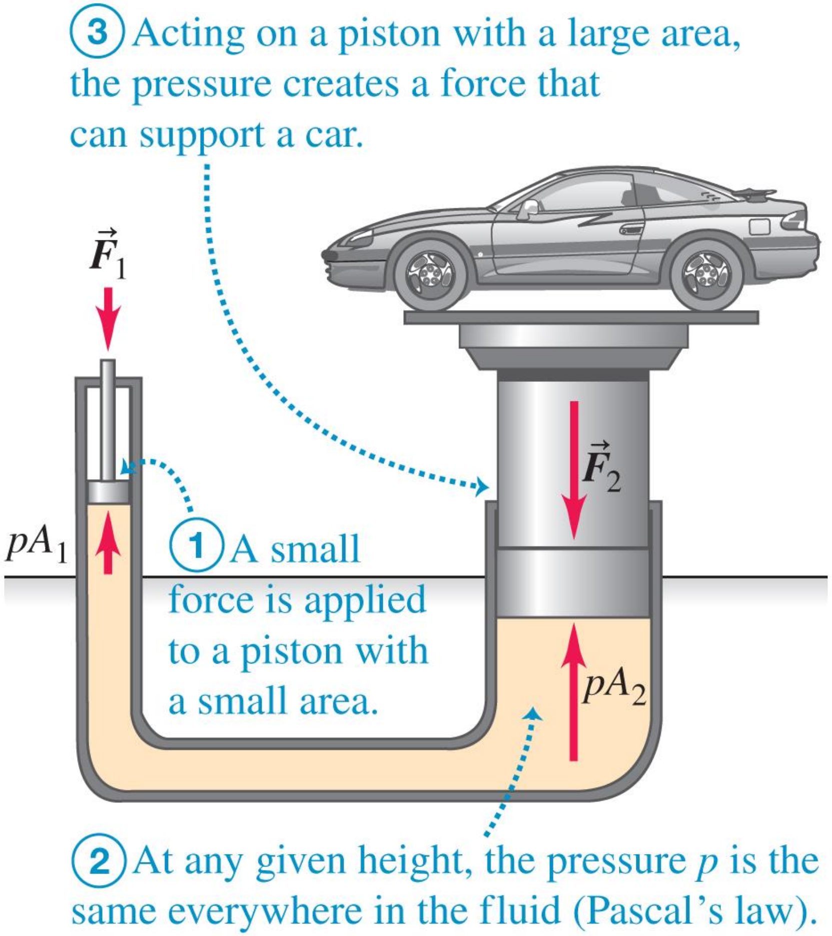
B) h_2

C) h3 and h2

D) The pressure at h_1 equals that at h_2.

Pascal's Law

Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel. The pressure depends only on the depth; the shape of the container does not matter.



Hydraulic Lift

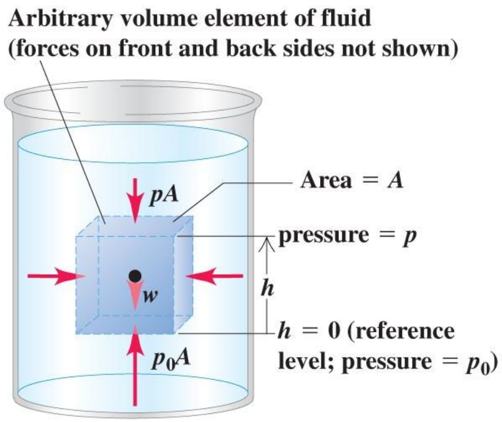
Neglect the pressure variation due to the weight or the fluid.

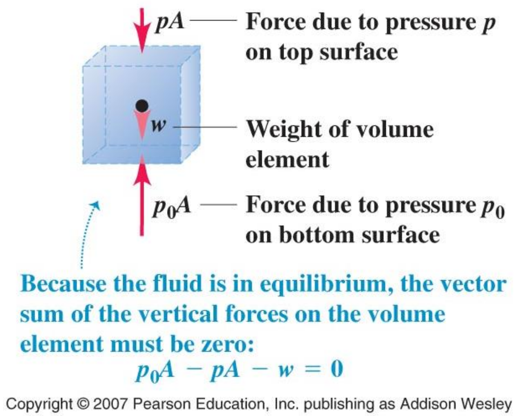
$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

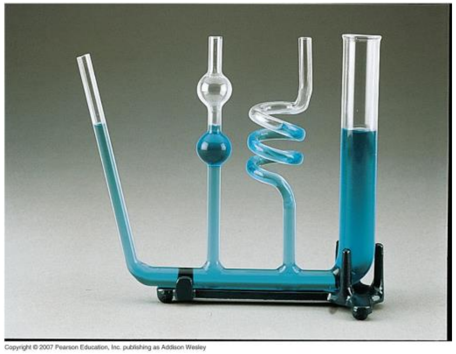
Therefore

$$F_2 = F_1 \frac{A_2}{A_1}$$

Pascal's Law

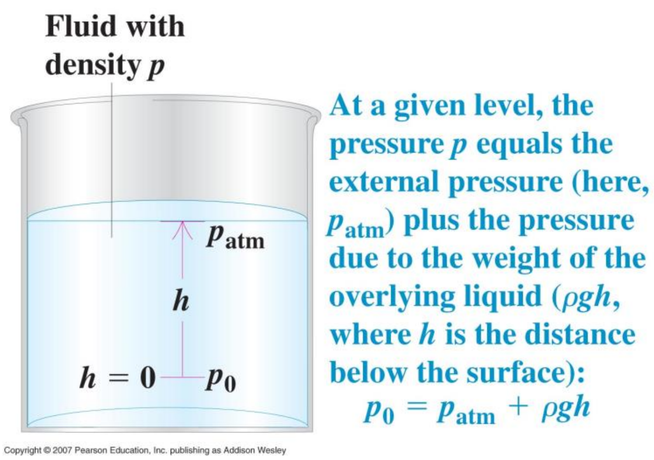






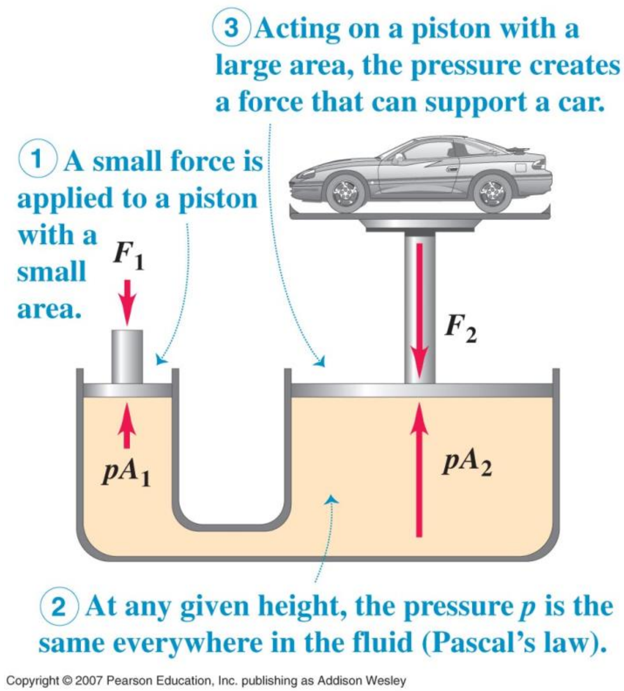
Communicating tubes:
Fluid has the same height at every height of the tubes, where the pressure is the same. $\boldsymbol{P = P_{atm} + \rho g h}$

Pascal law:
Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.

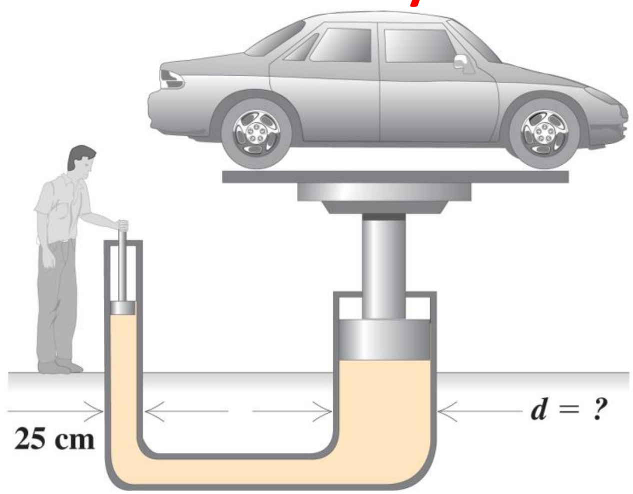


$$\boldsymbol{P = \frac{F_1}{A_1} = \frac{F_2}{A_2}}$$

$$\boldsymbol{\rightarrow F_2 = \frac{A_2}{A_1} F_1}$$



Hydraulic lift



Note: $F_1 = 100N$ and $F_2 = 3500kg * 9.8\frac{m}{s^2} = 3.43x10^4N$ volume displaced at each piston, when they move by d_1 and d_2 is the same. (Area) $A = \pi r^2$

Problem 13-27

Design a lift which can handle cars up to 3000kg, plus the 500kg platform. The worker should need to exert 100N.

a) What is the diameter of the pipe under the platform?
b) If the worker pushes down with a stroke 50cm long, by how much will he raise the car?

Use: Pascal law → $\frac{F_1}{A_1} = \frac{F_2}{A_2}$: pressure is the same everywhere in the fluid.

a) $\frac{F_1}{r_1^2} = \frac{F_2}{r_2^2}$; $r_2 = r_1\sqrt{\frac{F_2}{F_1}} = 0.125m\sqrt{\frac{3.43x10^4}{100}} = 2.32m$ → diameter (d_2)=4.64m

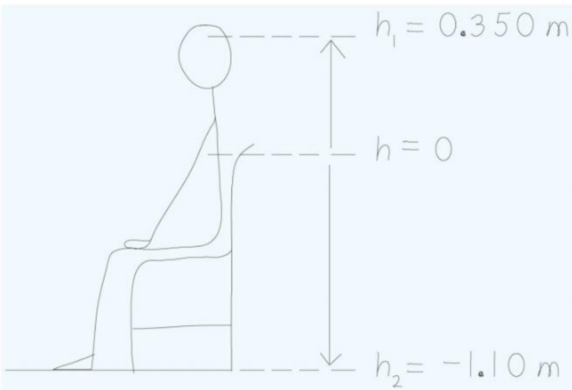
b) $d_1A_1 = d_2A_2$; $d_2 = d_1\frac{A_1}{A_2} = d_1\frac{\pi r_1^2}{\pi r_2^2} = 50cm(\frac{0.125m}{2.32m})^2 = 1.45mm$

Reflect: Work done by worker and on car must be the same.

$$F_1d_1 = 100N * 0.5m = 50\,J$$
$$F_2d_2 = 3.43x10^4N * 1.45x10^{-3}m = 50\,J$$

Medical instruments measure pressure
← Eye Heart →

A tonometer measures pressure within the eyeball to diagnose glaucoma.



Blood pressure varies with height:

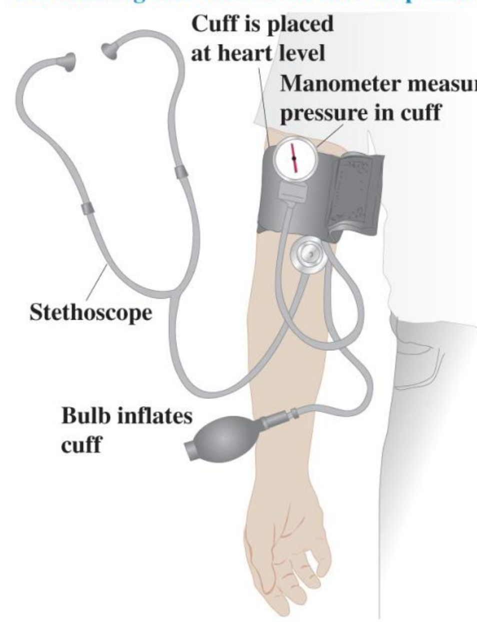
At the height of the heart $P_o = 1.3x10^4 Pa$, and $\rho_{blood} = 1.06x10^3 \frac{kg}{m^3}$

Use: $P = P_o - \rho g h$

a) Pressure at the level of the head:
$P_1 = P_o - \rho g h = 1.30x10^4 - 1.6x10^3 * 9.8 * 0.35 = 9.4x10^3\ Pa$
(light - headed)

b) Pressure at the level of the foot
$P_2 = 1.30x10^4 - 1.6x10^3 * 9.8 * (-1.1) = 2.44x10^4\ Pa$

To measure blood pressure, you listen through the stethoscope for arterial flow while letting the cuff deflate. Flow starts when the pressure in the deflating cuff reaches the arterial pressure.



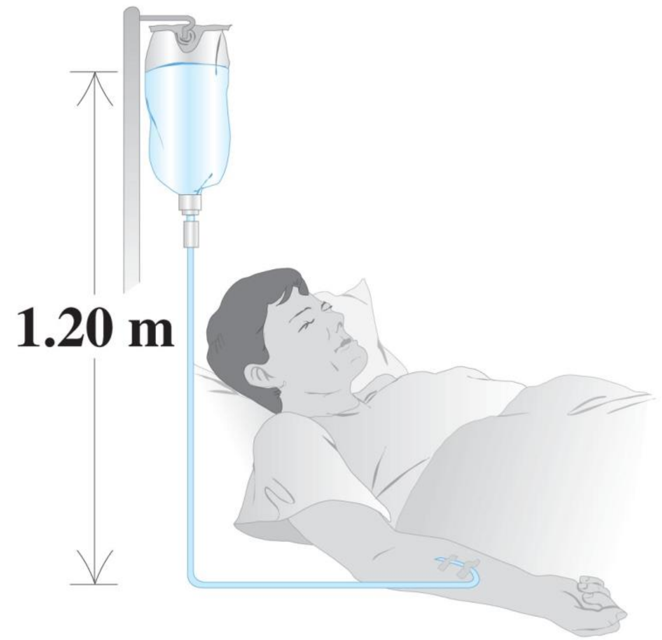
Healthy blood pressure is 120/80mm Hg.

Pressure is measured at the height of the heart

Systolic pressure:
Blood starts to spread discontinuously through the compressed vessel.

Distolic pressure:
Blood flows continuously

Intravenous feeding



Problem 13.17:

The liquid has a density of 1060 kg/m^3. The container hangs 1.2m above the patients arm.

What is the pressure this fluid exerts on the patient's vein in millimeters of mercury.

$$P = \rho g h = 1060\frac{kg}{m^3} * 9.8\frac{m}{s^2} * 1.2\ m = 1.25x10^4\ Pa$$

$$\therefore\ 1\ mm\ Hg = 133.3\ Pa$$

So; $\frac{1.25x10^4\ Pa}{133.3\ Pa} * 1\ mm\ Hg = 93.5\ mm\ Hg$

Problem 13.19

An electrical short cuts off all power to a submergible diving vessel when it is 30 m below the surface. the crew pushes out a hatch of area0.75m ^{2} and weight 300 N on the bottom of the escape. If the pressure inside is1.0 atm, what downward force must the crew exert on the hatch to open it ?

13.19. *Set Up:* $p = p_0 + \rho gh$. $F = pA$. For seawater, $\rho = 1.03 \times 10^3\ \text{kg/m}^3$

Solve: The force F that must be applied is the difference between the upward force of the water and the downward forces of the air and the weight of the hatch. The difference between the pressure inside and out is the gauge pressure, so

$$F = (\rho gh)\,A - w = (1.03 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(30\ \text{m})(0.75\ \text{m}^2) - 300\ \text{N} = 2.3 \times 10^5\ \text{N}$$

Reflect: The force due to the gauge pressure of the water is much larger than the weight of the hatch and would be impossible for the crew to apply it just by pushing.

Archimedes buoyancy

A solid aluminum ingot weights 89 N in air. a) What is its volume? b)The ingot is suspended by a rope and totally submerged in water what is the tension(the apparent weight) in the rope??

***13.29. Set Up:** The density of aluminum is $2.7\times10^3\,\text{kg/m}^3$. The density of water is $1.00\times10^3\,\text{kg/m}^3$. $\rho = m/V$. The buoyant force is $F_\text{B} = \rho_\text{water}V_\text{obj}g$.

Solve: **(a)** $T = mg = 89\ \text{N}$ so $m = 9.08\ \text{kg}$. $V = \dfrac{m}{\rho} = \dfrac{9.08\ \text{kg}}{2.7\times10^3\ \text{kg/m}^3} = 3.36\times10^{-3}\ \text{m}^3 = 3.4\ \text{L}$.

(b) When the ingot is totally immersed in the water while suspended, $T + F_\text{B} - mg = 0$.

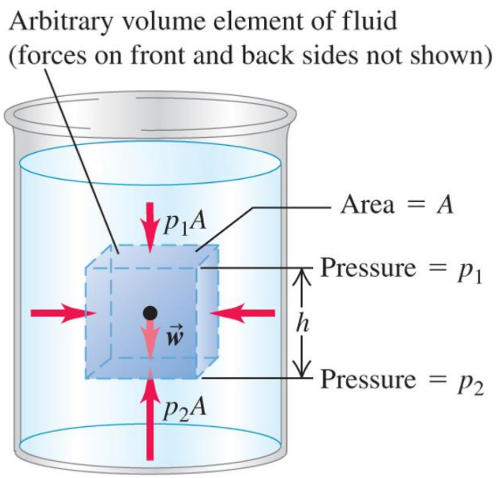
$$F_\text{B} = \rho_\text{water}V_\text{obj}g = (1.00\times10^3\ \text{kg/m}^3)(3.36\times10^{-3}\ \text{m}^3)(9.80\ \text{m/s}^2) = 32.9\ \text{N}$$

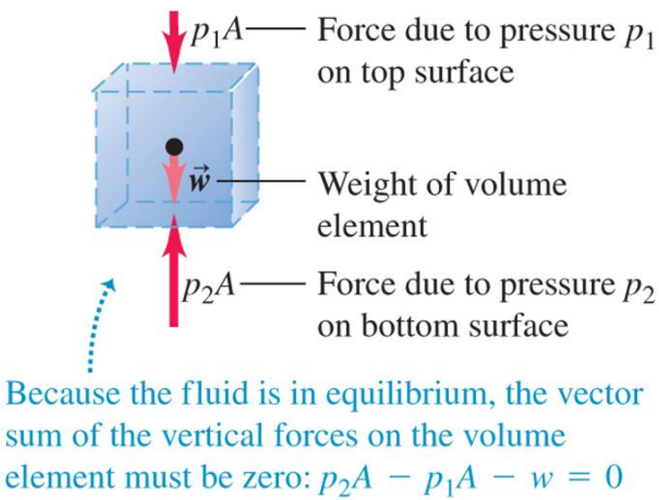
$$T = mg - F_\text{B} = 89\ \text{N} - 32.9\ \text{N} = 56\ \text{N}$$

Reflect: The buoyant force is equal to the difference between the apparent weight when the object is submerged in the fluid and the actual gravity force on the object.

13.3 Archimedes Principle: Buoyancy

- When an object is completely or partially immersed in a fluid, the fluid exerts an upward force on the object equal to the weight of the fluid that is displaced by the object.





Question: How does the pressure vary in a fluid?

In equilibrium:

$$F_2 - F_1 - mg = 0$$

$$p_2A - p_1A - \rho ghA = 0$$

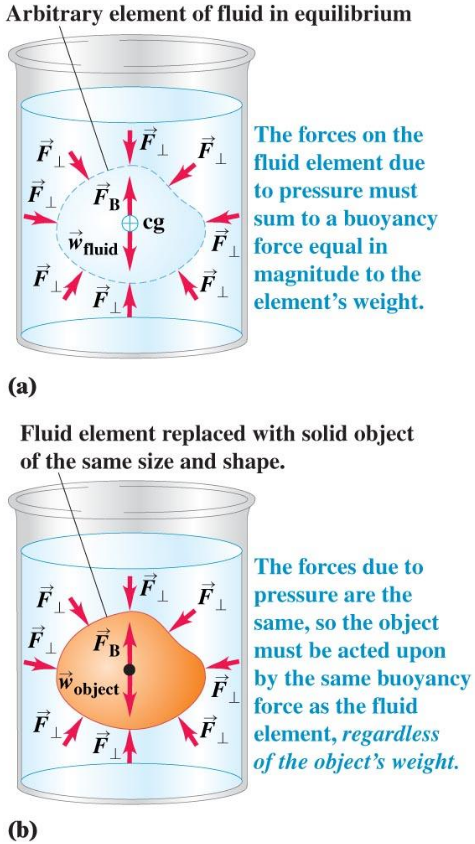
$$p_2 - p_1 - \rho gh = 0$$

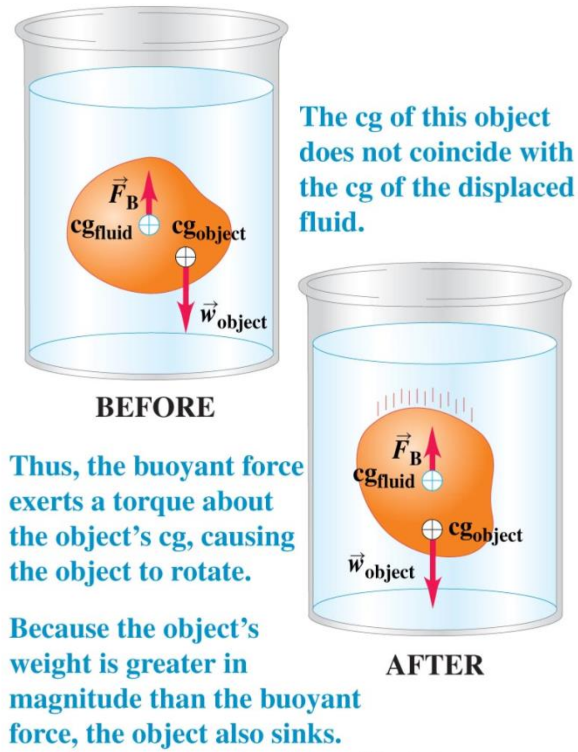
$$p_2A - p_1A - \rho ghA = 0$$

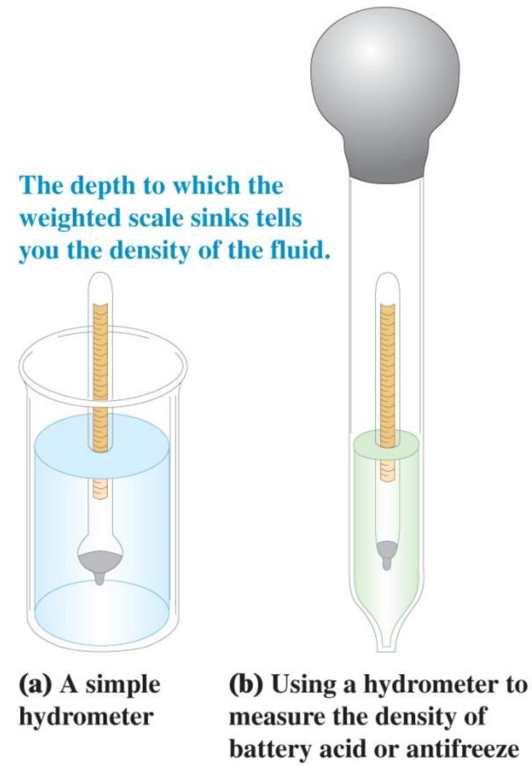
$$F_2 = F_1 + mg$$

$$F_2 = F_1 + \text{weight of the fluid displaced}$$

Archimedes's Principle and Buoyancy







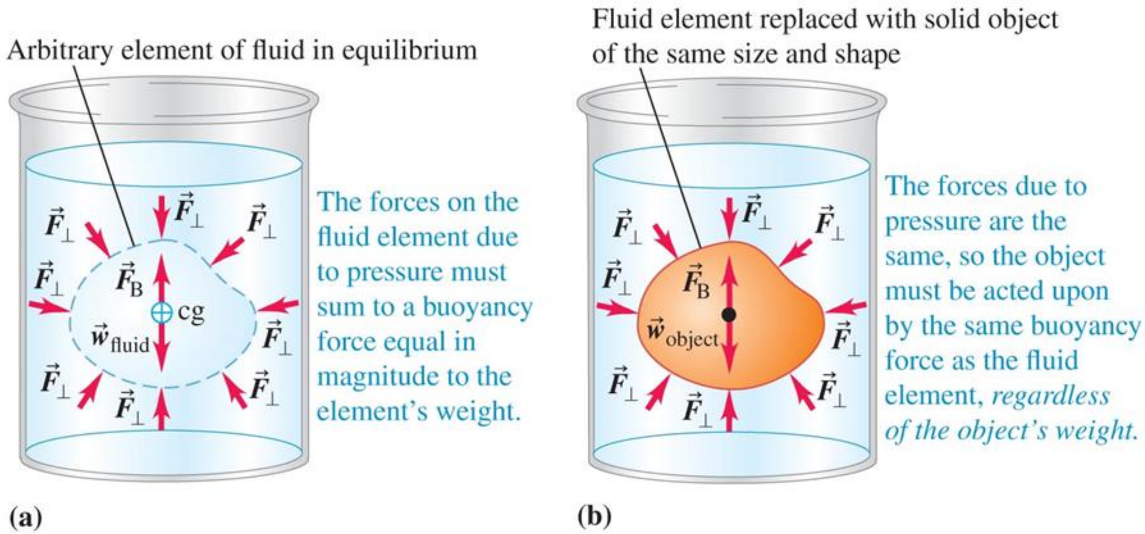
Note: hydrometer floats higher in denser fluids

Archimedes' principle:
When an object is immersed into a fluid, the fluid exerts an upward force on the object equal to the weight of the displaced fluid.

Note: The upward force is labeled the buoyant force

Archimedes's Principle – Figure 13.15

- An object submersed in a fluid experiences buoyant force equal to the mass of any fluid it displaces.
- An object can experience buoyant force greater than its mass and float. Even if it sinks, it would weigh measurably less.
- Refer to Example 13.7.



Clicker - Questions

Compare with an empty ship. Will a ship loaded with a cargo of Styrofoam float

a)lower in water?   b) higher in water?

Clicker - Questions

Buoyant force is greater on a empty steel barge when it is;

A) Floating on the surface

B) Capsized and sitting on the bottom

C) Same either way.

Buoyant force is greater on a submarine when it is;

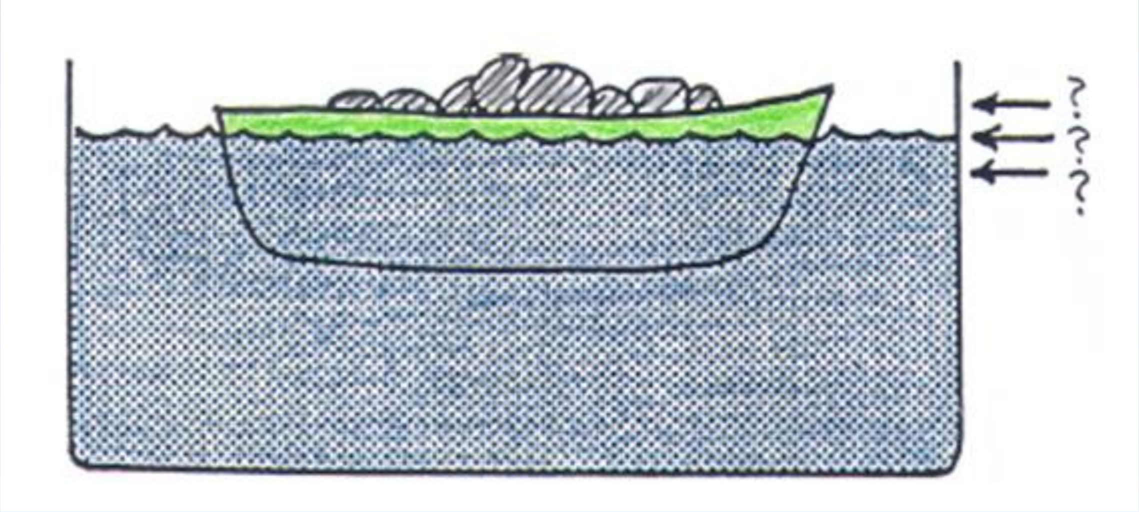
D) Floating

E) Submerged

D) Same either way.

Clicker - Questions

Consider a boat loaded with scrap iron in a swimming pool. If the iron is thrown overboard into the pool, will the water level at the edge of the pool a) rise, b)fall or c) remain unchanged?

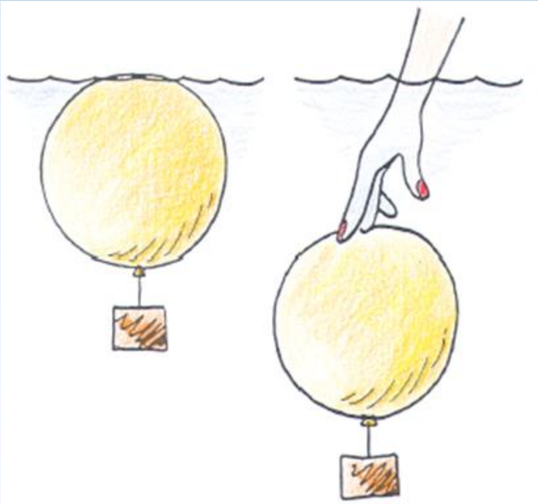


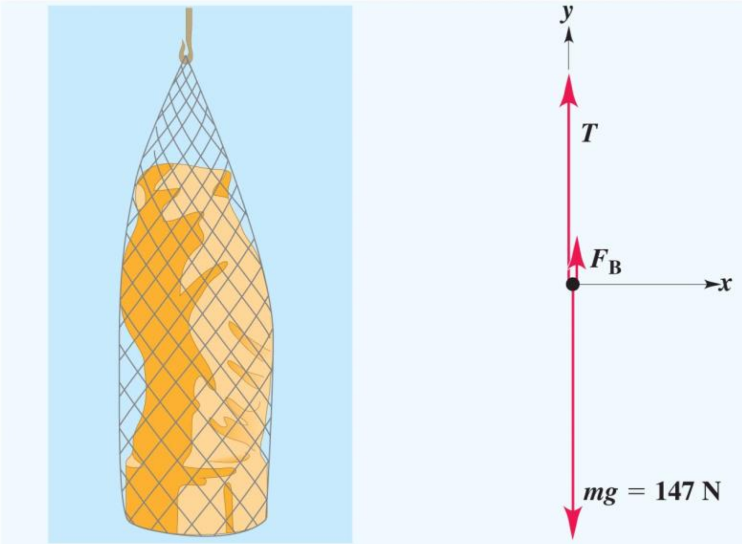
Clicker - Questions

Consider an air-filled balloon weighted so that it is on the verge of sinking – that is, its overall density just equals that of water.

Now if you push it beneath the surface. It will;

a) Sink
b) Return to the surface
c) Stay at the depth to which it is pushed.





(a) Immersed statue in equilibrium (b) Free-body diagram of statue

Example 13.7 Raising sunken treasure

Note: Weighing in water decreases the scale reading

A 15kg gold statue is raised from a sunken ship:

a) Find the tension in the hoisting cable while the statue is submerged

(Statue) $V = \frac{m}{\rho} = \frac{15\ kg}{19.3x10^3 \frac{kg}{m^3}} = 7.77x10^{-4} m^3$

Weight of equal volume of water is the buoyant force.

$$F_B = \rho_{water} * V_{water} * g = 1x10^3 * 7.77x10^{-4} * 9.8 = 7.61\ N$$

$$T + F_B - W = 0 \rightarrow T + 7.61 - 15 * 9.8 = 0 \quad \therefore T = 139\ N$$

b) Find the tension when the statue is out of the water

$$T = F_g = 15\ kg * 9.8 = 147\ N$$

Archimedes buoyancy

32. II An ore sample weighs 17.50 N in air. When the sample is suspended by a light cord and totally immersed in water, the tension in the cord is 11.20 N. Find the total volume and the density of the sample.

13.32. Set Up: $F_B = \rho_{water} V_{obj} g$. $w = mg = 17.50\ \text{N}$ and $m = 1.79\ \text{kg}$.

Solve: $T + F_B - mg = 0$. $F_B = mg - T = 17.50\ \text{N} - 11.20\ \text{N} = 6.30\ \text{N}$.

$$V_{obj} = \frac{F_B}{\rho_{water} g} = \frac{6.30\ \text{N}}{(1.00 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 6.43 \times 10^{-4}\ \text{m}^3$$

$$\rho = \frac{m}{V} = \frac{1.79\ \text{kg}}{6.43 \times 10^{-4}\ \text{m}^3} = 2.78 \times 10^3\ \text{kg/m}^3$$

Archimedes buoyancy

!3.35 A hollow plastic sphere is held below the surface of a lake by a cord anchored to the bottom of a lake. The sphere has a volume of 0.650 m^3 and the tension in the cord is is 900 N .

a) calculate the buoyant force.

B) what is the mass of the sphere?

C) The cord breaks and the sphere rises to the surface. When the sphere comes to rest, what fraction of its volume will be submerged??

Set Up: $F_B = \rho_{water} V_{obj} g$. The net force on the sphere is zero.

Solve: **(a)** $F_B = (1000\ \text{kg/m}^3)(0.650\ \text{m}^3)(9.80\ \text{m/s}^2) = 6.37 \times 10^3\ \text{N}$

(b) $F_B = T + mg$ and

$$m = \frac{F_B - T}{g} = \frac{6.37 \times 10^3\ \text{N} - 900\ \text{N}}{9.80\ \text{m/s}^2} = 558\ \text{kg}.$$

(c) Now $F_B = \rho_{water} V_{sub} g$, where V_{sub} is the volume of the sphere that is submerged. $F_B = mg$. $\rho_{water} V_{sub} = mg$ and

$$V_{sub} = \frac{m}{\rho_{water}} = \frac{558\ \text{kg}}{1000\ \text{kg/m}^3} = 0.558\ \text{m}^3$$

$$\frac{V_{sub}}{V_{obj}} = \frac{0.558\ \text{m}^3}{0.650\ \text{m}^3} = 0.858 = 85.8\%$$

Reflect: When the sphere is totally submerged, the buoyant force on it is greater than its weight. When it is floating, it needs to be only partially submerged in order to produce a buoyant force equal to its weight.

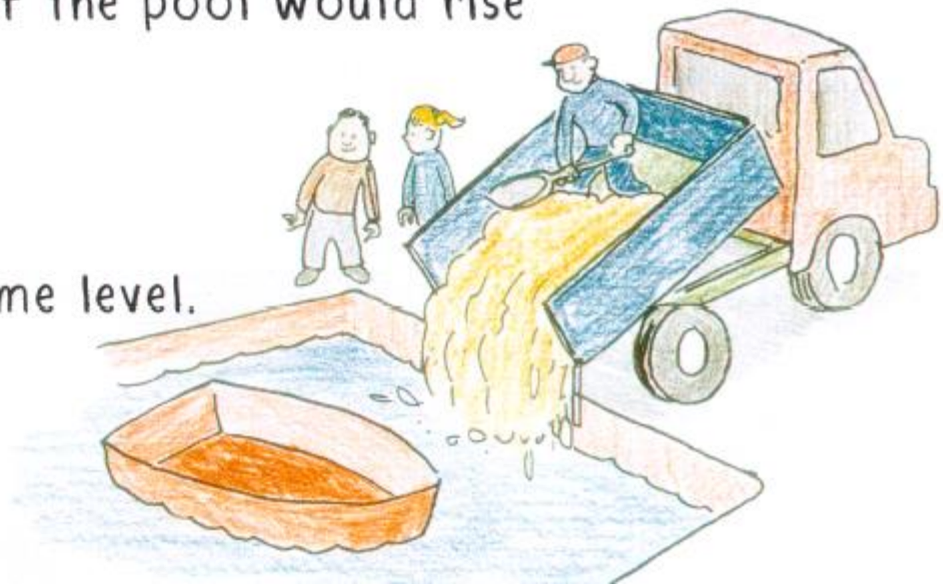
Clicker - Questions

A load of sand is poured into a pool to give it a new sandy bottom. It also raises the water level of the pool. If the sand were instead poured into a boat floating in the pool, the water level of the pool would rise

a) less.

b) more.

c) to the same level.



Archimedes buoyancy

A U-tube open on both ends to air contains some mercury. Water is poured into the left arm until the vertical height is 15cm.
a)What is the gauge pressure at the mercury-water interface?
b)Calculate the vertical distance h from the top of the mercury at the right arm of the tube to the top of the water at the left arm of the tube?

13.57. *Set Up:* $\rho_{\text{Hg}} = 13.6 \times 10^3 \text{ kg/m}^3$. Let x be the height of the mercury surface in the right arm above the level of the mercury-water interface in the left arm.

Solve: **(a)** $p - p_{\text{air}} = \rho_{\text{water}} gh = (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.150 \text{ m}) = 1.47 \times 10^3 \text{ Pa}$

(b) The gauge pressure a distance x below the mercury surface in the right arm equals the gauge pressure at the mercury-water interface, since these two points are at the same height in the mercury. $\rho_{\text{Hg}} gx = 1.47 \times 10^3 \text{ Pa}$ and

$$x = \frac{1.47 \times 10^3 \text{ Pa}}{(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 1.10 \text{ cm}$$

$h + x = 15.0$ cm, so $h = 13.9$ cm.

Reflect: The weight of the water pushes the mercury down in the left-hand arm. A 1.1-cm column of mercury produces the same pressure as a 15.0-cm column of water.

Archimedes buoyancy

A solid aluminum ingot weights 89 N in air. a) What is its volume? b)The ingot is suspended by a rope and totally submerged in water what is the tension(the apparent weight) in the rope??

***13.29. Set Up:** The density of aluminum is $2.7 \times 10^3\ \mathrm{kg/m^3}$. The density of water is $1.00 \times 10^3\ \mathrm{kg/m^3}$. $\rho = m/V$. The buoyant force is $F_B = \rho_{water} V_{obj} g$.

Solve: **(a)** $T = mg = 89\ \mathrm{N}$ so $m = 9.08\ \mathrm{kg}$. $V = \dfrac{m}{\rho} = \dfrac{9.08\ \mathrm{kg}}{2.7 \times 10^3\ \mathrm{kg/m^3}} = 3.36 \times 10^{-3}\ \mathrm{m^3} = 3.4\ \mathrm{L}$.

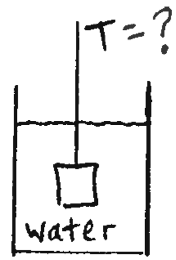
(b) When the ingot is totally immersed in the water while suspended, $T + F_B - mg = 0$.

$$F_B = \rho_{water} V_{obj} g = (1.00 \times 10^3\ \mathrm{kg/m^3})(3.36 \times 10^{-3}\ \mathrm{m^3})(9.80\ \mathrm{m/s^2}) = 32.9\ \mathrm{N}$$

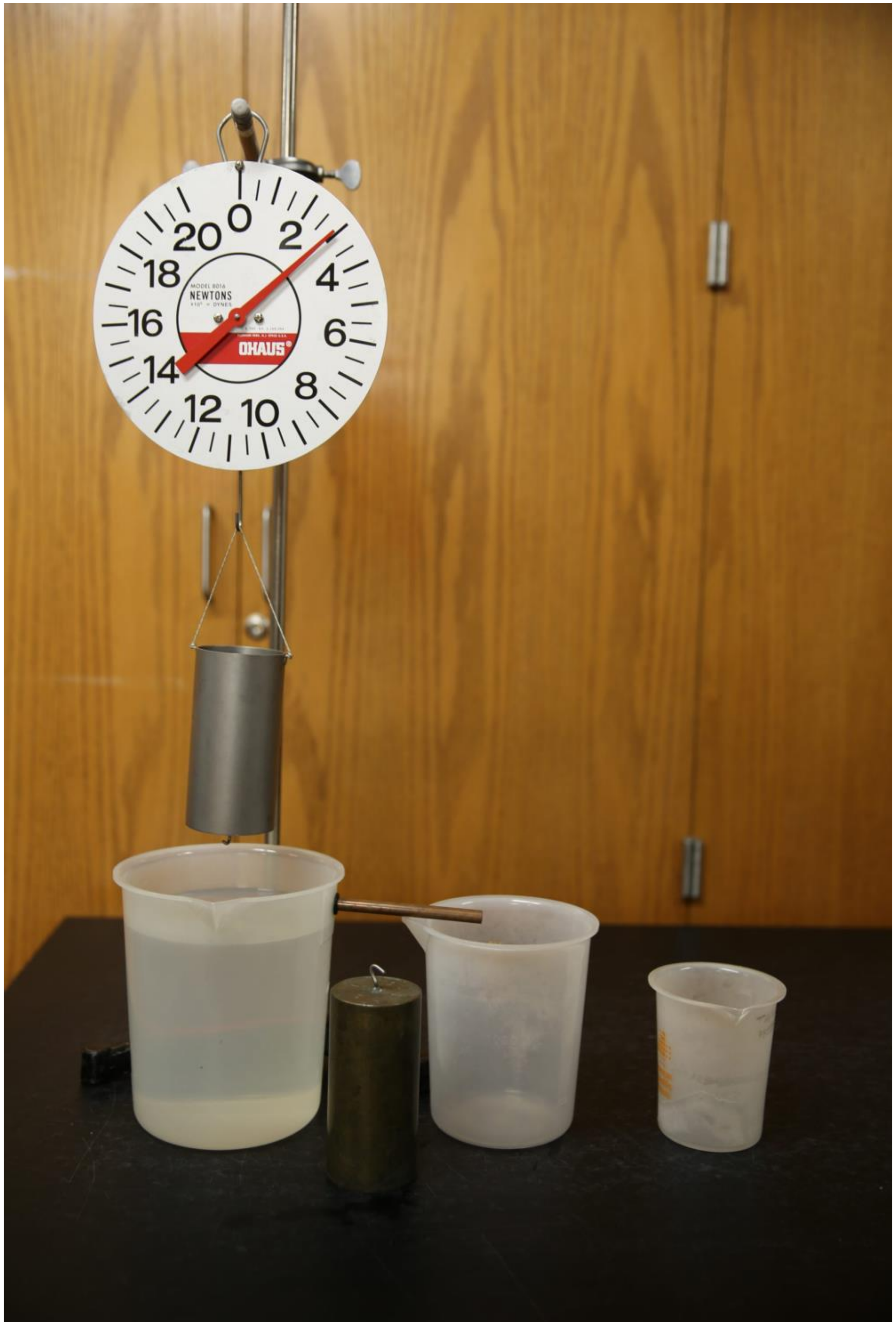
$$T = mg - F_B = 89\ \mathrm{N} - 32.9\ \mathrm{N} = 56\ \mathrm{N}$$

Reflect: The buoyant force is equal to the difference between the apparent weight when the object is submerged in the fluid and the actual gravity force on the object.

(5 pts) 3. A block of metal with density $4000\ \text{kg/m}^3$ is suspended from a light string. When the block ~~book~~ is in air, the tension in the string is 39.2 N. What is the tension in the string when the block is totally immersed below the surface of water that is in a bucket? The block is not touching the bucket. The density of water is $1000\ \text{kg/m}^3$.



(a) 4.9 N
(b) 9,8 N
d (c) 18.6 N
(d) 29.4 N
(e) none of these answers



Archimedes' Principle

In this demonstration, first find the initial weight of the can/weight system that is being submerged in the beaker of water by using the scale. Once the system is fully submerged there will be a drop in the reading on the scale. This is due to the buoyancy force opposing the force of gravity. The displaced water can later be poured into the empty can to show that its weight is equivalent to the bourancy force.

Ball and Ring

Initially the ball is just shy of being able to fit into the ring. But if the ring is heated up, it will expand slightly allowing the ball to fit.

Geyser

In a flask, water is heated by using a hot plate. As the water heats up and begins to evaporate, because gas takes up a greater volume than a liquid, pressure begins to increase inside the flask. Due to the increase in pressure, the liquid water is able to rise in the tube and expel from the other end.

Density Bowling Balls

In this demonstration, two identical looking bowling balls which differ in their mass and density are submerged in a tank of water. The lighter bowling ball is hollow inside, thus its overall density is less than water. The heavier bowling ball is fully solid and has a larger density than water. These characteristics result in the hollow bowling ball floating while the solid bowling ball sinks.

Cartesian Diver

When the bottle is squeezed, the air that is in the diver is allowed to compress. This compression of air in the diver makes it more dense than the water around it, so it sinks. When the bottle is released the diver becomes less dense and allows it to rise.